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The green paradox of the economics of exhaustible resources

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HIGHLIGHTS

- The green paradox is a direct application of Hotelling's rule from the economics of exhaustible resources.
- Hotelling's analysis was a profound contribution to economic thought but evidence for it is weak.
- Hotelling-style analysis assumes incorrectly that production can be rearranged at will among time periods.
- Technological and geological features of oil production make the prediction of the green paradox unlikely.

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ABSTRACT

The green paradox states that an increasing tax on emissions of carbon dioxide, consonant with the expected increase in their marginal damages, may induce oil producers to shift their production toward the present and thereby to exacerbate the problem of climatic change. The model is based on Hotelling models of resource use that do not take the natural and technical features of oil production into account. Natural features include the decline of production through time according to a decline curve. Technical features include the requirement to sink investment in productive capacity. A model of a profit-maximizing firm indicates that, if these features are taken into account, the prediction of the green paradox is unlikely.

1. Introduction: The meaning of the green paradox

In discussions of appropriate policy responses to climatic change, the role of fossil fuels, especially oil, takes centre stage. There is a current sense of urgency to begin to reduce consumption of these fuels. A method favoured by some economists is a tax on emissions of carbon dioxide, in essence on oil use. Since the marginal harm inflicted by emissions is expected to increase over several decades, a proposal consistent with much of environmental economics is that the tax should be announced as increasing through time, in step with the marginal damages.

Suppose that a global tax on fossil fuels were implemented, and that governments worldwide could commit to the future schedule of taxes deemed appropriate to balance the costs and the benefits of oil use. Would this development be salutary in the context of climatic change?

One theoretic development holds that it may not. The *green paradox* states that dynamic influences may thwart the intent of the tax by giving producers an incentive to shift production toward the present. It would thereby cause an increase in damages in the short and medium terms.

Oil is an exhaustible resource. The economics of exhaustible resources is expressed through Hotelling's rule. In its simplest form the rule states that in equilibrium the net price, the price net of marginal costs and marginal taxes, rises at the rate of interest. The argument for the green paradox is a direct application of the rule, which prescribes the optimal timing of the extraction and use of exhaustible resources. By changing the *relative* net values of a unit of oil at different future dates (as compared to the original equilibrium without the tax) the tax may induce producers "to tilt" their production toward the present. Greater emissions in the present and medium term may be induced. Since there is a fixed quantity of fossil fuels in the earth, the greater emissions come at the expense of emissions in the long-term future. (In the simplest models there is a one-to-one shift in production.) By then, other means to attack the climate problem may be available. Paradoxically, the tax, intended to help to solve the problem of climatic

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change in the short and medium runs, may exacerbate it, and yet provide only limited relief in the long run.

The green paradox merits attention from environmental economists because the theoretic issue is recast by climate change; it becomes the timing of a tax instead of the equity and apparent efficiency of having the "polluter pay" directly for the marginal damages caused. The policy issue is whether the tilt toward the present, described in theory, can be expected to play out in practice.

The present paper expresses doubts about the analysis that gives rise to the green paradox. These doubts are the product of doubts about the applicability of Hotelling's rule. The analysis draws attention to features of extraction such as sunk investments and production constraints, which are neglected in Hotelling models. The paper begins with a brief explanation of Hotelling models. Then it reviews the application of Hotelling analysis to effects of the tax on flows. Later, it interprets a survey of the empirical analysis related to Hotelling's rule. Finally, it considers technological and natural features of oil production. These features make the oil industry more complicated than envisaged in Hotelling models. Definitive answers are not possible, but indications are that the effects of a carbon tax may more likely be conventional than paradoxical.

2. Hotelling models

Hotelling's (1931) model of *the economics of exhaustible resources* is a profound contribution to economic thought. It provides five major insights:

- Exhaustible resources are a form of capital.
- The price of the resource is determined in a dynamic equilibrium that regulates both the flow of the resource to market and the holding of the resource as an asset.
- The timing of decisions is of central significance and warrants careful analysis.
- Resources are subject to the usual market failures, viz. monopoly, externality and informational asymmetry.
- Exhaustibility in itself does *not* entail a special form of market failure. In particular, competitive markets are not subject to a myopic inability to allocate an exhaustible resource in way that efficiently balances the interests of the present and the future.

In the Hotelling model, units of the resource are viewed as being available to society for extraction at any time, at a known cost that depends on the quantity extracted and possibly other factors. A striking analytic result of the model is *Hotelling's rule*: under certain assumptions, in a dynamic, competitive equilibrium the price net of marginal cost of an exhaustible resource rises at the rate of interest. (Under other assumptions the rule is more complicated.) The rule can be proved through optimal-control analysis and is mathematically incontrovertible.

The economic reasoning behind the rule is even more striking. Consider what is called herein a *type-one* Hotelling model of an exhaustible resource, in which extraction costs depend only on the quantity of the resource that is currently being extracted. In this case, the extraction cost of $q>0$ units of the resource is given by some function $c(q)$. This function is assumed to be increasing, so that extracting more units at a given time costs more, and to be convex, so that it becomes ever costlier to extract additional units.

