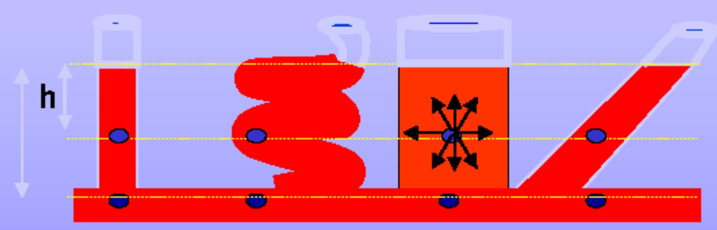
Rheology of blood circulation

1. Basic physical laws of liquids

Law of Pascal

Liquid column causes a pressure (hydrostatic pressure) that is directly proportional to the height of the liquid column (h), density of the liquid (and gravitational acceleration (g).

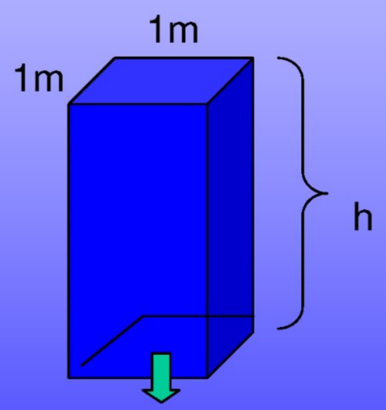


$p = h.\rho.g$

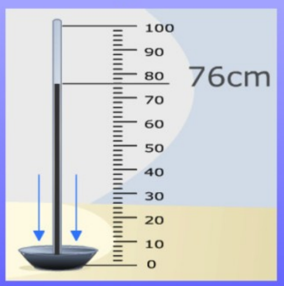
h = height

ρ = density

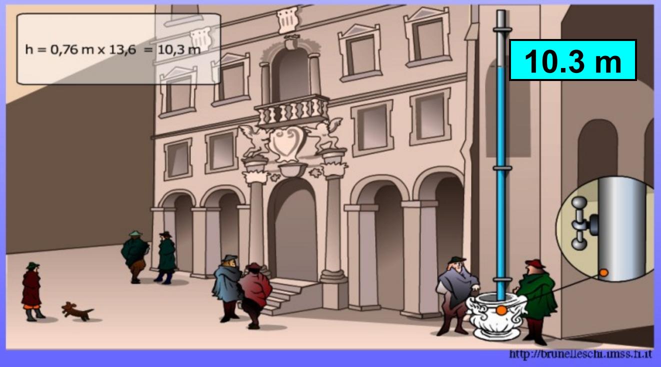
g = gravitational acceleration



Torricelli pokus:



Hg



H_2O

Pa

mm Hg

mm H_2O

133,322 Pa = 1 mm Hg

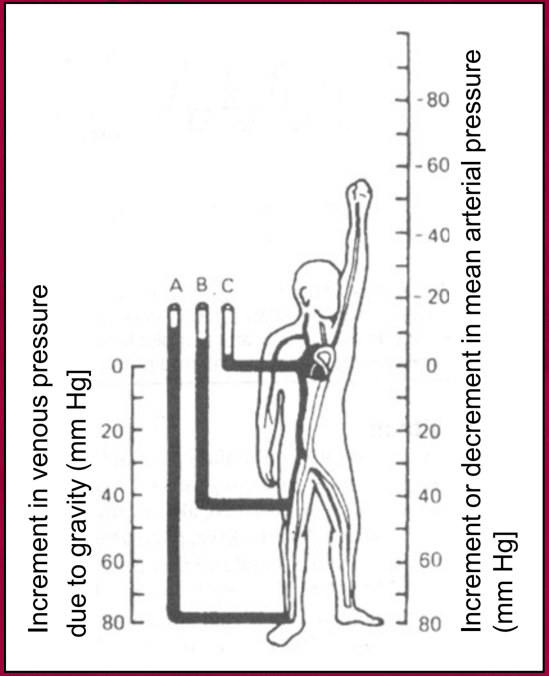
760 mmHg= 1 atm = 10.3 m H_2O

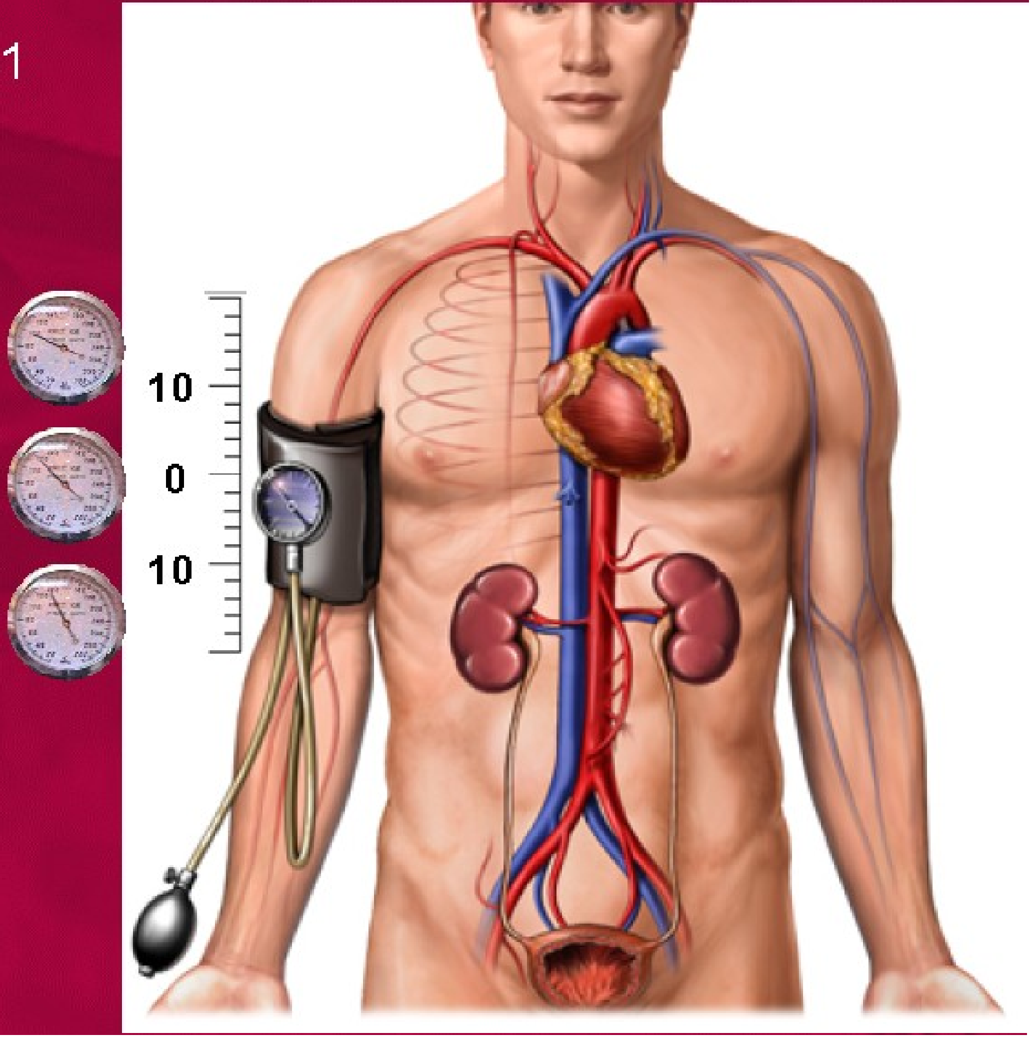
Effect of gravity on arterial and venous pressure

