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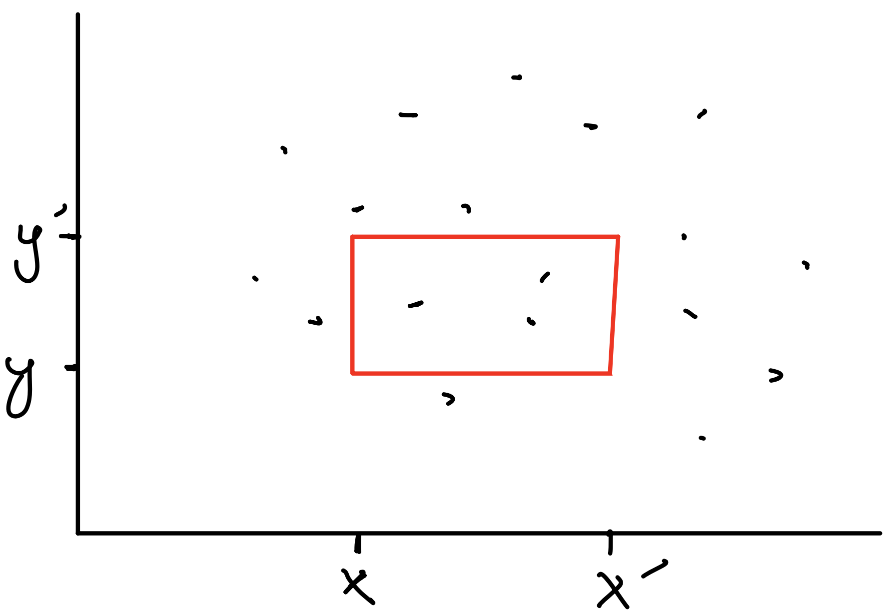
- Find points of P belonging to the range.



- Relevant to querying databases.

1-d range searching

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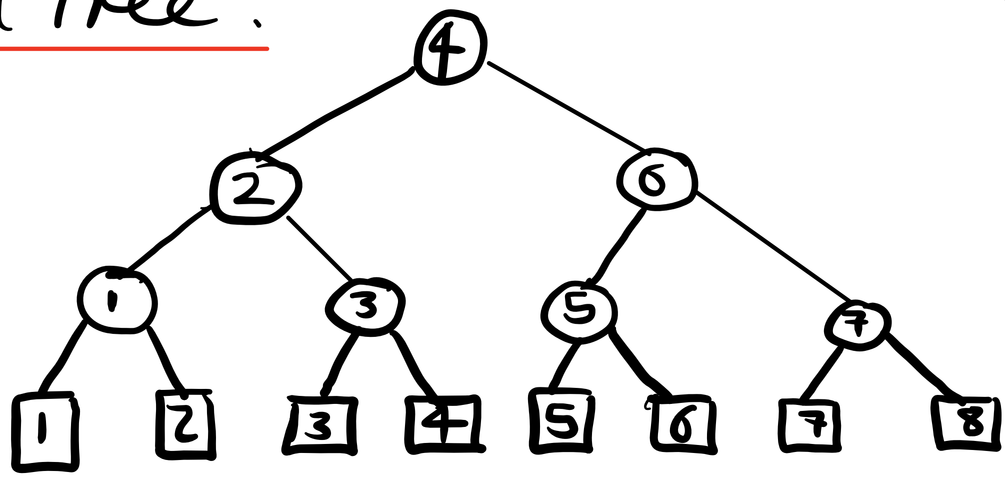

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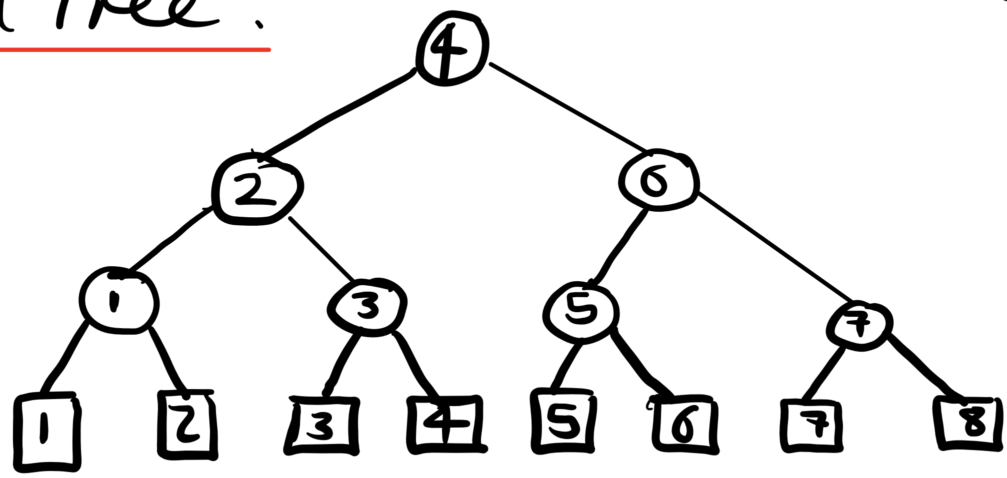

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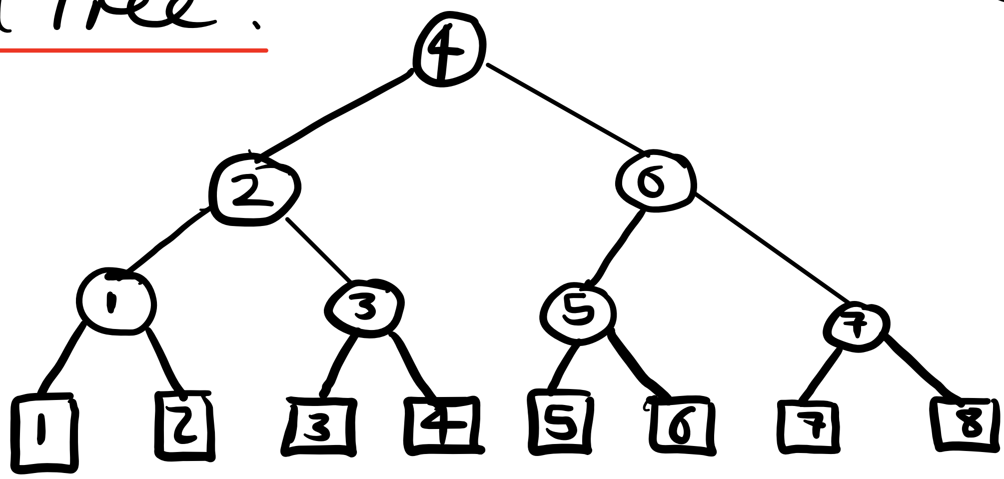

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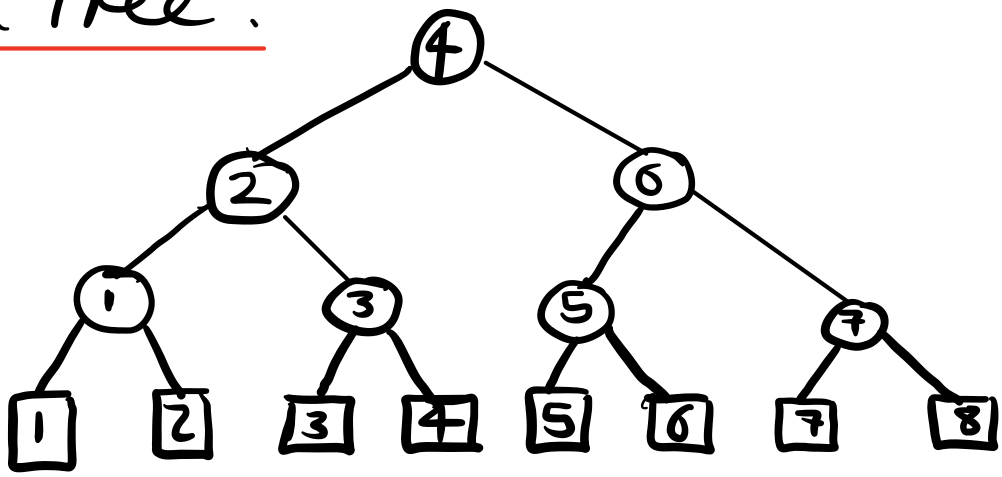

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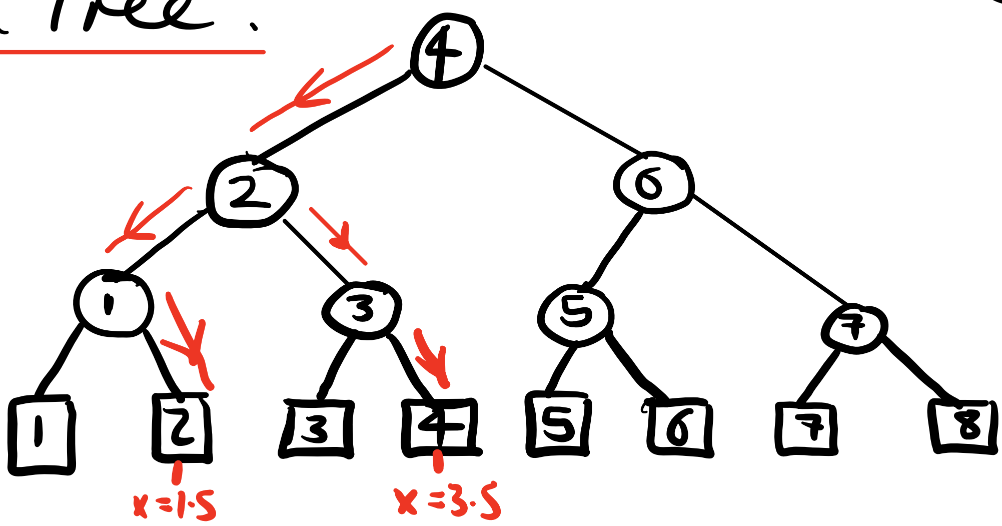

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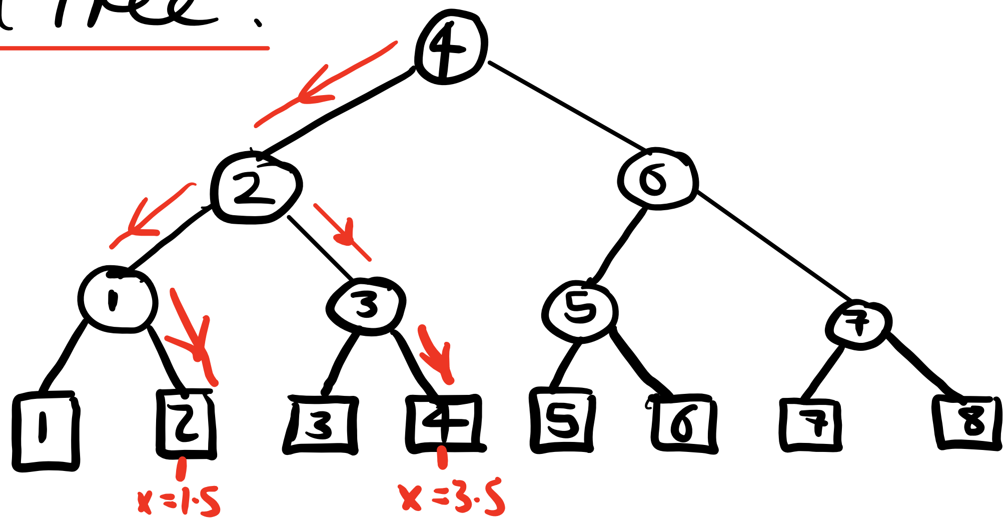