As argued by Solow (1974), the owner of a resource who wishes to maximize net present value is led to re-arrange extraction such that what is earned by the marginal unit in each time period is equal in present-value terms. If the marginal unit at one time gains less than at another time, present value can be increased by reallocating output from the period with the lower gain to the one with the higher gain. In symbols, let $p(t)$ represent the price at time t, r represent the prevailing rate of interest and $\mu(t)=[dc(q)/dq]_t$ represent the marginal cost of production. Suppose that a proposed path of extraction is such that, for times t and s during production

$$D(t,s)=\frac{p(t)-\mu(t)}{(1+r)^t}-\frac{p(s)-\mu(s)}{(1+r)^s}>0.$$

Then a unit of production can be re-allocated from time s to time t, at a net gain of $D(t,s)$.

The re-allocation can be done repeatedly, so long as an inequality holds, at a net gain each time. Ultimately, a constant, λ say, is determined such that for all times t and s during which extraction takes place

$$\frac{p(t)-\mu(t)}{(1+r)^t}=\lambda=\frac{p(s)-\mu(s)}{(1+r)^s}.$$

Another re-arrangement yields that

$$p(t)-\mu(t)=\lambda(1+r)^t \tag{1}$$

the *net* price (price net of marginal cost) rises at the rate of interest. Eq. (1) expresses Hotelling's rule. The discussion stresses that Hotelling's rule is an arbitrage condition for the use of an asset, through the allocation of extraction, over different periods of time.

Often it is assumed that the marginal cost is constant, so that for some number β, $c(q)=\beta q$. In this case, for any value of q or t, $\mu(t)=\beta$. The assumption allows for developing sharp mathematical results, including the early insights of the green paradox by Sinclair (1992) and Ulph and Ulph (1994), as well as some more-recent ones.

A remarkable feature of Hotelling's original paper is that he also considered what may be called a *type-two* model. In type-two models, cost depends on the total available reserve, Q say, as well as current output, and is written $C(q,Q)$. The properties of this cost function are that it is an increasing, convex function of output q and a decreasing, convex function of available reserves Q. (Costs are lower if reserves are higher.) Also, $C(0,Q)=0$ for any value of Q. A type-two model delivers less sharp theoretic results than a type-one model: There is still arbitrage among marginal units of the resource but the influence of the remaining reserve on cost yields a more complicated expression of Hotelling's rule. The rule is expressed in terms of the discounted sum, over the future of production from the resource, of the increases in cost that arise because current production affects future costs through depleting total reserves. Though harder to work with theoretically, type-two models are considered to be more realistic.

The function $C(q,Q)$ has been a workhorse of empirical research in economics since the late 1970s. Several theoretic analyses have also utilized it. In the main, however, theorists have resorted to the simpler function βq. The same observations are true of the green paradox: Although some authors have used the function $C(q,Q)$ in theoretic work, the simpler function is the foundation of the more striking conclusions.

A key point is that either function implies that at any time any level of extraction q is possible if one is willing to incur the current marginal cost. (If cost is βq, an unbounded level of output can be had at the constant marginal cost β). Consistently with the nature of arbitrage in Hotelling models, output can be shifted at will over time. There is no impediment to tilting output toward the present.

3. The green paradox

Even though the analysis of extraction with cost $C(q,Q)$ is more complicated than with cost βq, Sinn (2008) deftly uses arbitrage between *adjacent* periods (times t and $t+1$) to make the argument for the green paradox. Let $C(q,Q)=g(Q)q$, where $g'(Q)<0$ and $q=-dQ/dt$. This case is special but only slightly so: it assumes constant marginal cost at any point in time, but increasing costs as the total stock Q decreases through extraction. The proceeds from a single unit of resource extracted at time t and invested for one period are $p(t)-g(Q)$ of principal and $r[p(t)-g(Q)]$ of interest. If instead that unit is not extracted until $t+1$, and also if the change in Q over the period (which is $-q$) is neglected as being quite small compared to Q, the owner gains $p(t+1)-g(Q)$. Let the change in price over the period be represented by $\Delta p(t)\equiv p\ (t+1)-p(t)$. Arbitrage renders the gains in the two periods equal, so that

$$[p(t)-g(Q)](1+r)=p(t)+\Delta p(t)-g(Q).$$

Simple algebra yields a single-period generalization of Hotelling's rule:

$$\frac{\Delta p(t)}{p(t)-g(Q)}=r \tag{2}$$

Condition (2) holds on an equilibrium path for each pair of adjacent time periods and so links all periods.

If a tax on emissions is imposed and it rises through time, say being equal to the discounted value of the damages due to the marginal unit of emission, the result may not be what is expected by proponents of the tax. To illustrate what may happen while using minimal algebra, Sinn uses a special type of tax. Let the tax $\tau(t)$ be such that the net price received by the firm (net of the tax as well as marginal cost), $[1-\tau(t)][p(t)-g(Q)]$, is reduced through time at rate δ:

$$1-\tau(t)=\frac{[1-\tau(0)]}{(1+\delta)^t}. \tag{3}$$

The arbitrage condition is that

$$\frac{[1-\tau(0)]}{(1+\delta)^t}[p(t)-g(Q)](1+r)=\frac{[1-\tau(0)]}{(1+\delta)^{t+1}}[p(t)+\Delta p(t)-g(Q)].$$

