Integral calculus
Integrální počet

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- Write appropriate functions or numbers into blank fields and press Enter.
- Use functions and mathematical notation as explained in the file instrukce.pdf.
- The green boundary indicates correct answer, the red boundary indicates wrong answer.
- If you cannot solve the problem, click Ans to see the correct answer. If there are more fields to be filled, click repeatedly.

- Vepište do políček co tam patří a stiskněte Enter.
- Zápis funkcí provádějte tak, jak je vysvětleno v nápovědě v souboru instrukce.pdf.
- Zelený okraj obélníku znamená správnou odpověď, červený špatnou.
- Kliknutím na Ans se zobrazí správný výsledek – s případě že problém nejste schopni vyřešit. Je-li v otázce více políček, klikněte na Ans opakovaně.

1. Test1

Indefinite integrals by formulas | Užití vzorců

Quiz

1. $\int e^x \mathrm{d}x =$ $+ C$
2. $\int e^{2x} \mathrm{d}x =$ $+ C$
3. $\int \left(1 + 3e^{-x}\right) \mathrm{d}x =$ $+ C$
4. $\int \left(e^x + 1\right)^2 \mathrm{d}x =$
5. $\int \frac{1}{2}\left(e^x + e^{-x}\right) \mathrm{d}x =$ $+ C$
6. $\int \left(\frac{1 + 2e^x}{e^x}\right) \mathrm{d}x =$ $+ C$
7. $\int \frac{e^x}{1 + e^x} \mathrm{d}x =$ $+ C$
8. $\int \frac{e^{-2x}}{1 + e^{-2x}} \mathrm{d}x =$ $+ C$

9. $\int 3 \cdot 2^x \mathrm{d}x =$ $+ C$

10. $\int \frac{x^2 + x + 4}{x} \mathrm{d}x =$ $+ C$

11. $\int \frac{\sqrt{x} + 1}{x} \mathrm{d}x =$ $+ C$

12. $\int \left(2x^2 - x + 4\right) \mathrm{d}x =$ $+ C$

13. $\int \sqrt{x}(1 - \sqrt{x}) \mathrm{d}x =$ $+ C$

14. $\int \frac{(x+1)(x-1)}{x^2} \mathrm{d}x =$ $+ C$

15. $\int \frac{x}{x^2 + 6} \mathrm{d}x =$

16. $\int \frac{1}{x^2 + 6} \mathrm{d}x =$ $+ C$

17. $\int \frac{x^2 + 2}{x^2 + 1} \mathrm{d}x =$ $+ C$

18. $\int \frac{x + 5}{x^2 + 4} \mathrm{d}x =$ $+ C$

19. $\int \frac{1 - \cos^2 x}{\cos^2 x} \mathrm{d}x =$ $+ C$

20. $\int \frac{\sin x}{\cos x} \mathrm{d}x = \qquad + C$

21. $\int 2 \sin x \cos x \mathrm{d}x = \qquad + C$

22. $\int \sin\left(x - \frac{\pi}{2}\right) \mathrm{d}x = \qquad + C$

23. $\int \sin(\pi - x) \mathrm{d}x = \qquad + C$

24. $\int e^{-x} \mathrm{d}x = \qquad + C$

25. $\int e^{3x+1} \mathrm{d}x = \qquad + C$

26. $\int 2e^{x-2} \mathrm{d}x = \qquad + C$

27. $\int e^{5-3x} \mathrm{d}x = \qquad + C$

28. $\int \frac{1}{3 + x^2} \mathrm{d}x = \qquad + C$

29. $\int \frac{1}{\sqrt{3 + x^2}} \mathrm{d}x = \qquad + C$

30. $\int \frac{-4}{\cos^2(2x)} \mathrm{d}x = \qquad + C$

31. $\int \left(\frac{6}{x^3} + x\right) \mathrm{d}x =$ $+ C$

32. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) \mathrm{d}x =$ $+ C$

33. $\int (x+1)^2 \mathrm{d}x =$ $+ C$

34. $\int \frac{1}{3x+5} \mathrm{d}x =$

35. Write correct numbers inside the small colored rectangles and then write the primitive function (white field).

Vepište správná čísla do malých podbarvených políček a potom nalezněte primitivní funkci (bílé políčko).

