Of the Four Methods of Experimental Inquiry

§ 1. [Method of Agreement] The simplest and most obvious modes of singling out from among the circumstances which precede or follow a phenomenon, those with which it is really connected by an invariable law, are two in number. One is, by comparing together different instances in which the phenomenon occurs. The other is, by comparing instances in which the phenomenon does occur, with instances in other respects similar in which it does not. These two methods may be respectively denominated, the Method of Agreement, and the Method of Difference.

In illustrating these methods, it will be necessary to bear in mind the twofold character of inquiries into the laws of phenomena; which may be either inquiries into the cause of a given effect, or into the effects or properties of a given cause. We shall consider the methods in their application to either order of investigation, and shall draw our examples equally from both.

We shall denote antecedents by the large letters of the alphabet, and the consequents corresponding to them by the small. Let A, then, be an agent or cause, and let the object of our inquiry be to ascertain what are the effects of this cause. If we can either find, or produce, the agent A in such varieties of circumstances, that the different cases have no circumstance in common except A; then whatever effect we find to be produced in all our trials ^a, is indicated as^a the effect of A. Suppose, for example, that A is tried along with B and C, and that the effect is a b c; and suppose that A is next tried with D and E, but without B and C, and that the effect is a d e. Then we may reason thus: b and c are not effects of A, for they were not produced by it in the second experiment; nor are d and e, for they were not produced in the first. Whatever is really the effect of A must have been produced in both instances; now this condition is fulfilled by no circumstance except a. The phenomenon a cannot have been the effect of B or C, since it was produced where they were not; nor of D or E, since it was produced where they were not. Therefore it is the effect of A.

a-aMS, 43 must, it would seem, be

For example, let the antecedent A be the contact of an alkaline substance and an oil. This combination being tried under several varieties of ^bcircumstances^b, ^cresembling each other^c in nothing else, the results agree in the production of a greasy and detersive or saponaceous substance: it is therefore concluded that the combination of an oil and an alkali causes the production of a soap. ^dIt is thus we inquire, by the Method of Agreement, into^d the effect of a given cause.

In a similar manner we may "inquire into" the cause of a given effect. Let a be the effect. Here, as 'shown in the last chapter', we have only the resource of observation without experiment: we cannot take a phenomenon of which we know not the origin, and try to find its mode of production by producing it: if we succeeded in such a random "trial" it could only be by accident. But if we can observe a in two different combinations, a b c and a d e; and if we know, or can discover, that the antecedent circumstances in these cases respectively were A B C and A D E; we may conclude by a reasoning similar to that in the preceding example, that A is the antecdent connected with the consequent a by a law of causation. B and C, we may say, cannot be causes of a, since on its second occurrence they were not present; nor are D and E, for they were not present on its first occurrence. A, alone of the five circumstances.

For example, let the effect *a* be crystallization. We compare instances in which bodies are known to assume crystalline structure, but which have no other point of agreement; and we find them to have one, and as far as we can observe, only one, antecedent in common: the deposition of solid matter from a liquid state, either a state of fusion or of solution. We conclude, therefore, that the solidification of a substance from a liquid state is an invariable antecedent of its crystallization.

In this example we may go farther, and say, it is not only the invariable antecedent but the cause h; or at least the proximate event which completes the cause^h. For in this case we are able, after detecting the antecedent A, to produce it artificially, and by finding that *a* follows it, verify the result of our induction. The importance of thus reversing the proof was 'strikingly manifested' when by keeping a phial of water charged with siliceous particles undisturbed for years, a chemist (I believe Dr. Wollaston) succeeded in obtaining crystals of quartz; 'and' in the equally interesting experiment in which Sir James Hall produced artificial marble by the cooling of its materials

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<sup>b-b</sup>MS, 43, 46, 51 circumstance

<sup>o-c</sup>MS agreeing

<sup>d-d</sup>MS And thus we have ascertained, by the Method of Agreement,

<sup>o-s</sup>MS ascertain

<sup>f-f</sup>MS already shewn

<sup>f-f</sup>MS guess

<sup>h-h</sup>+51, 56, 62, 65, 68, 72

<sup>i-d</sup>MS, 43, 46 never more strikingly manifested than

<sup>j-j</sup>MS or
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from fusion under immense pressure: two admirable examples of the light which may be thrown upon the most secret processes of nature by wellcontrived interrogation of her.

