Úprava výrazů - Lomené výrazy II.

Příklad 1. Zjednodušte a pak dosaďte hodnoty, jsou-li uvedeny:

1. $\dfrac{\sqrt{\sqrt{x}+1}}{x\sqrt{x}+x+\sqrt{x}} : \dfrac{1}{x^2-\sqrt{x}}$

2. $\left((\sqrt[4]{p}-\sqrt[4]{q})+(\sqrt[4]{p}+\sqrt[4]{q})^{-2}\right) : \dfrac{\sqrt{p}+\sqrt{q}}{p-q}$

3. $\dfrac{(\sqrt{a^2+a\sqrt{a^2-b^2}}-\sqrt{a^2-a\sqrt{a^2-b^2}})^2}{2\sqrt{a^3b}} : \left(\sqrt{\dfrac{a}{b}}+\sqrt{\dfrac{b}{a}}-2\right) ; a>b>0$

4. $\left(\dfrac{(a+b)^{-n/4}c^{1/2}}{a^{2-n}b^{-3/4}}\right)^{1/3} : \left(\dfrac{b^3c^4}{(a+b)^2na^{26-8n}}\right)^{1/6}$

5. $\dfrac{2x^{-1/3}}{x^{2/3}-3x^{-2/3}} - \dfrac{x^{2/3}}{x^{5/3}-x^{2/3}} - \dfrac{x+1}{x^2-4x+3}$

6. $\dfrac{(\sqrt{a}+\sqrt{b})^2-4b}{(a-b):\left(\sqrt{\dfrac{1}{b}}+3\sqrt{\dfrac{1}{a}}\right)} : \dfrac{a+9b+6\sqrt{ab}}{\dfrac{1}{\sqrt{b}}+\dfrac{1}{\sqrt{a}}}$

7. $\dfrac{(\sqrt[4]{m}+\sqrt[4]{n})^2+(\sqrt[4]{m}-\sqrt[4]{n})^2}{2(m-n)} : \dfrac{1}{\sqrt[3]{m}-\sqrt[3]{n}} - 3\sqrt{mn}$

8. $\left(\left(\dfrac{2^{3/2}+27y^{3/5}}{\sqrt{2}+3\sqrt[5]{y}} + 3\sqrt[50]{32y^2} - 2\right)\cdot 3^{-2}\right)^5$

9. $\dfrac{2\sqrt{1+\dfrac{1}{4}\left(\sqrt{\dfrac{1}{t}}-\sqrt{t}\right)^2}}{\sqrt{1+\dfrac{1}{4}\left(\sqrt{\dfrac{1}{t}}-\sqrt{t}\right)^2}-\dfrac{1}{2}\left(\sqrt{\dfrac{1}{t}}-\sqrt{t}\right)^2}$

10. $t\dfrac{1+\dfrac{2}{\sqrt{t+4}}}{2-\sqrt{t+4}} + \sqrt{t+4} + \dfrac{4}{\sqrt{t+4}}$

11. $\left(\dfrac{1+\sqrt{x}}{\sqrt{1+x}}-\dfrac{\sqrt{1+x}}{1+\sqrt{x}}\right)^2 - \left(\dfrac{1-\sqrt{x}}{\sqrt{1+x}}-\dfrac{\sqrt{1+x}}{1-\sqrt{x}}\right)^2$

12. $\dfrac{x-1}{x+x^{1/2}+1} : \dfrac{x^{0,5}+1}{x^{1,5}-1} + \dfrac{2}{x^{-0,5}}$

13. $\left(\dfrac{1}{\sqrt{a}+\sqrt{a+1}} + \dfrac{1}{\sqrt{a}-\sqrt{a-1}}\right) : \left(1 + \sqrt{\dfrac{a+1}{a-1}}\right)$

14. $\dfrac{x-y}{x^{3/4}+x^{1/2}y^{1/4}} \cdot \dfrac{x^{1/2}y^{1/4}+x^{1/4}y^{1/2}}{x^{1/2}+y^{1/2}} \cdot \dfrac{x^{1/4}y^{-1/4}}{x^{1/2}-2x^{1/4}y^{1/4}+y^{1/2}}$

