Using visual and other aids for working with children with learning difficulties

Irena Budínová

When working with children who have difficulties in learning mathematics (brought about e.g. by various causes connected to deficiencies of partial functions of mathematical abilities or specific learning difficulties) we look for such teaching purposed that would appeal to them and mobilize the activity of their minds in such a way that the work of their hands would transform into the work of their brains.

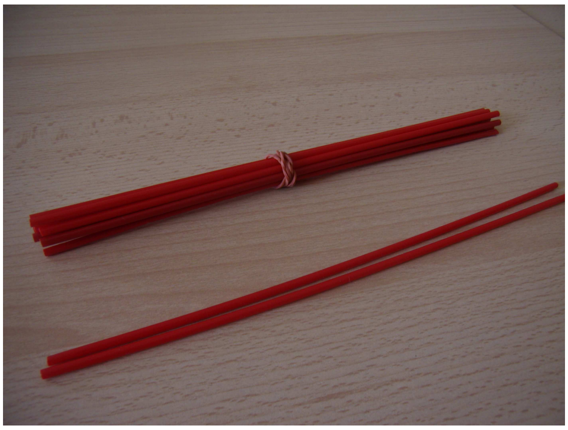
Within the individualized teaching, we are using a set of simple aids, from which we can form a portfolio for each pupil. We have had good experience with the pupils‘ participation on the production of the aids. The financial costs for the aids are minimal. The pupils then work with the aids in ways fitting for them – if they are not certain and need some support. or when they need concrete models. It shows that respecting multisensorial approach in constituting the individual notions, because when we make use of as many senses as possible, at least one of them is usually the constituent one for each pupil.

Some sets of visual aids:

1. **Aids suitable for constituting the notion of a natural number and constituting the operations with natural numbers:** concrete objects – smaller toys (cars, dolls), pebbles, cubes, sticks, PET-bottle lids of different colours and sizes, bigger buttons, etc. We can also make good use of cards with dots from a blank card to a card with twenty dots, a set of cubes, and a set of squares.

2. **Aids suitable for understanding the positional decimal system**:

   Sets of ten straws: Sets of ten straws or sticks are suitable for modelling and understanding two-digit numbers. The child sees ten units tied into a one block of ten. If the child for example takes 42 to mean 24 and vice versa (right-left orientation disability), the aid is very suitable.



Figue 1. Sets of straws

Cards with numberswith more dgitis,e.g.:

50	400	7 000	60 000	300 000	8 000 000	20 000 000

When putting the cards one on the other, the pupils have a better idea of a number, because they work with all the orders of magnitude (which are on the bottom card). The use of cards for calculations is described below.

Money model: Paper models of money are suitable when the children understand the value of the individual coins and banknotes. The so-called fairy-tale money are less suitable, since the children's attention is often drawn to the picture, and not to the value of the banknote.

3. **Abacus** (twenty-bead, hundred-bead, and order of magnitude abacus).