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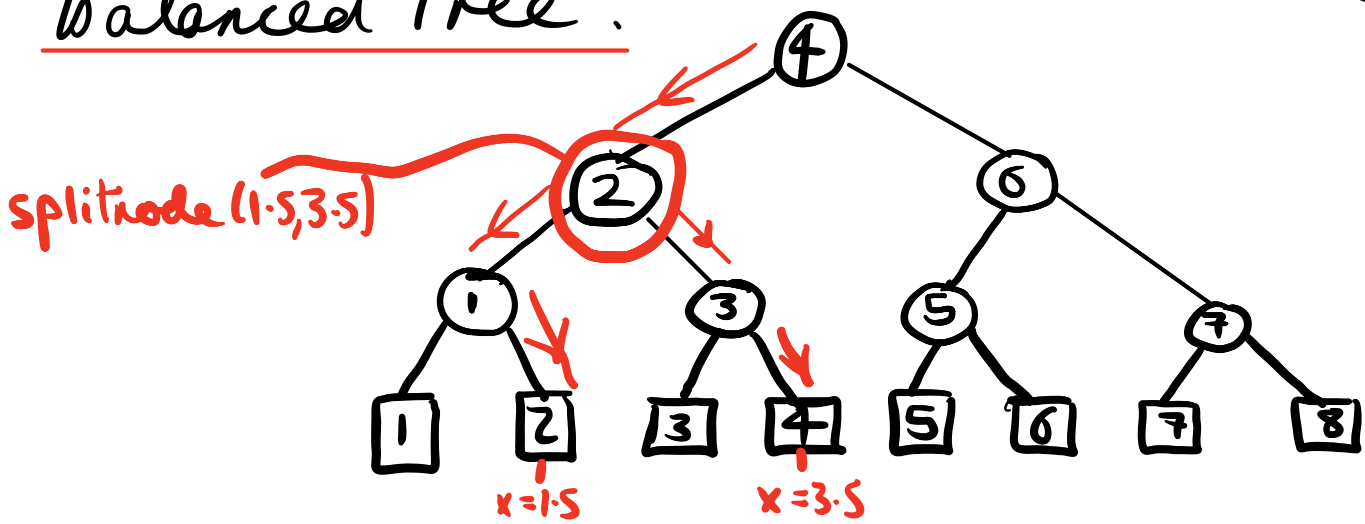

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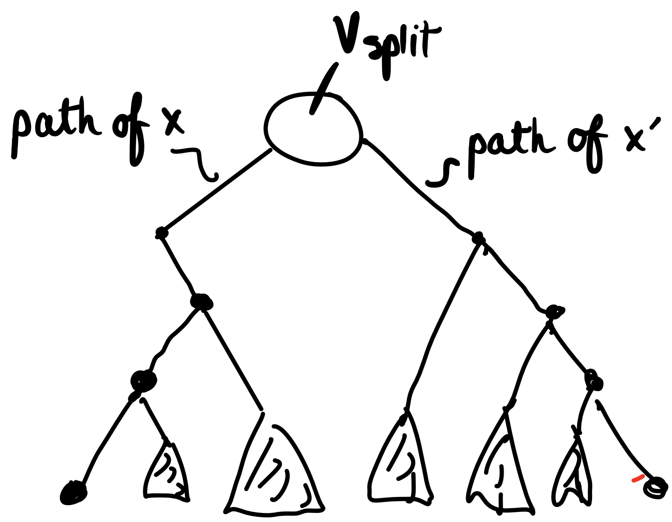

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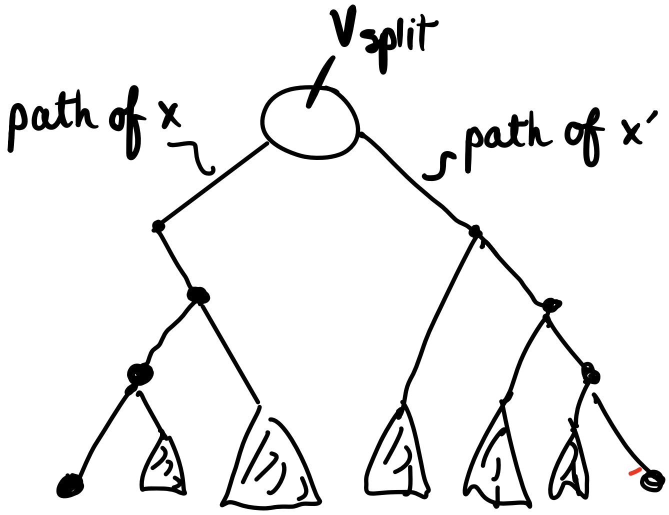

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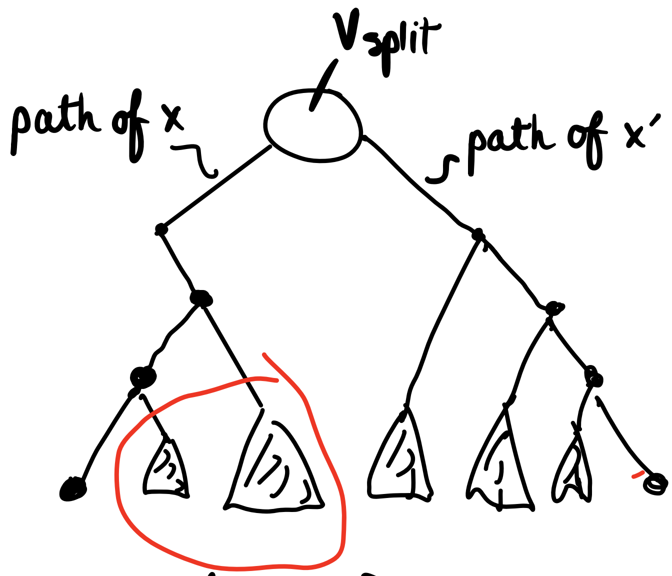

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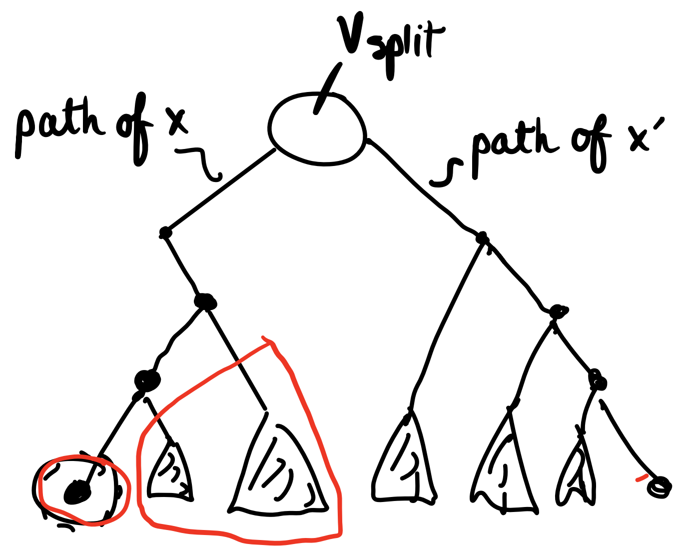

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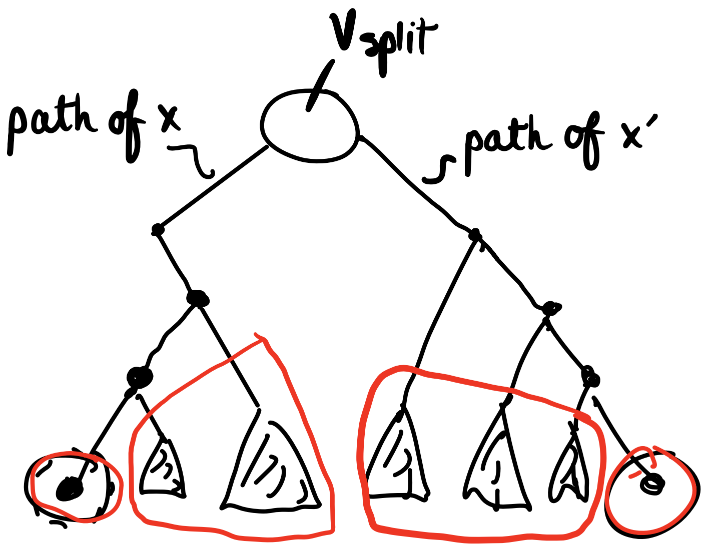

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- Follow path of x' from v_{split} to a leaf:

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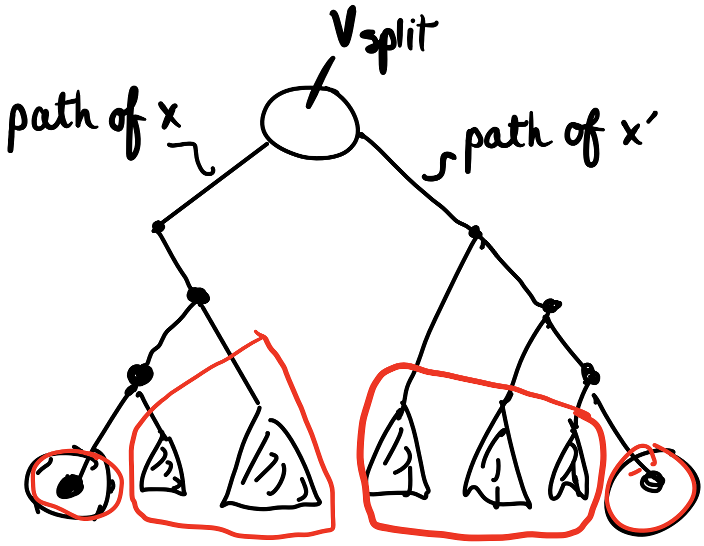
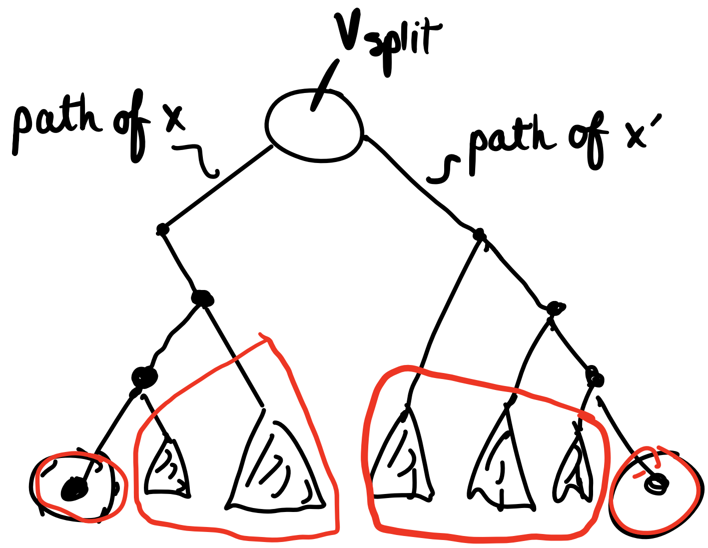


Why does this find all solutions?