15. $\sqrt[n]{y^{\frac{2n}{m-n}}} : \sqrt[m]{y^{\frac{(m-n)^2+4mn}{m^2-n^2}}}$

16. $\left(\dfrac{(z^{2/p}+z2/q)^2 - 4z^{2/p+2/q}}{(z^{1/p}-z^{1/q})^2 + 4z^{1/p+1/q}}\right)^{1/2}$

17. $\dfrac{x-1}{x^{3/4}+x^{1/2}} \cdot \dfrac{x^{1/2}+x^{1/4}}{x^{1/2}+1} \cdot x^{1/4}+1$

18. $\left(\dfrac{1+x+x^2}{2x+x^2} + 2\dfrac{1-x-x^2}{2x-x^2}\right)^{-1} \cdot (5-2x^2); x = \sqrt{3,92}$

19. $\dfrac{(x^2-y^2)\left(\sqrt[3]{x}+\sqrt[3]{y}\right)}{\sqrt[3]{x^5}+\sqrt[3]{x^2y^3}-\sqrt[3]{x^3y^2}-\sqrt[3]{y^5}} - \left(\sqrt[3]{xy}+\sqrt[3]{y^2}\right); x = 64$

20. $\sqrt{\dfrac{2a}{(1+a)\sqrt[3]{1+a}}} \cdot \sqrt[3]{\dfrac{4+\dfrac{8}{a}+\dfrac{4}{a^2}}{\sqrt{2}}}$

21. $\dfrac{4x(x+\sqrt{x^2-1})^2}{(x+\sqrt{x^2-1})^4 - 1}$

22. $\dfrac{\sqrt{(x+2)^2 - 8x}}{\sqrt{x}-\dfrac{2}{\sqrt{x}}}$

23. $\sqrt[4]{6x(5+2\sqrt{6})} \cdot \sqrt{3\sqrt{2x}-2\sqrt{3x}}$

24. $\sqrt[6]{4x(11+4\sqrt{6})} \cdot \sqrt[3]{4\sqrt{2x}-2\sqrt{3x}}$

25. $\dfrac{a^3-a-2b-\dfrac{b^2}{a}}{\left(1-\sqrt{\dfrac{1}{a}+\dfrac{b}{a^2}}\right) \cdot (a+\sqrt{a+b})} : \left(\dfrac{a^3+a^2+ab+a^2b}{a^2-b^2} + \dfrac{b}{a-b}\right); a = 23, b = 22$

26. $\dfrac{(\sqrt[5]{a^{4/3}})^{3/2}}{(\sqrt[5]{a^4})^3} \cdot \dfrac{\left(\sqrt{a\sqrt[3]{a^2b}}\right)^4}{\left(\sqrt[4]{a\sqrt{b}}\right)^6}$

27. $\dfrac{\sqrt[3]{x+\sqrt{2-x^2}} \cdot \sqrt[6]{1-x\sqrt{2-x^2}}}{\sqrt[3]{1-x^3}}$

28. $\dfrac{x(x^2-a^2)^{-1/2}+1}{a(x-a)^{-1/2}+(x-a)^{-1/2}} : \dfrac{a^2\sqrt{x+a}}{x-(x^2-a^2)^{1/2}} + \dfrac{1}{x^2-ax}$

29. $\dfrac{\left(\sqrt[3]{(r^2+4)\cdot\sqrt{1+\dfrac{4}{r^2}}} - \sqrt[3]{(r^2-4)\cdot\sqrt{1-\dfrac{4}{r^2}}}\right)^2}{r^2-\sqrt{r^4-16}}$

30. $\sqrt{\dfrac{\sqrt{2}}{a}+\dfrac{a}{\sqrt{2}}+2} - \dfrac{a^2\sqrt[4]{2}-2\sqrt{a}}{a\sqrt{2a}-\sqrt[4]{8a^4}}$

31. $\left(\dfrac{\sqrt[4]{a^3}-1}{\sqrt[4]{a}-1}+\sqrt[4]{a}\right)^{1/2} \cdot \left(\dfrac{\sqrt[4]{a^3}+1}{\sqrt[4]{a}+1}+\sqrt{a}\right) \cdot (a-\sqrt{a^3})^{-1}$

