Normál. rozdelenie $No(\mu, \sigma^2)$ - príklady

(PR. 8.24) pst, že zasadený strom sa ujme = 0,85
pst, že z 500 zasadnutých stromov sa ujme: a) aspoň 420
b) najviac 440

Aproximácia $Bi(N,p) \Rightarrow No(\mu, \sigma^2)$

$Bi(N,p)$: $EX = N \cdot p$, $DX = N \cdot p(1-p)$ $\Rightarrow$ $No(\mu, \sigma^2)$: $EX = N \cdot p = \mu$, $DX = N \cdot p(1-p) = \sigma^2$

$EX = \mu = N \cdot p = 500 \cdot 0,85 = 425$

$DX^2 = \sigma^2 = N \cdot p \cdot (1-p) = 425 \cdot 0,15 = 63,75 \Rightarrow \sqrt{DX} = \sqrt{63,75}$, $DX = \sqrt{63,75} = \sigma$

$\Phi(-u) = 1 - \Phi(u)$

a) $P(X \geq 420)$

$\frac{X-\mu}{\sigma}$

$$P(X \geq 420) = P\left(\frac{X-425}{\sqrt{63,75}} \geq \frac{420-425}{\sqrt{63,75}}\right) = P(U \geq \underbrace{-0,63}_{\Phi(-u)}) = 1 - \Phi(u) =$$

$$= 1 - \underbrace{0,7356527}_{\text{výsledok}} = 0,2643473 \quad \text{— } P(U < 0,63) \text{ pre } P(X < 420)$$

pre $P(X \geq 420)$

- s korekciou 0,5:

$$P\left(\frac{419,5-425}{\sqrt{63,75}} \geq \frac{(X-0,5)-425}{\sqrt{63,75}}\right) = P(-0,69 \geq U) = 1 - \Phi(u) =$$

$$= 1 - \underbrace{0,7549029}_{\text{výsledok}} = 0,2450971 \quad \text{— } P(U < 0,69) \text{ pre } P(X < 419,5)$$

pre $P(X-0,5 > 419,5)$

b) $P(X \leq 440) = P\left(U \leq \frac{440-425}{\sqrt{63,75}}\right) = P(U \leq 1,88) = \underline{0,9699460}$

- s korekciou $P\left(U \leq \frac{440,5-425}{\sqrt{63,75}}\right) = P(U \leq 1,94) = \underline{0,9738102}$

→

PR. 8.27 zhruba 75 % domácností má TV ... náhod. vybraných 400 domácností

a) pst, že 20 400 domácností má TV v $I \in (290 < X \leq 305)$

b) urč., v akých medziach (súmer. okolo EX) bude počet domácností s TV (20 400) s pst = 0,95

$EX = \mu = N \cdot p = 400 \cdot 0,75 = 300$ $\quad DX = \sqrt{75} = \sigma$

$DX^2 = \sigma^2 = N \cdot p(1-p) = 300 \cdot 0,25 = 75 \rightarrow \sqrt{DX} = \sqrt{75}$

a) $P(290 \leq X \leq 305) = P\left(\frac{290-300}{\sqrt{75}} \leq \frac{X-300}{\sqrt{75}} \leq \frac{305-300}{\sqrt{75}}\right) =$

$= P(\underbrace{-1,15}_{u_1} \leq U \leq \underbrace{0,58}_{u_2}) = \Phi(0,58) - (1 - \Phi(1,15)) = 0,7190427 - 1 +$

$+ 0,8749281 = 0,5939708$

$P(u_1 \leq U \leq u_2) = \Phi(u_2) - \Phi(u_1)$

• s korekciou: $P(289,5 \leq X \leq 305,5) = P\left(\frac{289,5-300}{\sqrt{75}} \leq U \leq \frac{305,5-300}{\sqrt{75}}\right) =$

$P(-1,21 \leq U \leq 0,64) = \Phi(0,64) - 1 + \Phi(1,21) = 0,7389137 - 1 +$

$+ 0,8868606 = 0,6257743$

b) $P(x_1 < X < x_2) = 0,95$

$x_1 = 300 - d$
$x_2 = 300 + d$

súmerné okolo EX, preto $300 \pm d$ $\quad d > 0$, konštanta

$P(-d < X - 300 < d)$

$\Rightarrow u_1 = \frac{x_1 - 300}{\sqrt{75}} = \frac{300 - d - 300}{\sqrt{75}} = \frac{-d}{\sqrt{75}}$

$u_2 = \frac{x_2 - 300}{\sqrt{75}} = \frac{300 + d - 300}{\sqrt{75}} = \frac{d}{\sqrt{75}}$

$P(-u < U < u) = 0,95$

$\Phi(u) - (1 - \Phi(u)) = 0,95$

$\Phi(u) - 1 + \Phi(u) = 0,95$

$2\Phi(u) = 1,95$

$\Phi(u) = 0,975$

$u = 1,96$

$\frac{x_1 - 300}{\sqrt{75}} = -1,96 \Rightarrow x_1 = -1,96 \cdot \sqrt{75} + 300$

$x_1 = 283$

$\frac{x_2 - 300}{\sqrt{75}} = 1,96 \Rightarrow x_2 = 1,96 \cdot \sqrt{75} + 300$

$x_2 = 316,97 \doteq 317$

medze: (283, 317)