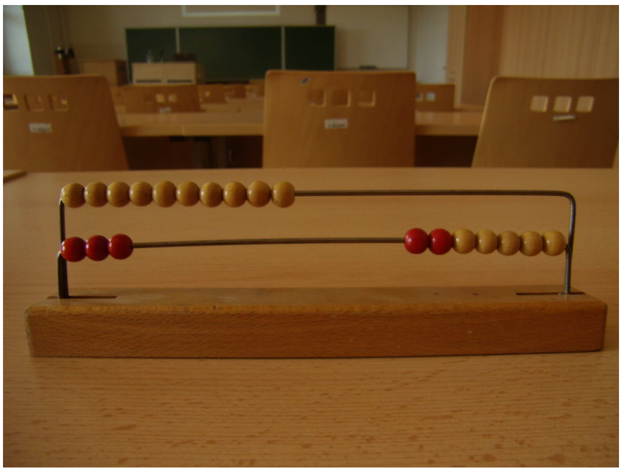


Figure 2. Twenty bead abacus

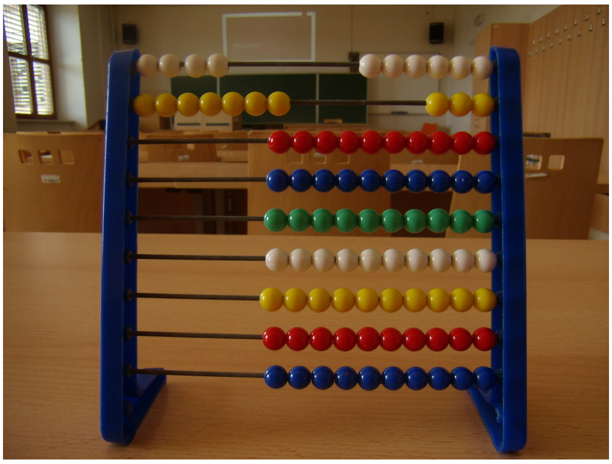


Figure 3 Hundred-beads abacus

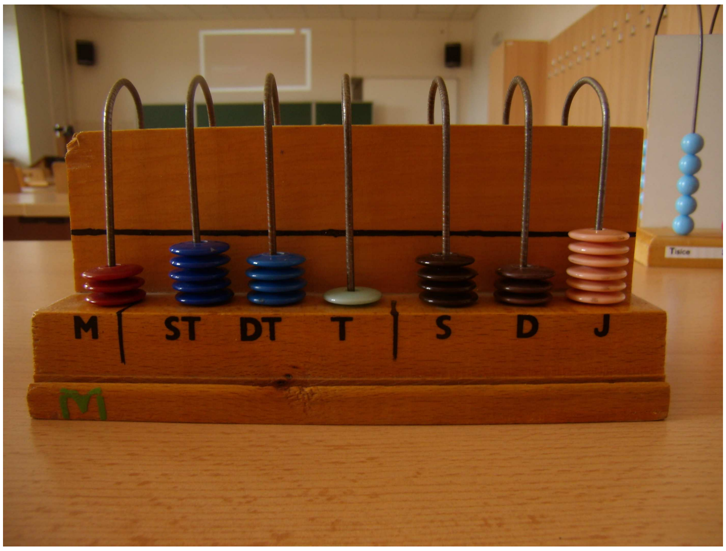


Figure 4. Counting frame

4. One-hundred table

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The one-hundred table is used by pupils to understand two-digit numbers (reading, writing, order, transfer between the tens) and also to identifiy multiples of numbers.

5. **Orderof magnitude tables for writing and reading numbers.**

MILLIONS	HT	TT	THOU-SANDS	HUN-DREDS	TENS	UNITS	,	tenths	hun-dredths	thou-sandths

6. **Frames for transfer between units**, models of clocks an various measuring devices. Frames are made of thick paper and are supplied with squares witht the individual digits that can be laid on the second line of the frame.

   Frame for transfer between length units

		km			**m**	dm	cm	mm
0	0	0	0	0	0	0	0	0

		km			**m**	dm	cm	mm
0	0	0	0	0	6	0	0	0

From the picture, the transfer is obvious: 6 m = 60 dm, 6 m = 600 cm, 6 m = 6 000 mm, 6 m = 0,006 km.

Frame for the transfer of area units:

				km^2		ha		a		**m^2**		dm^2		cm^2		mm^2	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

				km^2		ha		a		**m^2**		dm^2		cm^2		mm^2	
0	0	0	0	0	0	0	0	0	0	2	5	0	0	0	0	0	0

25 m^2 = 2 500 dm^2 = 250 000 cm^2, 25 m^2 = 0,25 a = 0,0025 ha

Frame for the transfer of volume units

m^3			dm^3			cm^3			mm^3		
			hl		*l*	*dl*	*cl*	*ml*			
0	0	0	0	0	0	0	0	0	0	0	0

.

m^3			dm^3			cm^3			mm^3		
			hl		*l*	*dl*	*cl*	*ml*			
0	0	0	0	3	4	0	0	0	0	0	0

34 dm^3 = 34 000 cm^3 34 dm^3 = 34 000 000 mm^3 34 dm^3 = 0,034 m^3

34 *l* = 340 *dl* = 3 400 *cl* = 34 000 *ml*.

7. **Paper board games –** loto, domino, memory, bingo, etc. (as a way to practice and consolidation of operations with numbers).
8. **Models of the fraction as a part of the whole** (the rectangle and circle model are especially useful, but there are also other models).
9. **Wooden sticks for crafts** of various lengths and colours to model straight line sections (comparing length of the sections, triangle inequality etc..).
10. **Models of geometrical shapes** from coloured paper of a foil (triangles, squares, rectangles, circles).
11. **Solid nets** made from items of daily use (boxes for tea, toothpaste, toothpicks, etc.).
12. Set of dice for Ludo.
13. Sets of solids (cuboid, cube, prism, roller, pyramid, cone, sphere).
14. Boxes and bottles for drinks in various volumes.
15. Models for the units of length and area (1 dm^2, 1 cm^2, 1 mm^2).