In this situation the modified rule becomes

$$\frac{\Delta p(t)}{p(t)-g(Q)}=r+\delta(1+r). \tag{4}$$

Sinn applies the following economic reasoning to this condition for different values of δ. When $\delta=0$, so that the tax is constant, condition (4) reduces to condition (2) throughout the period of production. Therefore, the tax is neutral (does not change the path of extraction and hence does not change the accumulation of carbon in the atmosphere). For given values of the other variables in condition (4), the change in price, Δp, is greater when $\delta>0$ than when $\delta=0$. (Price grows faster when the tax is increasing.) Sinn (2008: 374ff) makes technical assumptions that assure that the reserves are eventually completely exhausted in both scenarios, with and without the tax. When the reserves are exhausted (and hence output is zero), the price must be the intercept of demand. Since the price rises faster with the increasing tax, and since each scenario ends up with an equal total quantity extracted, the price with a tax must start out lower. In equilibrium at the current time t, therefore, output must be higher with the tax.

The economics behind the result is that the rising tax changes the *relative* gains to the producer over the life of the reserve, making present extraction comparatively more attractive at the margin. With greater present extraction, the industry is in equilibrium at a lower present price. Lower prices also prevail in the near future.

Consequently, global warming is exacerbated in the present. The green paradox is that a policy designed to tax the emission of carbon in a way that is strongly related to damages caused may lead to greater current emissions. Some have suggested that a better rule may be to have an initially high but deceasing tax (so that δ is negative). Such a tax would encourage the producer to shift output toward the future. The form of the tax, then, is central to its effects.

Sinn observes that other forms of policy, such as a subsidy to greener forms of energy, would be subject to a green paradox as well. His analysis, as summarized above, is a benchmark for understanding the phenomenon.[1]

4. Evidence of Hotelling's rule

For a mathematically incontrovertible result, Hotelling's rule has been subject to much controversy. Practitioners, whose conscious, rational decisions are supposed to implement the rule, flatly and (to this author's knowledge unanimously) deny its relevance. Strong challenges have come from academia.

An erudite review of empirical research on Hotelling's rule finds that the evidence does "not necessarily invalidate the conceptual message of the Hotelling Rule" (Livernois, 2009, pp. 37–38). The finding arises from a conviction "that mining firms think not just of the present but about the future and that they wish to maximize the value of their assets".

Thus, Hotelling's analytic framework of maximizing net present value ("the value of their assets") seems to be borne out. In the statements of the green paradox, however, as well as in the foundations of the economics of exhaustible resources, the thrust of Hotelling's message is not limited to the notion that extracting firms maximize net present value. This notion is an initial working hypothesis about conduct in any industry.[2] Hotelling's rule, his analytic result, is the message that is specific to exhaustible resources and is used by resource economists as the basis for thinking about the dynamics of resource prices (Livernois, 2009, p. 22).

Elsewhere, Livernois (2009, p. 22) describes Hotelling's rule as "a condition of inter-temporal arbitrage that ensures that the last unit extracted in any time period earns the same return (in present value terms)". As Solow's (1974) definitive treatment notes, the arbitrage consists of being able to move production of units of output freely among time periods in response to changes in prices and interest rates. Livernois (2009, pp. 37–38) finds that "overall one cannot conclude that the Hotelling Rule has been a significant force", and that "other factors, notably technological change, revisions to expectations regarding the resource base, and market structure, have had a more significant influence on the evolution of prices". Again, comparable influences affect all industries, not just exhaustible-resource industries.

Livernois's review suggests that, in spite of econometricians' efforts to control for the other influences, the arbitrage condition has not received broad support from empirical research. Instead, the evidence suggests that the arbitrage which underlies the predictions of Hotelling theory may not be being realized. Is there an explanation for what appears to be a troubling departure from an incontrovertible mathematical result?

5. Technological models

The key assumption, usually glossed over, is that output can be re-arranged as desired. It implies that the output decision is made

[1] Several other estimable economists have studied the paradox within the Hotelling canon but under somewhat different assumptions from Sinn's and have qualified his results.

[2] It is not a good hypothesis about behaviour of the national oil companies that are responsible for much of the world's output of oil. See Cairns and Calfucura (2012). However, it is accepted herein for the sake of argument.

period-by-period, with no restriction other than exhaustibility. The sole physical limit to the level of output in any period is held to be the level of remaining reserves. All costs are variable. As a result, Hotelling's rule is, in principle, observable; econometricians can try to tease evidence for (or against) it from the data.

The function $C(q,Q)$ of Hotelling models, however, is not a valid representation of the technology of oil production, let alone the simpler βq. In reality, the potential to produce is obtained by a high up-front cost in drilling wells that determines an output capacity, as reasoned by Campbell (1980) in introducing what may be called a *technological* analysis of non-renewable resources. Once capacity is installed, it is usually not changed for a significant period of time. Campbell's ideas were advanced by Crabbé (1982) and Lasserre (1985).

In the oil industry, after the period of capacity production, which may be short, the productivity of a well decreases through what is known as *natural decline*. Natural decline is the technological counterpart to the assumption in type-two models that costs increase as a result of decreases in available reserves. Other than for "ramping up" production, for maintenance, or for production problems, the level of output is restricted by capacity and by natural factors that produce what is known as a *decline curve* (Cairns, 2009).