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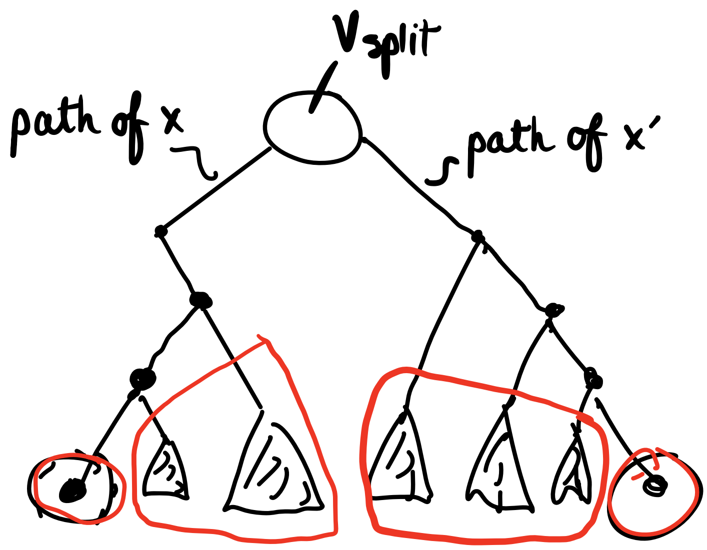

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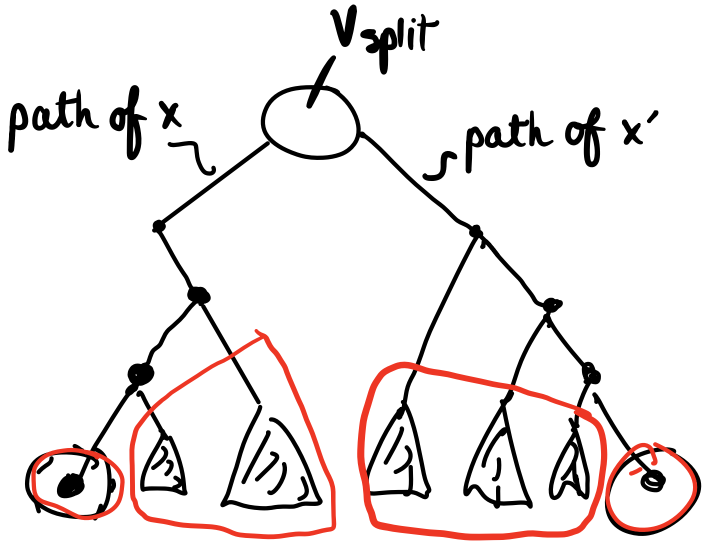

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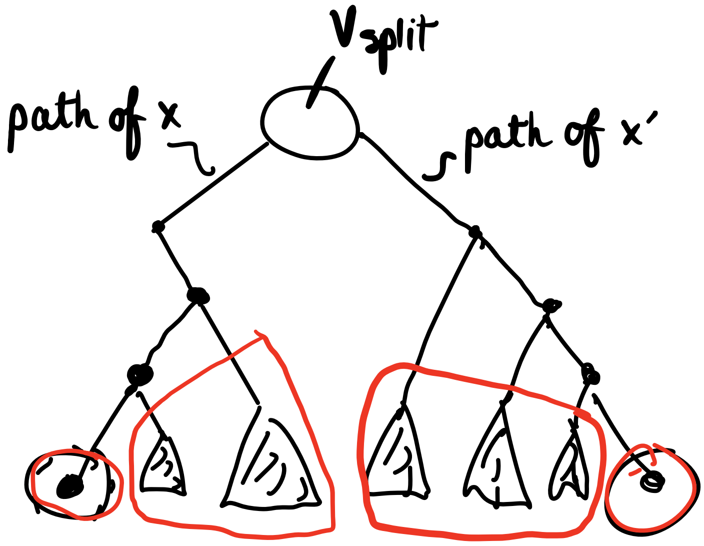

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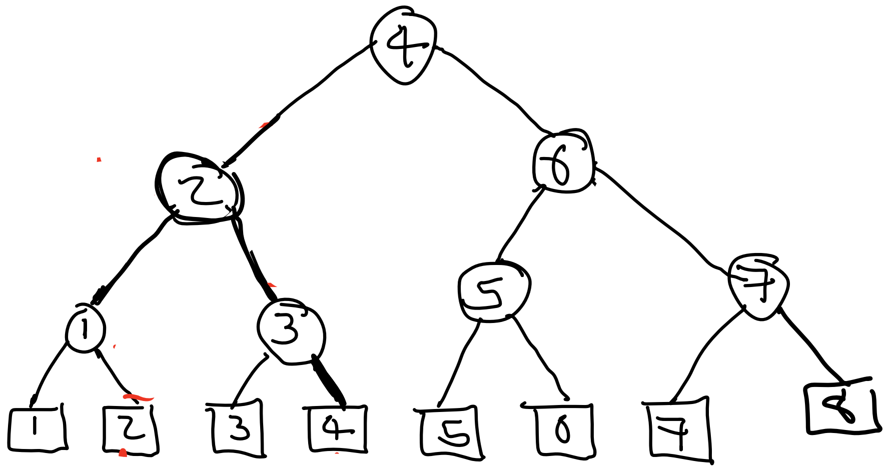
Complexity

- Time $O(\log n)$ to follow path of x.
- Likewise for x'.
- Time to report k solutions is $O(k)$.

Total complexity $O(\underbrace{\log n}_{\text{no of leaves}} + \underbrace{k}_{\text{no of solutions}})$

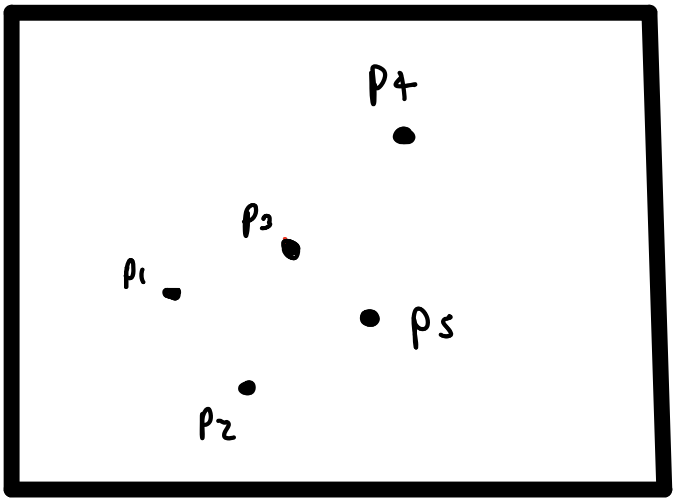
[5.5, 9]



Do in class.

2-d range searching

- $P \subseteq \mathbb{R}^2$ (assume no pts have same x or y coord)



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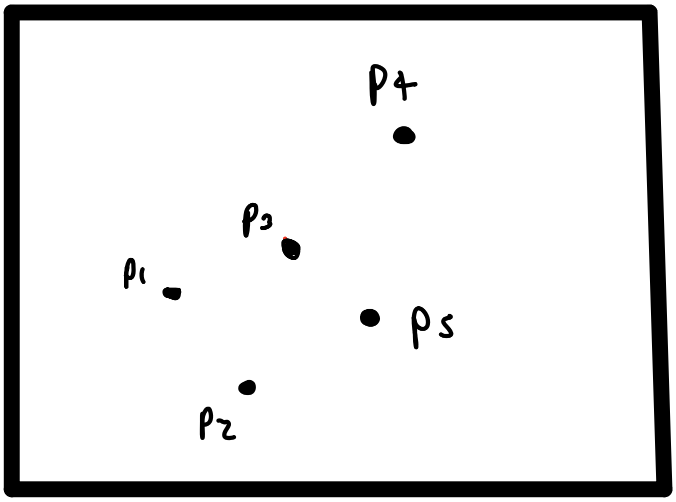

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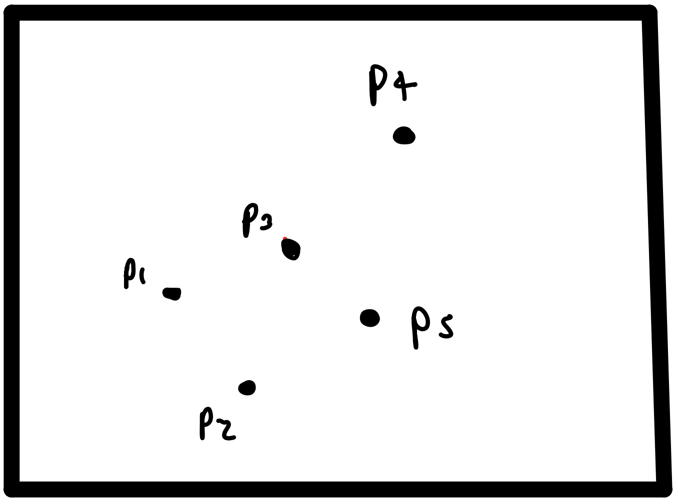

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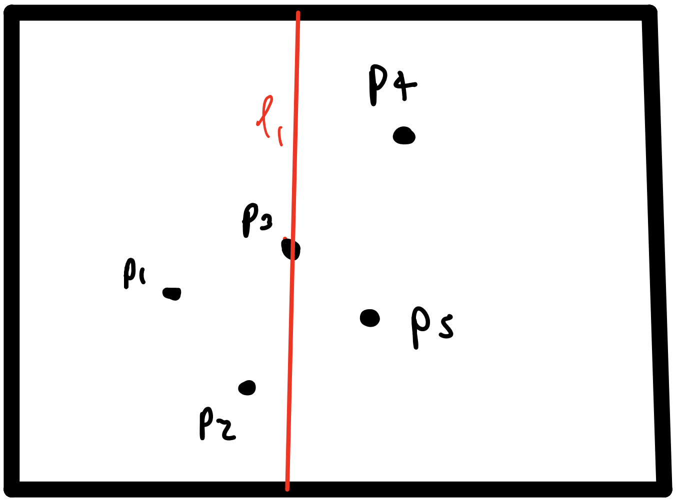

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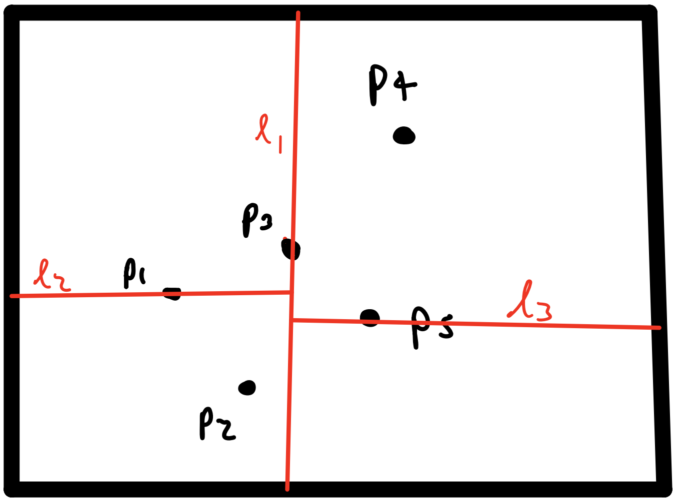

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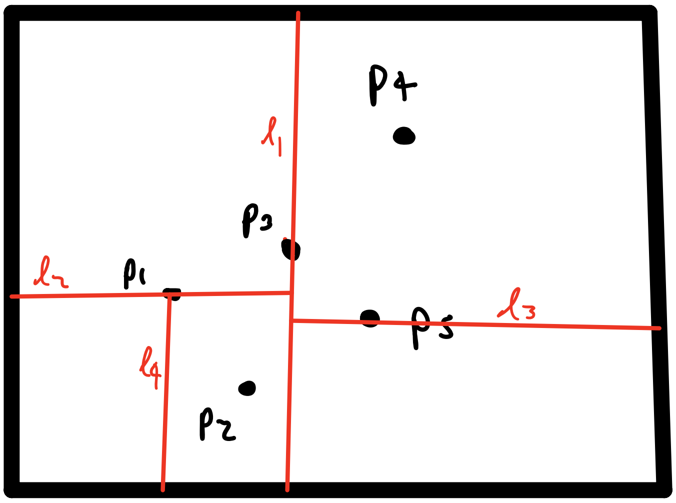

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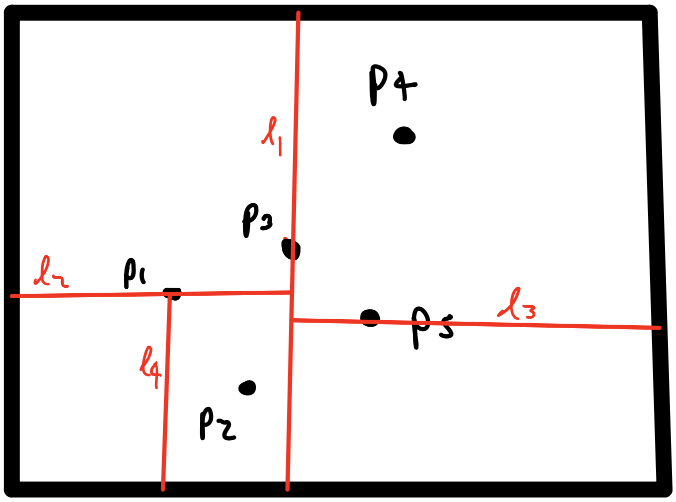

