Numbers

The way we count quantities and write numbers is important. Our decimal number system helps us to make calculations and handle money, while computers use a code based on the binary system.

Number

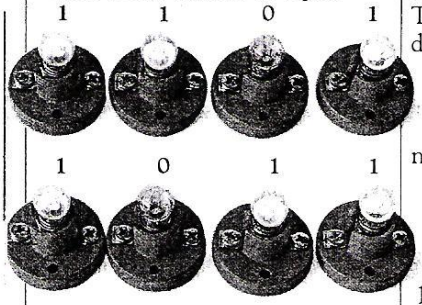
How many things exist in any quantity of things

A written number consists of **digits** or **numerals**, which are symbols such as 5, 0, 2, and 7. The number 313 has three digits, for example. A **positive number** is any number greater than zero, such as 6. A **negative number** is less than zero, such as –6. A whole number is called an **integer**. A whole number greater than 1 that can be divided only by 1 and by itself is a **prime number**, such as 2, 3, 5, 7, 11, 13, 17, 19, 23, and so on. A **base** is a number on which an entire number system is based. The decimal system uses 10 as a base. The binary system uses 2 as a base.

Binary number

A number made of the digits 0 and 1

The binary number 1101 means one eight, one four, no two, and one one. Each digit has twice the value of the digit to its right.



The binary system
The light bulbs represent, from left to right, eight, four, two and one. The binary number 1101 is 13 (8 + 4 + 0 + 1), and 1011 is 11 in decimal numbers (8 + 0 + 2 + 1).

Decimal number

A number made of the ten digits from 0 to 9

The number 24.06 means two tens, four ones, no tenths, and six hundredths. Each digit in a number has ten times the value of the digit to its right. The **decimal point** is a dot that shows which digits represent values greater than 1 and which represent values less than 1. The digits after the dot are called **decimal places**. A **recurring decimal** has one or more digits that repeat for ever, as in 0.1666666... (one sixth). A **rounded number** is an approximate number, so 0.1666666... can be rounded to 0.167, and 231,543 to 230,000.

Fraction

A number expressed as one number divided by another

Two-thirds ($\frac{2}{3}$ or 2/3) is a fraction. The top number (2) is the **numerator**, and the bottom number (3) is the **denominator**. The numerator is less than the denominator in a **proper fraction**, and more in an **improper fraction**, such as 22/7. A **mixed number** contains a whole number and a fraction, such as $2\frac{1}{2}$.

Rational number

A whole number or a fraction

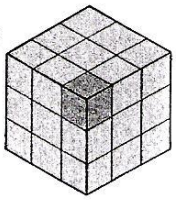
125, –6, 24/88, and –33/5 are all rational numbers. An **irrational number** is one that is not equal to one integer divided by another. If written as a decimal number, the digits never stop. For example, the square root of 2 is 1.414213... .

Square

A number multiplied by itself

The square of 5, or 5 squared (5^2), is 5 x 5, or 25. The **square root** ($\sqrt{}$) of a number is the number that, when squared, gives the first number. The square root of 36 is 6.

The cube
Each edge of this cube has three blocks, so it contains 3 cubed (3 x 3 x 3), or 27, blocks in all.



Cube

A number multiplied by itself twice

The cube of 2, or 2 cubed (2^3), is 2 x 2 x 2, or 8. The **cube root** ($\sqrt[3]{}$) of a number is the number that, when cubed, gives the first number. The cube root of 27 is 3.

Power

How many times a number is multiplied by itself

10 raised to the sixth power is 10 x 10 x 10 x 10 x 10 x 10, making 1,000,000 or 10^6. The **index**, or **exponent**, is the number of the power (in this case, 6). **Standard form**, or **scientific notation**, shows a large or small number as a number between 1 and 10 multiplied by a power of 10, so 434,000 is 4.34 x 10^5. Small numbers have negative powers, so 4.34 x 10^{-5} = 0.0000434.

Reciprocal

The result of dividing a number into 1

The reciprocal of 7 is 1/7. The reciprocal of a fraction is the fraction turned upside down, so the reciprocal of 1/7 is 7/1, or 7.