In essence, Hotelling models of both types one and two are short-run models. They neglect the decision to commit capital to oil production. They neglect the facts that the technology does not allow for unlimited output and that there are natural limits to production at any time. The option to shift production unimpeded from one point in time to another, the free arbitrage basic to Hotelling analysis, is not available.

Moreover, the limits or constraints have a shadow value that is confounded with the shadow value that gives rise to the r-percent rule in the net price. (The two shadow values sum to the net price.) The r-percent rule is not observable in principle.

Where empirical evidence of the path of price net of marginal cost has been sought, it has been found that this variable rises more slowly than at the rate of interest. The type-two model predicts that the net price rises at less than the interest rate. But comparatively few sectorial models incorporating behaviour at individual reserves have used even a type-two view of these markets. One, by Cairns and Quyen (1998, p. 181) for mineral exploration, argues that the modified rule for the rate of change of resource rent "involves a complicated convex sum, with endogenous weights, of the deposit-specific terms" so that the cost function $C(q,Q)$, involving aggregate magnitudes q and Q, does not generate the correct aggregate rule.

For analysis of the effects of policy, a long-run analysis is required. Technological models are reserve-based rather than sector-based, analysing conditions of production at the individual reserve. Unlike in most Hotelling models, decisions are disaggregated. The behaviour of the sector is an aggregate of the behaviour of individual producers. As a result, the industry cannot be fully represented in formal, analytical models. Numerical analysis or less formal, economic logic may be required to provide bridges and extensions.

Cairns and Davis (2001) observe that oil is produced from underground reservoirs that are under great pressure from the contained oil and gas. A well drilled into the formation, like a pin prick into a balloon, allows the pressure to be released, and with it the oil and gas. In some cases natural drive is sufficient to produce the oil, but at declining rates. In others, if pressure is not great enough, pumps are installed at the surface to lift the oil to the surface. For what is known as secondary production, wells can be installed at the periphery of the reservoir to inject water or gas that will drive the hydrocarbons toward the producing well. In all cases, the valuable natural product is oil (and gas). The scarce natural or artificial instrument of production is pressure. Technology allows for the augmentation of pressure in various ways.

The approach can be used to build more comprehensive views of the sector. First, a short-run model can depict a firm's behaviour once investment has been made. Second, one can model investments in development as contingent upon that behaviour. Finally, exploration can be viewed as looking forward to succeeding decisions.

6. Appropriate short-run analysis: the decline curve

An initial analysis is that of a *producing* property: all investments have been made and operation is in the short run. This discussion is the counterpart at the level of the reserve to a Hotelling analysis at the level of the industry. Following Adelman (1990, 1993) and Cairns and Davis (1998, 2001) let the oil be driven to the surface by pressure, $P(t)$, which is subject to *natural decline* at rate a, according to the equation

$$dP(t)/dt = -aq(t). \tag{5}$$

For some constant π, output depends on pressure according to the inequality

$$q(t) \le \pi P(t). \tag{6}$$

As is argued by Adelman and by Cairns and Davis, the growth in net price is typically less than the rate of interest. Producing at less than $\pi P(t)$ thus reduces net present value. Consequently,

$$q(t) = q(0)e^{-at}. \tag{7}$$

Eq. (7) is the decline curve of the reservoir.

Suppose that the reserve has been prepared for exploitation at time $t=0$. The (recoverable) reserve, initially available for production, is

$$R(0) = \int_0^\infty q(t)dt = \frac{q(0)}{a}.$$

Let the *net* price (price net of marginal cost) be represented by $\nu(t)$ and grow at rate $\gamma < r$. Then the *value* of the reserve is given by

$$V_0 = \int_0^\infty [\nu(0)e^{\gamma t}][q(0)e^{-at}]e^{-rt}\,dt = \nu(0)R(0)\frac{a}{a+r-\gamma}.$$

Since $\gamma < r$, the value V_0 is strictly less than $\nu(0)R(0)$, which is the value given by a condition derived from Hotelling's rule called the Hotelling Valuation Principle (Miller and Upton, 1985).[3]

Production in this model is fully determined by geological features and not economic choices. The formal optimization of its value is trivial. The economic meaning of the result, however, is not trivial. Since the reserve is extracted over time, with the net price per unit rising at less than the interest rate, the value must be lower than given by the Hotelling valuation principle. Adelman's perspective on the oil industry indicates that, at an individual reservoir and hence for the whole industry, a prediction of the Hotelling model cannot hold.

The mathematical reason is the shadow value of pressure. It arises from constraint (6). In an abstract sense, pressure is a *scarce resource* because an increase in pressure would allow the operator to extract more quickly and thereby to increase net present value. As mentioned above, a portion of the net price is attributable to pressure. The results hold, with minor modifications, under

[3] Adelman (1990) observes that a rule of thumb in the oil industry is that at any time t during a reserve's producing life, it is valued at about one half of net price applied to remaining reserves: $V_t = \nu(t)R(t)\left(a/(a+r-\gamma)\right) \approx \frac{1}{2}\nu(t)R(t)$. The approximation arises when $\gamma \approx 0$ and $a \approx r$, and has been called *Adelman's rule* (Cairns and Davis, 2001).

conditions of uncertainty as well (Davis and Cairns, 1999; Thompson, 2001).