(a) $\int \frac{x^2}{x^3+1}\mathrm{d}x = \int \qquad \frac{(x^3+1)'}{x^3+1}\mathrm{d}x$

$= \qquad + C$

(b) $\int \frac{3x}{x^2+4}\mathrm{d}x = \int \qquad \frac{(x^2+4)'}{x^2+4}\mathrm{d}x$

$= \qquad + C$

(c) $\int \frac{x^2-1}{x^2+1}\mathrm{d}x = \int \qquad + \frac{\qquad}{x^2+1}$

$= \qquad \mathrm{d}x + C$

(d) $\int \frac{x^2-2x+1}{x^2+2x+1}\mathrm{d}x = \int \qquad + \frac{x+\qquad}{x^2+2x+1}\mathrm{d}x$

$= \int \qquad + \qquad \frac{2x+2}{x^2+2x+1} + \frac{\qquad}{x^2+2x+1}$

$= \qquad + C$

(e) $\int \frac{x+5}{x^2+4} dx = \int \left(\qquad \frac{2x}{x^2+4} + \qquad \frac{1}{x^2+4} \right) dx$

$= \qquad\qquad + C$

(f) $\int \frac{1}{x^2+2x+5} dx = \int \frac{1}{\left(x+ \quad\right)^2 + \quad} dx$

$= \qquad\qquad + C$

(g) $\int \frac{1}{x^2-3x+4} dx = \int \frac{1}{\left(x- \quad\right)^2 + \quad} dx$

$= \qquad\qquad + C$

(h) $\int \frac{1}{\sqrt{x^2+x+1}} dx = \int \frac{1}{\sqrt{\left(x+ \quad\right)^2 + \quad}}$

$= \qquad\qquad + C$

(i) $\int \frac{x+1}{x^2+4x+6} dx = \int \qquad \frac{2x+4}{x^2+4x+6} dx$

$+ \int \left(\quad \right) \frac{1}{\left(x+ \quad\right)^2 + \quad} dx$

$= \qquad\qquad + C$

(j) $\int \sin x \cos x \mathrm{d}x \; = \int \qquad \sin\left(\qquad x\right) \mathrm{d}x$

$= \qquad\qquad\qquad\qquad + C$

Test1

Test2

Test3

Test4

2. Test2

Integration by parts

Integrace per-partés

When integrating by parts we use the formula

Pro integraci per-partés používáme následující vzorec

$$\int u(x)v'(x)\mathrm{d}x = u(x)v(x) - \int u(x)v'(x)\mathrm{d}x. \tag{Eq:1}$$

Quiz

We use the integration by parts especially for integrals of the type

Typicky používáme integraci per-partés pro integrály typu

$$\int p(x)f(ax+b)\mathrm{d}x, \tag{Eq:2}$$

where $p(x)$ is a polynomial and

kde $p(x)$ je polynom a

$$f(x) \in \{e^x, \sin x, \cos x, \operatorname{atan} x, \ln^m x\}$$

Here $\operatorname{atan}(x)$ is the usual arctangent functions.

Zde $\operatorname{atan}(x)$ je obvyklá funkce arkustangens.

Question: Are the following integrals like (Eq:2)? Are the integral convenient for integration by parts?

Otázka: Jsou následující integrály typu (Eq:2)? Je vhodné je integrovat metodou per-partés?

1. $\int e^{-x^2}\mathrm{d}x$ Yes No
2. $\int xe^{x^2}\mathrm{d}x$ Yes No
3. $\int x^2e^x\mathrm{d}x$ Yes No
4. $\int (3x+1)e^{-x+1}\mathrm{d}x$ Yes No

5. $\int (x+4)\operatorname{atan}\frac{x}{2}\mathrm{d}x$ Yes No

6. $\int x\sin x^2\mathrm{d}x$ Yes No

7. $\int x^2\ln x\mathrm{d}x$ Yes No

8. $\int \operatorname{atan} x\mathrm{d}x$ Yes No

9. $\int x\ln x\cos x\mathrm{d}x$ Yes No

10. $\int x\cos^3 x\mathrm{d}x$ Yes No

11. $\int (2+x)\cos(2x)\mathrm{d}x$ Yes No

12. $\int (x^3-1)\sin\left(\frac{\pi}{2}-x\right)\mathrm{d}x$ Yes No

Quiz Integrate | Integrujte

$I = \int (x^2 + x - 2)\sin x \,\mathrm{d}x.$

1. $u =$ $u' =$
   $v' =$ $v =$

2. $I =$ $- \int$ $\mathrm{d}x$

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3.

$$I = -(x^2+2x+1)\cos x + 2\int (x+1)\cos x\,\mathrm{d}x.$$

Now we have an expression which can be written as above(check it yourself). We integrate by parts in $\int (x+1)\cos x\,\mathrm{d}x$.