Infinity

A number that is too big to count

An **infinite** quantity is endless and cannot be measured. An **infinitesimal** object is too small to be measured.

Arithmetic & algebra

You use arithmetic whenever you count money or make calculations. Algebra is very useful in science because it enables us to write formulas, for example, and to write computer programs.

Arithmetic

The addition, subtraction, multiplication, and division of numbers

A **sum** of several numbers is their total when added together. For example, 15 is the sum of 7 and 8. A **product** is the result of multiplying numbers, so 15 is the product of 3 and 5. **Multiples** are numbers given by multiplying one number by others. The numbers 8, 12, and 16 are multiples of 4 (4 x 2 = 8; 4 x 3 = 12; 4 x 4 = 16). A **quotient** is the whole number result of a division, and the **remainder** is any amount left over. When 7 is divided by 3, it gives a quotient of 2 and a remainder of 1.

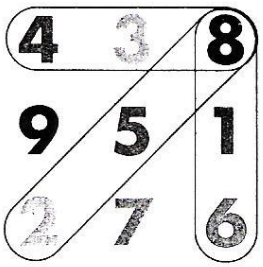
Sequence

A set of numbers that have a particular relationship

The numbers 2, 4, 6, 8, 10, and so on, form a sequence of even numbers, because all are multiples of 2. A sequence is also called a **series**. An **arithmetic progression** is a series in which you add or subtract the same number each time to get the next number, such as: 4, 7, 10, 13, 16, and so on (start at 4 and add 3 each time). In a **geometric progression**, you multiply or divide by the same number each time to get the next number, such as: 2, 4, 8, 16, 32, and so on (start at 2 and multiply by 2 each time). In the **Fibonacci series**, each number is the result of adding together the two previous numbers, such as: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, and so on.

Magic square

A square array of numbers in which the columns, rows, and diagonals each add up to the same total



Magic square
In this magic square, any three numbers in a straight line add up to 15.

Parentheses

The symbols (and)

Parentheses surround any operation in arithmetic, such as adding or multiplying, that must be carried out first. So 3 + (4 x 2) = 3 + 8 = 11, but (3 + 4) x 2 = 7 x 2 = 14.

Calculus

The use of algebra to calculate changing quantities

Calculus deals with quantities such as curving lines. It can, for example, calculate the slope of the line at any point and the area under any part of a curve.

Probability

The degree of chance that something might happen

Probability ranges from 0 (will not happen) to 1 (will happen). Throwing a die to get 5 has a probability of 1/6, as this is one result out of six possible results.

Percentage

A fraction given as a number that you have to divide by 100

$\frac{3}{4}$ is equal to 75 ÷ 100, so 3/4 is often shown as 75 per cent (75%).

Ratio

A comparison between two numbers or amounts

The masses of two objects of 10 kilograms and 2 kilograms are in the ratio 5:1 (5 to 1). You divide two numbers or amounts to get their ratio. Two amounts that vary are **proportional** if they always have the same ratio.

Algebra

The use of letters to represent quantities in calculations

The area of a rectangle is the length multiplied by the width. If we call the length l and the width w, the area of any rectangle is l x w, or lw. The letters l and w stand for numbers – in this case, the actual length of either side in units such as inches. l and w are **variables**, meaning that they can take any value. A calculation using variables, such as lw, is called an **expression**. Algebra often uses x and y as variables.

Equation

A mathematical statement that two quantities or expressions are equal

In your pocket are nine coins marked 10 and 5. If the total value is 65, how many of each kind are there? You can calculate the answer using an equation. Let the number of "10" coins be x. Their value is $10x$. The number of "5" coins is $9–x$, and their value is $5(9–x)$. As the values total 65, this gives us the equation: $65 = 10x + 5(9–x)$. Solving this equation shows that $x = 4$, so there must be four coins marked 10 and five coins marked 5.

Trigonometry

It is possible to find out how far away you are from something, or how high mountains and buildings are, without actually measuring these things. Trigonometry uses triangles to solve such problems.

Trigonometry

The branch of mathematics that studies triangles

A triangle has three sides and three angles where each pair of sides meets. If you know the size of any three of these sides and angles, you can work out the others using trigonometry. You make calculations using the length of one side and a trigonometrical ratio.

