Aspects of Geometry From the environment to the classroom

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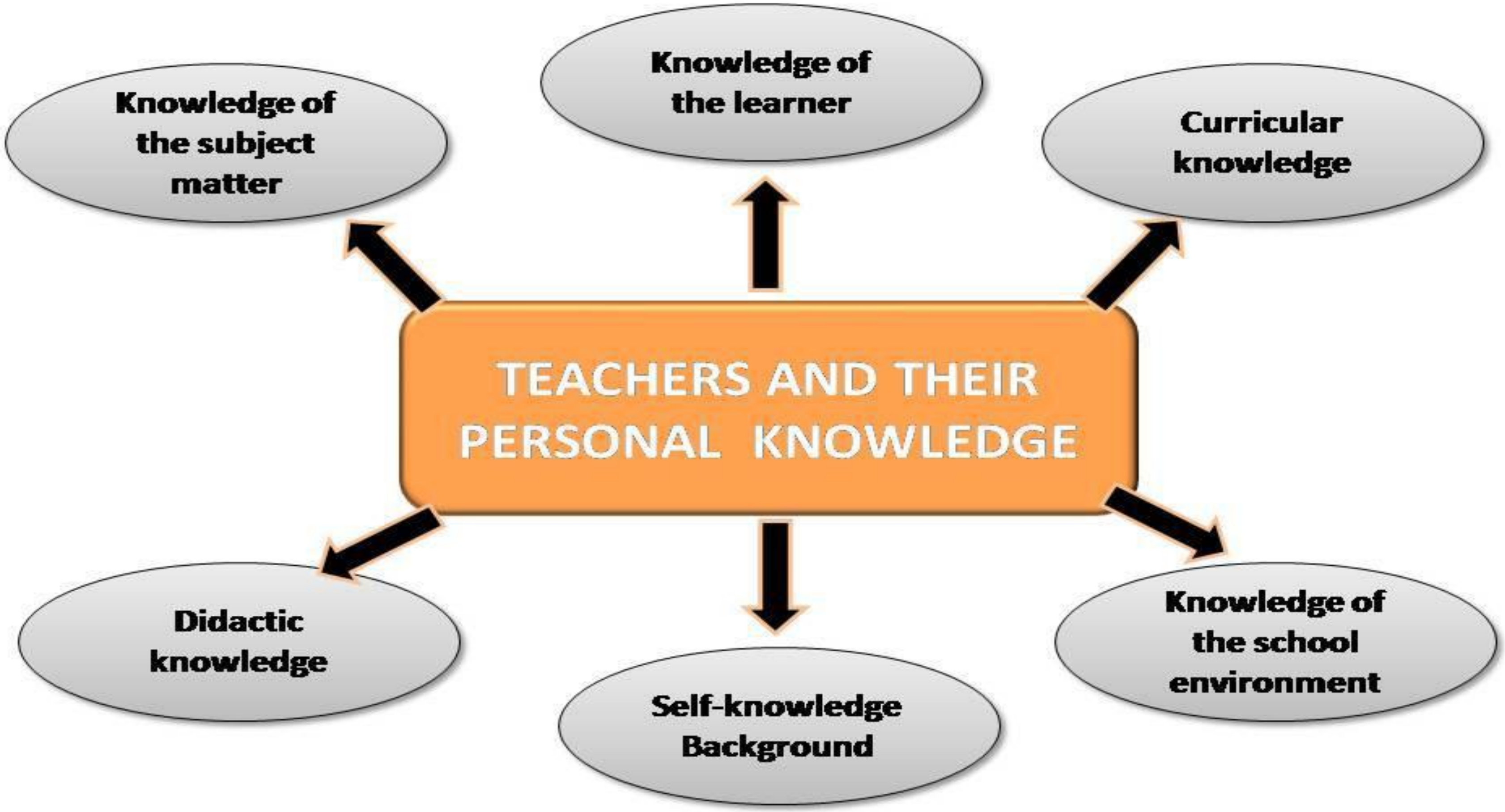
Our workshop

- Teaching and Learning mathematics in the 21 century - Promoting meaningful learning
- Geometry and the Van Hiele Theory
- Activities from the word around us
- Pedagogic and didactic functions of the activities
- The workshop
- Questions and conclusions'

“Adding creativity to daily instructional practices will ensure that students are given opportunities to develop all of their potential….”