^k But if we cannot artificially produce the phenomenon A, the conclusion that it is the cause of a remains subject to very considerable doubt. Though an invariable, it may not be the unconditional antecedent of a, but may precede it as day precedes night or night day. This uncertainty arises from the impossibility of assuring ourselves that A is the only immediate antecedent common to both the instances. If we could be certain of having ascertained all the invariable antecedents, we might be sure that the unconditional invariable antecedent, or cause, must be found somewhere among them. Unfortunately it is hardly ever possible to ascertain all the antecedents, unless the phenomenon is one which we can produce artificially. Even then, the difficulty is merely lightened, not removed: men knew how to raise water in pumps long before they adverted to what was really the operating circumstance in the means they employed, namely, the pressure of the atmosphere on the open surface of the water. It is, however, much easier to analyse completely a set of arrangements made by ourselves, than the whole complex mass of the agencies which nature happens to be exerting at the moment ¹of the production of a^{i} given phenomenon. We may overlook some of the material circumstances in an experiment with an electrical machine; but we shall, at the worst, be better acquainted with them than with those of a thunder-storm.

The mode of discovering and proving laws of nature, which we have now examined, proceeds on the following axiom: Whatever circumstance can be excluded, without prejudice to the phenomenon, or can be absent notwithstanding its presence, is not connected with it in the way of causation. The casual circumstances being thus eliminated, if only one remains, that one is the cause which we are in search of: if more than one, they either are, or contain among them, the cause; and so, *mutatis mutandis*, of the effect. As this method proceeds by comparing different instances to ascertain in what they agree, I have termed it the Method of Agreement: and we may adopt as its regulating principle the following canon:

FIRST CANON

If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree, is the cause (or effect) of the given phenomenon.

Quitting for the present the Method of Agreement, to which we shall almost immediately return, we proceed to a still more potent instrument of the investigation of nature, the Method of Difference.

^kMS [no paragraph]

¹⁻¹MS, 43, 46 when she produces any

§ 2. [Method of Difference] In the Method of Agreement, we endeavoured to obtain instances which agreed in the given circumstance but differed in every other: in the present method we require, on the contrary, two instances resembling one another in every other respect, but differing in the presence or absence of the phenomenon we wish to study. If our object be to discover the effects of an agent A, we must procure A in some set of ascertained circumstances, as A B C, and having noted the effects produced, compare them with the effect of the remaining circumstances B C, when A is absent. If the effect of A B C is a b c, and the effect of B C, b c, it is evident that the effect of A is a. So again, if we begin at the other end, and desire to investigate the cause of an effect a, we must select an instance, as a b c, in which the effect occurs, and in which the antecedents were A B C, and we must look out for another instance in which the remaining circumstances, b c, occur without a. If the antecedents, in that instance, are B C, we know that the cause of a must be A: either A alone, or A in conjunction with some of the other circumstances present.

It is scarcely necessary to give examples of a logical process to which we owe almost all the inductive conclusions we draw in daily life. When a man is shot through the heart, it is by this method we know that it was the gunshot which killed him: for he was in the fulness of life immediately before, all circumstances being the same, except the wound.

The axioms "implied" in this method are evidently the following. Whatever antecedent cannot be excluded without preventing the phenomenon, is the cause, or a condition, of that phenomenon ^b: Whatever^b consequent can be excluded, with no other difference in the antecedents than the absence of a particular one, is the effect of that "one". Instead of comparing different instances of a phenomenon, to discover in what they agree, this method compares an instance of its occurrence with an instance of its non-occurrence, to discover in what they differ. The canon which is the regulating principle of the Method of Difference may be expressed as follows:

SECOND CANON

If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance ^ain common save one^d, that one occurring only in the former; the circumstance in which alone the two instances differ, is the effect, or ^ethe^e cause, or ^fan indispensable^f part of the cause, of the phenomenon.

^{a-a}MS, 43, 46 which are taken for granted ^{b-b}MS , & whatever ^{c-c}MS antecedent ^{d-d}MS, 43 save one in common ^{e-e}+56, 62, 65, 68, 72 ^{f-f}MS, 43, 46, 51, 56 a necessary § 3. [Mutual relation of the Methods of Agreement and Difference] The two methods which we have now stated have many features of resemblance, but there are also many distinctions between them. Both are methods of elimination. This term (^a employed in the theory of equations to denote the process by which one after another of the elements of a question is excluded, and the solution made to depend on the relation between the remaining elements only) is well suited to express the operation, analogous to this, which has been understood since the time of Bacon to be the foundation of experimental inquiry: namely, the successive exclusion of the various circumstances which are found to accompany a phenomenon in a given instance, in order to ascertain what are those among them which can be absent consistently with the existence of the phenomenon. The Method of Agreement stands on the ground that whatever can be eliminated, is not connected with the phenomenon by any law. The Method of Difference has for its foundation, that whatever cannot be eliminated, is connected with the phenomenon by a law.