The reason that the Hotelling rule does not hold is that the output from a producing reservoir is constrained. Output can be reduced but not increased. There is no change in output as a result of a tax (unless the tax is so high as to put the firm out of business).

In sum, at any time at any producing property in the oil industry, output is constrained by a technological–geological constraint. The short-term increase in output assumed in the green paradox cannot be obtained from currently active reserves without some increase in investment.

There is no reason to believe in a green paradox if one bases the analysis on current abilities to increase supplies in response to a change in the time profile of net prices resulting from a tax. If anything, the tax may be expected to result in a reduction in production, and a reduction in emissions of greenhouse gases, since the tax may render some producing properties no longer profitable.

7. The development of known reserves

The discussion thus far has abstracted from some of the key decisions that are made by producers. In practice, some reserves are known but "held on the shelf" for later development. A more sophisticated analysis admits the possibility that the *aggregate* production constraint can be circumvented, to an extent, by investment at new reserves.

Development of a known reserve involves two major decisions, namely the timing of investment (often called *the order of entry* in resource economics) and its level. There is an option to develop immediately or to postpone development. If a given reserve is developed at a proposed time S, it has a present value as of that date. That present value depends on conditions in the market at S and throughout the projected productive lifetime of the reserve. Optimal development takes place when the present value of a development project, taken as of the current date t, is maximized with respect to $S \geq t$. The ordering criterion is that the rate of increase of the current net present value of the project falls to the rate of interest at the optimal strike time (Cairns and Davis, 2007; Davis and Cairns, 2012). Let the net present value of a reserve at the present date $t \leq S$ be represented by $V(a(S),S)$. Then the optimal value of S is the solution to the equation, $(dV(a(S),S))/dS = 0$. Usually, however, conditions are stated to hold as of the time decisions are made. Let $W(a(S),\ S) = e^{rS}V(a(S),S)$ denote the *forward* (as of time S) net present value of the project. The rule is that, at the optimal start-up time, the rate of increase of the forward net present value has fallen to the rate of interest:

$$\frac{1}{W(a(S),S)}\frac{dW(a(S),S)}{dS} = r. \tag{8}$$

The order of entry is determined, then, by a formula that takes into account the conditions throughout the exploitation of the reserve and not just the net price at a given instant.

A *level* of investment can be chosen for each possible value of the start-up date S. In a simple model, the choice of level can be represented as a choice of the value of a in Eq. (5) (Adelman, 1990; Smith, 2012). Once made, the investment "locks in" the maximal rate and, in practice, the actual rate of production in the short run.

Consequently, instead of there being an economically meaningful choice of the timing of the production of individual units of a stock from the given reserve as portrayed in Hotelling models, timing is expressed through the choice of the date of the development investment. Any change arising from a tax depends on both the timing (S) and level (a) of investment. The effect of the imposition of a tax on the "tilt" of production is given by the level of investment in the reserve and by the decline curve.

Consider the extension of the analysis of the tax introduced in Eq. (3) to a reserve in which such an investment is made. Postulating this simple form of tax again facilitates the analysis, in this case in drawing the distinction between Hotelling and technological models in their implications for the timing of extraction of oil and the emission of carbon dioxide.

The analysis is easier in continuous time. In the expression of the path of the tax through time, the factor $(1-\tau)/(1+\delta)^t$ gives way to $(1-\tau)e^{-\delta t}$. Because the net price at reserve i depends on costs, which are specific to the reserve, the net price and its rate of growth are specific to the reserve. Let the net price at time t be represented very simply by $\nu_i e^{\gamma_i t}$ and the discounted net price (net of the tax as well as marginal cost) to the producer by $\nu_i(1-\tau)\ e^{(\gamma_i-\delta-r)t}$.

For simplicity, the analysis abstracts from the possibility of an initial period of capacity production by combining the constraints of capacity and natural decline into the variable a_i. At producing property i having a remaining, recoverable reserve R_{it}, the level of output at time t is given by $q_{it} = a_i R_{it}$. Because of natural decline, output at time $s > t$ is given by $q_{is} = a_i R_{is} = a_i R_{it} e^{-a_i(s-t)} = q_{it} e^{-a_i(s-t)}$. The cost of investment is also important. Smith (2012) states that, for some parameter k_i that is specific to the reserve, the investment cost for (onshore) development is approximately equal to $k_i a_i^{5/3}$.

Now let the subscripts i be suppressed to reduce clutter. Under the above assumptions the net present value of the initial reserve R at the present date $t \leq S$ is

$$\begin{aligned} V(a(S),S) &= \int_S^\infty aRe^{-a(t-S)}\nu(1-\tau)e^{(\gamma-\delta)t}e^{-rt}\,dt - ka^{5/3}e^{-rS} \\ &= \left[\frac{aR\nu(1-\tau)}{a+r+\delta-\gamma}e^{-(\delta-\gamma)S} - ka^{5/3}\right]e^{-rS}. \end{aligned}$$

Even under the strong assumptions of this model, including that of the simple tax from Eq. (3) as modified, the comparative-statics analysis is inconclusive. It involves finding conditions on the values of parameters, applicable to individual reserves, for which complicated expressions have a particular sign. It is not possible to obtain a definitive solution for the whole industry.