Per each 10 cm

$\Delta p = \Delta h . \rho_{krve} . g = 0,1 \; . \; 1\,065 \; . \; 9,81$

$= 1\,045 Pa = $ 7.8 mm Hg





Law of Laplace

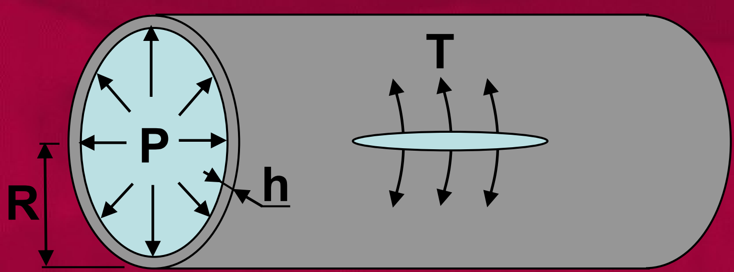
Relation between distending pressure (P [N/m^2]) and tension in the wall of hollow object (T [N/m]) :

$$T = \frac{P}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

R_1 and **R_2** are the biggest and the smallest radii of curvature

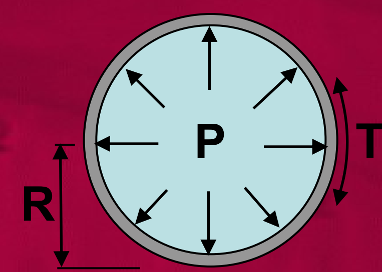
For vessel:

$R_2 =$ **T= P·R**



For sphere:

$R_1 = R_2$ **T= P·R/2**



Considering thickness of vessel wall (h [m]): **T=P·R/h [N/m^2]**

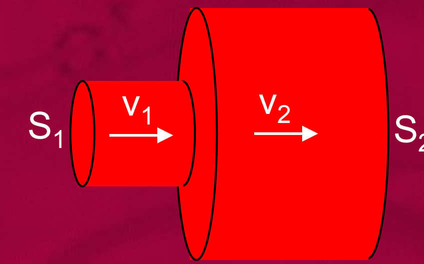
Characteristics of vessels

	P	R	P•R	h	P•R/h
vessel	**P [kPa]**	**radius**	**tension (N/m)**	**wall thickness**	**tension (N/m²)**
aorta	13,3	13 mm nebo méně	170	2 mm	**85000**
arteries	12	5 mm	60	1 mm	**60000**
arterioles	8	150–62 μm	1,2–0,5	20 μm	**40000**
capillaries	4	4 μm	$1{,}6.10^{-2}$	1 μm	**16000**
venules	2,6	10 μm	$2{,}6.10^{-2}$	2 μm	**13000**
veins	2	200 μm a více	0,4	0,5 mm	**800**
vena cava	1,33	16 mm	21	1,5 mm	**14000**

Continuity equation

The volume of fluid flowing through a tube (vessel) per unit of time (Q [l/s]) is constant.

$$Q = S_1 \cdot v_1 = S_2 \cdot v_2 = constant$$



v – velocity S – area

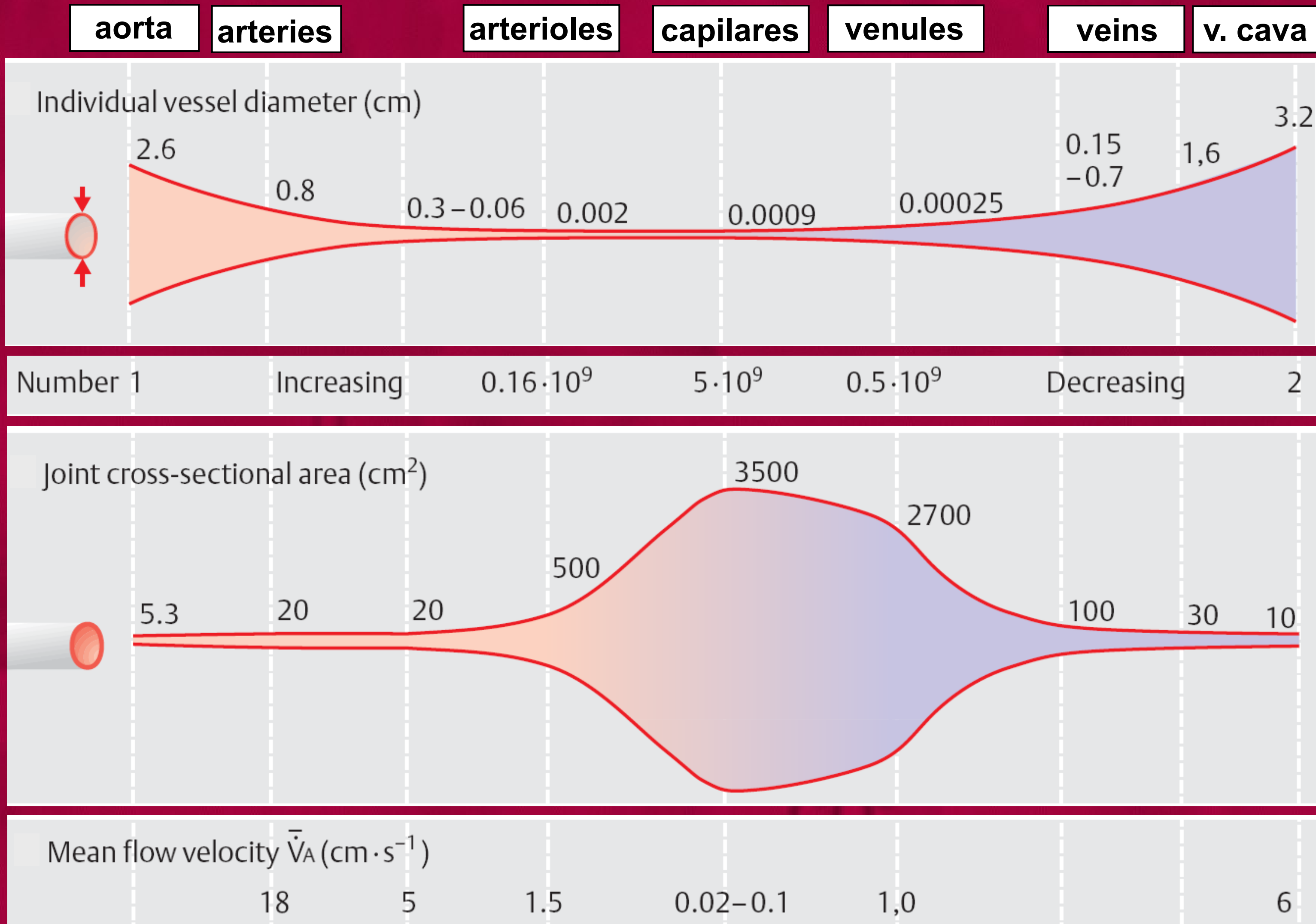
Average blood velocity in vessels

$$v = \frac{Q}{S}$$

$Q_{rest} \approx 5.6$ l/min

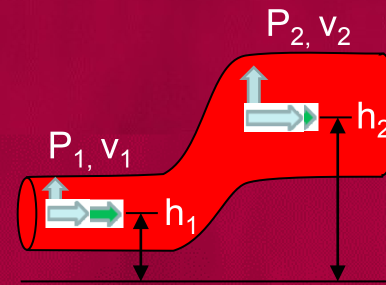
vessel	diameter	number	total area	velocity
aorta	2.6 cm	1	5.3 cm²	~ **18 cm/s**
arterioles	20-50			~ 1.5 cm/s
capilaries	4-9 μm	5×10^9	2000 cm²	~ **0.04 cm/s**
venules				~ 1 cm/s
vena cava		2		~ 7 cm/s

Relation between total cross-sectional area of vessels and mean blood flow velocity