32. $\dfrac{\sqrt{\dfrac{abc+4}{a}+4+\sqrt{\dfrac{bc}{a}}}}{\sqrt{abc}+2} ; a=0,04$

33. $\dfrac{\sqrt{(2p+1)^3}+\sqrt{(2p-1)^3}}{\sqrt{4p+2\sqrt{4p^2-1}}}$

34. $1 - \dfrac{\dfrac{1}{\sqrt{a-1}}-\sqrt{a+1}}{\dfrac{1}{\sqrt{a+1}}-\dfrac{1}{\sqrt{a-1}}} : \dfrac{\sqrt{a+1}\cdot\sqrt{a^2-1}}{(a-1)\sqrt{a+1}-(a+1)\sqrt{a-1}}$

35. $\left(\dfrac{a+2}{\sqrt{2a}}-\dfrac{a}{\sqrt{2a}+2}+\dfrac{2}{a-\sqrt{2a}}\right)\cdot\dfrac{\sqrt{a}-\sqrt{2}}{a+2}$

36. $\left(\sqrt[4]{36mn^2p}+m\sqrt{\dfrac{3n}{m}}+\sqrt{3np}\right)\cdot\left(\sqrt[4]{36mn^2p}-\sqrt{3mn}-p\sqrt{\dfrac{3n}{p}}\right)$

37. $\dfrac{1-x^{-2}}{x^{\frac{1}{2}}-x^{-\frac{1}{2}}} - \dfrac{2}{x^{\frac{3}{2}}} + \dfrac{x^{-2}-x}{x^{\frac{1}{2}}-x^{-\frac{1}{2}}}$

38. $\left(\dfrac{\sqrt{a}}{2}-\dfrac{1}{2\sqrt{a}}\right)^2 \cdot \left(\dfrac{\sqrt{a}-1}{\sqrt{a}+1}-\dfrac{\sqrt{a}+1}{\sqrt{a}-1}\right)$

39. $\dfrac{9b^{\frac{4}{3}}-\dfrac{a^{\frac{3}{2}}}{b^2}}{\sqrt{a^{\frac{3}{2}}b^{-2}+6a^{\frac{3}{4}}b^{-\frac{1}{3}}+9b^{\frac{4}{3}}}} ; b=4$

40. $\dfrac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}} \cdot (1 + \frac{b^2+c^2-a^2}{2bc}) : \frac{a-b-c}{abc}; a = 0.002, b = -11.05, c = 1.07$

41. $\dfrac{1}{2(1+\sqrt{a})} + \dfrac{1}{2(1-\sqrt{a})} - \dfrac{a^2+2}{1-a^3}$

42. $\dfrac{\sqrt{2}(x-a)}{2x-a} - ((\dfrac{\sqrt{x}}{\sqrt{2x}+\sqrt{a}})^2 + (\dfrac{\sqrt{2x}+\sqrt{a}}{2\sqrt{a}})^{-1})^{\frac{1}{2}}; a = 0.32, x = 0.08$

43. $\dfrac{(m^2-\frac{1}{n^2})\cdot(n+\frac{1}{m})^{n-m}}{(n^2-\frac{1}{m^2})^n\cdot(m-\frac{1}{n})^{m-n}}$

44. $(\dfrac{\sqrt{x-a}}{\sqrt{x+a}+\sqrt{x-a}} + \dfrac{x-a}{\sqrt{x^2-a^2}-x+a}) : \sqrt{\dfrac{x^2}{a^2}-1}; x > a > 0$

45. $(\dfrac{\sqrt[4]{x^3}-\sqrt[4]{x}}{1-\sqrt{x}} + \dfrac{1+\sqrt{x}}{\sqrt[4]{x}})^2 \cdot (1 + \dfrac{2}{\sqrt{x}} + \dfrac{1}{x})^{-\frac{1}{2}}$

46. $\dfrac{\sqrt{1-x^2}-1}{x} \cdot (\dfrac{1-x}{\sqrt{1-x^2}+x-1} + \dfrac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}})$

47. $\dfrac{2b+a-\frac{4a^2-b^2}{a}}{b^3+2ab^2-3a^2b} \cdot \dfrac{a^3b-2a^2b^2+ab^3}{a^2-b^2}$