Although these aids are already at schools for demonstration, it is useful for the pupils to have their own set of these aids so that they can use it in a suitable moment.

From the point of view of didactics, **Montessori material** is very suitable. We will show their possible utilization in the following text.

Pupils‘ success in mathematics can be limited by their inability to count with natural and decimal numbers. As these calculations are needed in mathematics throughout, this handicap is very serious for the concerned pupils. Although some teachers are benevolent to numerical mistakes, the fact that they are unable to perform the calculation correctly is usually very frustrating for the pupils

When counting with **natural numbers**, we often encounter mistakes related to written operations, which are usually grounded in some specific disability – this may be dyslexia, dysgraphia, or discalculia. We will now introduce the most common mistakes encountered in written addition and subtraction of natural numbers and the possibility to introduce Montessori material when teaching these pupils. This material can be helpful in overcoming their problem.

Written addition of natural numbers

1. **Not respecting the order of magnitude within the natural number:** Pupils do not write the numbers down with the corresponding orders of magnitude below each other, but they write the highest order of each number below each other:

$$\begin{array}{l} 2\,582 \\ \underline{392\quad} \\ 6\,502 \end{array}$$

2. **Not understanding the written form of the number in the decimal positional system:** Pupils make various mistakes in additions connected with counting up to 20. They do not hold the „one“ and do not add it to the higher order of magnitude. For example, when calculating 8+9=17, they are unable to add ten units of the lower order of magnitude to the higher one..

$$\begin{array}{r} 2\,582 \\ \underline{\quad 392} \\ 2\,874 \end{array}$$

The pupil writes the numbers as they come up in the calculation.

$$\begin{array}{r} 2\,582 \\ \underline{\quad 392} \\ 28\,174 \end{array}$$

We encounter a number of other mistakes connected with the base 10 positiona notation (decimal positional notation).

To eliminate the above-mentioned mistakes and also other ones, Montessori material Bank may successfully be used. This material contains small beads, representing units, then rows of ten beads, representing tens, ten rows of ten beads representing one hundred, and ten one-hundred tables piled on top of each other – thousands. The material further contains two sets of cards – a small one for calculations and the big one for results. Children can work with it already in the kindergarten age, where they use the Bank to exchange different quantities, they e.g. exchange 10 tens for 1 hundred. In primary school, they use the Bank for addition, subtraction, simple multiplication and division of natural numbers.

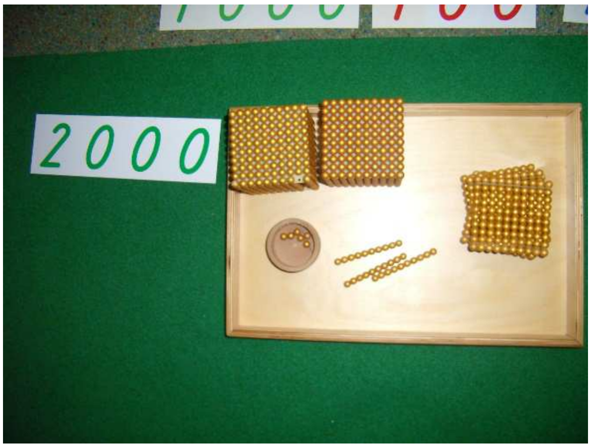
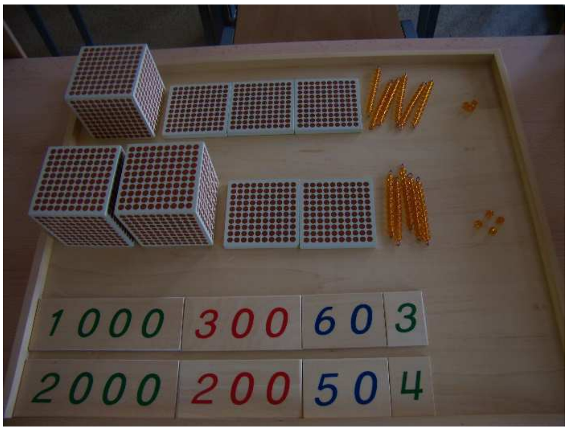
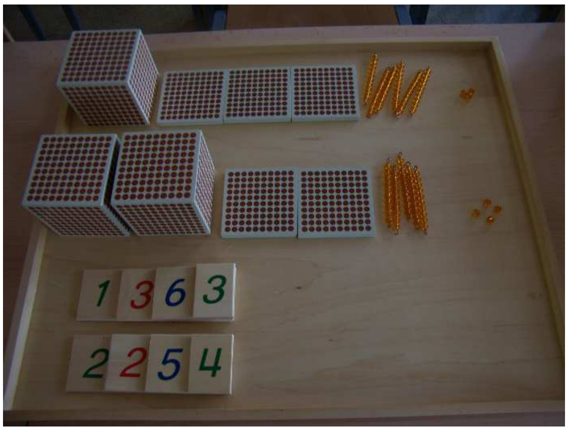