Nyní máme něco, co se dá přepsat do výše uvedeného tvaru (zkontrolujte si) do tvaru. Integrujeme výraz $\int (x+1)\cos x\,\mathrm{d}x$. Použijeme opět metodu per-partés.

$u =$ $\qquad$ $u' =$

$v' =$ $\qquad$ $v =$

4.

$$I = -(x^2+2x+1)\cos x$$
$$+2\Big(\qquad - \int \qquad \mathrm{d}x \Big)$$

5.

$$\begin{aligned} I &= -(x^2+2x+1)\cos x + 2\Big((x+1)\sin x - \int \sin x \mathrm{d}x\Big) \\ &= -(x^2+2x+1)\cos x + 2\Big((x+1)\sin x - \qquad\qquad\qquad \Big) \\ &= \Big(\qquad\qquad\qquad \Big)\sin x + \Big(\qquad\qquad\qquad \Big)\cos x + C \end{aligned}$$

Quiz Integrate | Integrujte

$I = \int \operatorname{atan} x \, \mathrm{d}x.$

1. $u =$ $u' =$

$v' =$ $v =$

2. $I =$ $- \int$ $\mathrm{d}x$

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3. Now we have an expression which can be written in the form (check it yourself). Find out the number which has to be in the first colored field. When you find out this number, the integration is easy.

Nyní máme něco, co se dá přepsat (zkontrolujte si) do tvaru. Zjistíte-li, jaké číslo je potřeba zapsat do prvního podbarveného obdélníčku, je integrace snadná.

$$\begin{aligned} I &= x \operatorname{atan} x - \int \frac{x}{x^2+1} \mathrm{d}x \\ &= x \operatorname{atan} x - \left(\qquad \right) \int \frac{2x}{x^2+1} \mathrm{d}x \\ &= x \operatorname{atan} x - \end{aligned}$$

4. The result is

Výsledek je

$$\int \operatorname{atan} x \, \mathrm{d}x = x \operatorname{atan} x - \frac{1}{2} \ln(1 + x^2) + C$$

Quiz Integrate | Integrujte

$$I = \int (x^2 - 1)e^x \mathrm{d}x$$

1. We integrate by parts with $u(x) = (x^2 - 1)$. With this notation we have (use zero constant of integration in responses)

Integrujeme per-partés při volbě $u(x) = x^2 - 1$

$u = x^2 - 1$ $\qquad$ $u' =$

$v' =$ $\qquad$ $v =$

2. Integration by parts gives . . .

Po použití vzorce pro integraci per–partés máme . . .

$$I = \underbrace{\qquad\qquad} - \int \underbrace{\qquad\qquad} \mathrm{d}x$$

3. We integrate once more by parts

Budeme integrovat ještě jednou per–partés

$u = 2x \qquad u' =$

$v' = \qquad v =$

4. The second integration by parts gives ...

Opětovné použití vzorce per–partés dává ...

$$I = \qquad - \Big[\qquad - \int \qquad \mathrm{d}x \Big]$$

5. The result after the last integration and simplifications is ...

Po poslední integraci a po snadné úpravě obdržíme ...

$$I = \qquad + C$$

Quiz Integrate | Integrujte

$$I = \int x \ln(x+1) \mathrm{d}x$$

1. We integrate by parts with $u(x) = \ln(x+1)$. | Budeme integrovat per–partés při volbě $u(x) = \ln(x+1)$.

$u = \ln(x+1)$ $\quad u' =$

$v' =$ $\quad v =$

2. Integration by parts gives ... | Aplice vzorce per–partés dává ...

$$I = \underbrace{\qquad\qquad}_{} - \int \underbrace{\qquad\qquad}_{A} \mathrm{d}x$$

3. The expression denoted by A is a rational function which is not proper. Divide the numerator by the denominator and write this functionas a sum of polynomial and proper function. Write the polynomial into the first field and the proper function into the second one.