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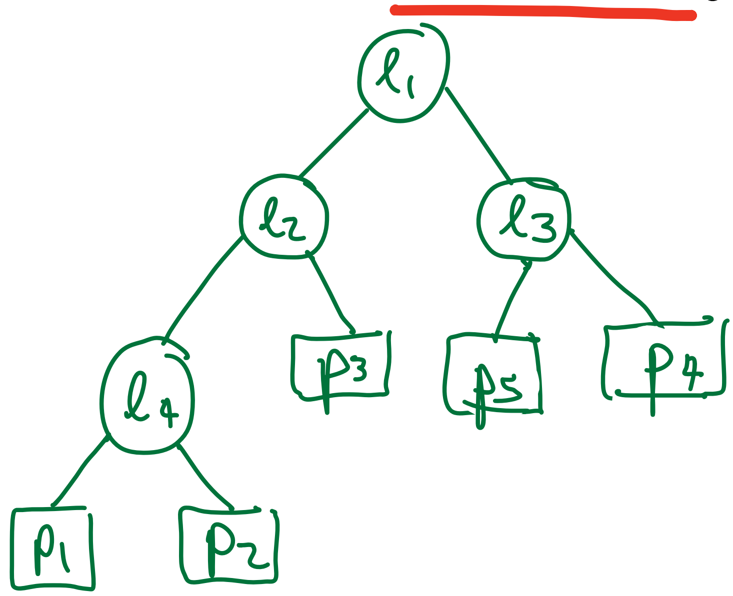

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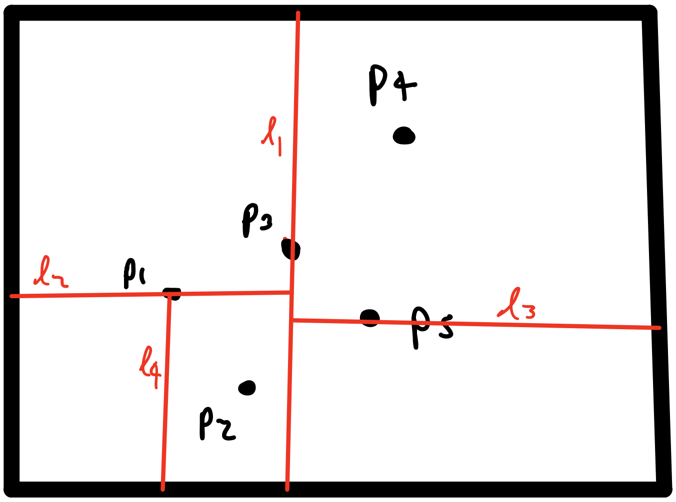

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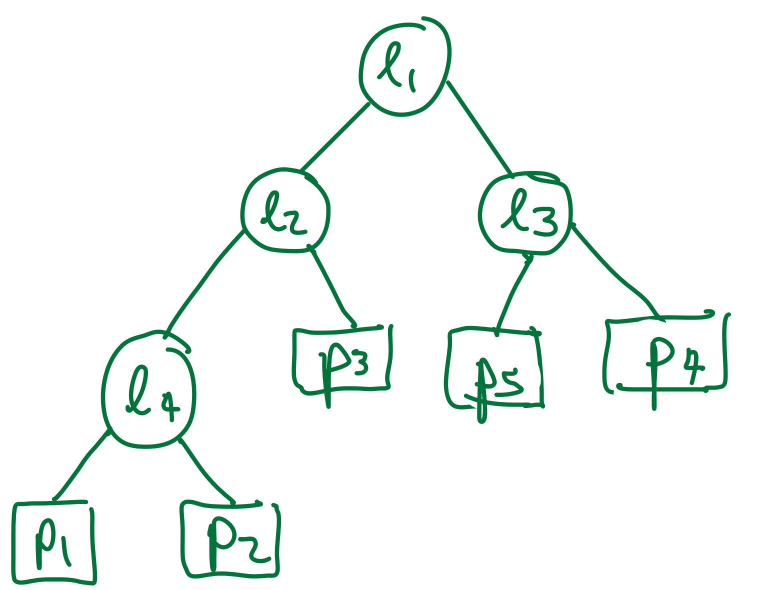
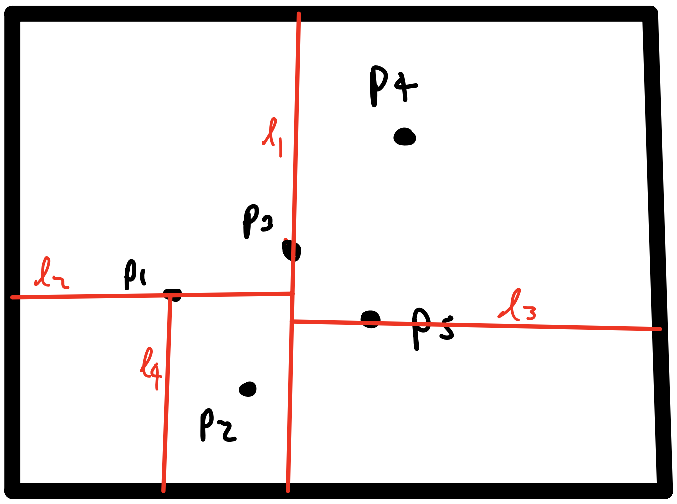
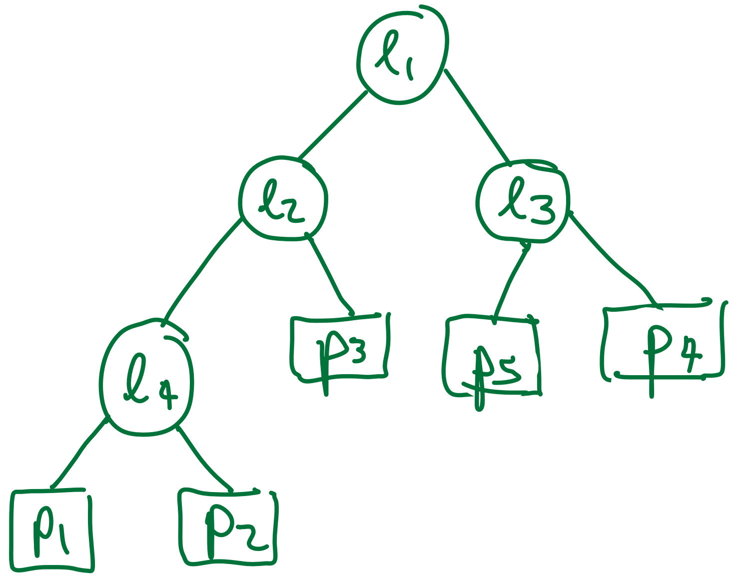


KD Trees





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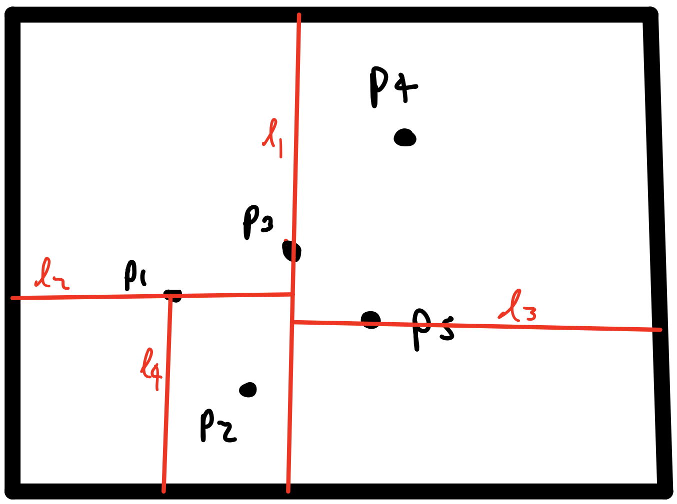

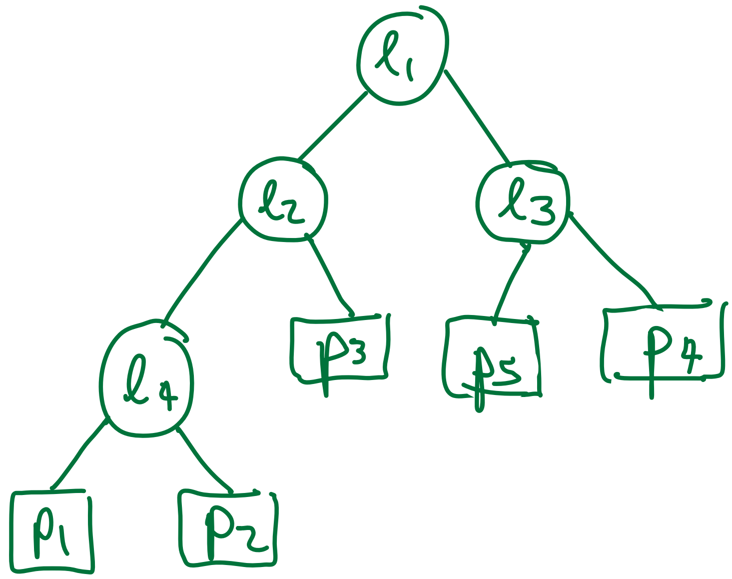

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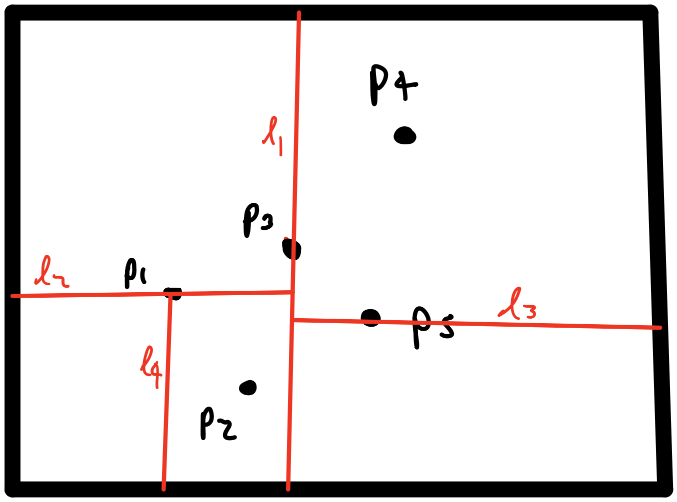