One suggestive contrast with results from Hotelling models can be obtained. The optimal level of investment is such that, for the chosen value of S,

$$\frac{\partial V(a,S)}{\partial a} = \left[\frac{R\nu(1-\tau)(r+\delta-\gamma)}{(a+r+\delta-\gamma)^2}e^{-(\delta-\gamma)S} - \frac{5}{3}ka^{2/3}\right]e^{-rS} \leq 0, \tag{9}$$

with equality holding when $a > 0$. It is easily checked that $V(a,S) > 0$ when there is equality in Eq. (9). The positive value $V(a,S)$ is the resource rent for the reserve under study, its scarcity value. It is a present value of the reserve, optimized over a and S, as opposed to the current value per unit that is found in Hotelling models.[4]

Using the fact that there is equality in condition (9) when $a > 0$ yields that

$$\begin{aligned} \frac{\partial V(a,S)}{\partial S} &= \left[\frac{-a(r+\delta-\gamma)R\nu(1-\tau)}{a+r+\delta-\gamma}e^{-(\delta-\gamma)S} + rka^{5/3}\right]e^{-rS} \\ &= a(a+r+\delta-\gamma)\left[\frac{-a(r+\delta-\gamma)R\nu(1-\tau)}{(a+r+\delta-\gamma)^2}e^{-(\gamma-\delta)S} \right. \\ &\quad \left. + \frac{r}{a+r+\delta-\gamma}ka^{2/3}\right]e^{-rS} \\ &= -\frac{5}{3}ka^{5/3}e^{-rS}\left[(a+\delta)-(\gamma-\frac{2}{5}r)\right]. \end{aligned}$$

[4] It is all the rent from the reserve. Rent due to quality of the reserve ("Hotelling" and "Ricardian") is optimized in the choice of S (rate of growth of net present value falls to r) and of scale in the choice of a.

If $\delta<\gamma-\frac{2}{5}r$, then $\partial V/\partial S>0$. In this case the reserve is never exploited because its value continues to increase indefinitely. This result is an artefact of the simplicity of the assumptions: in a dynamic equilibrium of the industry, the net price would not continue to rise at the constant rate γ as assumed but price would adjust so that the reserve would eventually be exploited.

In an internal solution (one in which $S>t$, $a>0$ and $\partial V/\partial S=0$), $a=(\gamma-\frac{2}{5}r)-\delta>0$. The optimal value of a decreases as the value of δ increases until $\delta=\gamma-\frac{2}{5}r$. It is plausible that the timing of investment is such that the reserve is developed sooner when there is a tax. For example, if $\gamma-\frac{2}{5}r>0>\gamma-\frac{2}{5}r-\delta$ there is an internal solution for the reserve when there is no tax ($S>t$ when $\delta=0$) but a corner solution when there is a tax ($S=t$ when $\delta>0$). The timing of investment is affected in the direction predicted by the green paradox. At the time that investment takes place, however, its level, a, is reduced. As a result, output is reduced throughout the life of the reserve according to the decline curve. This effect counters that of the timing of investment in the time path of oil production.

For those reserves which, with no tax, are under development or being studied for development (for which $\delta=0$ and $\gamma-\frac{2}{5}r<0$), the imposition of the tax has no effect on timing: $S=t$. In this case, $(\partial^2 V/\partial a^2)<0$ and $(\partial^2 V/\partial\delta\partial a)<0$, so that $(da/d\delta)<0$: as the carbon tax increases, the level of investment a and the initial output aR from the reserve decrease.[5] The prediction can be said to be contrary to that of the green paradox.[6]

In this model, $\nu>0$ and hence $\nu(1-\tau)e^{-\delta t}>0$. The tax of Eq. (3) operates on the net price ν but leaves the net cash flow always positive, so that a developed reserve is always fully exploited. Moreover, there is always a value of $a>0$ for which $V(a,S)>0$. A reserve is always developed.

Rather than being imposed on the net price, a carbon tax in reality would more likely be imposed on emissions, broadly speaking on output.[7] The discounted, instantaneous net cash flow to a producer facing tax *per unit output* of $\tau' e^{\delta' t}$ at time t is $aR(\nu e^{\gamma t}-\tau' e^{\delta' t})e^{-a(t-S)}e^{-rt}$. This tax is akin to a royalty that increases at rate δ'. For a marginal reserve, the imposition of the tax may reduce the net present value to less than zero, so that the reserve is not developed at all. Moreover, production at a developed reserve ceases at the time s when $\nu e^{\gamma s}-\tau' e^{\delta' s}=0$. Cessation is a drag on investment.