Výraz označený jako A je racionální funkce a je nutno ji intgerovat tak, že nejprve vydělíme čitatel jmenovatelem. Napište do prvního políčka podíl a do druhého zbytek po dělení.

$$A = \underbrace{\qquad\qquad}_{\text{polynomial}} + \underbrace{\qquad\qquad}_{\text{remainder}}$$

4. The integration and simplification give ...

Finální integrací a úpravou získáváme ...

$$I = \qquad\qquad\qquad\qquad + C$$

Quiz Find the following integral: $I = \int (x+1)e^{-x}\mathrm{d}x$

1. We integrate by parts with $u(x) = (x+1)$. With this notation we have (use zero constant of integration in responses):

$u'(x) =$

$v'(x) =$

$v(x) =$

2. Integration by parts gives

$I = \qquad\qquad - \int \qquad\qquad \mathrm{d}x$

3. Integration gives the indefinite integral

$I = \qquad\qquad + C$

Quiz Find the following integral: $I = \int (x^2 - 1)\sin x \mathrm{d}x$

1. We integrate by parts with $u(x) = (x^2 - 1)$. With this notation we have (use zero constant of integration in responses):
$u'(x) =$
$v'(x) =$
$v(x) =$

2. Integration by parts gives
$I = \qquad - \int \qquad \mathrm{d}x$

Go to the next page.

3. Now you have $I = -(x^2-1)\cos(x) + 2\int x\cos(x)\mathrm{d}x$. We integrate by parts with $u(x) = x$. With this notation we have (use zero constant of integration in responses):

$u'(x) =$

$v'(x) =$

$v(x) =$

4. Integration by parts gives

$$I = -(x^2-1)\cos x + \\ +2\Big[\qquad\qquad - \int \qquad\qquad \mathrm{d}x \Big]$$

5. Integration gives the indefinite integral

$$I = \qquad\qquad\qquad + C$$

Quiz Find the following integral: $I = \int \ln x \mathrm{d}x$

1. We integrate by parts with $u(x) = \ln x$. With this notation we have (use zero constant of integration in responses):
   $u'(x) =$
   $v'(x) =$
   $v(x) =$
2. Integration by parts gives
   $I = \qquad - \int \qquad \mathrm{d}x$
3. Integration gives the indefinite integral
   $I = \qquad + C$

Quiz Find the following integral: $I = \int x^2 \operatorname{atan} x \mathrm{d}x$ (we use "$\operatorname{atan}(x)$" for the usual arctangent function).

1. We integrate by parts with $u(x) = \operatorname{atan} x$. With this notation we have (use zero constant of integration in responses):
   $u'(x) =$
   $v'(x) =$
   $v(x) =$
2. Integration by parts gives
   $I = \qquad - \int \qquad \mathrm{d}x$
3. Integration gives the indefinite integral
   $I = \qquad + C$

Quiz Find the following integral: $I = \int (x+3)e^{2x}\,\mathrm{d}x$

1. We integrate by parts with $u(x) = (x+3)$. With this notation we have (use zero constant of integration in responses):

$u'(x) =$

$v'(x) =$

$v(x) =$

2. Integration by parts gives

$I = \qquad\qquad - \int \qquad\qquad \mathrm{d}x$

3. Integration gives the indefinite integral

$I = \qquad\qquad + C$

3. Test3

Integration by substitution | Integrace substitucí

When integrating by substitution we use the formula

$$\int f(\phi(x))\phi'(x)\mathrm{d}x = \int f(t)\mathrm{d}t \tag{Eq:3}$$

(i.e. we substitute $\phi(x) = t$ and $\phi'(x)\mathrm{d}x = \mathrm{d}t$)or

$$\int f(x)\mathrm{d}x = \int f(\phi(t))\phi'(t)\mathrm{d}t \tag{Eq:4}$$

(i.e. we substitute $x = \phi(t)$ and $\mathrm{d}x = \phi'(t)\mathrm{d}t$).