(Burke Adams 2007)



Knowledge of the subject matter
Knowledge of the learner
Curricular knowledge
TEACHERS AND THEIR PERSONAL KNOWLEDGE
Didactic knowledge
Self-knowledge Background
Knowledge of the school environment

Our Goals

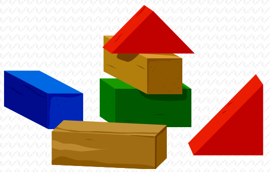
To develop mathematical thinking "outside the box".

To integrate other disciplines in the teaching of mathematics.

To develop literacy in general, and mathematics literacy in particular. (Reading Writing and Speaking)

Geometry

- Geometry is one of the major subjects in the curriculum and is perceived as one of the most complex fields.
- Students frequently experience a sense of travelling to "an isolated island" where everything is structured in a "logical" or "unusual" way, without any relation to daily life.



The Van-Hiele Theory

- There are various theories dealing with the development of pupils' geometric thinking.
- One of them is the Van-Hiele theory.
- According to this theory, development in the study of geometry progresses in a hierarchical order along levels of mastery.
- Where partial mastery at a particular level is necessary, but not sufficient for understanding on a higher level.

The Van Hiele theory

- The van Hiele theory, **unlike that of Piaget**,is based on the assumption that **Moving from one level of thinking to another depends more on experience and teaching than on age or biological maturity of learners**

(Geddes, Fuys, Lovett & Tischler, 1982; van Hiele, 1999).

Van Hiele –Level 1

- **Level I: Recognition or visualization**

At this level learners can identify geometric shapes and distinguish between them. Each of the concepts or the shapes is perceived as a whole, in the way it is seen.

Learners are capable of distinguishing between similar shapes as well as name them. At this level, learners are unable to specify the properties of those shapes.

Van Hiele –Level 2

Level 2: Analysis or description:

At this level learners can analyze properties of shapes but are unable to attribute properties of a particular item to the properties of the group to which it belongs.

For example: learners know that the opposite sides of a rectangle are equal or that the diagonals of a rectangle have the same length. However they cannot infer from it about some of the square's properties.

Learners are unable to deduct that also in a square, being a special rectangle; the diagonals are equal

Van Hiele –Level 3

Level 3: Order or informal deduction:

- At this level learners identify a hierarchical order of connection between groups of different shapes according to their properties and definitions.
- For example: At this level learners understand the relationship of connection between a rectangle and a square – every square is a rectangle but not every rectangle is a square.
- At this level learners are still unable to prove that the diagonals of a rectangle are the same.

Van Hiele –Level 4

Level 4: Formal deduction and rigor:

- **At this level learners understand the roles of basic concepts, axioms, definitions, theorems and proofs and their interrelations.**
- **They can use assumptions in order to prove theorems and understand the meaning of necessary and sufficient conditions.**

Van Hiele –Level 4

- **At this level learners are able to provide reasons and arguments for the various levels of the proving process.**
- **Moreover, they comprehend the importance of discussing the proofs, the deduction from the particular to the general and even the need for a proof of any kind**

(Patkin & Levenberg, 2010).

BASIC SKILLS

- This induced Hoffer (1981) to define 5 **Basic skill areas** which greatly determine the extent of succeeding to enhance the learners' levels of thinking.
- These basic skills are: **visual skills, verbal skills, logical skills, drawing skills and applied skills.**

Drawing skills

- For study geometry, more than for many other subjects, learners need drawing skills in order to understand properties of shapes.

Applied skills

- Applicability: Geometry is a theoretical model of the physical world around us.
- A student can apply what is learnt in geometry lessons to the surrounding world and
- Can draw from the world in order to understand geometry.

