

# Algebra 1

## Domaci' cviceni' 7

66.  $f(x) = 2x^3 + ax^2 + bx + 12$ ,  $c = 2$  asponi dvojnai.

$$f'(x) = 6x^2 + 2ax + b$$

$$f(2) = 16 + 4a + 2b + 12$$

$$f'(2) = 24 + 4a + b$$

---


$$28 + 4a + 2b = 0$$

$$24 + 4a + b = 0 \quad | \cdot (-1)$$

---


$$\underline{\underline{b = -4}}, \quad \underline{\underline{a = -5}}$$

$$f(x) = \underline{\underline{2x^3 - 5x^2 - 4x + 12}}$$

zk.: Hornerovo schéma

	2	-5	-4	12
2	2	-1	-6	0
2	2	3	0	

$$f(x) = (x-2)^2(2x+3)$$

67.  $f(x) = 4x^5 + 20x^4 + 25x^3 - 10x^2 - 20x + 8$

$$f'(x) = 20x^4 + 80x^3 + 75x^2 - 20x - 20$$

$$(4x^5 + 20x^4 + 25x^3 - 10x^2 - 20x + 8) : (4x^4 + 16x^3 + 15x^2 - 4x - 4) = x + 1$$

$$(zb. -6x^3 - 21x^2 - 12x + 12)$$

$$-(4x^5 + 16x^4 + 15x^3 - 4x^2 - 4x)$$

$$4x^4 + 10x^3 - 6x^2 - 16x + 8$$

$$-(4x^4 + 16x^3 + 15x^2 - 4x - 4)$$

$$-6x^3 - 21x^2 - 12x + 12$$

$$\rightsquigarrow 2x^3 + 7x^2 + 4x + 4$$

$$(4x^4 + 16x^3 + 15x^2 - 4x - 4) : (2x^3 + 7x^2 + 4x - 4) = 2x + 1$$

$$-(4x^4 + 14x^3 + 8x^2 - 8x)$$


---


$$2x^3 + 7x^2 + 4x - 4$$

$$-(2x^3 + 7x^2 + 4x - 4)$$


---


$$0$$

$$\text{NSD}(f, f') = 2x^3 + 7x^2 + 4x - 4$$

Výběříme racionální kořeny:

$$c = \frac{p}{q}, \quad p | (-4), \quad q | 2$$

$$\boxed{f(-1) = -3}$$

$$\boxed{f(1) = 9}$$

$\frac{p}{q}$	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{2}{1}$	$\frac{4}{1}$	$-\frac{1}{1}$	$-\frac{1}{2}$	$-\frac{2}{1}$	$-\frac{4}{1}$
$p+q$	2	3	3	5	0	1	-1	-3
$p-q$	0	-1	1	3	-2	-3	-3	-5

kandidáti na kořeny:  $\frac{1}{2}, 2, -\frac{1}{2}, -2$

	4	20	25	-10	-20	8
$\frac{1}{2}$	4	22	36	8	-16	0
$\frac{1}{2}$	4	24	48	32	0	
<del><math>\frac{1}{2}</math></del>	<del>4</del>	<del>26</del>	<del>64</del>			
-2	4	16	16	0		

$$f(x) = (x - \frac{1}{2})^2 \cdot (x+2) \cdot (4x^2 + 16x + 16) = (2x-1)^2 \cdot (x+2) \cdot (x^2 + 4x + 4) = (2x-1)^2 \cdot (x+2)^3$$

$c = \frac{1}{2}$  je dvojnásobným kořenem a  $c = -2$  je trojnásobným kořenem.

$$68. f(x) = 4x^3 - 4x^2 - 11x + 6$$

$$c = \frac{p}{q} \quad p|6 \quad q|4$$

$$p: 1, 2, 3, 6$$

$$q: 1, 2, 4$$

$$f(-1) = 9$$

$$f(1) = -5$$

$\frac{p}{q}$	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{2}{1}$	$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{4}$	$\frac{6}{1}$	$-\frac{1}{1}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{2}{1}$	$-\frac{3}{1}$	$-\frac{3}{2}$	$-\frac{3}{4}$	$-\frac{6}{1}$
$p+q$	2	3	5	3	4	5	7	7	0	1	3	-1	-2	-1	1	-5
$p-q$	0	-1	-3	1	2	1	-1	5	-2	-3	-5	-3	-4	-5	-7	-7

kandidáti:  $\frac{1}{2}, 2, -\frac{1}{4}, -\frac{3}{2}$

	4	-4	-11	6
$\frac{1}{2}$	4	-2	-12	0
$\frac{1}{2}$	4	0		
2	4	6	0	

$$f(x) = (x - \frac{1}{2}) \cdot (x - 2) \cdot (4x + 6) = (2x - 1) \cdot (x - 2) \cdot (2x + 3)$$

racionalní kořeny:  $\frac{1}{2}, -2, -\frac{3}{2}$