Pro integraci pomocí substituce používáme výše uvedené vzorce.

Quiz Find the following integral: $I = \int \frac{x+\sqrt{x-4}}{(x+1)\sqrt{x-4}} \mathrm{d}x$

1. We use the substitution $x - 4 =$

2. With this substitution we have
$\mathrm{d}x = \qquad \mathrm{d}t \qquad x =$

$t =$

3. Susbtitution gives
$I = \int \qquad \mathrm{d}t$

4. We have to divide the numerator by the denominator. This gives a sum of polynomial and proper rational fraction (which is also a partial fraction). Write this polynomial into the first and the partial fraction into the second field.
$I = \int \qquad + \qquad \mathrm{d}t$

5. Integration in t gives
$I =$

6. The back substitution gives the result in the variable x
$I = \qquad + C$

Quiz Find the following integral: $I = \int \frac{\sin(x)\cos(x)}{\sin(x)+1} \mathrm{d}x$

1. We use the substitution $t =$
2. With this substitution we have
   $\mathrm{d}t = \qquad \mathrm{d}x$
3. Susbtitution gives
   $I = \int \qquad \mathrm{d}t$
4. We have to divide the numerator by the denominator. This gives a sum of polynomial and proper rational fraction (which is also a partial fraction in our particular example). Write this polynomial into the first and the partial fraction into the second field.
   $I = \int \qquad + \qquad \mathrm{d}t$
5. Integration in t gives
   $I =$
6. The back substitution gives the result in the variable x
   $I = \qquad + C$

Quiz Convert the following integral by substitution into an integral of rational function: $I = \int x\sqrt{\frac{x+1}{x-1}}\mathrm{d}x$

1. We use the substitution $t^2 = \frac{x+1}{x-1}$ With this substitution we have

$x =$

$\mathrm{d}x = \qquad \mathrm{d}x$

2. Susbtitution and simpification give

$I = \int \qquad \mathrm{d}t$

Quiz Find the following integral: $I = \int x^2 e^{-x^3} \mathrm{d}x$

1. With substitution $-x^3 = t$ we have

$$\cdot \,\mathrm{d}x = \mathrm{d}t$$

2. Substitution gives

$$I = \int \qquad \qquad \mathrm{d}t$$

3. Integration in t gives the indefinite integral

$$I = \qquad \qquad + C$$

4. In the original variable x we have

$$I = \qquad \qquad + C$$

Quiz Find the following integral: $I = \int \sin^5 x \mathrm{d}x$

1. With substitution $\cos x = t$ we have

$$\cdot \,\mathrm{d}x = \mathrm{d}t$$

2. Substitution gives

$$I = \int \qquad \mathrm{d}t$$

3. Integration in t gives the indefinite integral

$$I = \qquad + C$$

4. In the original variable x we have

$$I = \qquad + C$$

Quiz Substitute $\tan x = t$ in the integral $I = \int \frac{\sin x - \cos x}{\sin^3 x + \cos^3 x} \mathrm{d}x$

1. With substitution $\tan x = t$ we have (write expression in t)
$x =$

2. Differentiating we get
$\mathrm{d}x = \qquad \cdot \mathrm{d}t$

3. From the right triangle with angle x, opposite side t, adjacent side 1 and hypotenuse $\sqrt{1+t^2}$ (draw such an triangle) we have the following relations between $\sin(x)$, $\cos(x)$ and new variable t:
$\sin(x) =$ (write expression in t)
$\cos(x) =$ (write expression in t)

4. Substitution gives
$I = \int \qquad \mathrm{d}t$

5. Now we stop. However, you can evaluate this integral using partial fractions.

Quiz Evaluate integral $I = \int \frac{1}{1+e^x} \mathrm{d}x$ by substitution.