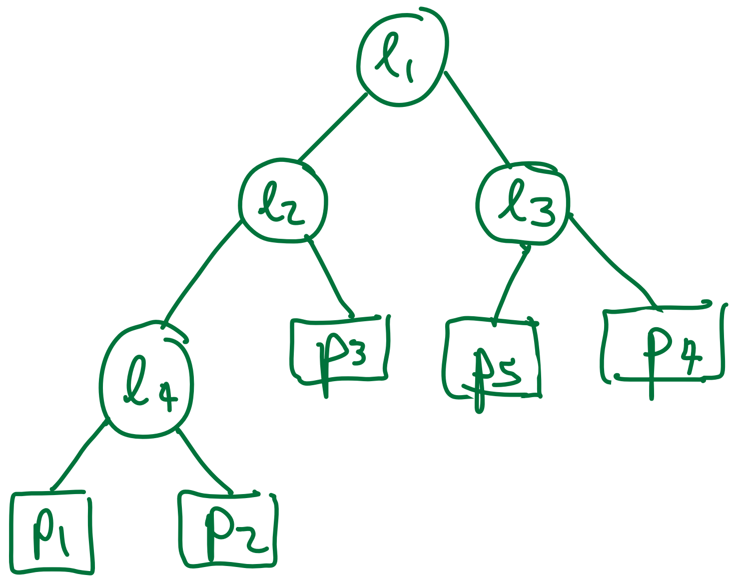

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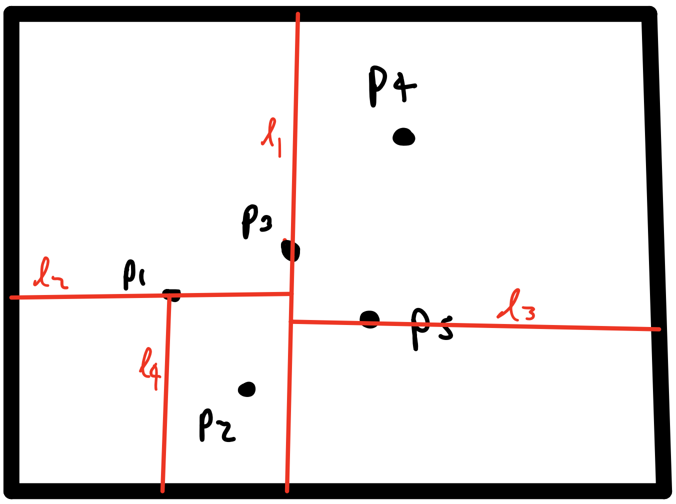

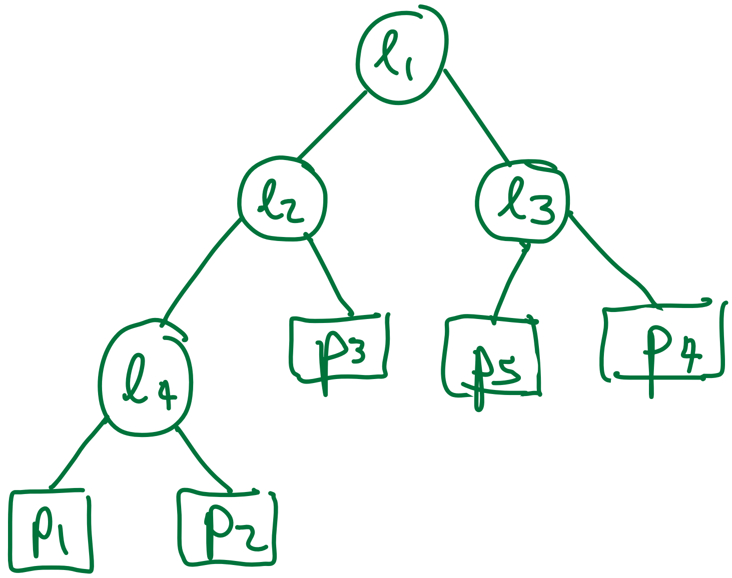

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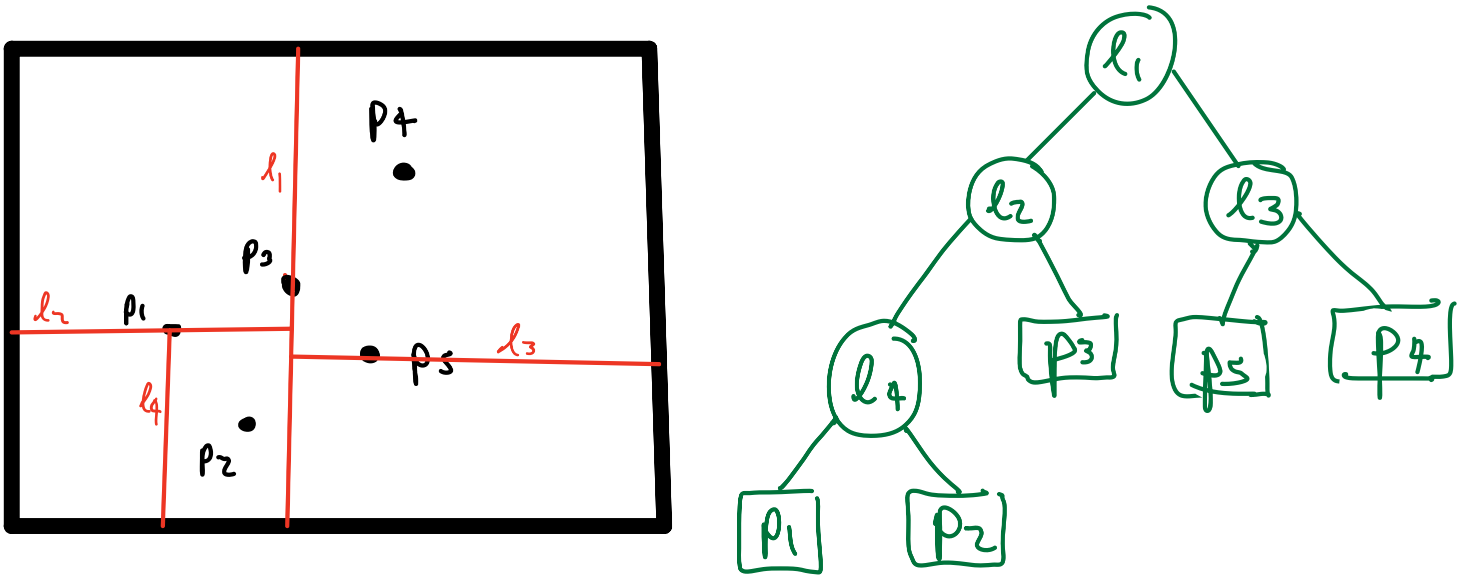

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- $O(n \log n)$ to create kd tree.
- To search kd-tree, need concept of region of a node.

Region of a node

- Region of node v :
rectangular region bounded by ancestors of v
(i.e. if v represents a line, reg(v) is the area which the line divides in two)

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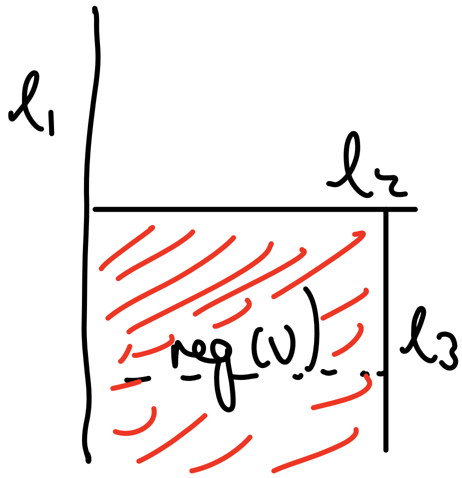

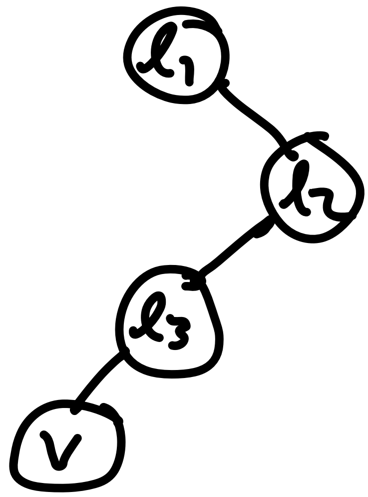

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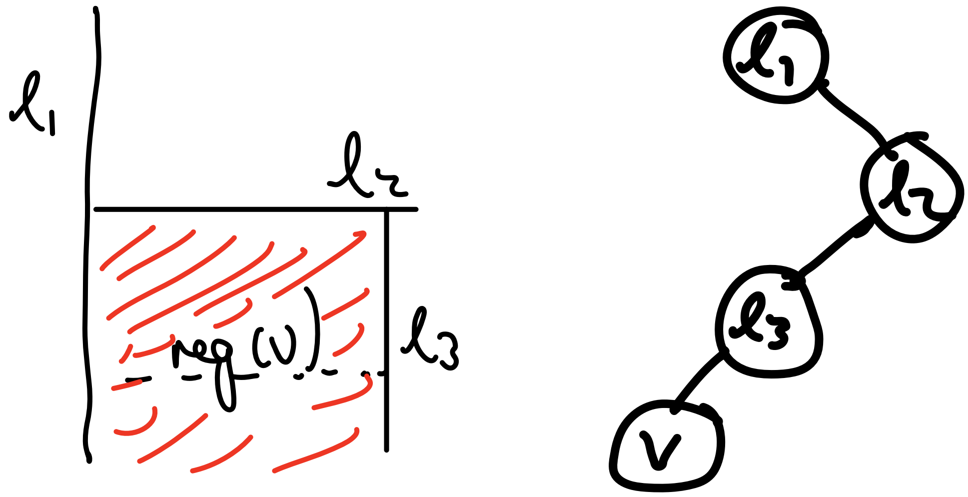

Recursively,

- Region (root) $= \mathbb{R}^2$
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  left child
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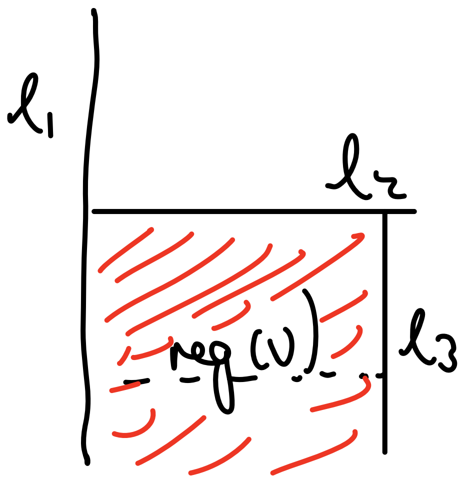

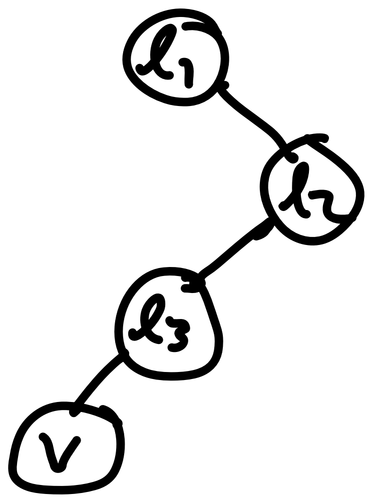

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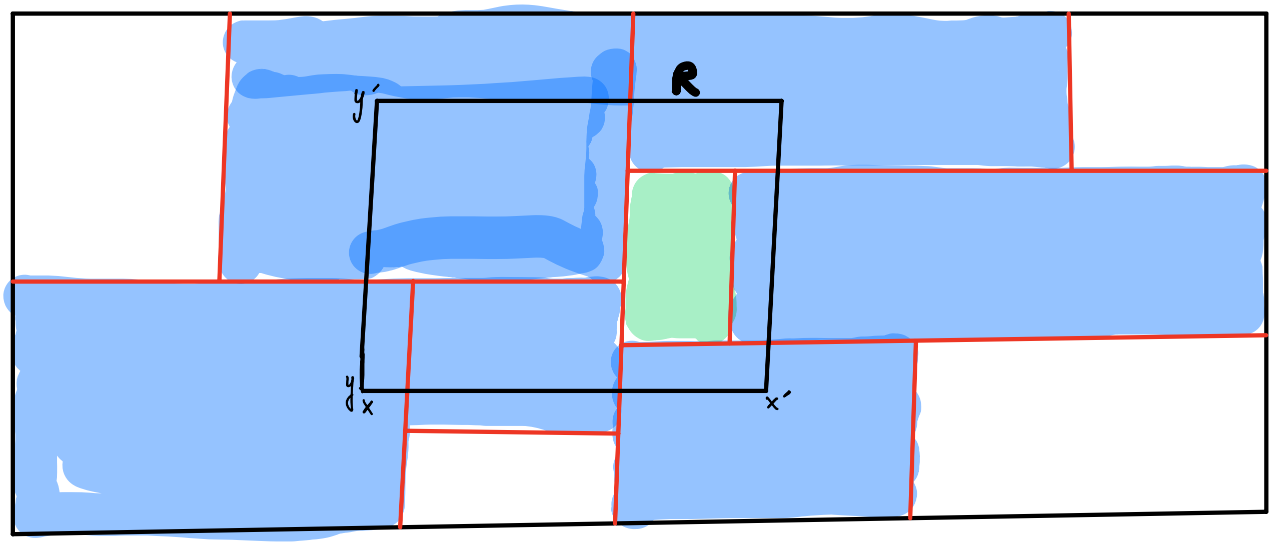
Fact: a point p of P belongs to region(v) $\Longleftrightarrow$ p belongs to subtree under v.

