Metaprogramming for math library

Jiří Chmelík, Marek Trtík
PA199

Metaprogramming

- **Metaprogram** is a code whose **compilation** generates a **new code** whose functionality we really want.

- First metaprogram:
  - Erwin Unruh, 1994.
  - During compilation of his program the **compiler computed** prime numbers.

- Our first metaprogram will be much simpler…

Our first metaprogram

- Let's make a C++ compiler to compute 3^N during **compilation**.
- Our metaprogram:

```
template<int N> struct Pow3 {
    enum { result = 3 * Pow3<N-1>::result };  // Recursion rule
};
template<> struct Pow3<0> {
    enum { result = 1 };   // Base case
};
std::cout << Pow3<4>::result;
```

- Resulting program:

```
std::cout << 3*3*3*3*1;
```

Unrolling loops: Motivation

- Let's consider this implementation of the dot product:

```
float dot_product(int dim, float* u, float* v) {
    float result = 0.0f;
    for (int i=0; i<dim; ++i)
        result += u[i] * v[i];
    return result;
}
float u[3] = {1,2,3};   // A 3D vector
float v[3] = {4,5,6};   // A 3D vector
std::cout << dot_product(3, u, v);  // Print their dot product.
```

- Faster code woud be: `std::cout << u[0]*v[0] + u[1]*v[1] + u[2]*v[2];`

Unrolling loops: Solution

```
template<int DIM> struct DotProduct {  // Recursion rule
   static inline float result(float* u, float* v) {
      return (*u) * (*v) + DotProduct<DIM-1>::result(u+1, v+1);
   }
};

template<> struct DotProduct<1> {   // Base case
   static inline float result(float* u, float* v) {
      return (*u) * (*v);
   }
};
```

Unrolling loops: Solution

```
template<int DIM> inline float dot_product(float* u, float* v) {
    return DotProduct<DIM>::result(u, v);
}
```

- The compiler then converts the call

```
    dot_product<3>(u,v)
```

  into code:

  - `DotProduct<3>::result(u,v);`
  - `(*u) * (*v) + DotProduct<2>::result(u+1,v+1);`
  - `(*u) * (*v) + ((*(u+1)) * (*(v+1)) + DotProduct<1>::result(u+2,v+2));`
  - `(*u) * (*v) + ((*(u+1)) * (*(v+1)) + (*(u+2)) * (*(v+2)));`

Expression templates: Motivation

- When designing a math library, it is desirable to provide this syntax:

```
Vector u(1000), v(1000);        // Define vectors
…                               // Initialise them
u = 1.2f * u + u * v;           // Perform operations.
```

- In C++ we can achieve this syntax via **operator overloading**.
- Question: Could there be a performance issue with such approach?
- Let's start with a naïve solution…

Expression templates: Motivation

```
class SVector {  // "Simple" Vector of any dimension.
    int size_;
    float* data_;
    void copy(SVector const& o) { for (int i=0; i<size(); ++i) data_[i] = o.data_[i]; }
public:
    explicit SVector(int size) : size_(size), data_(new float[size_]) {}
    ~SVector() { delete [] data_; }
    SVector(SVector const& o) : size_(o.size()), data_(new float[size_]) { copy(o); }
    SVector& operator=(SVector const& o) { copy(o); return *this; }
    int size() const { return size_; }
    float operator[](int const i) const { return data_[i]; }
    float& operator[](int const i) { return data_[i]; }
};
```

Expression templates: Motivation

```
SVector operator+(SVector const& u, SVector const& v) {
   SVector result(u.size()); // A temporary for result computation.
   for (int i=0; i<u.size(); ++i)
      result[i] = u[i] + v[i];
   return result;
}

SVector operator*(SVector const& u, SVector const& v) {
   SVector result(u.size()); // A temporary for result computation.
   for (int i=0; i<u.size(); ++i)
      result[i] = u[i] * v[i];
   return result;
}
```

Expression templates: Motivation

```
SVector operator*(float const a, SVector const& u) {
   SVector result(u.size()); // A temporary for result computation.
   for (int i=0; i<u.size(); ++i)
      result[i] = a * u[i];
   return result;
}
```

Expression templates: Motivation

- Let's now use the code:

```
SVector u(1000), v(1000);
u = 1.2f * u + u * v;
```

- How efficient is this code?
  - `tmp1 = 1.2f * u;     // tmp1 == "result" variable in operator*.`
    - operator* creates "result" (allocation of "data_"). Then 1000 iterations in the loop.
  - `tmp2 = u * v;        // tmp2 == "result" variable in operator*.`
    - operator* creates "result" (allocation of "data_"). Then 1000 iterations in the loop.
  - `tmp3 = tmp1 + tmp2; // tmp3 == "result" variable in operator+.`
    - operator+ creates "result" (allocation of "data_"). Then 1000 iterations in the loop.
  - `u = tmp3;`
    - operator= performs 1000 iterations in the loop.
    - Then 3 deallocations of arrays "data_" in tmp1, tmp2, and tmp3.