Of these methods, that of Difference is more particularly a method of artificial experiment; while that of Agreement is more especially the resource ^bemployed^b where experimentation is impossible. A few reflections will prove the fact, and point out the reason of it.

It is inherent in the peculiar character of the Method of Difference, that the nature of the combinations which it requires is much more strictly defined than in the Method of Agreement. The two instances which are to be compared with one another must be exactly similar, in all circumstances except the one which we are attempting to investigate: they must be in the relation of A B C and B C, or of a b c and b c. It is true that this similarity of circumstances needs not extend to such as are already known to be immaterial to the result. And in the case of most phenomena we learn at once, from the ^ecommonest^e experience, that most of the coexistent phenomena of the universe may be either present or absent without affecting the given phenomenon; or, if present, are present indifferently when the phenomenon does not happen and when it does. Still, even limiting the identity which is required between the two instances, A B C and B C, to such circumstances as are not already known to be indifferent; it is very seldom that nature affords two instances, of which we can be assured that they stand in this precise relation to one another. In the spontaneous operations of nature there is generally such complication and such obscurity, they are mostly either on so overwhelmingly large or on so inaccessibly minute a scale, we are so ignorant of a great part of the facts which really take place, and even those of which we are not ignorant are so multitudinous, and therefore so seldom exactly alike in any two cases, that a spontaneous experiment, of the kind required by the

^aMS, 43, 46 which is b-bMS, 43, 46 we employ

c-cMS, 43 most ordinary

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Method of Difference, is ⁴commonly not⁴ to be found. When, on the contrary, we obtain a phenomenon by an artificial experiment, a pair of instances such as the method requires is obtained almost as a matter of course, provided the process does not last a long time. A certain state of surrounding circumstances existed before we commenced the experiment; this is B C. We then introduce A; say, for instance, by merely bringing an object from another part of the room, before there has been time for any change in the other elements. It is, in short (as M. Comte observes), the very nature of an experiment, to introduce into the pre-existing state of circumstances a change perfectly definite.[*] We choose a previous state of things "with which we are well acquainted^e, so that no unforeseen alteration in that state is likely to pass unobserved; and into this we introduce, as rapidly as possible, the phenomenon which we wish to study; so that 'in general we' are entitled to feel complete assurance that the pre-existing state, and the state which we have produced, differ in nothing except ^g the presence or absence of that phenomenon. If a bird is taken from a cage, and instantly plunged into carbonic acid gas, the experimentalist may be fully assured (at all events after one or two repetitions) that no circumstance capable of causing suffocation had supervened in the interim, except the change from immersion in the atmosphere to immersion in carbonic acid gas. There is one doubt, indeed, which may remain in some cases of this description; the effect may have been produced not by the change, but by the means ^h employed to produce the change. The possibility, however, of this last supposition generally admits of being conclusively tested by other experiments. It thus appears that in the study of the various kinds of phenomena which we can, by our voluntary agency, modify or control, we can in general satisfy the requisitions of the Method of Difference; but that by the spontaneous operations of nature those requisitions are seldom fulfilled.

The reverse of this is the case with the Method of Agreement. We do not here require instances of so special and determinate a kind. Any instances whatever, in which nature presents us with a phenomenon, may be examined for the purposes of this method; and if all such instances agree in anything, a conclusion of considerable value is already attained. We can seldom, indeed, be sure that 'the' one point of agreement is the only one; but 'this' ignorance does not, as in the Method of Difference, vitiate the conclusion; the certainty of the result, as far as it goes, is not affected. We have ascertained one invariable antecedent or consequent, however many other invariable ante-

[*See, e.g., Cours, Vol. III, p. 321.]