Figure 5. Montessori material: the Bank

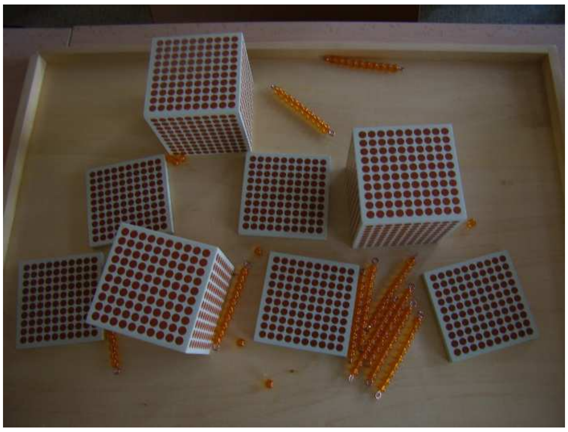
Let us show the work with the Bank step by step, using the example $1\,363 + 2\,254$.



Step 1: We model numbers $1\,363$ and $2\,254$ from the material. We find the corresponding numbers from the set of small cards. W leave the cards laid out next to each other,



Step 2: We manipulate the cards in sucha way that the notation for that number in decimal positional notation be visible.



Step 3: If the children are comfortable with this, we can turn the material into chaos. That means that we mingle the material regardless of the orders of magnitude.

Step 4: We arrange the material according to the orders of magnitude. We have 7 units, 11 tens, 5 hundreds, and 3 thousands

Step 5: We exchange ten tens for one hundred, and we can count from the units and write down the result:

$1\ 363 + 2\ 254 = 3\ 617$.

An experience: Two students of social pedagogy of the branch teacher’s assistent, Jana and Linda worked with the Bank. One of them, Jana, had troubles with writing numbers when she was a child. The other one, Linda, loved order. They first started to discuss whether to go through the step of chaos, or not. Jana liked the idea, but Linda did not like it. They agreed to leave out the step of chaos at first, and Linda was happy. They worked properly with the material as well as with the number cards. Next time, they thus decided to go through chaos, but Linda was very much looking forward to having everything organized. When they first reached the stage of counting over 10, Jana said, „Shall we go to the bank?“ Later, I could see that they could not wait for the point of going to the bank and when that happened, they loudly rejoiced.

Both students were excited about the work with the Bank. Jana said that the Bank would probably have been helpful for her as a child, because she did not care for the orders of magnitude and claimed that „zero does not matter“. Her father was quite upset about that, but could not change anything, because for her, the orders of magnitude meant nothing.

Written subtraction of natural numbers

We will most often encounter the following mistakes in performing written subtraction:

1. **Misunderstanding substraction when going over the base of 10:** The pupil always subtracts lesser number from the greater one; e.g. when calculating $43 - 19$, they calculate $49 - 13$ instead:

$$\begin{array}{r} 43 \\ \underline{-19} \\ 36 \end{array}$$

2. **When subtracting, they do not add „one“:**
They calculate the task $43 - 19$ as follows: 9 and how any are 13? 4, they do not hold the one. The next step is 1 and how many is 4? 3.

$$\begin{array}{r} 43 \\ \underline{-19} \\ 34 \end{array}$$

We also encounter a number of other mistakes.

The Bank material can help the pupils also in this case. On one hand, it facilitates the understanding of the order of magnitude and respecting them, and on the other, it does not allow the pupils to turn around the digits so that they always subtract the smaller number from the grater one.