Bernoulli's principle

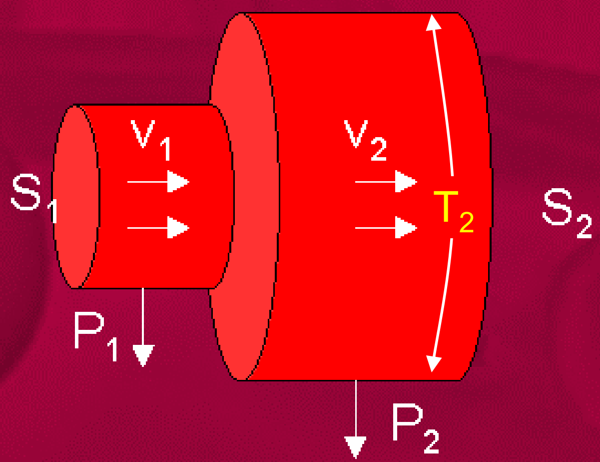
Law of energy conservation for fluid :



$$E_k + E_p + E_{ps} = \text{const.} \Rightarrow \frac{1}{2}\rho v^2 + h.\rho.g + P = \textbf{costant}$$

$[J/m^3] = [Pa]$

dynamic pressure + static pressure = total pressure



$T_2 = P_2 \cdot R_2$

Implication for aortic aneurysm

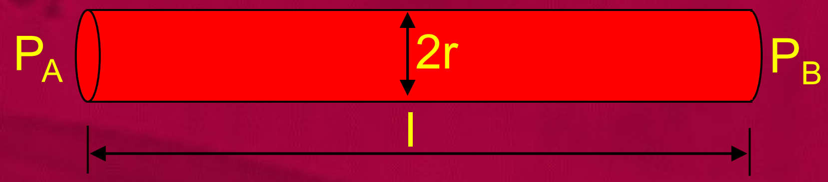
$S_1 v_1 = S_2 v_2$ a je-li $S_1 < S_2$, musí platit: $v_1 > v_2$

$$\frac{1}{2}\rho v_1^2 + h.\rho.g + P_1 = \frac{1}{2}\rho v_2^2 + h.\rho.g + P_2$$

$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2$$

For $v_2 < v_1 \Rightarrow P_2 > P_1$

Poiseuille – Hagen equation



$$Q = \frac{\pi \cdot \Delta P \cdot r^4}{8 \cdot l \cdot \eta}$$

The flow of liquid in the cylindrical tube (Q) is directly proportional to the pressure difference between two ends of the tube ($\Delta P = P_A - P_B$), to the fourth power of the tube radius (r) and inversely proportional to tube length (*l*) and to the viscosity of liquid (η)

Limitation:

- For stationary flow in Newtonian fluids where viscosity is constant and independent on flow velocity.

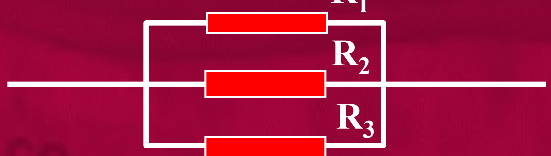
$$Q = \frac{\pi \cdot \Delta P \cdot r^4}{8 \cdot l \cdot \eta}$$

$$Q = \frac{\Delta P}{R_v}$$

Vascular resistance (R_v): a consequence of the friction between fluid and vessel wall.