^{d-d}MS scarcely ever ^{e-e}MS which we exactly know ^aMS, 43 in ⁱ⁻⁴MS, 43, 46 this

¹⁻¹MS, 43, 46 we in general ^hMS, 43, 46 we ^{j-j}MS, 43, 46 our cedents or consequents may still remain unascertained. If A B C, A D E, A F G, are all equally followed by a, then a is an invariable consequent of A. If $a \ b \ c$, $a \ d \ e$, $a \ f \ g$, all number A among their antecedents, then A is connected as an antecedent, by some invariable law, with a. But to determine whether this invariable antecedent is a cause, or this invariable consequent an effect, we must be able, in addition, to produce the one by means of the other; or, at least, to obtain that which alone constitutes our assurance of having produced anything, namely, an instance in which the effect, a, has come into existence, with no other change in the pre-existing circumstances than the addition of A. And this, if we can do it, is an application of the Method of Difference, not of the Method of Agreement.

It thus appears to be by the Method of Difference alone that we can ever, in the way of direct experience, arrive ^k with certainty^k at causes. The Method of Agreement leads only to laws of phenomena '(as some writers call them, but improperly, since laws of causation are also laws of phenomena): that is, to uniformities, which either are not laws of causation, or' in which the question of causation must for the present remain undecided. The Method of Agreement is chiefly to be resorted to, as a means of suggesting applications of the Method of Difference (as in the last example the comparison of A B C, A D E, A F G, suggested that A was the antecedent on which to try the experiment whether it could produce a; or as an inferior resource, in case the Method of Difference is impracticable; which, as we before showed, generally arises from the impossibility of artificially producing the phenomena. And hence it is that the Method of Agreement, though applicable in principle to either case, is more emphatically the method of investigation on those subjects where artificial experimentation is impossible: because on those it is, generally, our only resource of a directly inductive nature; while, in the phenomena which we can produce at pleasure, the Method of Difference generally affords a more efficacious process, which will ascertain causes as well as mere laws.

§ 4. [Joint Method of Agreement and Difference] "There are, however," many cases in which, though our power of producing the phenomenon is complete, the Method of Difference either cannot be made available at all, or not without a previous employment of the Method of Agreement. This occurs when the agency by which we can produce the phenomenon is not that of one single antecedent, but ^b a combination of antecedents, which we

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^k-k+43, 46, 51, 56, 62, 65, 68, 72

¹⁻¹MS, 43 , as Mr. Whewell calls them [*Philosophy of the Inductive Sciences*, Vol. II, pp. 260ff.], but which (since laws of causation are also laws of phenomena) I prefer to designate as uniformities] 46 , as some writers call them ... as MS

 $a \rightarrow a$ MS, 43, 46 Our next remark shall be, that there are b = 51 of

have no power of separating from each other, and exhibiting apart. For instance, suppose the subject of inquiry to be the cause of the double refraction of light. We can produce this phenomenon at pleasure, by employing any one of the many substances which are known to refract light in that peculiar manner. But if, taking one of those substances, as Iceland spar for example, we wish to determine on which of the properties of Iceland spar this remarkable phenomenon depends, we can make no use, for that purpose, of the Method of Difference; for we cannot find another substance precisely resembling Iceland spar except in some one property. The only mode, therefore, of prosecuting this inquiry is that afforded by the Method of Agreement; by which, in fact, through a comparison of all the known substances which chavec the property of doubly refracting light, it was ascertained that they ^dagree in the^d circumstance of being crystalline substances; and though the converse does not hold, though all crystalline substances have not the property of double refraction, it was concluded, with reason, that there is a real connexion between these two properties; that either crystalline structure, or the cause which gives rise to that structure, is one of the conditions of double refraction.

Out of this employment of the Method of Agreement arises a peculiar modification of that method, which is sometimes of great avail in the investigation of nature. In cases similar to the above, in which it is not possible to obtain the precise pair of instances which our second canon requires—instances agreeing in every antecedent except A, or in every consequent except a; we may yet be able, by a double employment of the Method of Agreement, to discover in what the instances which contain A or a, differ from those which do not.

If we compare various instances in which a occurs, and find that they all have in common the circumstance A, and (as far as can be observed) no other circumstance, the Method of Agreement, so far, bears testimony to a connexion between A and a. In order to convert this "evidence" of connexion into proof of causation by the direct Method of Difference, we ought to be able, in some one of these instances, as for example A B C, to leave out A, and observe whether by doing so, a is prevented. Now supposing (what is often the case) that we are not able to try this decisive experiment; yet, provided we can by any means discover what would be its result if we could try it, the advantage will be the same. Suppose, then, that as we previously examined a variety of instances in which a occurred, and found them to agree in containing A, so we now observe a variety of instances in which a does not occur, and find them agree in not containing A; which establishes, by the

^{c-c}MS, 43, 46 had ^{d-d}MS, 43, 46 agreed in the single ^{e-e}MS, 43 proof Method of Agreement, the same connexion between the absence of A and the absence of a, which was before established between their presence. As, then, it had been shown that whenever A is present a is present, so it being now shown that when A is taken away a is removed along with it, we have by the one proposition A B C, a b c, by the other B C, b c, the positive and negative 'instances' which the Method of Difference requires."