Angle

The space between two lines that meet or cross

The angle formed where two lines meet or cross is expressed as the amount of rotation needed to move one of the lines to the position of the other line. This movement is measured in degrees (°) or radians. An angle of 0° is a **null** or **zero angle**. Where the lines make a square corner, the angle is 90° and is called a **right angle**. An **acute angle** is less than 90°, and an angle that is between 90° and 180° is an **obtuse angle**. A **reflex angle** is between 180° and 360°, while an angle of 360° is a **round angle**. A **solid angle** is the amount by which a cone spreads from its point, or **vertex**.

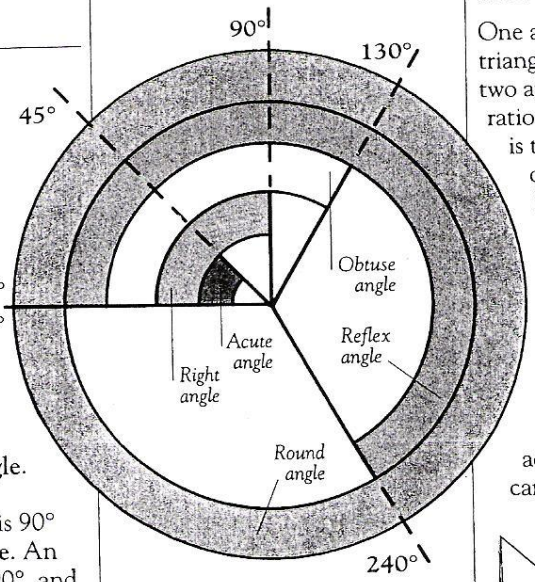
See also

Reciprocal 170

Degree (°)

A unit used to measure angles

A degree can be divided into 60 units called **minutes** (′), while a minute is made up of 60 smaller units called **seconds** (″). A **radian** (rad) is another unit of angle equal to 57.296°, or 180°/pi (pi = 3.14159). The word degree also refers to other small units, such as degrees of temperature.

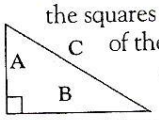


Rotation and angles
Two lines lying on top of each other have an angle of 0°. If one line rotates about one end, the angle between the lines increases. An angle of 45° is an acute angle. A right angle is formed when the line makes a quarter turn and rotates 90°, while an angle of 130° is an obtuse angle. Any angle formed by a rotation of more than 180°, such as 240°, is a reflex angle. The line makes a complete rotation in 360° and returns to its original position, giving a round angle.

Pythagoras

Greek philosopher (c.580–c.500 BC)

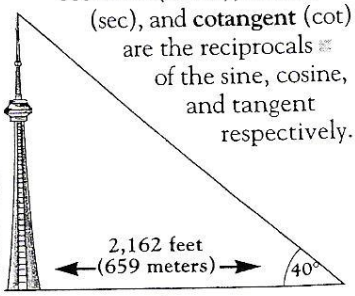
Pythagoras proposed that everything is governed by relationships of numbers. He is best known for the **Pythagorean theorem**. This states that the square of the length of the hypotenuse of a right-angled triangle (side C in the diagram below) is equal to the sum of the squares of the lengths of the other two sides ($C^2 = A^2 + B^2$).



Trigonometrical ratio

The ratio of the lengths of two sides of a right-angled triangle

One angle of a right-angled triangle is 90°. Either of the other two angles has six trigonometrical ratios. The **sine** (sin) of the angle is the length of the side opposite the angle divided by the hypotenuse (the side opposite the right angle). The **cosine** (cos) of the angle is the length of the adjacent side (next to the angle) divided by the hypotenuse. The **tangent** (tan) is the length of the opposite side divided by the adjacent side. Calculators can give these ratios. The **cosecant** (cosec), **secant** (sec), and **cotangent** (cot) are the reciprocals of the sine, cosine, and tangent respectively.



Making use of trigonometry
The height of the tower equals the distance (2,162 feet) multiplied by the tangent of 40° (0.839) = 1,814 feet (553 meters).