$$R_v = \frac{\Delta P}{Q} = \frac{8 \cdot l \cdot \eta}{\pi \cdot r^4}$$

Parallel arrangement of vessels

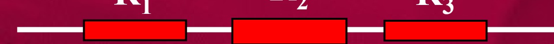


$$\frac{1}{R_c} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

pro $R_1=R_2=R_3=R_n$

$R_c=R/n$

Series arrangement of vessels

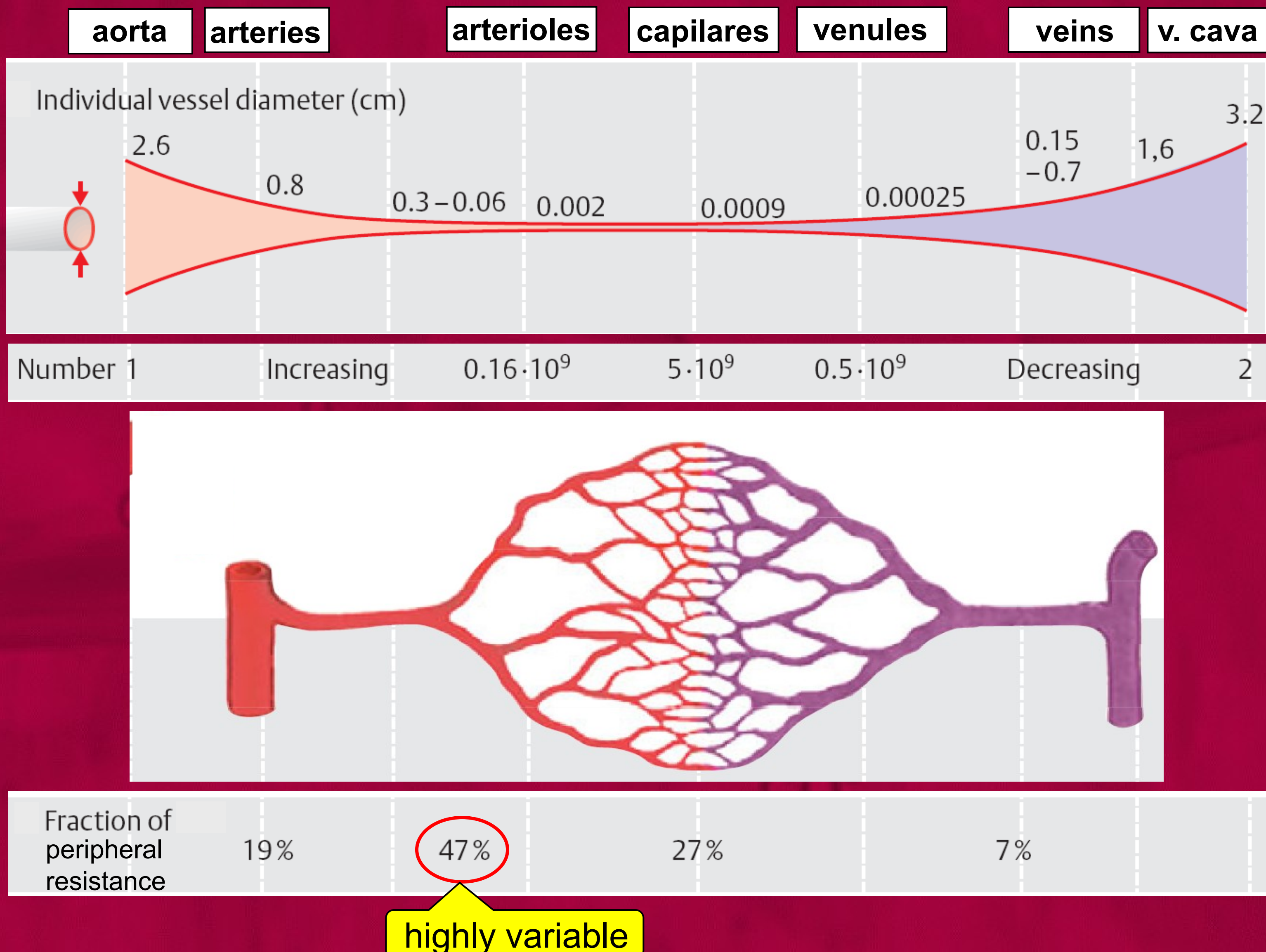


$$R_c = R_1 + R_2 + \ldots$$

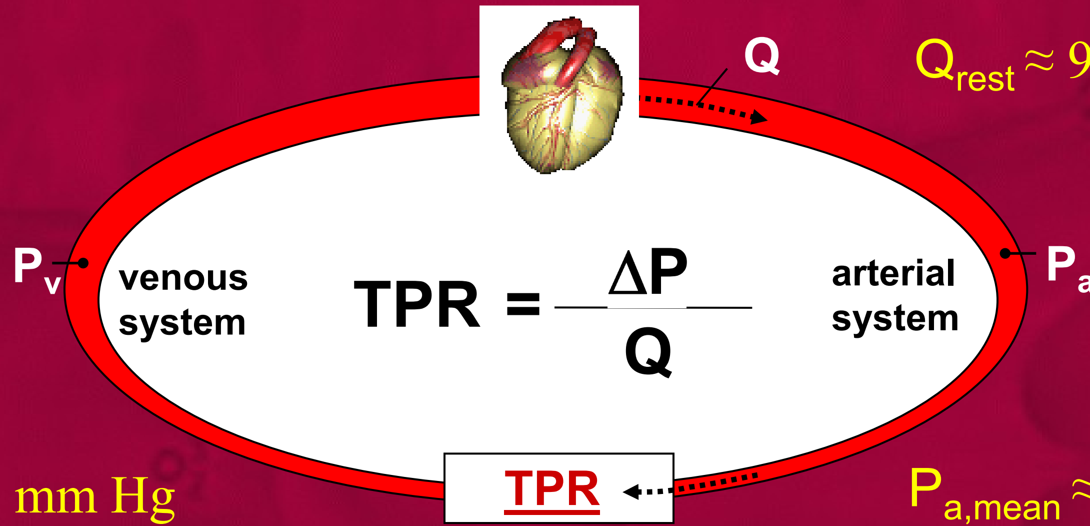
pro $R_1=R_2=R_3=R_n$

$R_c=R \cdot n$

Relation between vessel radius and peripheral resistance



Total peripheral resistance (TPR) of vascular system



$Q_{rest} \approx 90$ ml/s

$P_{v,mean} \approx 3$ mm Hg

$P_{a,mean} \approx 93$ mm Hg

$$TPR = \frac{\Delta P}{Q} = \frac{P_a - P_v}{Q} \approx \frac{P_a}{Q} = \frac{93}{90} \approx 1 \frac{\text{mmHg s}}{\text{ml}}$$

$$P_a \approx TPR \times Q$$

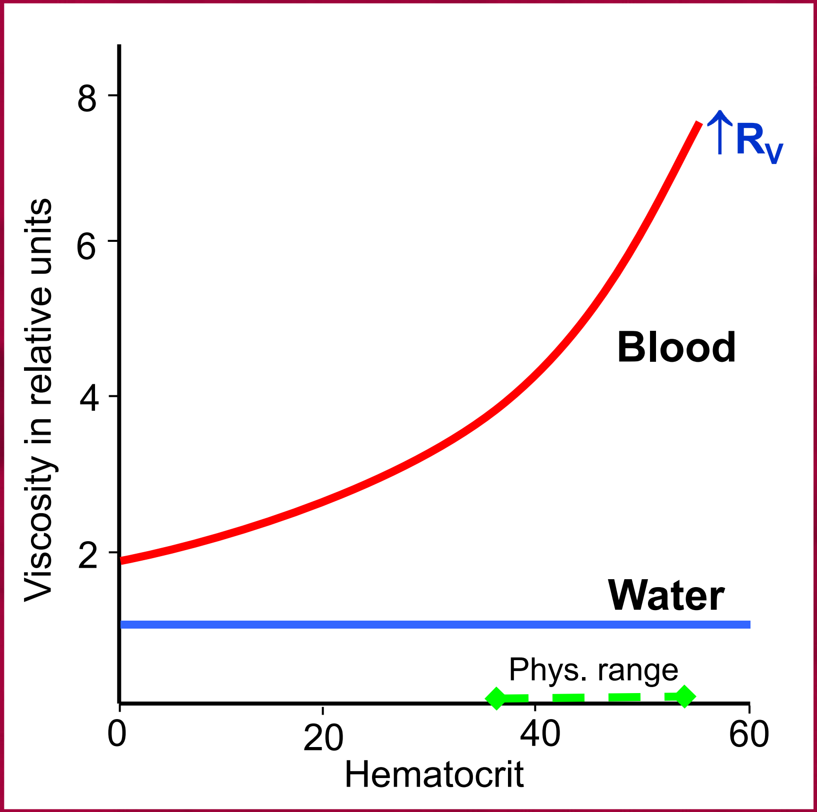
For constant Q: PR P_a ypertension,…cardiac disease.

2. Rheological features of blood and vessels

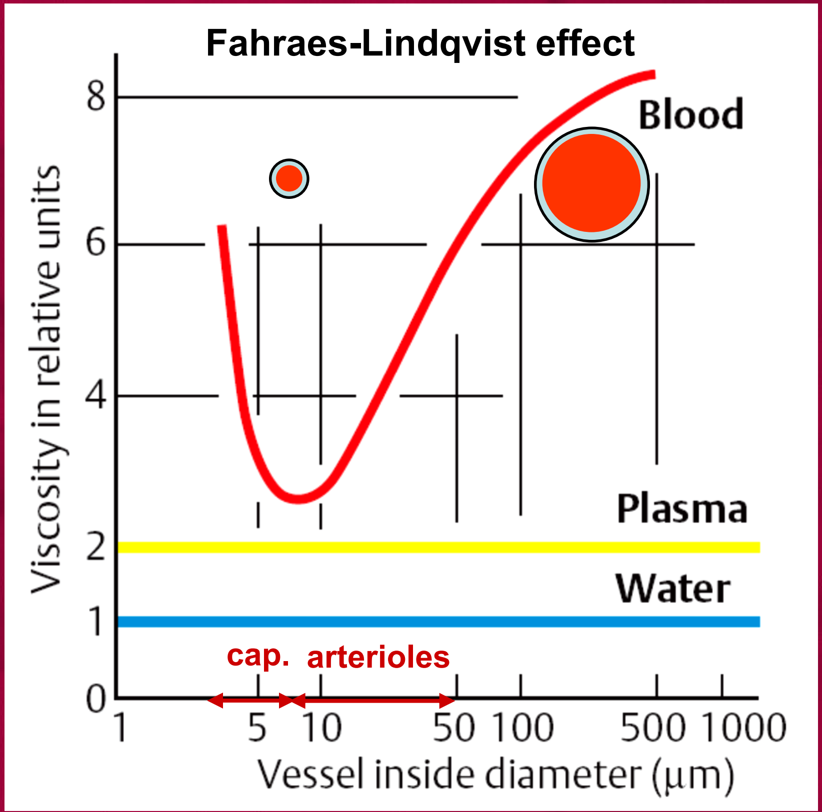
Blood viscosity

$$R_v = 8 \cdot l \cdot \eta / (\pi \cdot r^4)$$

Effect of hematocrit



Effect of diameter in small vessels

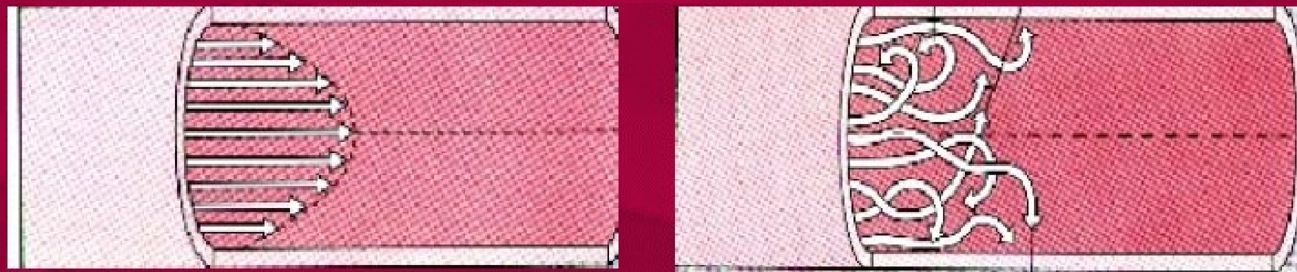


Other factors causing increase of viscosity:

- decrease of blood flow velocity
- elevation of plasma proteins

Laminar and turbulent flow

Velocity profile in laminar and turbulent flow



The character of the flow is determined by Reynolds number

$$R_e = \frac{v \cdot \rho \cdot r}{\eta}$$

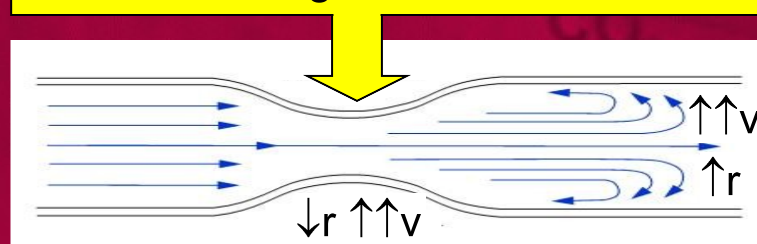
laminar flow

Re<2000

turbulent flow

Re>3000

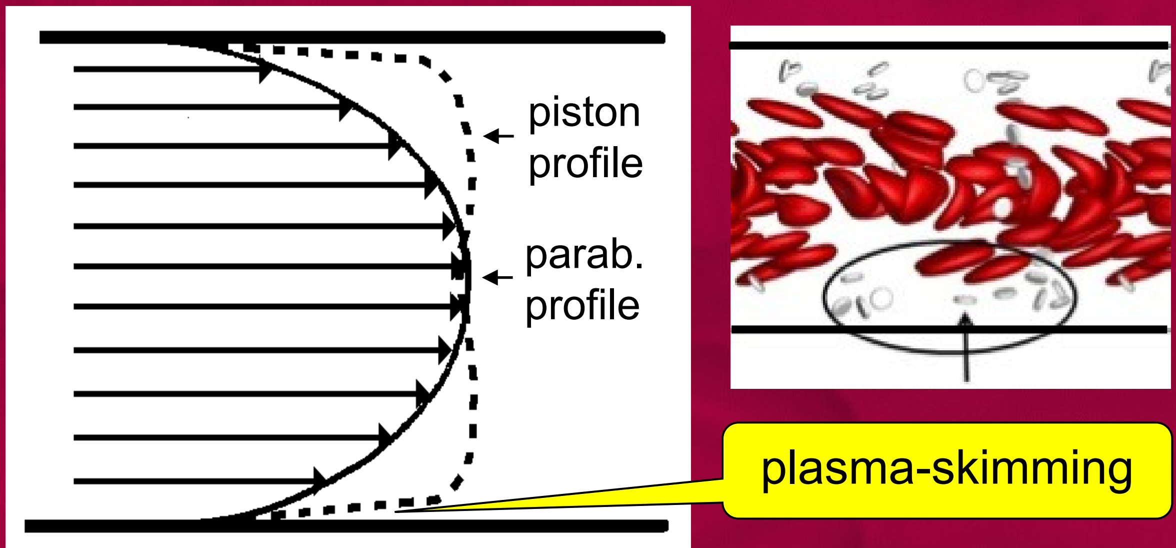
Sudden change of vessel diameter

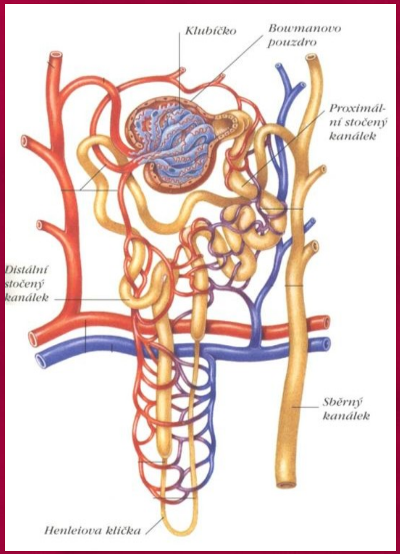


↑R_e ⇒ `R_V

Pathological states causing turbulent flow: aneurisma, stenosis, arteriosclerosis, decreased blood viscosity, .

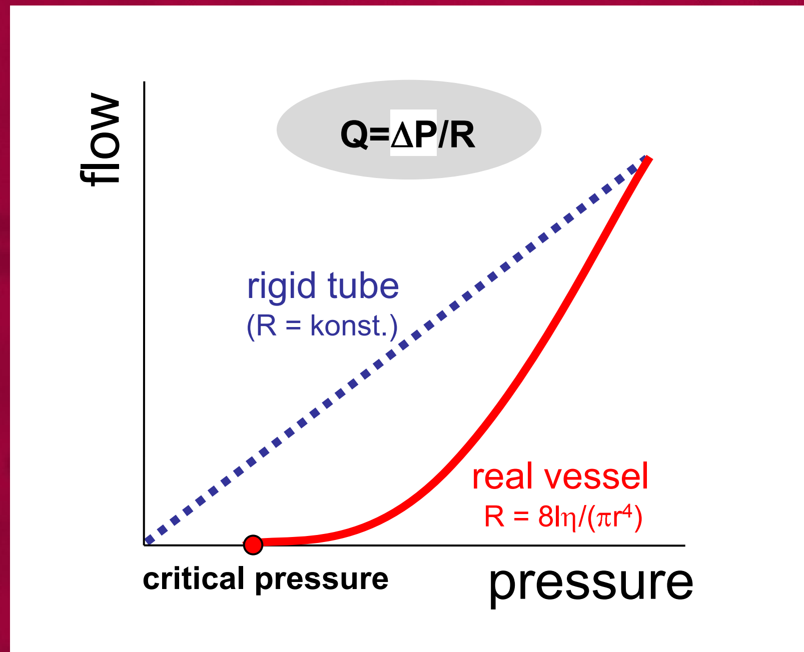
Velocity profile of the blood flow in vessels

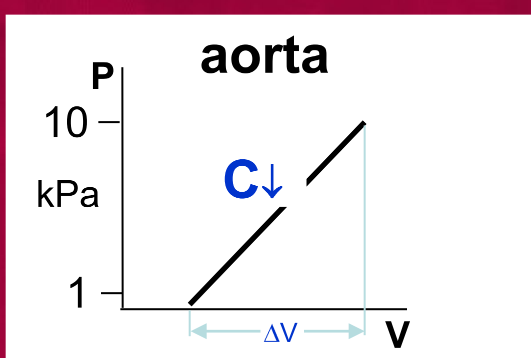




- In small arteries (at r < 100 μm), the central movement of erytrocytes causes a piston-like profile of the blood flow.
- In bigger arteries (at r > 500 μm), the laminar flow prevails and the velocity profile of the blood flow has a parabolic shape.
- In big arteries (especially in aorta), a higher cardiac output causes a turbulent flow ($R_e > 3000$) and the parabolic profile of the blood flow changes to the piston one.

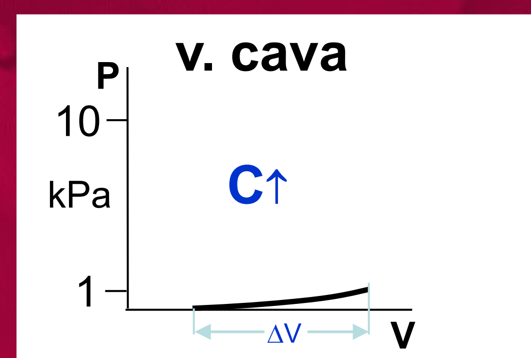
Elasticity of vessels



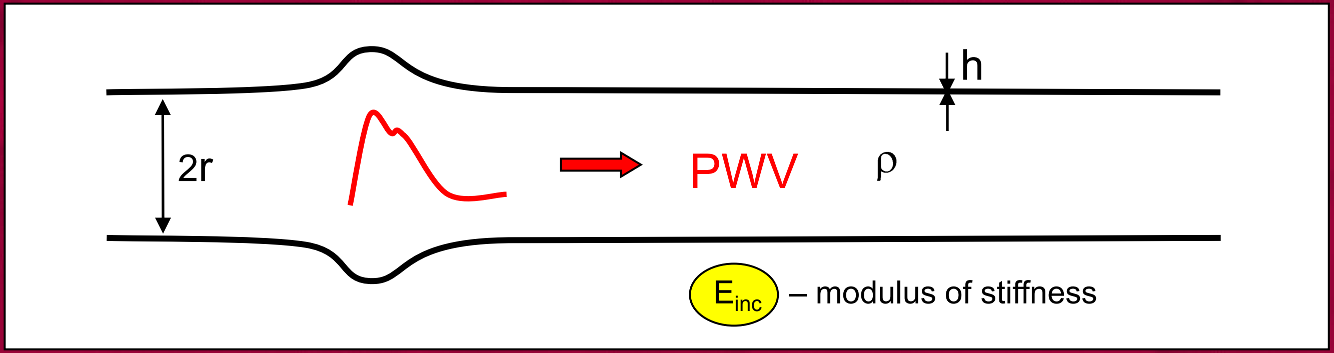


compliance

$$C = \frac{\Delta V}{\Delta P}$$



Pulse wave velocity (PWV)

