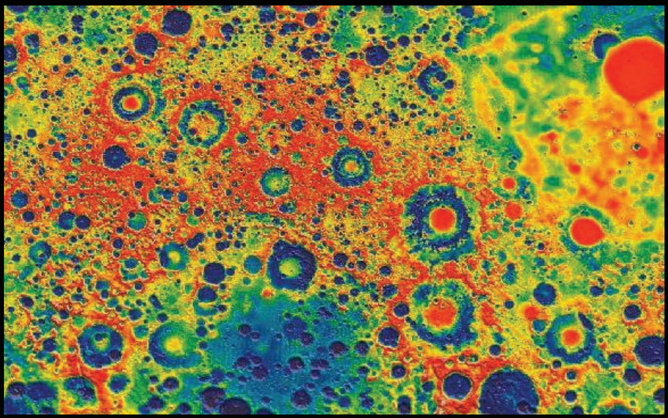
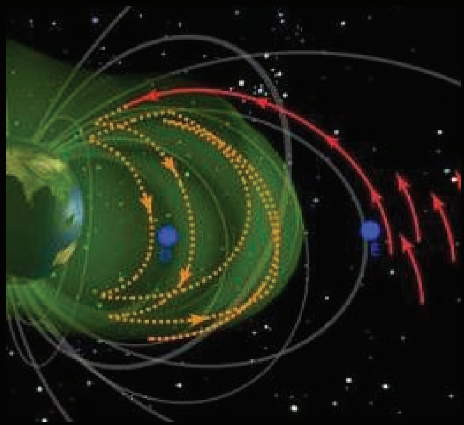
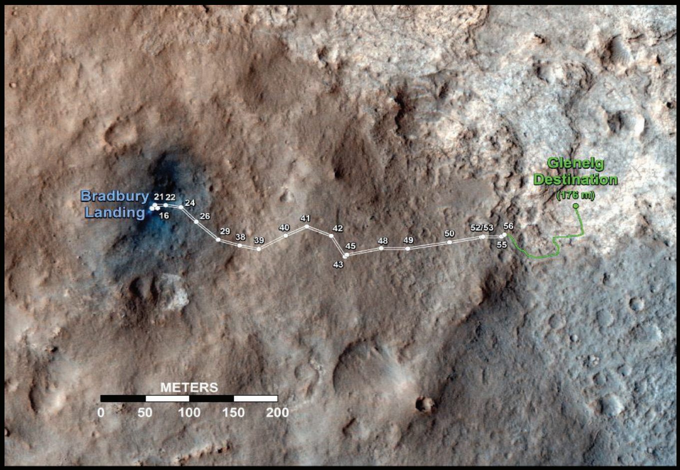
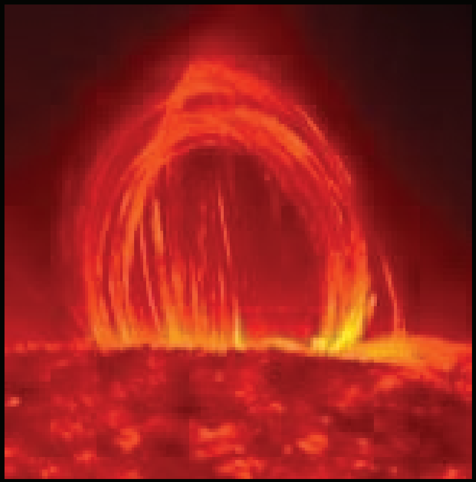
National Aeronautics and Space Administration

Space Math IX









This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2012-2013 school year. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 5 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

For more weekly classroom activities about astronomy and space visit the NASA website,
http://spacemath.gsfc.nasa.gov

Add your email address to our mailing list by contacting Dr. Sten Odenwald at
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Front and back cover credits: Front) Grail Gravity Map of the Moon -Grail NASA/ARC/MIT; Dawn Chorus - RBSP/APL/NASA; Erupting Prominence - SDO/NASA; Location of Curiosity - Curiosity/JPL./NASA; Chelyabinsk Meteor - WWW; LL Pegasi Spiral - NASA/ESA Hubble Space Telescope. Back) U Camalopardalis (Courtesy ESA/Hubble, NASA and H. Olofsson (Onsala Space Observatory)