48. $\dfrac{(2p-q)^2 + 2q^2 - 3pq}{2p^{-1}+q^2} : \dfrac{4p^2-3pq}{2+pq^2}; p = 78, q = \dfrac{7}{25}$

49. $\left(\dfrac{pq^3}{(p+q)^{5/2}} - \dfrac{2pq^2}{(p+q)^{3/2}} + \dfrac{pq}{\sqrt{p+q}}\right) : \left(\dfrac{p^2}{(p+q)^{5/2}} - \dfrac{p^2q}{(p+q)^{1/2}}\right)$

50. $\dfrac{2(x^4+4x^2-12)+x^4+11x^2+30}{x^2+6}$

51. $\dfrac{(a^2-b^2)(a^2+\sqrt[3]{b^2}+a\sqrt[3]{b})}{a\sqrt[3]{b}+a\sqrt{a}-b\sqrt[3]{b}-\sqrt{ab^2}} : \dfrac{a^3b}{a\sqrt[3]{b}-\sqrt[6]{a^3b^2}-\sqrt[3]{b^2}+a\sqrt{a}}; a = 4.91, b = 0.09$

52. $\left((1-x^2)^{-1/2}+1+\dfrac{1}{(1-x^2)^{-1/2}-1}\right)^{-2} : (2-x^2-2\sqrt{1-x^2})$

53. $\left((1-p^2)^{-1/2}-(1+p^2)^{-1/2}\right)^2 + 2(1-p^4)^{-1/2}$

54. $\dfrac{3a^2+2ax-x^2}{(3x+a)(a+x)} - 2 + 10 \cdot \dfrac{ax-3x^2}{a^2-9x^2}$

55. $\left(\dfrac{\sqrt[3]{x+y}}{\sqrt[3]{x-y}} + \dfrac{\sqrt[3]{x-y}}{\sqrt[3]{x+y}} - 2\right) : \left(\dfrac{1}{\sqrt[3]{x-y}} - \dfrac{1}{\sqrt[3]{x+y}}\right)$

56.
$$\left(\frac{4}{4a + \dfrac{1}{b + \dfrac{1}{c}}} : \frac{1}{a + \dfrac{1}{b}} - \frac{4}{b(abc + a + c)} \right)^{-1/2}$$

57.
$$\left(\left(\frac{x}{y - x} \right)^{-2} - \frac{(x + y)^2 - 4xy}{x^2 - xy} \right) \cdot \frac{x^4}{x^2 y^2 - y^4}$$

58.
$$\left(\left(\frac{1}{a} + \frac{1}{b + c} \right) : \left(\frac{1}{a} - \frac{1}{b + c} \right) \right) : \left(1 + \frac{b^2 + c^2 - a^2}{2bc} \right) ; a = 1\frac{33}{40}, b = 0.625, c = 3.2$$

59.
$$\left(\left(\frac{x^2}{y^2} + \frac{1}{x} \right) : \left(\frac{x}{y^2} - \frac{1}{y} + \frac{1}{x} \right) \right) : \frac{(x - y)^2 + 4xy}{1 + \dfrac{y}{x}}$$

60.
$$\left(\frac{3}{2x - y} - \frac{2}{2x + y} - \frac{1}{2x - 5y} \right) : \frac{y^2}{4x^2 - y^2}$$

61.
$$\left(x^2 + 2x - \frac{11x - 2}{3x + 1} \right) : \left(x + 1 - \frac{2x^2 + x + 2}{3x + 1} \right) ; x = 7, (3)$$

62.
$$\left(6a^2 + 5a - 1 + \frac{a + 4}{a + 1} \right) : \left(3a - 2 + \frac{3}{a + 1} \right)$$

63.
$$\frac{x^{-6} - 64}{4 + 2x^{-1} + x^{-2}} \cdot \frac{x^2}{4 - \dfrac{4}{x} + \dfrac{1}{x^2}} - \frac{4x^2(2x + 1)}{1 - 2x}$$

64.
$$\frac{2b + a - \dfrac{4a^2 - b^2}{a}}{b^3 + 2ab^2 - 3a^2 b} \cdot \frac{a^3 b - 2a^2 b^2 + ab^3}{a^2 - b^2}$$