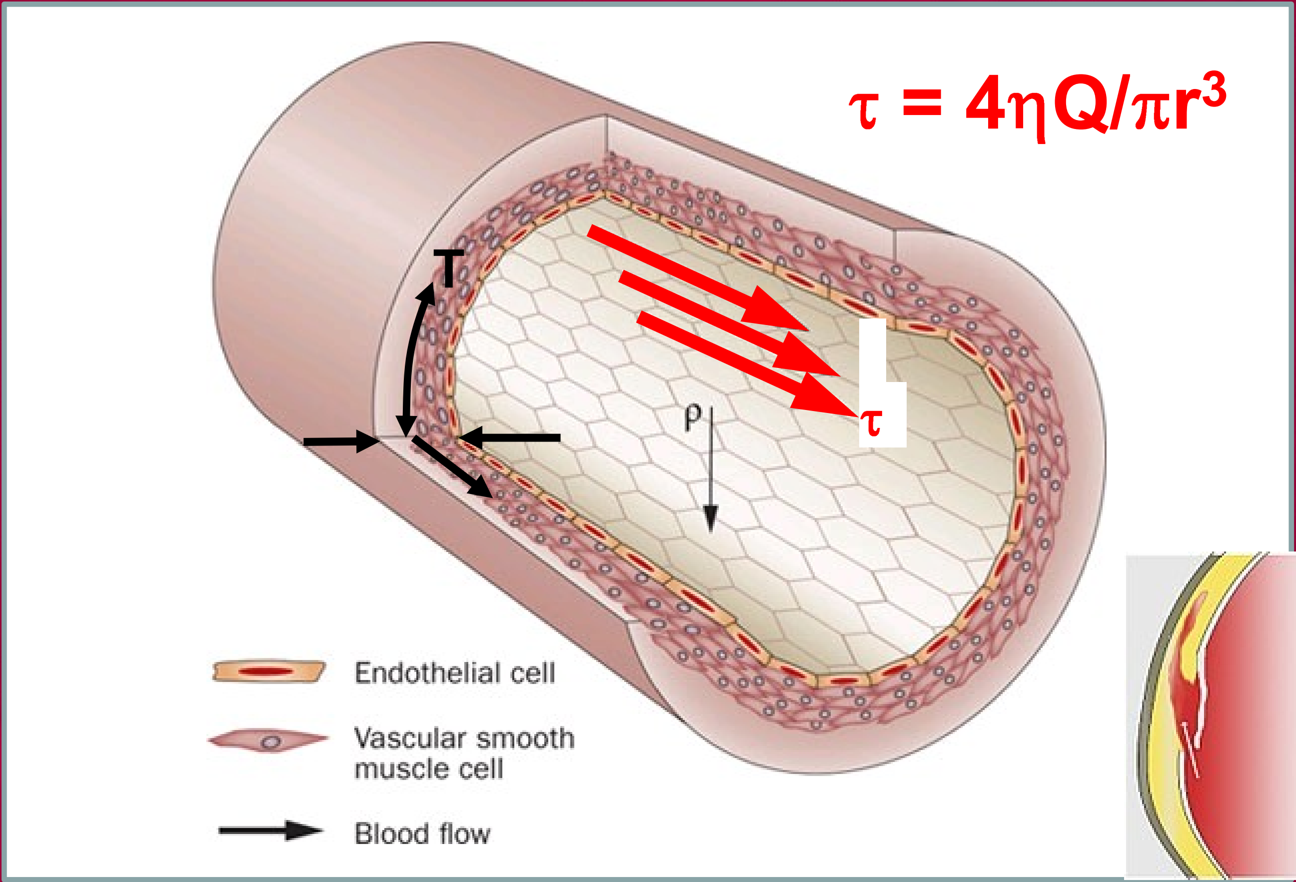


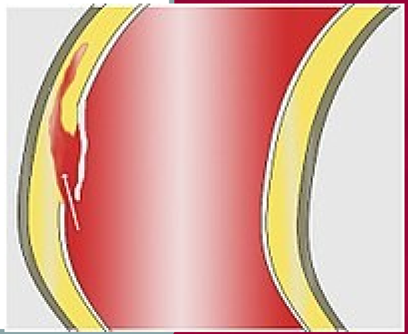
Moens-Korteweg (1878)