Interior Illustrations: All images are courtesy NASA and specific missions as stated on each page, except for the following: 20) Chelyabinsk Meteor and classroom (chelyabinsk.ru); 32) diffraction figure (Wikipedia); 39) Planet accretion (Alan Brandon, Nature magazine, May 2011); 44) Beatrix Mine (J.D. Myers, University of Wyoming); 53) Mars interior (Uncredited ,TopNews.in); 89) Earth Atmosphere (NOAA); 90, 91) Lonely Cloud (Henriette, The Cloud Appreciation Society, 2005); 101, 103) House covered in snow (The Author);

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Mathematics Topic Matrix

Topic	Problem Numbers																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry																															
Technology, rulers																															
Numbers, patterns, percentages	X	X		X	X															X							X				X
Averages					X																										
Time, distance, speed				X						X			X					X			X	X	X	X		X					
Density, mass, volume																	X		X						X			X			
Areas and volumes						X											X			X							X	X	X		
Scale Drawings, proportions	X	X	X				X	X		X	X	X	X	X	X	X	X	X				X	X	X		X	X				
Geometry, Pythagorean Theorem			X				X	X	X				X	X	X	X		X													
Scientific Notation																			X	X	X			X	X		X	X	X		
Unit Conversions						X													X								X	X	X		
Fractions		X																													
Graph or Table Analysis							X		X		X	X									X	X	X								
Solving for X									X																					X	
Evaluating Fns											X	X								X	X									X	X
Modeling							X				X	X																		X	
Probability																				X											
Rates/Slopes															X																
Logarithmic Fns																															
Polynomials																														X	
Power Fns																															
Exponential Fns																															
Conics																															
Piecewise Fns																															
Trigonometry																															
Integration																															
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
Inquiry																															
Technology, rulers																															
Numbers, patterns, percentages										X				X		X													X		
Averages																															
Time, distance, speed		X				X				X						X													X		
Density, mass, volume																						X						X			
Areas and volumes							X										X	X				X								X	
Scale drawings	X										X	X	X		X		X	X			X									X	
Geometry, Pythagorean Theorem		X				X					X	X			X		X	X			X										
Scientific Notation				X	X			X											X			X	X	X	X	X		X			
Unit Conversions		X	X					X							X		X													X	
Fractions																															
Graph or Table Analysis	X			X			X																				X				
Solving for X				X	X			X											X												
Evaluating Fns		X	X	X	X		X	X															X	X	X	X					
Modeling	X			X	X			X	X				X					X			X				X	X					
Probability																															
Rates/Slopes			X										X	X											X	X					
Logarithmic Fns																			X	X											
Polynomials							X																								
Power Fns																															
Exponential Fns					X																										
Conics																															X
Piecewise Fns																															
Trigonometry	X		X						X																						
Integration							X	X																							
Differentiation						X		X	X																						
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
Inquiry																															
Technology, rulers	X																				X										
Numbers, patterns, percentages								X											X												
Averages																															
Time, distance, speed	X									X																					
Density, mass, volume																				X					X		X			X	
Areas and volumes							X	X			X	X	X	X			X	X		X	X	X		X			X			X	
Scale drawings	X						X	X		X	X	X	X				X	X			X	X		X			X	X	X	X	
Geometry, Pythagorean Theorem	X																	X										X	X	X	
Scientific Notation																				X			X	X	X		X				X
Unit Conversions		X															X								X						
Fractions																															
Graph or Table Analysis		X							X					X												X					X
Solving for X																							X								
Evaluating Fns				X	X	X					X	X	X		X	X							X								
Modeling		X		X	X	X												X					X								
Probability																															
Rates/Slopes																			X												X
Logarithmic Fns		X																													
Polynomials																															
Power Fns																															
Exponential Fns																															
Conics			X	X	X	X									X	X															
Piecewise Fns																															
Trigonometry			X	X		X																									
Integration																															
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers											
	94	95	96	97	98	99	100	101	102	102	104	105	106
Inquiry													
Technology, rulers													
Numbers, patterns, percentages			X										
Averages			X										
Time, distance, speed													
Density, mass, volume	X								X	X			
Areas and volumes	X					X	X		X				
Scale drawings					X	X	X						
Geometry, Pythagorean Theorem										X			
Scientific Notation	X												
Unit Conversions													
Fractions													
Graph or Table Analysis				X				X				X	X
Solving for X			X										
Evaluating Fns			X	X							X		
Modeling													
Probability													
Rates/Slopes		X					X						
Logarithmic Fns				X									
Polynomials													
Power Fns													
Exponential Fns													
Conics													
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Integration													
Differentiation													
Limits													

How to use this book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Math offers math applications through one of the strongest motivators-Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for all of the Space Math applications.