Visualization

- **Visual competence constitutes an important factor when examining learners already at the first stage of thinking.**
- **Developing visual competence aims to increase learners' mathematical power and promote the ability to solve mathematical problems.**

Geometry Lessons

- **During geometry lessons, the use of all types of visual displays, pictures, presentations and movies, which show geometry in the pupils‘ environment,**

Constitutes a Bridge

between the Concrete and the Abstract.

Geometry Lessons

- **Every object around us can serve as an illustration means for the subject.**
- **Even a football or an orange can be the beginning of a fascinating lesson.**
- **It all depends on the extent of imagination and creativity of teachers wanting to evoke interest and improve their pupils' understanding of geometry.**

The Activities

- **The purpose of this workshop is to share suggestions and examples of activities to promote and develop geometric thinking.**

- **The activities based on visual illustrations taken from the learners environment.**

The Activities

- The activities incorporate both natural and man-made examples.

These activities can serve as :

1. Introduction to a studied subject
2. The core of the studied subject
3. Introduced in enrichment advanced lessons
4. As a summary of a chapter.

The pedagogical and didactic functions of the activities

- **Offer interesting and unusual mathematical activities to the pupils.**
- **Develop mathematics studies out of experience.**
- **Develop the learners' ability to cope with problems taken from their daily environment.**

The pedagogical and didactic functions of the activities

- **Present the relation between mathematics and other disciplines, such as biology, architecture, etc.**
- **Reduce anxiety of the subject.**
- **Create opportunities for mathematical activity also for pupils who find the subject difficult.**

(Levenberg& Patkin , 2012)

Examples for activities based on nature

"Natural compass"

- **The picture shows sandy soil and a desert plant, the ends of its leaves "drawing a circle" by means of the wind.**
- **The plant, of course, is the center of the circle and the size of the drawn circles depends on the length of the plant leaves and the force of the wind.**

The plant as a Natural compass

Recommended questions

Try describing in <u>Words</u> or in <u>Drawing</u>

- Why do some plants draw small circles while others draw big ones?
- Are there other phenomena which form shapes similar to those presented in the picture?
- If you examine two plants which are close to each other, try drawing the circles created by the wind. Specify several options.

Levels of questions

- **The above questions relating to a circle comply with the two first levels of the Van-Hiele theory.**
- **In the first question, pupils have to identify shapes (1st level) and the other questions relate to the features of the shape (2nd level).**
- **Like in nature, people too form circles as can be seen in the photo of a square at a street junction.**

Butterflies

- **Photos of various butterflies were presented to the pupils.**
- **Butterflies with different wing shapes**

Recommended questions:

1. **Try describing in words the exhibit in the photo.**
2. **Please note what is similar and what is different in the butterflies in the photo.**
3. **Try examining the term "symmetry" in relation to the different butterfly shapes.**