$$PWV = \sqrt{\frac{E_{inc} \cdot h}{2 \cdot r \cdot \rho}}$$

In aorta PWV = 4 - 6 m/s

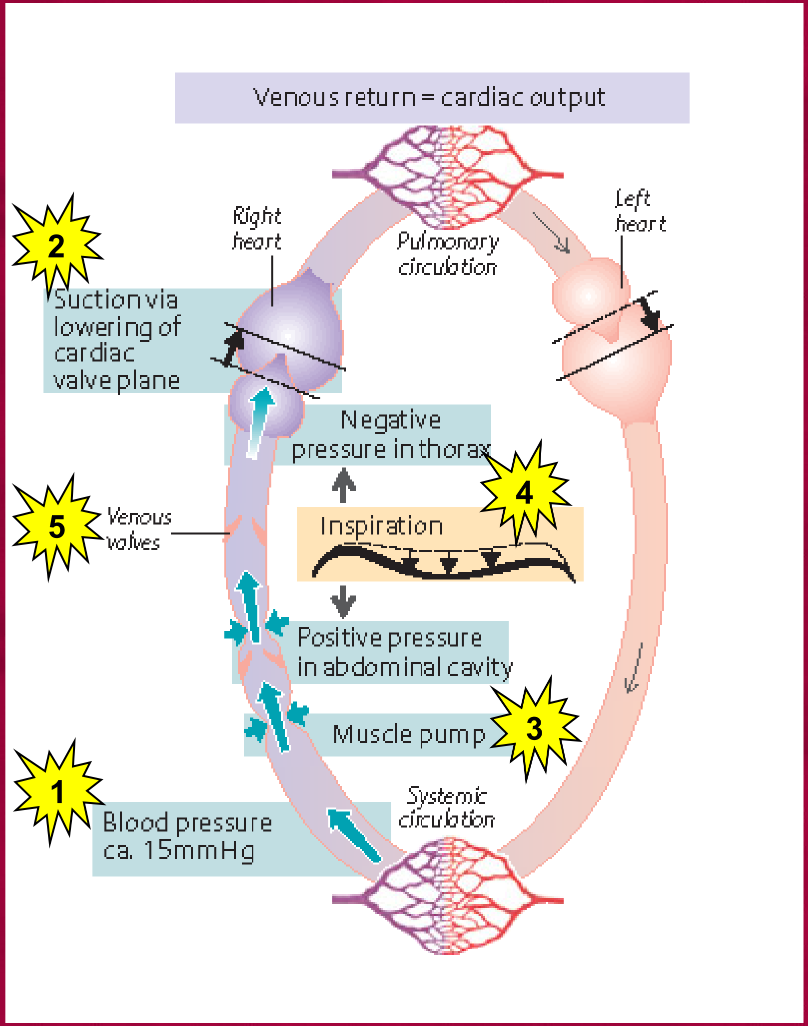
Share stress in vessel wall





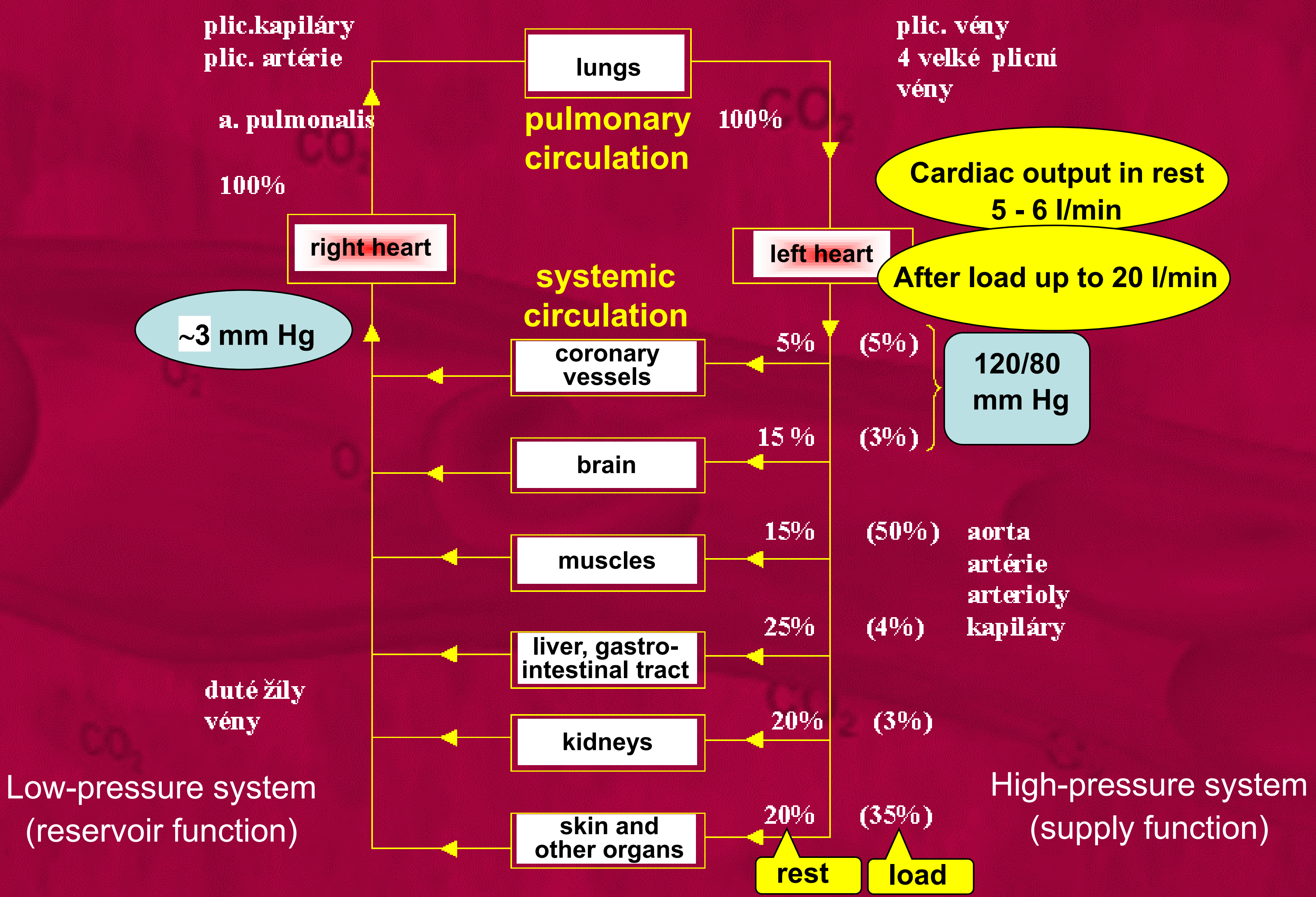
- Share stress in vessel wall may lead to a tear in endothelial layer and to arterial dissection.

Mechanisms of venous return



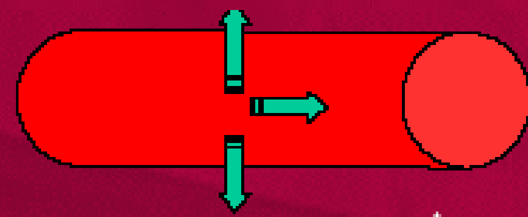
3. Blood circulation and pressure

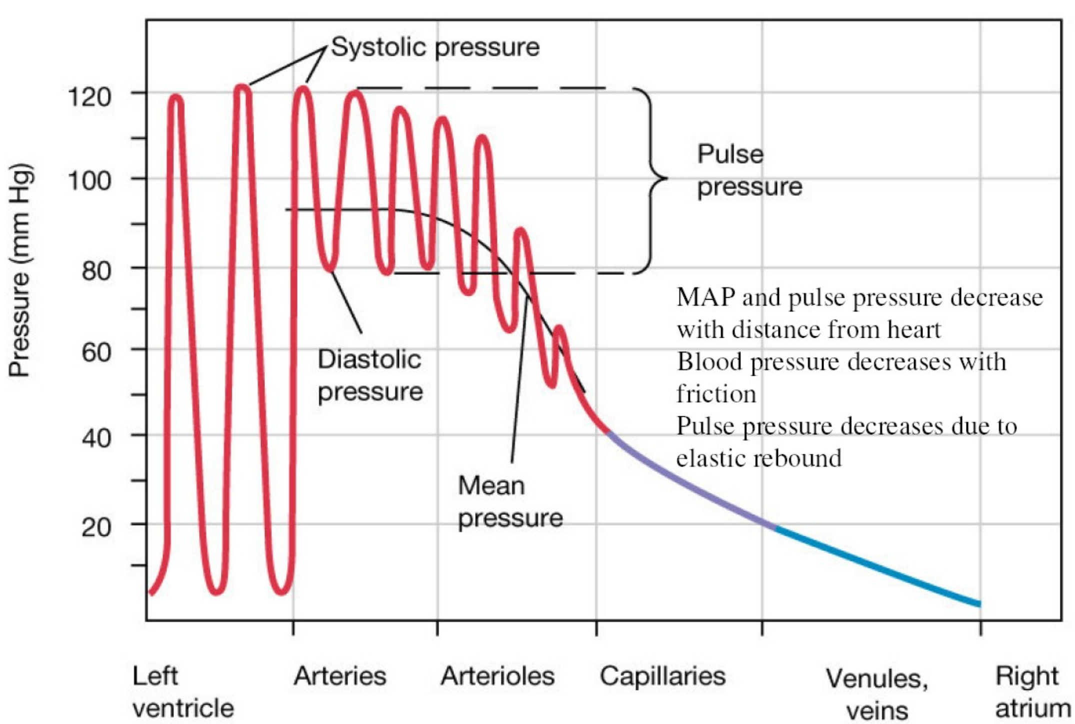
Blood circulation

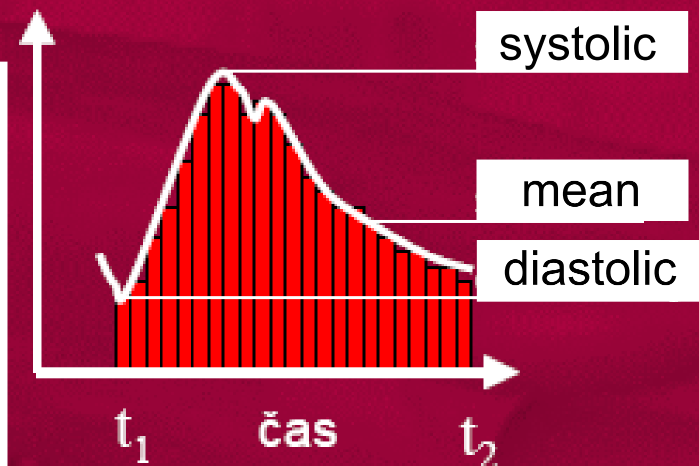


Blood pressure

Blood pressure (BP) is the pressure exerted by circulating blood upon the walls of blood vessels.