Search algorithm - idea



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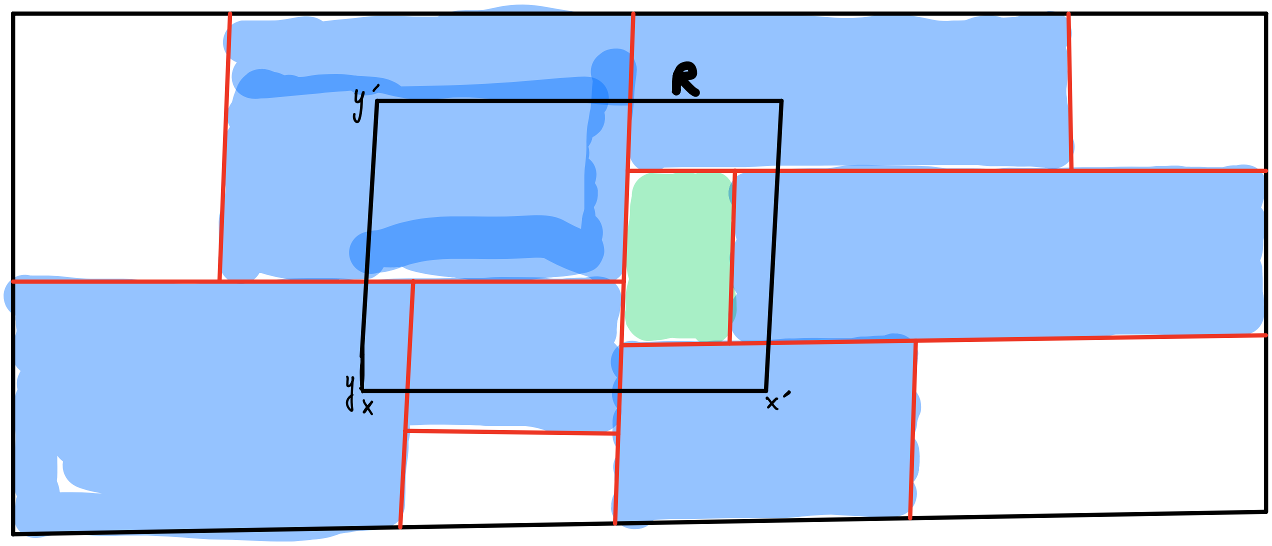

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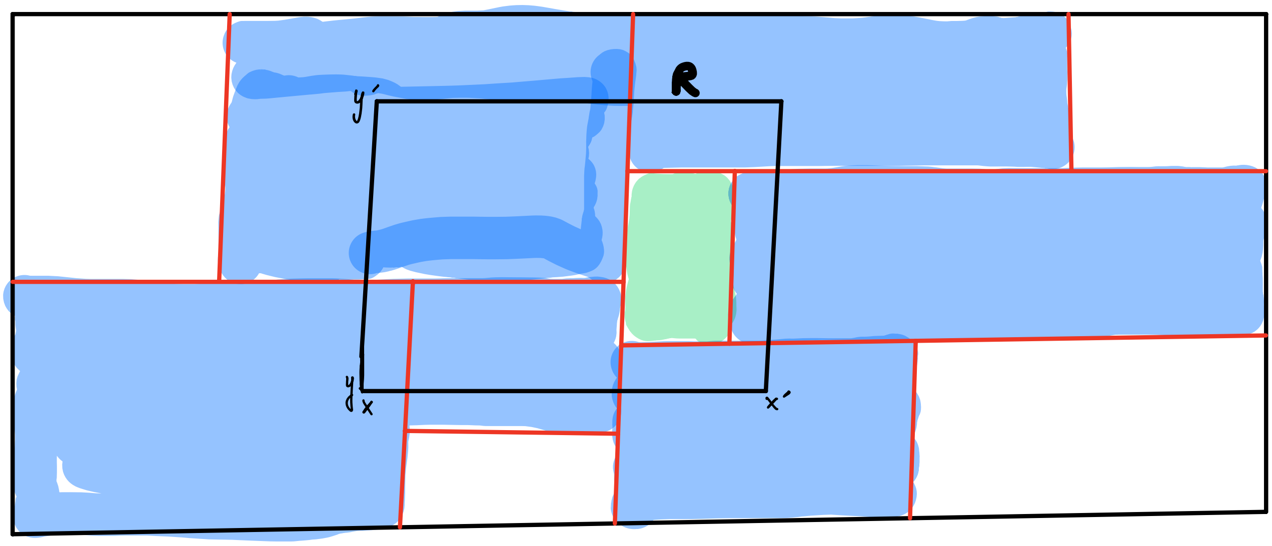
1) region(v) $\subseteq$ R (green above)

Search algorithm - idea



Idea of algorithm

- Want to find all points from P in R.
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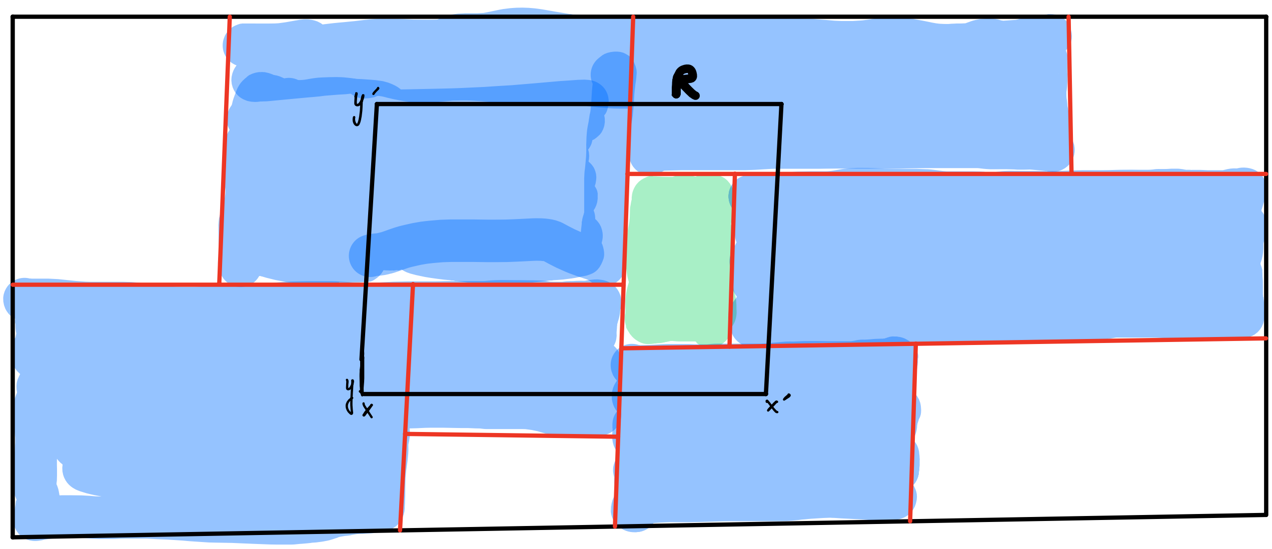

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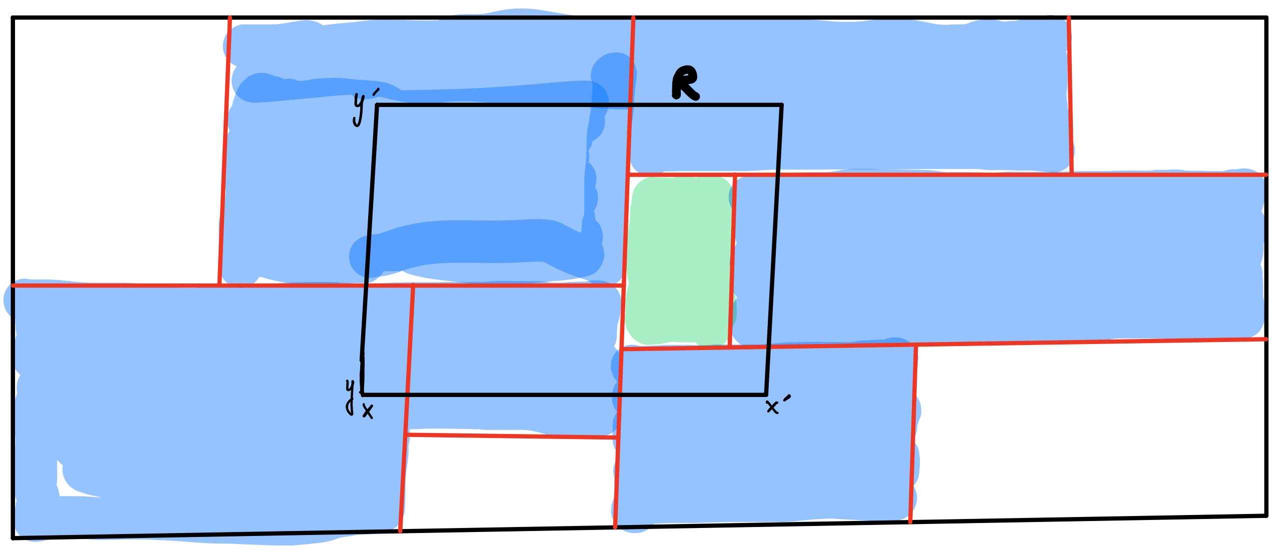

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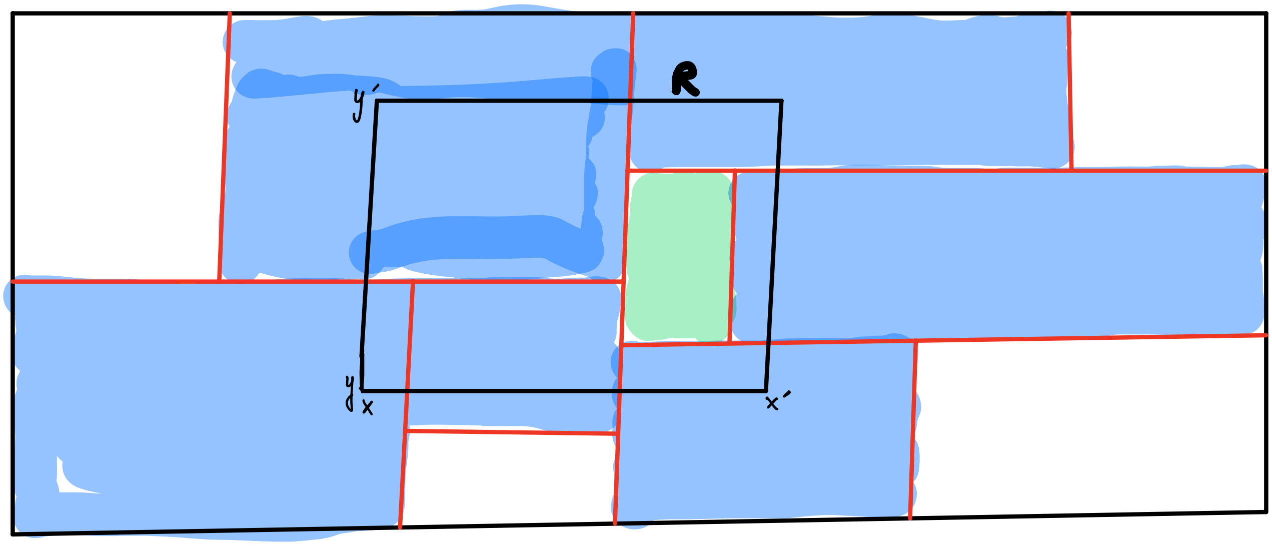