For other reserves, each of ν and $V(a,S)$ may pass from negative to positive at some future time S. Other things being equal, the time at which the net price (net of tax as well as current cost) becomes positive is pushed forward (toward the future) as compared to the case with no tax (with $\tau'=0$). The determination of the optimal start-up date S is delicate because it obeys an r-percent rule, Eq. (8), that depends on the dynamic equilibrium of the oil market.[8]

This type of tax is much more difficult to study analytically than the one studied above. The tax could reduce net present value more significantly, having greater effects on the chosen level of a, or leading to the abandonment of otherwise viable prospects. It is difficult to predict the start-up date. Simulations of the sort performed by Smith may be more revealing than trying to pursue the analytic approach. The assumption that other things are equal, however, is almost surely not valid. Any simulation would be even more difficult than Smith's since the effects of the tax on the equilibrium level of γ, involving the decisions of all current and future producers, would have to be determined rather than simply assumed. The results for individual reserves would again depend on the values assumed for the various parameters. Much that matters is not easily modelled analytically or simulated. It is noteworthy that Smith considers his sophisticated, subtle analysis to be “highly simplified”.

Some economists may object that the model does not envisage an attempt to reduce the green paradox through tax design or even through a deliberate choice of δ or τ. In a Hotelling model it is relatively easy to fold any form of tax into a maximization of social welfare and thereby to define an optimal tax. In this way a carbon tax can be made socially efficient. (It still may have a paradoxical effect on the timing of emissions.) In an aggregate technological model, an optimal tax would have to be

- optimized specifically for given reserve, not a “carbon tax” at all but an optimally varying royalty, or else
- optimized *across* and common to all of the reserves (developed, undeveloped and undiscovered), and not optimized for any one of them.

Sinn's simple analysis of the green paradox, though not a welfare analysis, makes the vital observation that the effects of a policy, especially a dynamic policy, may be contrary to predicted effects. His strategy of studying a given, non-optimized tax makes sense in discussing the effects of a carbon tax, both at a given reserve and in the aggregate.

8. Undiscovered reserves and other forms of capital

Exploration is a response to an incentive provided by a positive expected net present value of discoveries (Cairns and Quyen, 1998). Exploration cost is a “set-up cost” (Hartwick et al., 1986), as opposed to an investment in productive capacity (Campbell, 1980). It involves a sunk cost but no constraint on arbitrage of the output from a reserve that has been “set up”.

An emissions tax would shift the distribution of net present values from investments in exploration in any mineral province “to the left” (toward lower values). A tax increasing continually through time would entail a continual shift to the left. A net increase or decrease in exploration may result. As predicted by the green paradox, there may be a “black-gold rush” to realize the value of exploration provinces earlier than in the original equilibrium. There may also be a holding back of exploration in the face of lower returns to the investment. Smith (2012) finds that a decline in expected net present value due to a tax reduces the number of wells an exploring firm is willing to drill in a given area.

Any current increase in exploration depends on whether equipment (e.g. drilling rigs) and professionals are available. In equilibrium, spare capacity is low. After all, investments are made

[5] Also, $da/d\tau<0$.

[6] Another source of “tilt” may be investment in secondary or tertiary (enhanced) recovery. Smith (2012) models the additional investment in increasing the decline factor to $a^e>a$ during the productive life of the reserve, at time $S^e>S$. (See Amit, 1986 for a discussion of the determination of S^e.) Smith demonstrates numerically how the level and timing of such investments complicate the analysis of the effects of taxes. One might encounter conditions under which the tax would induce a decrease in S^e. The likelihood of inducing a reduction of a^e remains.

[7] The tax is on emissions. For any particular reserve the tax rate depends on properties of the reserve (loosely, on “carbon content”) but for purposes of analysis of the exploitation of any particular reserve the tax can be assumed in a first instance to be proportional to output.

[8] Nystad (1985, 1987, 1988) outlines models with both natural decline and investment. The 1987 paper considers the possibility that long-run recovery from oil in place is a decreasing function of the choice of the initial rate of extraction. The effect would provide a drag on investment that would depend on its magnitude. If this effect were to hold, a tax such as the one modelled by Sinn, by reducing initial investment, could lead to an *increase* in the recovery of oil, and in emissions, in the long term. An increasing royalty per unit would have a more complicated effect because the reserve would be abandoned once the net price dropped to zero. Mentioned, but not modelled, is the *maximum efficient rate* of production. If the rate

(*footnote continued*)
of production exceeds the maximum efficient rate, m say, the level of recovery of reserves in place decreases severely. It would provide a stronger disincentive to investment beyond $a=m$.

to reap quasi rents, not to sit idle in anticipation of a possible tax and the consequent reduction of both scarcity and quasi rents. The response to the tax would have to include changes in investment sunk in exploration capacity. Because there is a long lead time from the start of exploration through development to production, any change due to a tax is at least a decade forward.

Investments in knowledge – in basic and applied research in exploration, development and extraction (set-up and capacity costs) and in the training of professionals (comparable to capacity costs) – are also sunk and have long lead times before coming into service. Because investments in knowledge are likely to be embodied in new vintages of capital, there is only a limited effect of such investments on wells currently in service or under development. The returns to these investments are likely to be reduced as compared to those in the original equilibrium before the tax. Prospective professionals, especially more promising minds, may shy away from training in an industry that is expected to be subject to increasing taxation, reduced rents, and societally mandated attempts to develop substitutes for its product.

The tax would not have an immediate effect on capacity in refining and transportation. It is difficult to perceive an incentive to increase investment in these activities in a way that would help to facilitate the realization of the green paradox.