This book is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Space Math IX.** Read the scenario that follows:

Ms. Green decided to pose a new activity using Space Math for her students. She challenged each student team with math problems from the **Space Math IX** book. She copied each problem for student teams to work on. She decided to have the students develop a factious space craft. Each team was to develop a set of criteria that included reasons for the research, timeline and budget. The student teams had to present their findings and compete for the necessary funding for their space craft. The students were to use the facts available in the **Space Math IX** book.

Space Math IX can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

Alignment with Standards

x

AAAS: Project:2061 Benchmarks

(3-5) - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1 **(6-8)** Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1 **(9-12)** - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

NCTM: Principles and Standards for School Mathematics

Grades 6–8 :

- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; .
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- use graphs to analyze the nature of changes in quantities in linear relationships.
- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system.

Grades 9–12 :

- judge the reasonableness of numerical computations and their results.
- generalize patterns using explicitly defined and recursively defined functions;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- draw reasonable conclusions about a situation being modeled.

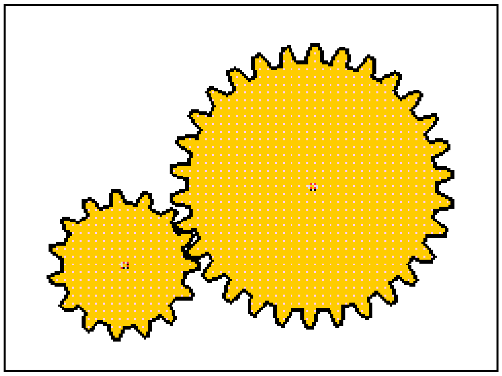
Teacher Comments

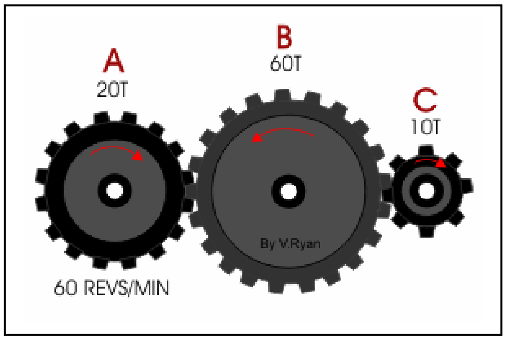
"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than "Consider a particle of mass 'm' and speed 'v' that…" (Associate Professor of Physics)





Gears are a common mechanical object used to change the rate of rotation of one shaft into a faster or slower rotation of a second shaft.

The top diagram shows two gears on two different shafts. The large gear has 30 teeth and the small gear has 15 teeth. As the small gear rolls along the circumference of the large gear, the shaft of the small gear completes 30/15 = 2 rotations for every 1 rotation of the large gear shaft. This is called a gear reduction, or gear ratio of 1:2.

The bottom diagram shows a more complicated gear 'chain' with 3 shafts. Gear A has 20 teeth, Gear B has 60 teeth and Gear C has 10 teeth. Gear A is turning at a rate of 60 revolutions per minute.

Problem 1 – When Gear A completes one revolution, how many revolutions does Gear B make?

Problem 2 – When Gear B makes one complete revolution, how many revolutions does Gear C make?

Problem 3 – What is the chain of reductions (example 1/2 x 1/3 x 4/6) that lets you calculate how many times Gear C rotates in revolutions per minute?

Problem 4 - An astronomer wants to build a 'clock drive' that will allow his telescope to keep up with the rotation of Earth so that stars will not move as he is looking at the sky with his telescope. To do this, the polar axis of the telescope mounting has to rotate exactly once each day, which lasts about 1437 minutes. He has a motor whose shaft rotates once every minute. The astronomer has the following gears with the indicated number of teeth: 1395, 309, 20 and 15. What combination will give him a speed reduction close to one shaft rotation every 1437 minutes?

Answer Key

Problem 1 – When Gear A completes one revolution, how many revolutions does Gear B make?

Answer: 20/60 = 1/3

Problem 2 – When Gear B makes one complete revolution, how many revolutions does Gear C make?

Answer: 60/10 = 6

Problem 3 – What is the chain of reductions (example 1/2 x 1/3 x 4/6) that lets you calculate how many times Gear C rotates in revolutions per minute?