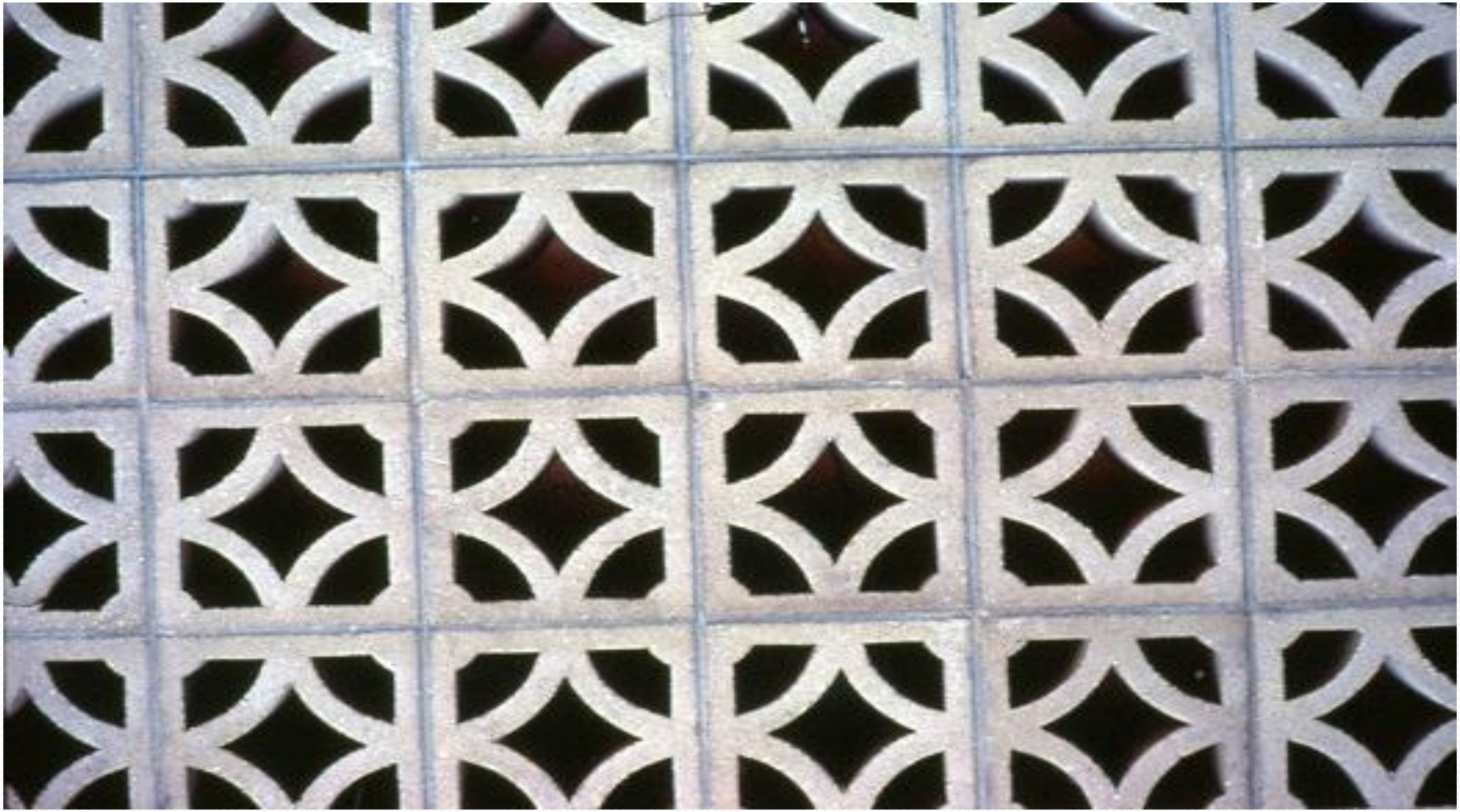
Man-made symmetry

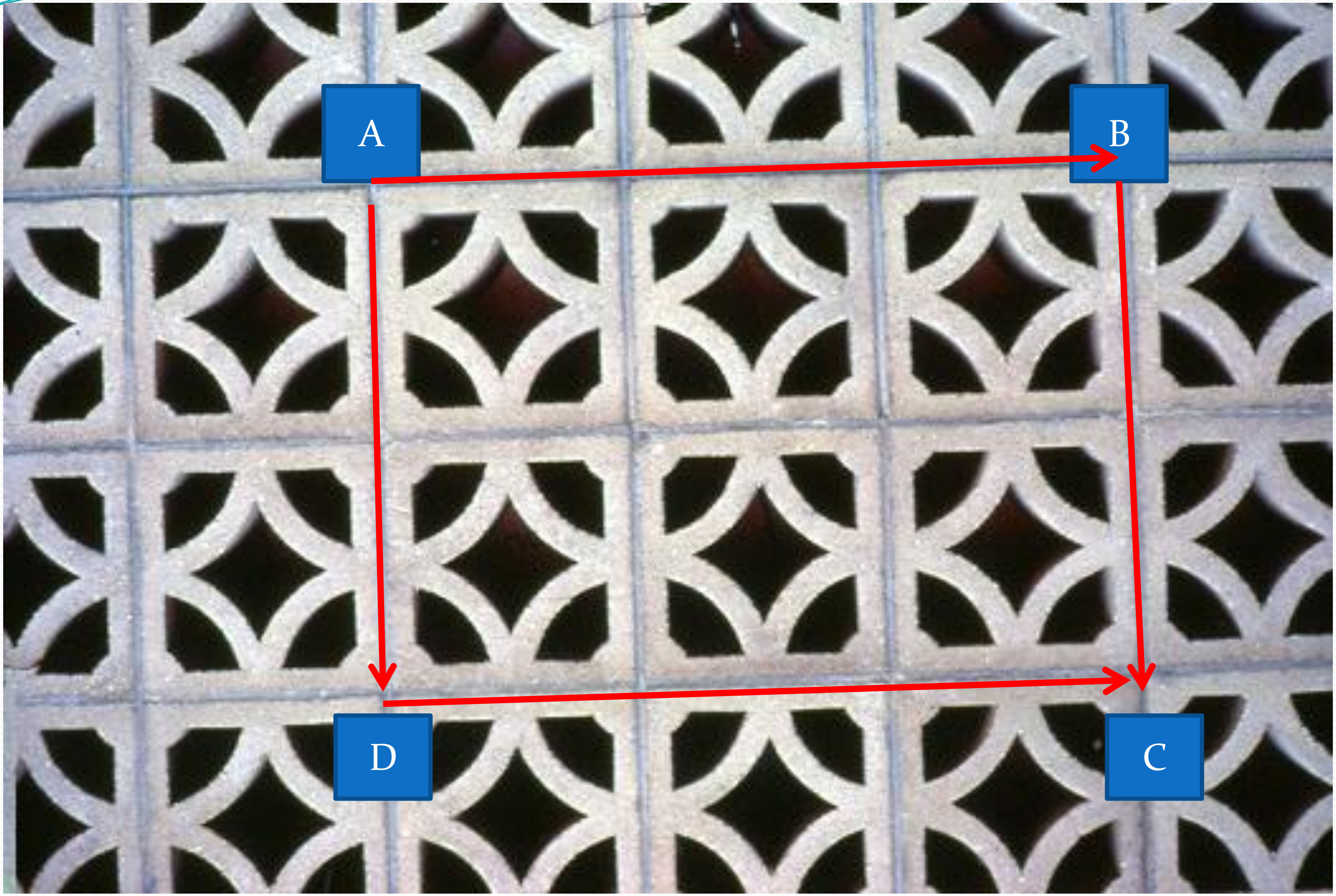
- **Man-made symmetry can be seen already in ancient periods. These are various mosaics works displayed in historical sites all over the world and can be a source of mathematical activity integrated with historical information.**

Alhambra Palace in Spain

Porf. Ilana Levenberg - 2019

CONCRETE DESIGN BLOCKS




A
B
D
C

CONCRETE DESIGN BLOCKS

- **Which geometrical forms can you recognize?**
- **Recognize the mutual relation between the circles.**
- **What can you say about the lines that connect the centers of 2 circles?**
- **How would you determine the area of the central shape in every square?**

תפוח זהב

AN ORANGE

1. **What can you say about the shape of an orange?**
2. **What can you say about the section of the orange?**
3. **How can we describe the slices? How many slices can you see?**
4. **If the slices were the same size, what will be the measure of the central angle of each "slice"?**

Car Logos

Car Logos

The car industry uses geometric shapes as logo.

- Create a logo for a company by usings three different geometric shapes.
- Describe the chosen logo in words and give reasons for choosing it.
- The logo can be designed only with a ruler and compass– as part of exercising the topic of geometrical structures.

