

$$1. \lim_{h \rightarrow \infty} \left(1 - \frac{3}{2h}\right)^{-4h+2} = \lim_{h \rightarrow \infty} \left(1 - \frac{3}{2h}\right)^{-4h} \cdot \lim_{h \rightarrow \infty} \left(1 - \frac{3}{2h}\right)^2 = \left[\frac{1}{k} = -\frac{3}{2h} \Rightarrow 2h = -3k \right. \\ \left. h = -\frac{3}{2} \cdot k \right]$$

$$= \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k}\right)^{-\frac{3}{2} \cdot (-\frac{3}{2}k)} = \lim_{k \rightarrow \infty} \left[\left(1 + \frac{1}{k}\right)^k\right]^{\frac{9}{2}} = e^{\frac{9}{2}}$$

$$2. \lim_{x \rightarrow 0} \frac{\sin 3x}{\sqrt{x+2}-\sqrt{2}} \cdot \frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}} = \lim_{x \rightarrow 0} \frac{\sin 3x \cdot (\sqrt{x+2}+\sqrt{2})}{(x+2)-2} = \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \cdot \frac{\sqrt{x+2}+\sqrt{2}}{1} \cdot \frac{3}{3}$$

L'Hosp

$$= \lim_{x \rightarrow 0} \frac{3 \cos 3x}{\frac{1}{2}(x+2)^{-\frac{1}{2}}} = \lim_{x \rightarrow 0} \frac{3 \cos 3x}{\frac{1}{2\sqrt{x+2}}} = \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \cdot 3 \cdot (\sqrt{x+2} + \sqrt{2}) = 1 \cdot 3 \cdot 2\sqrt{2} = 6\sqrt{2}$$

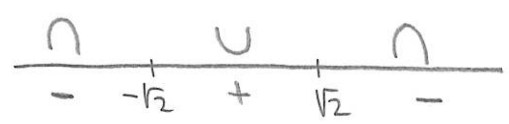
$$= 3 \cdot 2 \cdot \sqrt{2} = 6\sqrt{2}$$

3. $f(x) = \ln(2x^2+4)$, $D(f) = \mathbb{R}$, protože $2x^2+4 > 0$ vždy

$$f'(x) = \frac{1}{2x^2+4} \cdot 4x = \frac{2x}{x^2+2}$$

$$f''(x) = \frac{2(x^2+2) - 2x \cdot 2x}{(x^2+2)^2} = \frac{4-2x^2}{(x^2+2)^2}, D(f'') = \mathbb{R}, f''(x) = 0 \Leftrightarrow 4-2x^2 = 0$$

$$4 = 2x^2 \Rightarrow 2 = x^2 \Leftrightarrow x = \pm\sqrt{2}$$



Pro $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ je $f(x)$ konkávní. Pro $x \in (-\sqrt{2}, \sqrt{2})$ je $f(x)$ konvexní.
Inflexní body: $x = \pm\sqrt{2} \rightarrow f(-\sqrt{2}) = \ln 8 = 2,08$
 $f(\sqrt{2}) = \ln 8 = 2,08$

4. $f(x) = x + e^{-x^2}$, $x_0 = 0$, $f(\frac{1}{2}) = ?$

$$f(x_0) = f(0) = 0 + e^0 = 1$$

$$f'(x) = 1 + e^{-x^2} \cdot (-2x) \rightarrow f'(x_0) = f'(0) = 1 + 0 = 1$$

$$f''(x) = e^{-x^2} \cdot (-2x) \cdot (-2x) - 2 \cdot e^{-x^2} = e^{-x^2} \cdot (4x^2 - 2) \rightarrow f''(x_0) = f''(0) = -2$$

$$f(x) \approx T_2(x_0) = f(x_0) + f'(x_0) \cdot (x-x_0) + \frac{f''(x_0)}{2!} \cdot (x-x_0)^2$$

$$= 1 + 1 \cdot x - \frac{2}{2} \cdot x^2 = 1 + x - x^2$$

$$f(\frac{1}{2}) \approx 1 + \frac{1}{2} - \frac{1}{4} = \frac{5}{4} = 1,25$$

6. $\lim_{(x,y) \rightarrow (1,2)} \frac{y \cdot (x-1)}{2x-y} = \left[\frac{0}{0} \right]$

1. Postupně limity:

$$L_1 = \lim_{y \rightarrow 2} \left[\lim_{x \rightarrow 1} \frac{y \cdot (x-1)}{2x-y} \right] = \lim_{y \rightarrow 2} \frac{0}{2-y} = 0$$

$$L_2 = \lim_{x \rightarrow 1} \left[\lim_{y \rightarrow 2} \frac{y \cdot (x-1)}{2x-y} \right] = \lim_{x \rightarrow 1} \frac{2 \cdot (x-1)}{2x-2} = \lim_{x \rightarrow 1} \frac{2(x-1)}{2(x-1)} = 1$$