Expression templates: Motivation

- We can certainly do better (optimal code):

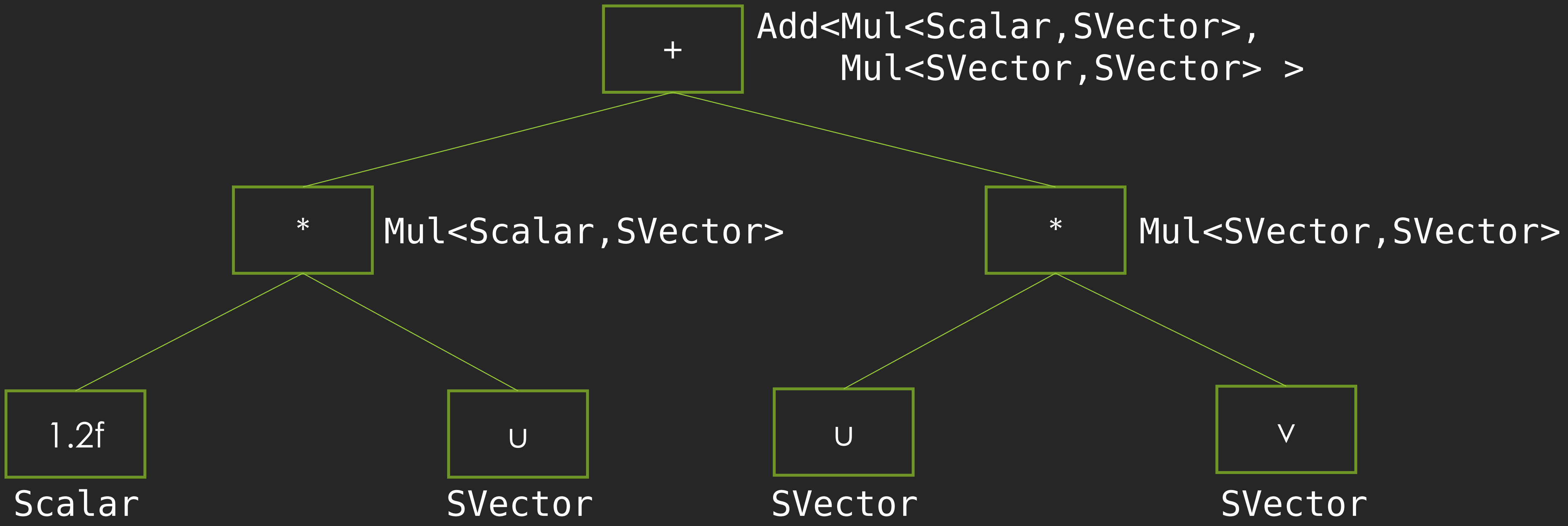
```
for (int i=0; i<u.size(); ++i)
   u[i] = 1.2f * u[i] + u[i] * v[i]; // Temporaries are CPU registers!
```

- How efficient is this code?
  - **NO RAM TEMPORARIES!** (No allocations, no redundant RAM loads/stores).
  - Just 1000 iterations in the loop.

- Can we achieve this efficiency while preserving the syntax?
  - YES – using **EXPRESSION TEMPLATES.**

Expression templates: Idea

- We let the compiler to transform the **syntax tree** of the expression “`u = 1.2f * u + u * v`” by **hierarchy of <u>simple</u> template classes**:



Expression templates: Scalar

```
class Scalar {
    float const& s_;
public:
    Scalar(float const& s) : s_(s) {}
    int size() const { return 0; }
    float operator[](int const) const { return s_; }
};
```

Expression templates: ArgType

- Inside operator classes Add and Mul we must distinguish how we reference operands:
  - Scalar operand – by value.
  - Other operands – by reference.

```
template<typename T> struct ArgType {
   typedef T const& result;
};
template<> struct ArgType<Scalar> {
   typedef Scalar result;
};
```

Expression templates: Mul

```
template<typename OP1, typename OP2>
class Mul {
    typename ArgType<OP1>::result op1_;
    typename ArgType<OP2>::result op2_;
public:
    Mul(OP1 const& op1, OP2 const& op2) : op1_(op1), op2_(op2) {}
    int size() const { return op1.size()!=0? op1.size() : op2.size(); }
    float operator[](int const i) const { return op1_[i] * op2_[i]; }
};
```

Expression templates: Add

```
template<typename OP1, typename OP2>
class Add {
   typename ArgType<OP1>::result op1_;
   typename ArgType<OP2>::result op2_;
public:
   Add(OP1 const& op1, OP2 const& op2) : op1_(op1), op2_(op2) {}
   int size() const { return op1.size()!=0? op1.size() : op2.size(); }
   float operator[](int const i) const { return op1_[i] + op2_[i]; }
};
```