Traffic Signs

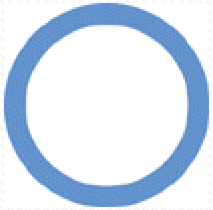
- **The first traffic signs placed at the side of the roads existed already during the centuries B.C. and these were the "milestones" erected by the Romans.**
- **On these "milestones", located in all countries of the Roman Empire, travelers could read information about the distance from their current position to the capital – Rome**

Traffic Signs

- **The first traffic signs of warning or instruction were created only at the end of the 19th century.**
- **The need for these signs resulted from the development of swift and quiet vehicles which endangered pedestrians and bicycle riders.**
- **Starting from the 20th century, traffic signs have become the international "language" which every civilized person should know.**

Traffic signs

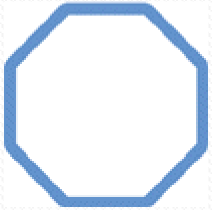
Circular signs.



Triangular signs

Rectangular signs

Octagonal signs.



Getting acquainted with traffic signs

- **All traffic signs are represented by geometric figures. The shapes are various polygons like triangle, square and rectangle.**
- **In the field, the geometric figure (mainly of triangles) is not always "pure geometric". The reason is that, on the production line (made of metal) of traffic signs, the triangle vertices are sometimes "rounded".**
- **Hence, they do not comply with the formal definition of triangle. Nevertheless, they are presented and defined as triangles in the guidebook of the "Caution on the Roads" Council.**

RED - stop

GREEN - direction

YELLOW - general warning

BLACK&WHITE - regulation

BLUE - motorist service (e.g., gas, food, hotels)

BROWN - recreational, historic, or scenic site

ORANGE - construction or maintenance warning

Traffic Signs

- **Within the framework of "caution on the roads" studies at school, pupils have to learn the traffic signs which are indications how to behave on the road.**
- **In fact, this is a combination of geometry studies and traffic signs as an international "language".**



STOP!

- The Traffic sign -“**Stop!**”is shaped like an **octagon** **and it is the only one in this shape**.
- **The traffic sign is usually placed at a junction or a point where streets meet, instructing drivers to fully brake their vehicle, advancing only if the road is free.**
- **Using the Stop signs started in the United States at the beginning of the 20th century.**
- **These traffic signs were smaller than the ones used today and the word "stop" was written in color. In Israel the traffic sign portrays a drawing of a palm.**

Give way!

- **The traffic sign -" Give way" is one of the triangle-shaped signs.**
- **Like the "stop" sign, it too is displayed so that it can be identified from every direction at the junction.**
- **It is presented “like” an upside-down triangle. It is the only triangle-shaped sign displayed in this way.**


60
STOP
k a s i a &j a n o

EXAMPELS OF RESPONSES

- ***“I did not think that we can learn geometry in this way”.***
- ***“I did not realize that traffic sign shape can be a polygon”.***
- ***“There is a very large difference between the traffic signs in the notebook and the traffic sign in the reality”.***
- ***״Never taught symmetry relating to traffic signs״.***
- ***״When I ride the bus I immediately notice the geometrical shape of the traffic signs״.***

Conclusions

- **The activities presented to you provide a rich environment for the purpose of developing mathematical thinking, developing logical thinking skills, using intuitions and developing spatial orientation and acquaintance with the environment in our reality.**
- **Those are but a few of the examples of numerous options of sharing activities at school between mathematics teachers and instructors of the "caution on the roads" subject.**
- **Learning in parallel those two subjects – significance of traffic signs and mathematics – will only enhance knowledge thereof.**
- **Perhaps in this way, as a my country struggling to reduce road traffic accidents, we will even manage to save lives.**

TO SUM UP!

- **We hope that these activities we have suggested provide contexts for meaningful exploration and learning,**
- **While at the same time enhancing mathematical communication and engaging pupils with the beauty of geometry.**

Papers
GEOMETRY AROUN US

- **Patkin, D.,& Levenberg, I**. (2012). Geometry from the world around us. *Learning and Teaching Mathematics. A Journal of AMESA, 13*, 14-18.
- **Levenberg, I**.& **Patkin, D.** (2014). Promoting meaningful learning - Studying mathematics with traffic signs. *International Journal of Learning and Development (IJLD), 4(2)*,1-8.

Thanks to you all

Ilana