Algebra 1

Domácí úkol 8

73. a) $2x^3 - 3x^2 - 3x + 2 = 0$

rec. rovnice 1. druhu, 3. stupně $\Rightarrow$ má kořen $c = -1$

	2	-3	-3	2
-1	2	-5	2	0

$(x+1)(2x^2 - 5x + 2) = 0$

$x_1 = -1$

$x_{2,3} = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4}$

$K = \{-1; \frac{1}{2}; 2\}$

b) $2x^3 + 7x^2 - 7x - 2 = 0$

rec. rovnice 2. druhu $\Rightarrow$ má kořen $c = 1$

	2	7	-7	-2
1	2	9	2	0

$(x-1)(2x^2 + 9x + 2) = 0$

$x_1 = 1$

$x_{2,3} = \frac{-9 \pm \sqrt{81-16}}{4} = \frac{-9 \pm \sqrt{65}}{4}$

$K = \{\frac{-9-\sqrt{65}}{4}; \frac{-9+\sqrt{65}}{4}; 1\}$

$\doteq -4{,}25 \quad \doteq -0{,}25$

c) $6x^4 + 17x^3 + 17x^2 + 17x + 6 = 0$

rovnice 4. stupně $\Rightarrow$ subst. $x + \frac{1}{x} = \mu$

$x^2 + \frac{1}{x^2} = \mu^2 - 2$

$6\left(x^2 + \frac{1}{x^2}\right) + 17\left(x + \frac{1}{x}\right) + 17 = 0$

$6 \cdot (\mu^2 - 2) + 17\mu + 17 = 0$

$6\mu^2 + 17\mu + 5 = 0$

$\mu_{1,2} = \frac{-17 \pm \sqrt{17^2 - 120}}{12} = \frac{-17 \pm \sqrt{169}}{12} = \frac{-17 \pm 13}{12} = \begin{cases} -\frac{5}{2} \\ -\frac{1}{3} \end{cases}$

- $\mu = -\frac{5}{2} \Rightarrow x + \frac{1}{x} = -\frac{5}{2} \quad / \cdot x \qquad x \neq 0$

$x^2 + \frac{5}{2}x + 1 = 0$

$x_{1,2} = \frac{-\frac{5}{2} \pm \sqrt{\frac{25}{4} - 4}}{2} = \frac{-\frac{5}{2} \pm \sqrt{\frac{9}{4}}}{2} = \frac{-\frac{5}{2} \pm \frac{3}{2}}{2} = \begin{cases} -\frac{1}{2} \\ -2 \end{cases}$

- $\mu = -\frac{1}{3} \Rightarrow x + \frac{1}{x} = -\frac{1}{3} \quad / \cdot x$

$x^2 + \frac{1}{3}x + 1 = 0$

$x_{3,4} = \frac{-\frac{1}{3} \pm \sqrt{\frac{1}{9} - 4}}{2} = \frac{-\frac{1}{3} \pm \sqrt{\frac{-35}{9}}}{2} = \frac{-\frac{1}{3} \pm \frac{1}{3}i\sqrt{35}}{2}$

$K = \underline{\underline{\left\{-2; -\frac{1}{2}; -\frac{1}{6} - \frac{1}{6}i\sqrt{35}; -\frac{1}{6} + \frac{1}{6}i\sqrt{35}\right\}}}$

d) $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

$6\left(x^2 + \frac{1}{x^2}\right) - 35\left(x + \frac{1}{x}\right) + 62 = 0$

$6 \cdot (\mu^2 - 2) - 35\mu + 62 = 0$

$6\mu^2 - 35\mu + 50 = 0$

$\mu_{1,2} = \frac{35 \pm \sqrt{(-35)^2 - 1200}}{12} = \frac{35 \pm \sqrt{25}}{12} = \frac{35 \pm 5}{12} = \begin{cases} \frac{10}{3} \\ \frac{5}{2} \end{cases}$

- $x+\frac{1}{x}=\frac{10}{3} \quad /\cdot x \qquad x\neq 0$

$x^2-\frac{10}{3}x+1=0$

$x_{1,2}=\frac{\frac{10}{3}\pm\sqrt{\frac{100}{9}-4}}{2}=\frac{\frac{10}{3}\pm\sqrt{\frac{64}{9}}}{2}=\frac{\frac{10}{3}\pm\frac{8}{3}}{2}=$

$=\begin{cases}3\\ \frac{1}{3}\end{cases}$

- $x+\frac{1}{x}=\frac{5}{2} \quad /\cdot x$

$x^2-\frac{5}{2}x+1=0$

$x_{3,4}=\frac{\frac{5}{2}\pm\sqrt{\frac{25}{4}-4}}{2}=\frac{\frac{5}{2}\pm\frac{3}{2}}{2}=\begin{cases}\frac{1}{2}\\ 2\end{cases}$

$K=\underline{\underline{\left\{\frac{1}{3};\frac{1}{2};2;3\right\}}}$

74. $2x^3-3x^2-3x+2=0$

$2\cdot(x^3+1)-3\cdot(x^2+x)=0$

$2\cdot(x+1)\cdot(x^2-x+1)-3\cdot(x+1)\cdot x=0$

$(x+1)\cdot[2\cdot(x^2-x+1)-3x]=0$

$(x-1)\cdot(2x^2-5x+2) \qquad =0$

$K=\underline{\underline{\left\{-1;\frac{1}{2};2\right\}}}$