Expression templates: Expression class

- How do we define operators? How about this:

```
template<typename T1, typename T2>
Mul<T1,T2> operator*(T1 const& u, T2 const& v) { … }      // (1)
```

- Not good, parameters are too general, e.g.:

```
struct A {} a;
struct B : public A {} b;
template<typename T>
A operator*(T const& s, A const& a) { … }                 // (2)
3 * a;   // Always calls (2)
3 * b;   // Calls (2) only when (1) is not defined.
```

Expression templates: Expression class

- We also do not want define all possible versions (too much work):

```
Mul< SVector, SVector > operator*(SVector const& u, SVector const& v);
template<typename T> Mul<SVector,Mul<T> > operator*(SVector const& u, Mul<SVector,T> const& v);
template<typename T> Mul<Mul<T>, SVector> operator*(Mul<Svector,T> const& u, SVector const& v);
template<typename T> Mul< SVector,Add<T> > operator*(SVector const& u, Add<SVector,T> const& v);
…
```

- Solution: We introduce a class "Expr" for **wrapping** our expressions:

```
template<typename Rep> class Expr;
```

- We can safely declare the operator as:

```
template<typename OP1, typename OP2>
Expr<Mul<OP1,OP2> > operator*(
        Expr<OP1> const& u, Expr<OP2> const& v);
```

Expression templates: Expression

```
template<typename Rep>
class Expr { // Expression
    Rep rep_;
public:
    explicit Expr(int size) : rep_(size) {}
    Expr(Rep const& rep) : rep_(rep) {}
    Rep const& rep() const { return rep_; } // Unwrap the instance.
    int size() const { return rep_.size(); }
    float operator[](int const i) const { return rep_[i]; }
    // Defined later:
    template<typename Rep2> Expr& operator=(Expr<Rep2> const& o);
};
```

Expression templates: Operators

```
template<typename OP1, typename OP2>
Expr<Mul<OP1,OP2> > operator*(Expr<OP1> const& u, Expr<OP2> const& v) {
    return Expr<Mul<OP1,OP2> >(Mul<OP1, OP2>(u.rep(), v.rep()));
}
```

Wrap the result: `Expr<Mul<OP1,OP2> >` — Actual operation: `(Mul<OP1, OP2>` — Unwrap operands: `(u.rep(), v.rep())`

```
template<typename OP2>
Expr<Mul<Scalar,OP2> > operator*(float const& s, Expr<OP2> const& u) {
    return Expr<Mul<Scalar,OP2> >(Mul<Scalar, OP2>(Scalar(s), u.rep()));
}

template<typename OP1, typename OP2>
Expr<Add<OP1,OP2> > operator+(Expr<OP1> const& u, Expr<OP2> const& v) {
    return Expr<Add<OP1,OP2> >(Add<OP1, OP2>(u.rep(), v.rep()));
}
```

Expression templates: Expr::operator=

```
Add<Mul<Scalar,SVector>,
    Mul<SVector,SVector> >
```

```
template<typename Rep>
template<typename Rep2>
Expr<Rep>& Expr<Rep>::operator=(Expr<Rep2> const& o) {
   for (int i=0; i<size(); ++i)
      rep_[i] = o[i];
   return *this;
}
```

Compiler converts ‘`o[i]`’ to ‘`1.2f*u[i] + u[i]*v[i]`’
=> we get the efficient version!

```
▶ o[i] ==> Add[i]    (Expr<Add<…> > forwards operator[] to ‘rep’)
       ==> Add::op1[i]                + Add::op2[i]
       ==> Mul::op1[i] * Mul::op2[i] + Mul::op1[i] * Mul::op2[i]
       ==> Scalar[i]   * SVector[i]  + SVector[i]  * SVector[i]
       ==> 1.2f        * u[i]        + u[i]        * v[i]
```

Expression templates: Notes & Limitations

- We must declare our vector variables as:

```
Expr<SVector> u(1000), v(1000);     // Not nice!
```

- Therefore, we usually define:

```
typedef Expr<SVector> Vector;
```

- Limitation: Does not work for operations where RAM temporaries are really needed, e.g., ‘x = A*x’, where x is a vector and A is a matrix.
  - But ‘y = A*x’ would work just fine (assuming x,y are **different** vectors).

- Usage rule of thumb:
  - For **small fixed-size** vectors use **metaprogramming**, e.g., unrolling loops.
  - For **large dynamic-size** vectors use **expression templates**.

References

[1] D.Vandevoorde, N.M.Josuttis; C++ *Temlates, The Complete Guide*; Addison-Wesley,2006.