This method may be called the Indirect Method of Difference, or the Joint Method of Agreement and Difference; and consists in a double employment of the Method of Agreement, each proof being independent of the other, and corroborating it. But it is not equivalent to a proof by the direct Method of Difference. For the requisitions of the Method of Difference are not satisfied, unless we can be quite sure either that the instances affirmative of a, agree in no antecedent whatever but A, or that the instances negative of a agree in nothing but the negation of A. Now if it were possible, which it never is, to have this assurance, we should not need the joint method; for either of the two sets of instances separately, would then be sufficient to prove causation. This indirect method, therefore, can only be ^hregarded^h as a great extension and improvement of the Method of Agreement, but not as participating in the more cogent nature of the Method of Difference. The following may be stated as its canon:

THIRD CANON

If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance; the circumstance in which alone the two sets of instances differ, is the effect, or 'the' cause, or 'an indispensable' part of the cause, of the phenomenon.

We shall presently "see" that the Joint Method of Agreement and Difference constitutes, in another respect not yet adverted to, an improvement upon the common Method of Agreement, namely, in being unaffected by a characteristic imperfection of that method, the nature of which still remains to be pointed out. But as we cannot enter into this exposition without introducing

^{h-h}MS, 43, 46 viewed ⁱ⁻⁴+56, 62, 65, 68, 72 ^{j-j}MS, 43, 46, 51, 56 a necessary ^{k-k}MS, 43 show

¹⁻¹MS instance

⁹MS, 43, 46 Thus, if it be true that all animals which have a well-developed respiratory system, and therefore aërate the blood perfectly, agree in being warm-blooded, while those whose respiratory system is imperfect, do not maintain a temperature much exceeding that of the surrounding medium, we may argue from this twofold experience, that the change which takes place in the blood by respiration is the cause of animal heat.

a new element of complexity into 'this long and intricate discussion', I shall postpone it to "a subsequent" chapter, and shall at once proceed to "a" statement of two other methods, which will complete the enumeration of the means which mankind possess for exploring the laws of nature by specific observation and experience.

§ 5. [Method of Residues] The first of these has been aptly denominated the Method of Residues.^[*] Its principle is very simple. Subducting from any given phenomenon all the portions which, by virtue of preceding inductions, can be assigned to known causes, the remainder will be the effect of the antecedents which had been overlooked, or of which the effect was as yet an unknown quantity.

Suppose, as before, that we have the antecedents A B C, followed by the consequents a b c, and that by previous inductions (founded, we will suppose, on the Method of Difference) we have ascertained the causes of some of these effects, or the effects of some of these causes; and are "thence" apprised that the effect of A is a, and that the effect of B is b. Subtracting the sum of these effects from the total phenomenon, there remains c, which now, without any fresh ^bexperiments^b, we may know to be the effect of C. This Method of Residues is in truth a peculiar modification of the Method of Difference. If the instance A B C, a b c, could have been compared with a single instance A B, a b, we should have proved C to be the cause of c, by the common process of the Method of Difference. In the present case, however, instead of a single instance A B, we have had to study separately the causes A and B, and to infer from the effects which they produce separately, what effect they must produce in the case A B C where they act together. Of the two instances, therefore, which the Method of Difference requires,-the one positive, the other negative,-the negative one, or that in which the given phenomenon is absent, is not the direct result of observation and experiment, but has been arrived at by deduction. As one of the forms of the Method of Difference, the Method of Residues partakes of its rigorous certainty, provided the previous inductions, those which gave the effects of A and B, were obtained by the same infallible method, and provided we are certain that C is the only antecedent to which the residual phenomenon c can be referred; the only agent of which we had not already calculated and subducted the

[*See Whewell, Novum Organon Renovatum, p. 216. JSM cancelled in MS a simple reference to Whewell.]

^{L-I}MS a discussion already, I fear, sufficiently fatiguing to the reader ^{m-m}MS, 43, 46 the next ^{n-m}MS, 43, 46, 51, 56 the ^{a-a}MS, 43, 46, 51 by this means ^{b-b}MS, 43, 46, 51, 56, 62 experiment 397