$L_1 \neq L_2 \Rightarrow$ limita neexistuje

2. Přibližování po přímkách:

$$y = k \cdot (x-1) + 2$$

$$\lim_{(x,y) \rightarrow (1,2)} \frac{y \cdot (x-1)}{2x-y} \stackrel{\text{subst.}}{=} \lim_{x \rightarrow 1} \frac{[k \cdot (x-1) + 2] \cdot (x-1)}{2x - [k \cdot (x-1) + 2]}$$

$$= \lim_{x \rightarrow 1} \frac{[k \cdot (x-1) + 2] \cdot (x-1)}{2x - 2 - k \cdot (x-1)} = \lim_{x \rightarrow 1} \frac{[k \cdot (x-1) + 2] \cdot (x-1)}{2 \cdot (x-1) - k \cdot (x-1)}$$

$$= \lim_{x \rightarrow 1} \frac{[k \cdot (x-1) + 2] \cdot (x-1)}{(x-1) \cdot (2-k)} = \lim_{x \rightarrow 1} \frac{k \cdot (x-1) + 2}{2-k} = \frac{2}{2-k}$$

Limita závisí na param. $k \Rightarrow$ neexistuje

3. přibližování po parabolech:

$$y = k(x-1)^2 + 2$$

$$\lim_{(x,y) \rightarrow (1,2)} \frac{y \cdot (x-1)}{2x-y} \stackrel{\text{subst.}}{=} \lim_{x \rightarrow 1} \frac{[k \cdot (x-1)^2 + 2] \cdot (x-1)}{2x - [k \cdot (x-1)^2 + 2]}$$

$$= \lim_{x \rightarrow 1} \frac{[k \cdot (x-1)^2 + 2] \cdot (x-1)}{2x - 2 - k \cdot (x-1)^2} = \lim_{x \rightarrow 1} \frac{[k \cdot (x-1)^2 + 2] \cdot (x-1)}{(x-1) \cdot [2 - k \cdot (x-1)^2]}$$

$$= \lim_{x \rightarrow 1} \frac{k \cdot (x-1)^2 + 2}{2 - k \cdot (x-1)^2} = 1 \Rightarrow \text{o existenci limity nelze rozhodnout}$$

4. Polárním souřadnicemi:

$$x = 1 + \rho \cdot \cos \varphi, y = 2 + \rho \cdot \sin \varphi$$

$$\lim_{(x,y) \rightarrow (1,2)} \frac{y \cdot (x-1)}{2x-y} \stackrel{\text{subst.}}{=} \lim_{\rho \rightarrow 0} \frac{(2 + \rho \sin \varphi) \cdot (\rho \cos \varphi)}{2 \cdot (1 + \rho \cos \varphi) - 2 - \rho \sin \varphi}$$

$$= \lim_{\rho \rightarrow 0} \frac{(2 + \rho \sin \varphi) \cdot \rho \cos \varphi}{2 \rho \cos \varphi - \rho \sin \varphi} = \lim_{\rho \rightarrow 0} \frac{(2 + \rho \sin \varphi) \cdot \cos \varphi}{2 \cos \varphi - \sin \varphi}$$

$$= \frac{2 \cos \varphi}{2 \cos \varphi - \sin \varphi}$$

Limita závisí na param $\rho \Rightarrow$ neexistuje

$$5. \times f(x, y) = x^3 - 4xy + y^2 + 4x + 1$$

$$f'_x = 3x^2 - 4y + 4 = 0$$

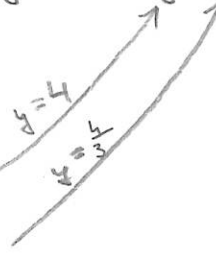
$$f'_y = -4x + 2y = 0 \Rightarrow 2y = 4x \Rightarrow y = 2x$$

$$3x^2 - 4 \cdot (2x) + 4 = 0$$

$$3x^2 - 8x + 4 = 0$$

$$d = 64 - 4 \cdot 3 \cdot 4 = 16$$

$$x_{1,2} = \frac{8 \pm 4}{6} = \begin{cases} 2 \\ \frac{2}{3} \end{cases}$$



$$S_1[2, 4]$$

$$S_2\left[\frac{2}{3}, \frac{4}{3}\right]$$

$$\left. \begin{array}{l} f''_{xx} = 6x, \quad f''_{xy} = -4 \\ f''_{yx} = -4, \quad f''_{yy} = 2 \end{array} \right\} H(x, y) = \begin{vmatrix} 6x & -4 \\ -4 & 2 \end{vmatrix} = 12x - 16$$

$$H(2, 4) = 12 \cdot 2 - 16 = 8 > 0, \quad S_1[2, 4] \text{ lok. extrém. Protože } f''_{xx}(2, 4) = 6 \cdot 2 = 12 > 0,$$

$S_1[2, 4]$ lok. minimum

$$H\left(\frac{2}{3}, \frac{4}{3}\right) = 12 \cdot \frac{2}{3} - 16 = -8 < 0, \quad S_2\left[\frac{2}{3}, \frac{4}{3}\right] \text{ je sedlový bod}$$