1. Differentiating $e^x = t$ we get

$$\cdot\, \mathrm{d}x = \mathrm{d}t$$

2. From $e^x = t$ we have (write x as a function of t)

$x =$

Differentiating this relation we have

$\mathrm{d}x = \qquad \cdot\, \mathrm{d}t$

3. After substitution we have

$I = \int \qquad \mathrm{d}t$

4. Decomposition into partial fraction and integration give the integral in the variable t:

$I = \qquad + C$

5. We return to the original variable x. We have

$I = \qquad + C$

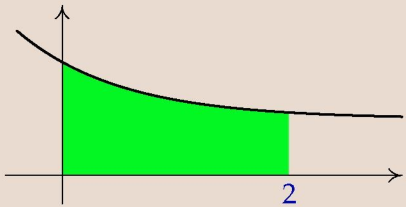
4. Test4

Definite integral in geometry

Aplikace v geometrii

Quiz The function on the picture is the function $y = e^x$ reflected about the y-axis and moved by one above. (In notation of this document the function e^x can be written as `exp(x)`, or `e^(x)`.) The green region correponds to the interval $x \in [0, 2]$.

Na obrázku je funkce $y = e^x$ převrácená okolo osy y a posunutá o jedničku nahoru. (V notaci tohoto dokumentu je možno funkci e^x zapsat jako `exp(x)`, nebo `e^(x)`.) Označený region odpovídá intervalu $x \in [0, 2]$.



1. Write an analytical formula for the function.

Napište analytický tvar funkce.

$y =$.

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2. Express the area of the green region as the definite integral.

Vyjádřete obsah vybarveného regionu jako určitý integrál.

$$S = \int \qquad\qquad \mathrm{d}x$$

3. Complete the following formula. This formula may be used later for integration.

Doplňte vzorec, který potom použijte pro integraci.

$$\int e^{-x}\mathrm{d}x = \qquad\qquad + C.$$

4. Integrerate and use the Newton–Leibniz formula.

Integrujte a použijte Newtonovu–Leibnizovu formuli.

$$S = \Big[\qquad\qquad \Big] \qquad .$$

5. Substitute the limits and evaluate the integral.

Dosaďte meze a dopočíteje integrál.

$$S = \qquad .$$

6. Write the volume of the of the solid of revolution formed by revolving the green region about the x-axis as a definite integreal.

Vyjádřete jako určitý integrál objem tělesa, které vznikne rotací tohoto obrazce okolo osy x.

$$V = \pi \int \qquad \mathrm{d}x.$$

7. Simplifying and integrating we get (use zero constant of integration) …

Po umocnění integrandu a po integraci (volte nulovou integrační konstantu) máme pro objem vztah …

$$V = \pi \Big[\qquad \Big] \qquad .$$

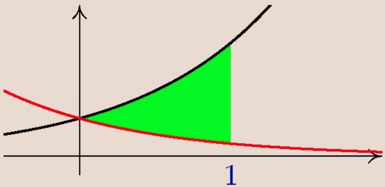
8. The volume is …

Výsledný objem je …

$$V = \qquad \pi.$$

Quiz The functions on the picture are $y = e^x$ and $y = e^{-x}$ (In notation of this document we can write the function e^x as `exp(x)` or `e^(x)` and the function e^{-x} as `exp(-x)` or `e^(-x)`.) The green region corresponds to $x \in [0, 1]$.

Na obrázku jsou funkce $y = e^x$ a $y = e^{-x}$ (V notaci tohoto dokumentu je možno funkci e^x zapsat jako `exp(x)`, nebo `e^(x)` a funkci e^{-x} jako `exp(-x)`, nebo `e^(-x)`.) Označený region odpovídá intervalu $x \in [0, 1]$.



1. The black curve is

Černá funkce je

$y =$.

2. The red curve is

Červená funkce je

$y =$.

3. Area of the green region can be evaluated as a definite integral ...

Obsah vybarveného regionu je možno vyjádřit jako určitý integrál ...

$$S = \int \qquad \mathrm{d}x$$

4. Integration gives

Po integraci dostaneme

$$S = \Big[\qquad \Big] \qquad .$$

5. Substituting limits and simplifying we obtain

Po dosazení mezí a výpočtu dostáváme

$$S = \qquad .$$

6. The volume of the solid of revolution which can be obtained by revolving the green region about the x-axis can be evaluated as the definite integral . . .

Objem tělesa, které vznikne rotací tohoto obrazce okolo osy x je možno vyjádřit jako určitý integrál . . .

$$V = \pi \int \qquad \mathrm{d}x.$$

7. Algebraic simplifications and integration give (use a zero constant of integration) . . .

Po umocnění integrandu a po integraci (volte nulovou integrační konstantu) máme pro objem vztah . . .

$$V = \pi \Big[\qquad \Big] \quad .$$

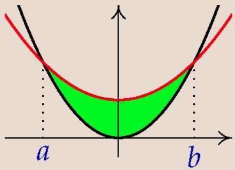
8. The volume is . . .