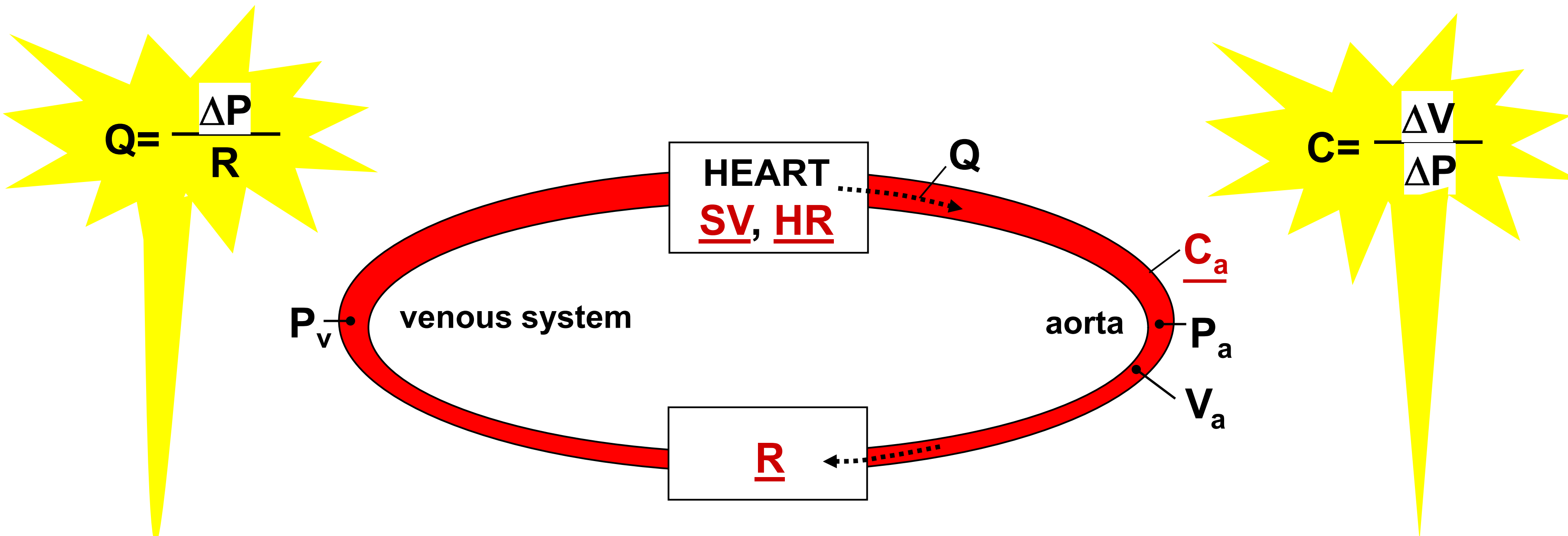




$$P_{mean} = \int_{t_1}^{t_2} \frac{P dt}{t_2 - t_1}$$

$$P_{mean} \cong Pd + \frac{1}{3}(Ps - Pd)$$

Dependence of blood pressure on cardiac output and vascular parameters



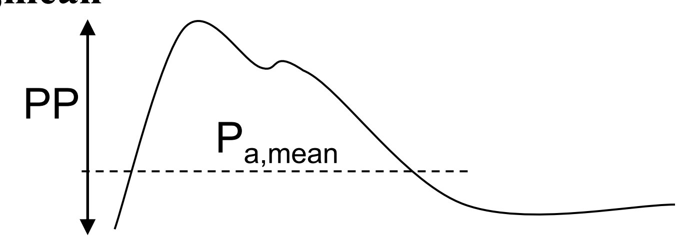
$$P_{a,mean} - P_{v,mean} = Q \cdot R$$

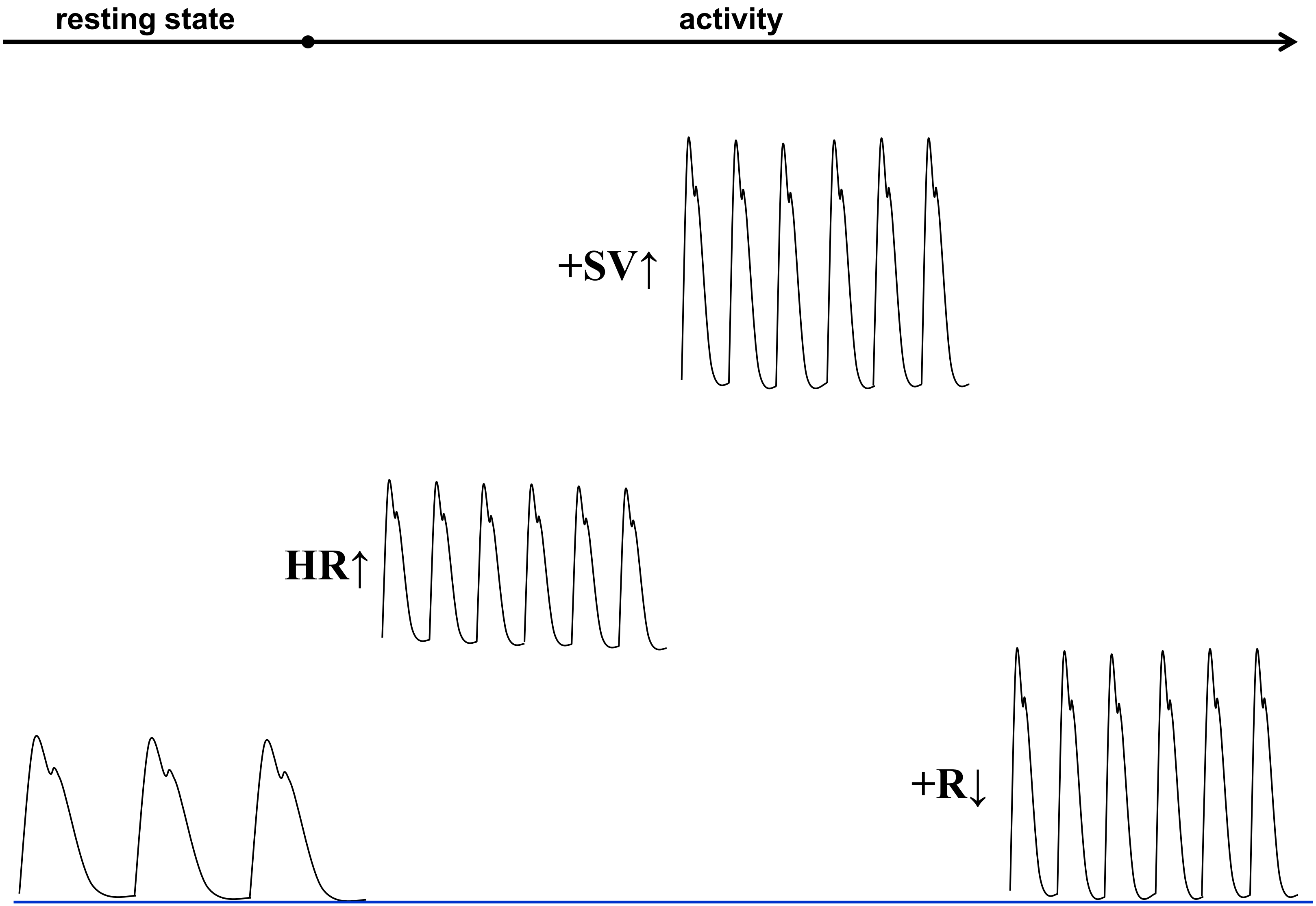
$$\Delta V \cong SV$$

$$P_{a,mean} = SV \cdot HR \cdot R + P_{v,mean}$$

$$PP \cong \frac{SV}{C}$$

$$P_{a,mean} \cong SV \cdot HR \cdot R$$





$$P_{a,mean} \cong SV \cdot HR \cdot R$$

$$PP \cong \frac{SV}{C}$$

Model of blood pressure changes in aorta