98.
$$\frac{1 - \sqrt{2t}}{\dfrac{1 - \sqrt[4]{8t^3}}{1 - \sqrt[4]{2t}} - \sqrt{2t}} \cdot \left(\frac{\sqrt[4]{\dfrac{1}{2t}} + \sqrt[4]{4t^2}}{1 + \sqrt[4]{\dfrac{1}{2t}}} - \sqrt{2t} \right)^{-1}$$

99.
$$\frac{(x^{2/3} + 2\sqrt[3]{xy} + 4y^{2/3})}{(\sqrt[3]{x^4} - 8y\sqrt[3]{x}) : \sqrt[3]{xy}} \cdot \left(2 - \sqrt[3]{\frac{x}{y}} \right)$$

100.
$$\frac{(z - z\sqrt{z} + 2 - 2\sqrt{z})^2 \cdot (1 + \sqrt{z})^2}{z - 2 + \dfrac{1}{z}} - z\sqrt{z} \cdot \sqrt{\frac{4}{z} + 4 + z}$$

101.
$$\left(\frac{1}{a + \sqrt{2}} - \frac{a^2 + 4}{a^3 + 2\sqrt{2}} \right) \cdot \left(\frac{a}{2} - \frac{1}{\sqrt{2}} + \frac{1}{a} \right)^{-1}$$

102. $\left(\dfrac{(a-1)^{-1}}{a^{-3}} - (1-a)^{-1}\right) \cdot \dfrac{1+a(a-2)}{a^2-a+1} \cdot \sqrt{\dfrac{1}{(a+1)^2}}$

103. $\left(\sqrt{ab} - ab(a+\sqrt{ab})^{-1}\right) : \left(2((ab)^{1/2}-b)\cdot(a-b)^{-1}\right)$

104. $\left(\dfrac{a}{b}\sqrt[3]{b-\dfrac{4a^6}{b^3}} - a^2\sqrt[3]{\dfrac{b}{a^6}-\dfrac{4}{b^3}} + \dfrac{2}{ab}\sqrt[3]{a^3b^4-4a^9}\right) : \dfrac{\sqrt[3]{b^3-2a^3}}{b^2}$

105. $\left(\dfrac{1+\sqrt{1-x}}{1-x+\sqrt{1-x}} - \dfrac{1-\sqrt{1+x}}{1+x-\sqrt{1+x}}\right)^2 \cdot \dfrac{x^2-1}{2} - \sqrt{1-x^2}$

106. $\dfrac{4a^2-b^2}{a^6-8b^6} \cdot \sqrt{a^2-2b\sqrt{a^2-b^2}} \cdot \dfrac{a^4+2a^2b^2+4b^4}{4a^2+4ab+b^2} \cdot \sqrt{a^2+2b\sqrt{a^2-b^2}}; a=4/3, b=0.25$

107. $\dfrac{1+(a-x)^{-1}}{1-(a-x)^{-1}} \cdot \left(1-\dfrac{1-(a^2+x^2)}{2ax}\right); x = \dfrac{1}{a-1}$

108. $\left(\dfrac{a}{b}+\dfrac{b}{a}+2\right)\left(\dfrac{a+b}{2a}-\dfrac{b}{a+b}\right) : \left(\left(a+2b+\dfrac{b^2}{a}\right)\cdot\left(\dfrac{a}{a+b}+\dfrac{b}{a-b}\right)\right); a=0.75, b=4/3$

109. $\left(-4a\sqrt[3]{\dfrac{\sqrt{ax}}{a^2}}\right)^3 + \left(-10a\sqrt{x}\sqrt{(ax)^{-1}}\right)^2 + \left(-2\left(\sqrt[3]{a\sqrt[4]{\dfrac{x}{a}}}\right)^2\right)^3; a=3\dfrac{4}{7}, x=0.28$

110. $\dfrac{\sqrt{c-d}}{c^2\sqrt{2c}} \cdot \left(\sqrt{\dfrac{c-d}{c+d}}+\sqrt{\dfrac{c^2+cd}{c^2-cd}}\right); c=2, d=1/4$

111. $\dfrac{(ab^{-1}+a^{-1}b+1)(a^{-1}-b^{-1})^2}{a^2b^{-2}+a^{-2}b^2-(ab^{-1}+a^{-1}b)}$