Výsledný objem je . . .

$$V = \qquad \pi.$$

Quiz The functions on the picture are $y = x^2$ and $y = \frac{x^2}{2} + 2$ (In the notation of this document you can write something like `y=x^2` and `y=x^2/2+2`).

Na obrázku jsou funkce $y = x^2$ a $y = \frac{x^2}{2} + 2$ (v notaci tohoto dokumentu lze tyto funkce zapsat např jako `y=x^2` a `y=x^2/2+2`).



1. The black curve is:

Černá křivka je grafem funkce:

$y =$.

2. The red curve is:

Červená křivka je grafem funkce:

$y =$.

3. Find the intercepts of both curves.

Najděte průsečíky křivek.

$a =$, $b =$.

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4. Express the area of the shaded region as an definite integral.

Vyjádřete obsah vyšrafované plochy pomocí určitého integráluje.

$$S = \int \qquad \qquad \mathrm{d}x.$$

5. The function inside integral is a polynomial. Find the coefficinets of this polynomial.

Integrand lze zapsat jako polynom. Doplňte koeficienty tohoto polynomu.

$$S = \int \qquad \Big(\qquad x^2 + \qquad \Big) \mathrm{d}x.$$

6. Integrate and use the Newton–Leibniz formula.

Integrujte a použijte Newtonovu–Leibnizovu formuli

$$S = \Big[\qquad \qquad \Big] \qquad = \qquad .$$

7. Write the integral which express the volume of the solid obtained by a revolution of the shaded region about the x-axis.

Rotuje-li vyšrafovaná plocha okolo osy x, získáme rotační těleso, jehož objem je možno zapsat ve tvaru určitého integrálu. Napište tento intgerál.

$V = \pi \int$ $\mathrm{d}x.$

8. The function in the integral can be expressed as a polynomial. Complete the coefficients of the polynomial.

Integrand lze vyjádřit jako polynom (doplňte čísla)

$$V = \pi \int \quad \Big(\quad x^4 + \quad x^2 + \quad \Big) \mathrm{d}x.$$

9. Integrate and use the Newton–Leibniz formula.

Integrujte a použijte Newtonovu–Leibnizovu formuli.

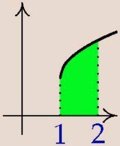
$$V = \pi \Big[\qquad \Big] \quad .$$

10. Substitute the limits and evaluate the integral.

Dosaďte horní a dolní mez a vypočtěte integrál.

$$V = \qquad \pi.$$

Quiz Na obrázku je funkce $y = \sqrt{x}$ posunutá o jedničku nahoru a o jedničku doprava. (V notaci tohoto dokumentu je možno funkci $\sqrt{x}$ zapsat jako `sqrt(x)`, nebo `x^(1/2)`.)



1. Analytický tvar funkce je $y =$.

2. Obsah vybarveného regionu je možno vyjádřit jako určitý integrál

$$S = \int \qquad \mathrm{d}x$$

3. Pro integraci lze použít vzorec

$$\int \sqrt{x}\mathrm{d}x = \int x^{\frac{1}{2}}\mathrm{d}x = \qquad + C.$$

4. Po aplikaci tohoto vzorečku dostáváme

$$S = \Big[\qquad \Big] \qquad .$$

5. Po dosazení mezí a výpočtu dostáváme $S =$.

6. Objem tělesa, které vzikne rotací tohoto obrazce je možno vyjádřit jako určitý integrál

$$V = \pi \int \qquad \mathrm{d}x.$$

7. Po umocnění integrandu a po integraci (volte nulovou integrační konstantu) máme pro objem vztah

$$V = \pi \Big[\qquad \Big] \qquad .$$

8. Výsledný objem je $V = \qquad \pi$.

That's all. The user is kindly asked to send his comments to these quizzes to my E-mail address.

Tot' vše. Prosím uživatele, aby své případné komentáře a náměty zasílali na moji E-mailovou adresu.