Answer: 60 rev/min x (20/60) x (60/10) = 120 revolutions/minute

Problem 4 - An astronomer wants to build a 'clock drive' that will allow his telescope to keep up with the rotation of Earth so that stars will not move as he is looking at the sky with his telescope. To do this, the polar axis of the telescope mounting has to rotate exactly once each day, which lasts about 1437 minutes. He has a motor whose shaft rotates once every minute. The astronomer has the following gears with the indicated number of teeth: 1395, 309, 20 and 15. What combination will give him a speed reduction close to one shaft rotation every 1437 minutes?

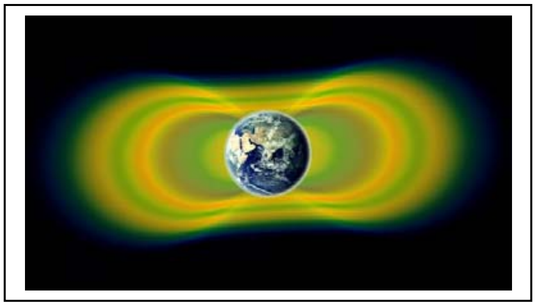
Answer: The astronomer needs a gear reduction of **1:1437**

(309/15) x (1395/20) = (20.6)(69.75) = **1436.85 or 1437**.

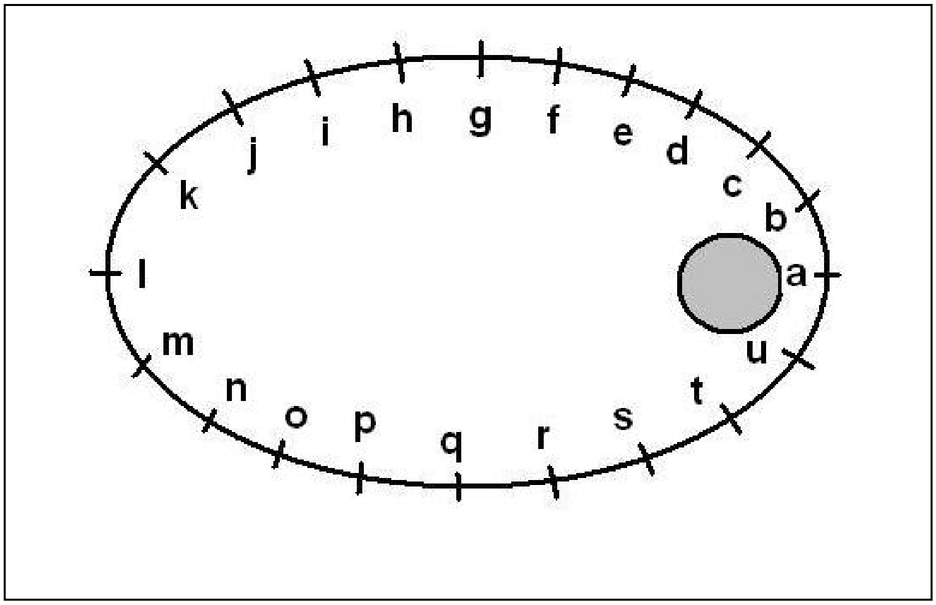
So the gear reductions look like this:

1 revolution/minute x (15/309) x (20/1395) = 1 rpm x (1/1437) = 1 revolution/day.

A New Belt for the Van Allen Belts!



Soon after launch, NASA's Van Allen Probes detected a new Third Belt in the Van Allen Radiation Belts that encircle Earth. Located between the Inner and Outer Belts, which have been known to scientists for decades, the third belt is very temporary. It only appears for a few weeks at a time when conditions in space are just right!



Problem 1 – The Van Allen Probe satellites travel along the elliptical orbit shown above, and in the sequence shown by the letters. At each point, the instruments make the following measurements:

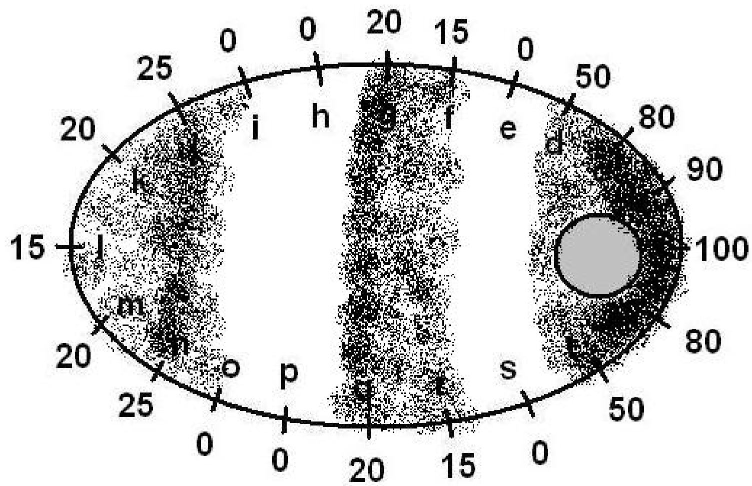
Point	Value	Point	Value	Point	Value	Point	Value
a	100	g	20	m	20	s	0
b	90	h	0	n	25	t	80
c	80	i	0	o	0	u	90
d	50	j	25	p	0		
e	0	k	20	q	20		
f	15	l	15	r	15		