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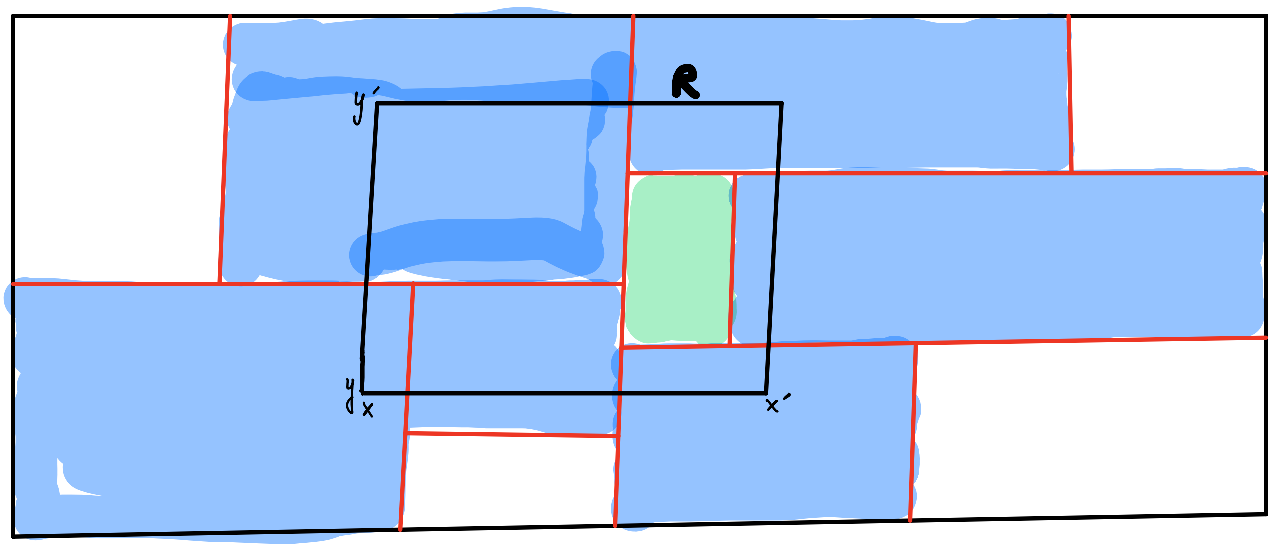

Idea of algorithm

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- In case ①, report all points in region.
- In case ②, don't search below v.
- In case ③, search regions for left & right child of v.

Algorithm SearchKDtree(v, R)

Input: range R + KD-tree of points P, node v in KD-tree.

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Complexity

$$O(\sqrt{n} + k)$$

no of pts in P

no of solutions

Removing assumption that no points in P have same x or y-coordinate

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elements of form $(a \mid b)$

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$$\hat{R} = [(x|-\infty), (x'|\infty)] \times [(y|-\infty), (y'|\infty)]$$

- Then $p \in R \iff \hat{p} \in \hat{R}$

i.e. $(p_x|p_y) \in [(x|-\infty), (x'|\infty)]$

$\iff (x|-\infty) < (p_x|p_y) < (x'|\infty)$.

First ineq. $x < p_x$ or $x = p_x$ ~ $x \leq p_x$

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- Then $p \in R \iff \hat{p} \in \hat{R}$ so only need to run our original alg (gen. to a totally ord. set) on $(\hat{P}, \hat{R})$ instead

i.e. $(p_x|p_y) \in [(x|-\infty), (x'|\infty)]$

$\iff (x|-\infty) < (p_x|p_y) < (x'|\infty)$.

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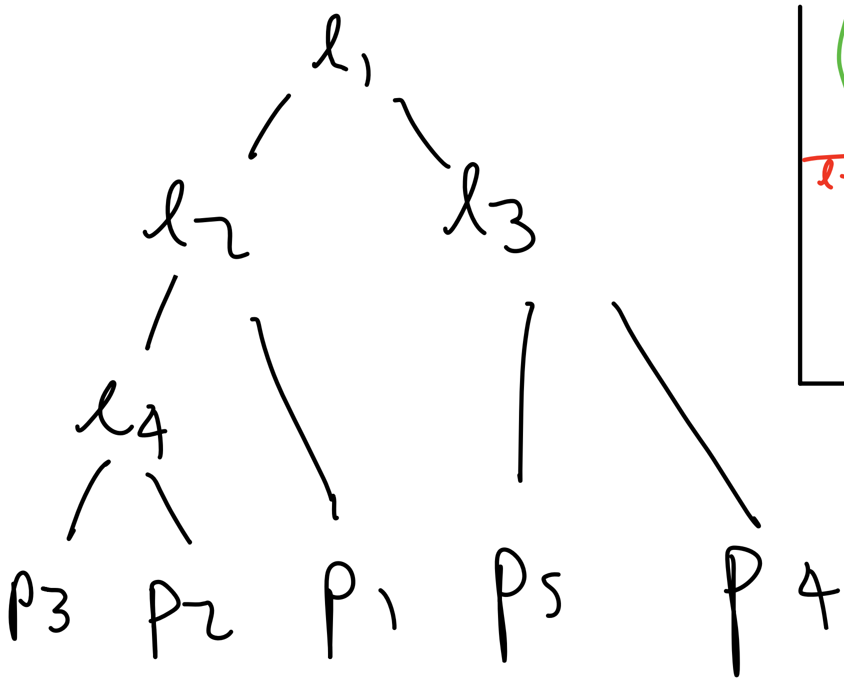
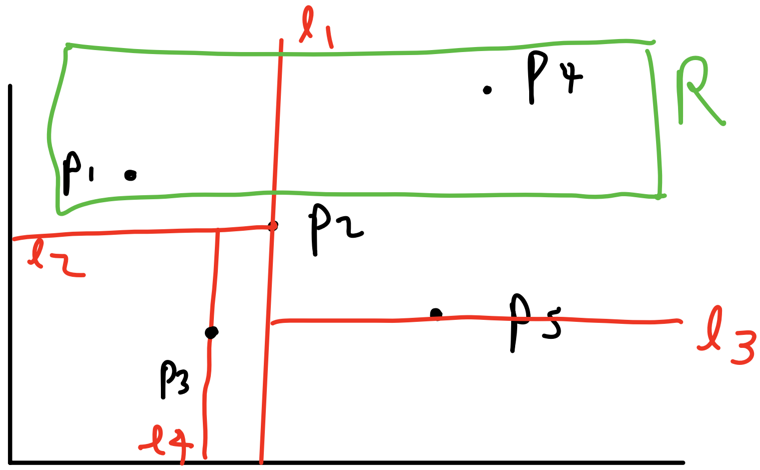
Example





- At l_1, look at l_2, l_3

→ Both $reg(l_2), reg(l_3) \cap R$, but not cont. in

At l_2, look at l_4, p_1.

- $reg(l_4) \cap R = \emptyset$.
- $p_1 \in R \Rightarrow$ report p_1.

At l_3, look at p_5, p_4.

$p_4 \in R$.

Second approach - range trees

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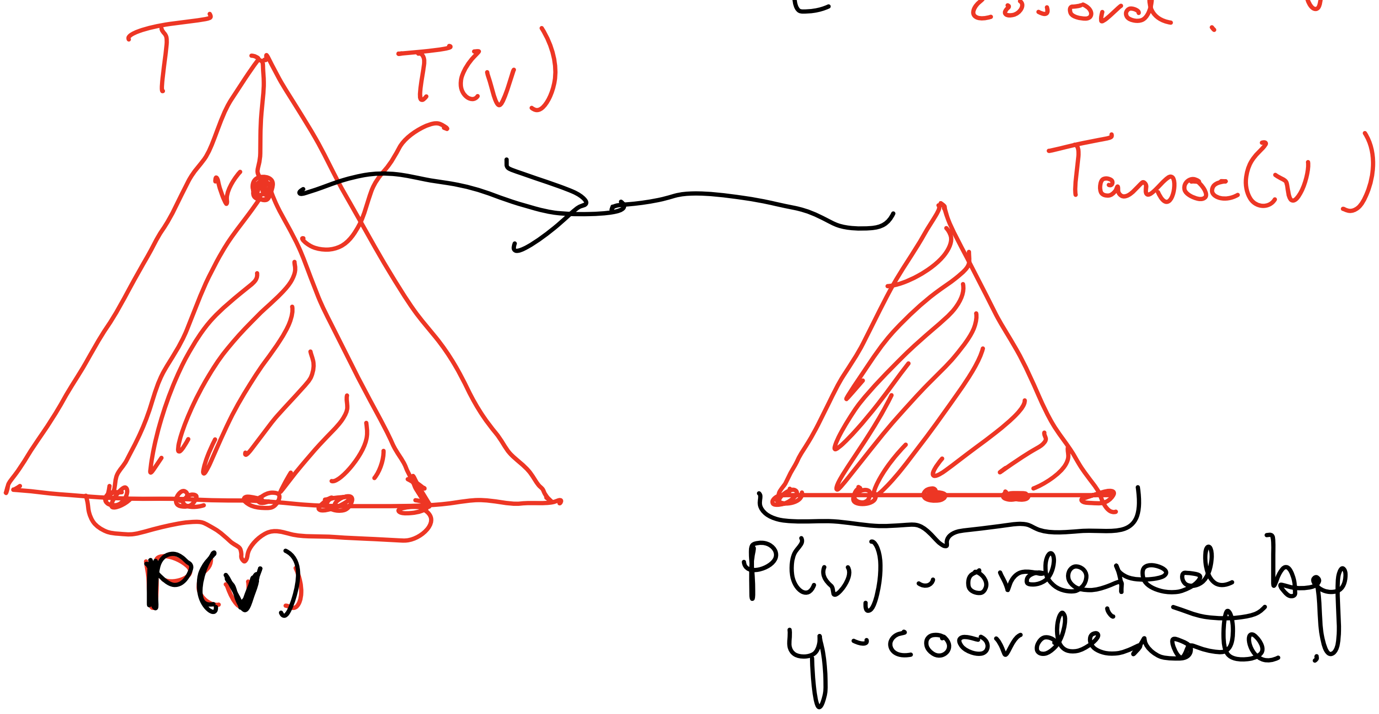