An example of subtraction without going over the base of ten: We may present the kuds with the following situation: there were 7 254 crowns in the Bank. A robber came and stole 3 142 crowns. The kids will then take 3 142 beads from the original amount. They will place these beads on a tray, so that they are on the side (which evokes the situation when the money is in fact somewhere else), but keep them so that they are able to check the task later. The pupils gradually learn to write the calculation in a column. In the algorithm for written subtraction, we start with units, and therefore we also start with units when using the material.

$$\begin{array}{r} 7\ 254 \\ \underline{-3\ 142} \\ 4\ 112 \end{array}$$

An example of calculation involving transfers from one order of magnitude to another. In the next phase, we deal with substraction involving one or more transfers from one order of magnitude to another. From the original amount of 7 254 crowns, the robber stole 3 461 crowns. This example involves two transfers between orders of magnitude.From 4 units, the pupils take 1 unit and put it on the tray. We cannot take 6 tens out of 5 tens, and thus the pupils first have to exchange 1 hundred for 10 tens. Now they take 6 tens from the 15 tens, and 9 tens remain. They are left with 1 hundred, from which 4 hundred cannot be taken. They therefore exchange 1 thousand for 10 hundred, and take 4 hundred from 11 hundred; thus 7 hundred remain. Finally, they take 3 thousand from the 6 thousand and 3 thousand are left. The final result is: $7\ 254 - 3\ 461 = 3\ 793$. The pupils realise this way what it means to „hold one“ and to which number we need to add 1.

Addition and subtraction of decimal numbers

1. **Children add or subtract digits of different orders,** , e.g.:
   $0{,}5 + 0{,}03 = 0{,}8$
   $0{,}08 - 0{,}1 = 0{,}7$
2. **They do not respect the transfer between the orders**, e.g.:
   $1{,}5 + 5{,}8 = 6{,}13$
   $6{,}2 - 2{,}9 = 4{,}7$
3. **They always subtract the lesser number from the greater one**, e.g.
   $7{,}35 - 5{,}90 = 2{,}65$
4. **They do not respect the rules of the positional decimal system**, e.g.:
   $0{,}8 + 0{,}2 = 0{,}10$
5. **When performing written subtraction, they do not write the digits in the corresponding columns**, e.g.:
   $87{,}2$
   $\underline{-4{,}12}$
   $4{,}60$

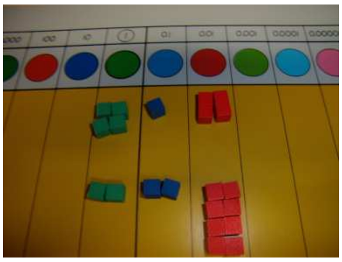
   And many other mistakes may be encountered in mental and written calculations.

Montessori materia Table for decimal numbers is an aid that helps children eliminate mistakes and understand the principle of calculating with decimal numbers. We will show how it works on the example $3{,}24 + 2{,}3$ (the first number contains non-zero number of hundreths, while the second one does not).

Step 1: We ask the pupil to enter the example onto the board. The pupil can immediately see that in the order of units, we now have 5 cubes, in the order of tenths also 5 cubes, and in the order of hundreths 4 cubes. This can now be modelled with the help of cards with numbers.

Step 2: The pupil puts the cubes together and puts the cards with numbers on each other. He can write down the result: $3{,}24 + 2{,}3 = 5{,}54$.

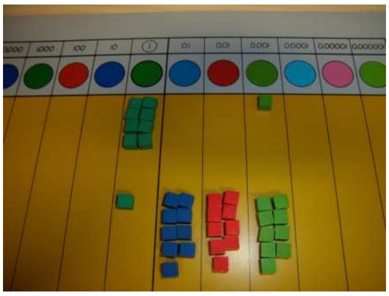
In the next phase, we can can try examples for addition involving transfer between orders of magnitude. The children that already know the Bank or the table for division do not have problems with exchanging the cubes between different orders. Let us show how to solve the task 4,14+2,28.

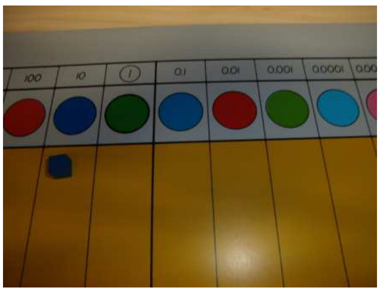


Step 1: The pupil enters the numbers with the use of cubes.

Step 2: He will move the cubes towards each other. He now has 12 cubes in the order of hundredths. He thus takes 10 of the hundredths cubes and exchanges it for one tenth-cube. He now has the result: $4{,}14 + 2{,}28 = 6{,}42$. He writes the result with the help of number cards and also copies the result into his exercise book.

Later, we can also choose more demanding and interesting tasks, as 8,001+1,999. After the pupil performs all the exchanging between orders, he will arrive to the result: 10.





References

Feez, S.: *Montessori and Early Childhood.* SAGE, 2010