9. Synopsis

The green paradox predicts oil producers' response to a carbon tax that increases through time in step with marginal social damages. The oil market is in a dynamic equilibrium that balances at the margin the contributions to firms' discounted net cash flows from the outputs of all future periods. A new tax affects the balance. If the tax increases through time, there is a greater effect on later than earlier marginal flows. Whatever the model, there is an incentive to change the timing of the relevant decision (extraction in a Hotelling model or investments in exploration and development in the present model). The reallocation across time is comparable to that of reallocation in a static model in response to a tax that varies across jurisdictions.

In a Hotelling model the reallocation is unrestricted. The industry changes the "tilt" of its output (the slope of the path of output through time) by increasing current production.

In a technological model, the extent to which the industry can act upon the incentive is limited. Contrary to a Hotelling model, it is not possible to increase production from reserves that are currently in production: the capacities are fixed by previous investment. Moreover, for development investments that are currently in progress (those that will be realized in the next 5 or so years), the incentive cannot move production toward the present because it is already being implemented in the present. For the great bulk of production in the near term, then, the tendency of the tax is not to increase but, if anything, to decrease production. Responding to the incentive requires irreversible investments at new reserves.

New investments are made continually to respond to increases in demand and to replace production that decreases through natural decline. The response may be to shift toward the present the development of some known but undeveloped reserves, as well as exploration for new reserves. This response is subject to its own constraints and may or may not be important in reality.

The tax also provides an incentive to reduce the level of development investment at those reserves. The investment affects the variable a, which is set at time S. A reduction in a means that the reserve is exploited more slowly (with a gentler decline, falling exogenously as $ae^{-a(t-S)}R$), rather than more quickly and at an endogenous rate as predicted in a Hotelling model.

Unlike in Sinn's model, there are two hurdles, exploration and development expenditures. The maximized value of V net of these expenditures must be positive. Looking forward from the present, there are incentives to hasten exploration but also to reduce the expenditure (current and future) when any region is being explored. Consequently, near-term discoveries may rise or fall, but there is a medium-term tendency for less to be discovered and hence exploited. Also, exploration takes time; any inducement to earlier exploration can lead to an increase in production only one to two decades forward, not immediately as predicted by the green paradox. In reality, exploration is constrained by the availability of personnel and equipment. With exploration and development, less may be eventually discovered and extracted, but the commitment of these sunk costs may be moved toward the present.

If the technology of extraction is not modelled, the sole investment hurdle is a set-up cost. There is no constraint on extraction once the decision to make the set-up cost is made. There may be lower levels of reserves, but for what is discovered, the same pattern of extraction is seen as in Sinn's model.

In a technological model, a further effect works against the green paradox. There is no change in the extraction pattern for reserves that are in production. The tax reduces new investment levels, represented by the variable a. The reduction in a, with the consequent "flattening" of the decline curve, acts "to tilt" output in the opposite direction to that predicted by the green paradox. The tilt of the decline curve is a constraint, not a choice. As it may with reductions in output alleged to be an exertion of market power (Cairns and Calfucura, 2012), natural decline may appear to the naked eye, as well as to the naked econometric analysis, to be consistent with a Hotelling response.

In short, Hotelling models of the green paradox are oversimplified. There are in reality *two* incentives that work in opposite directions, in complicated ways. The resultant of the two depends on assumptions. If the prediction of Hotelling models is right and the equilibrium price does fall in the near to medium term, then a price-taking firm has an even greater incentive to reduce exploration and development than under the assumption, made above, that the price path is not affected. The microresponse of producers appears to be inconsistent with the macroeffect on price, and hence on output and emissions.

10. Conclusion

Hotelling's rule is a result from a simple model that teaches that non-renewable resources are a form of capital and that they should be analysed as such. His important insight concerning far-sighted decision making reigns as a foundation of non-renewable resource economics.

Hotelling may reign but he does not rule. Models in his tradition assume free allocation of resources over time. The rule is an arbitrage condition relating the values of net price over the productive life of the reserve. Empirical evidence suggests that allocation is subtler than in the Hotelling model. The operative constraint in oil industry is that allocation over time is capped in one of a number of ways, so that arbitrage among periods is constrained. Calculations and comparisons are not simply of current costs at different time periods but of commitments, especially sunk costs, predicated on the entire future of operations.

At each instant in Hotelling models, decisions are made about the level of flow of units of the resource. Technological models break qualitatively from type-one and -two Hotelling models concerning the form of decisions in the oil industry. Decisions about flows are atrophied. Extraction requires a combination of a discovered reserve with fixed capital. The fundamental decisions are about the timing and level of investments in exploration and development.

The present analysis is still highly simplified. Features of several aspects of oil production must be stitched together if one wishes to begin to analyse the dynamics of the industry in a way that is relevant to policy. Some of these features have been pointed to herein. Some are becoming more fully understood. Each, to have credibility for policy analysis, requires long and deep research.

It is not possible to demonstrate a green paradox given the current limitations of mathematical analysis of a complicated, multi-faceted industry. The weight of many influences discussed in the present paper is inimical to its predictions. The paradox does not have an adequate foundation in the conditions of production in the oil industry to affect policy or the timing of policy respecting climatic change.

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