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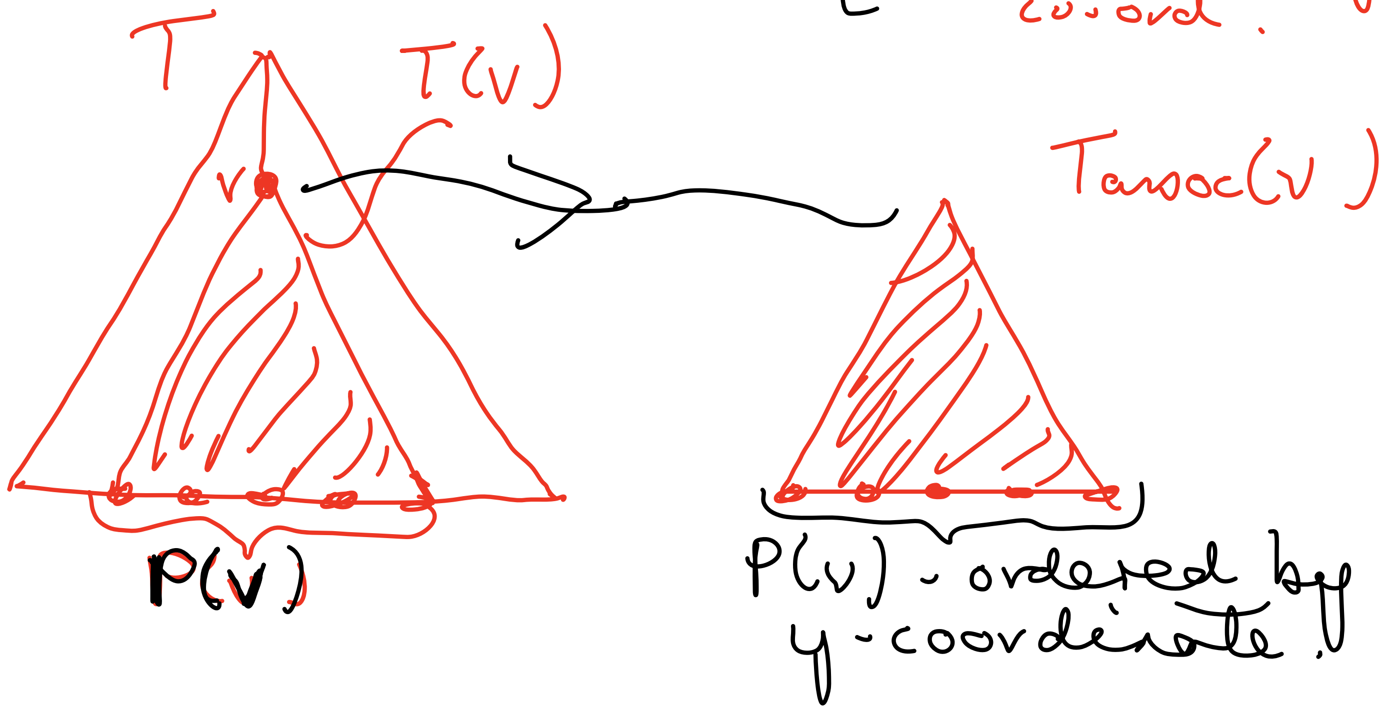

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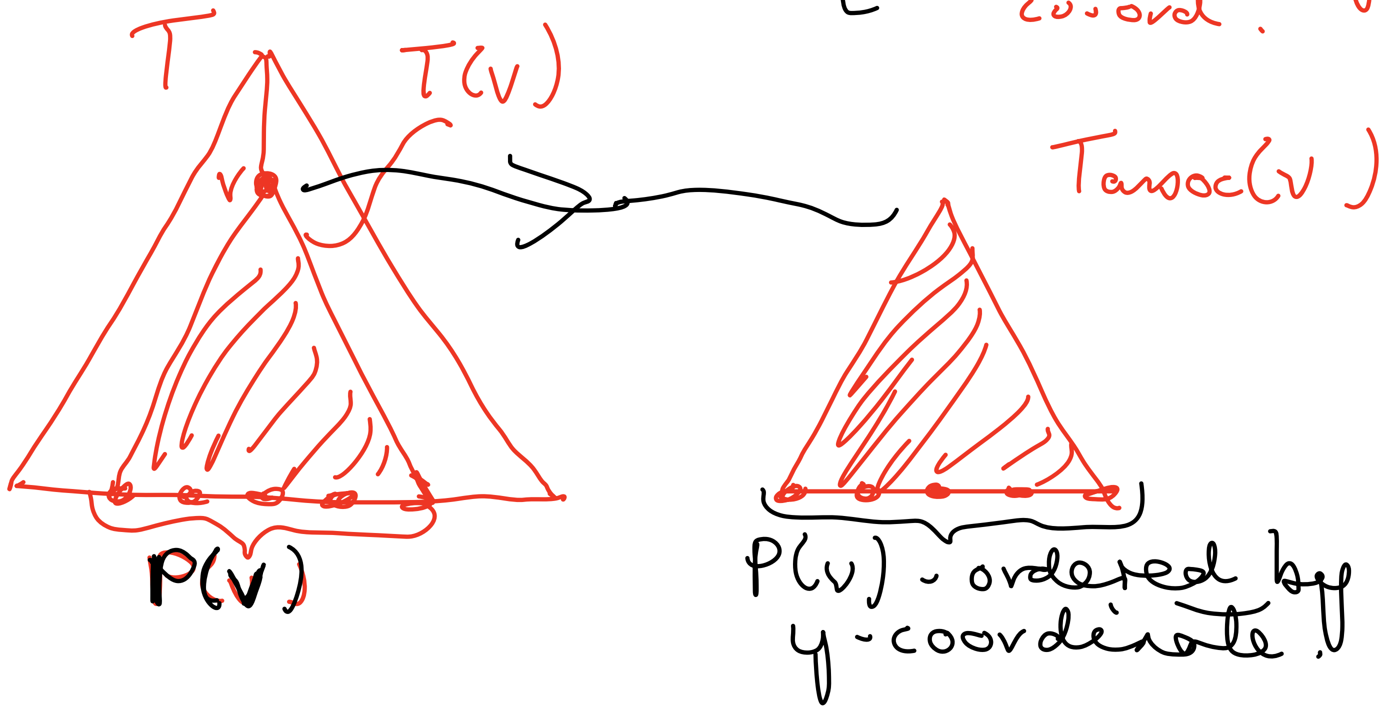

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- Storage $O(n \log n)$

Searching a range tree

$R = [x, x'] \times [y, y']$

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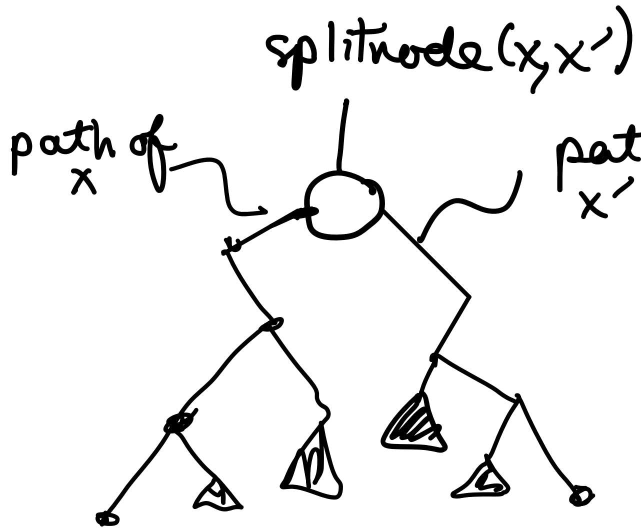
- Look at tree ordered by x-coord, find split node of x & x'



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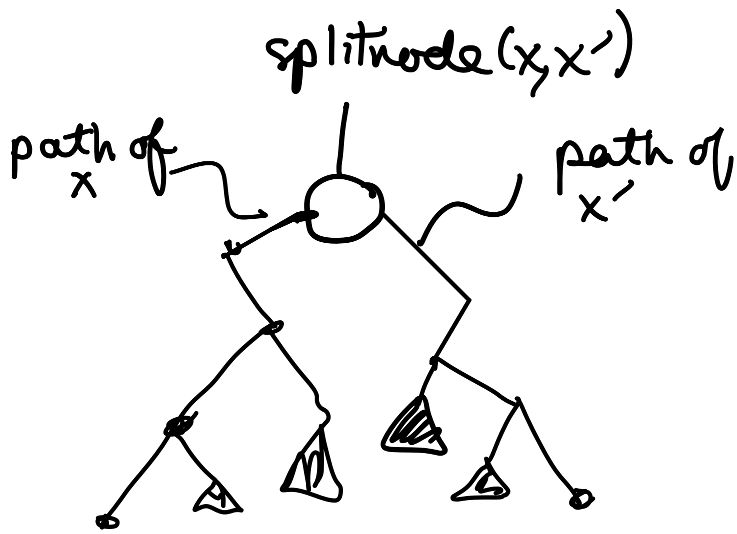

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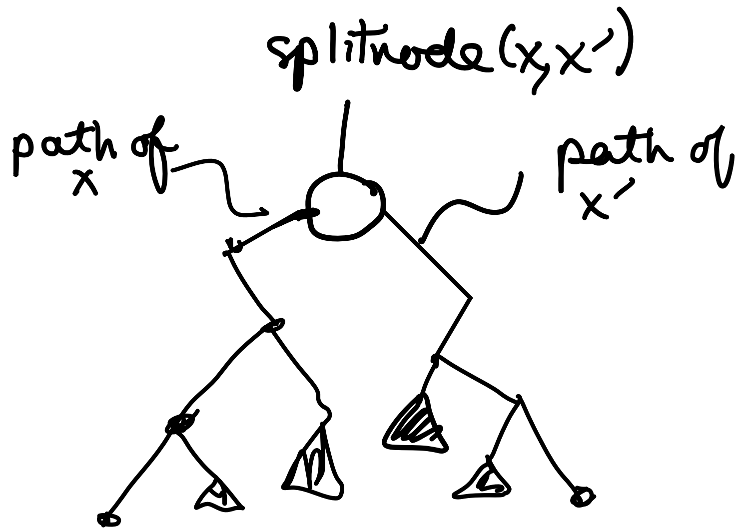

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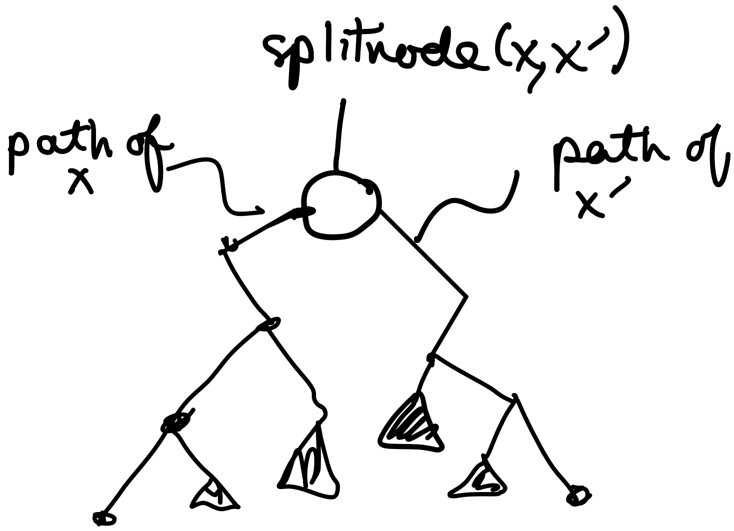

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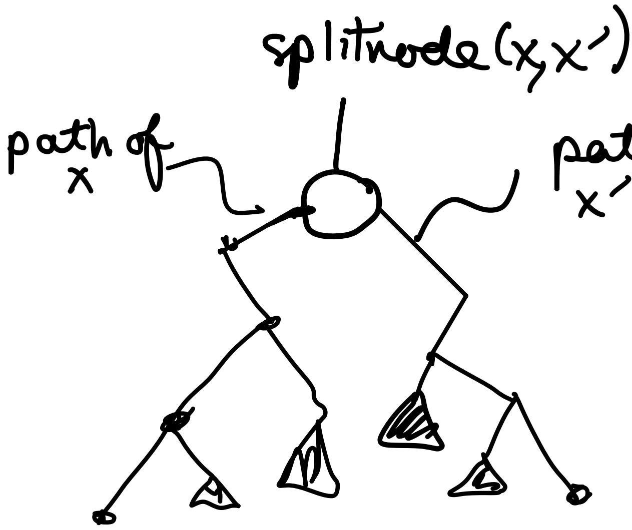

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- Similarly search path of x' below split node.

Complexity

$O(\log n^2 + k)$

no. of points

k no. of solutions

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no. of points — k no. of solutions

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Comparison in d dimensions

	kd-trees	range trees
Storage	$O(n)$	$O(n \log^{d-1} n)$
Construction	$O(n \log n)$	$O(n \log^{d-1} n)$
Search	$O(n^{1-1/d} + k)$	$O(n \log^{d} n + k)$