Shade-in the shapes of the belts by connecting data measured at points A through L with the corresponding points M through U. (Hint: J and N are connected because they have similar values and are opposite each other in the orbit). Use dark colors for large values and light colors for small values.

Problem 2 – About what percentage of the elliptical path is covered by each of the belts that you discovered?

Note: The twin satellites orbit Earth every 9 hours. A very sensitive instrument on the satellite called the Relativistic Electron and Proton Telescope (REPT) can detect electrons and protons in space along the orbit of the satellite at points A, B, C etc. After numerous orbits, these data tracks can be stitched together to build up an image of the belts, and capture the fleeting existence of the third belt.

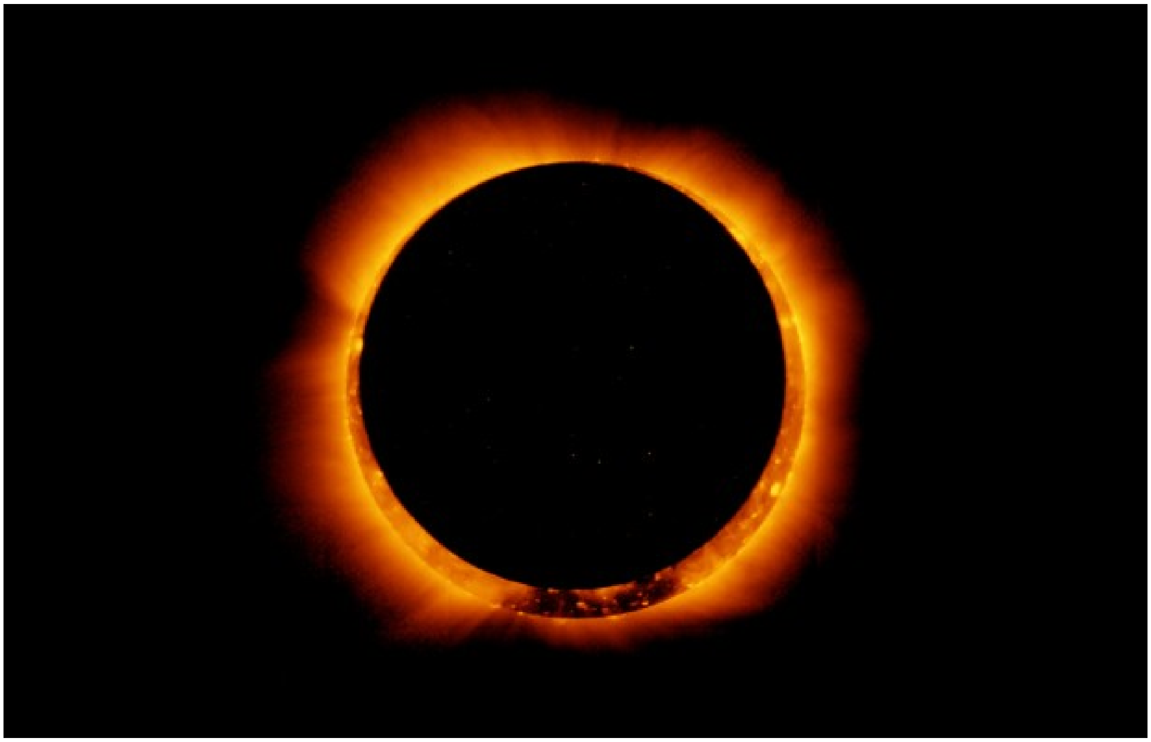
Problem 1 – The Van Allen Probe satellites travel along the elliptical orbit shown above, and in the sequence shown by the letters. At each point, the instruments make the following measurements. Shade-in the shapes of the belts by connecting data on opposite halves of the orbit. Use dark colors for large values and light colors for small values.



Problem 2 – About what percentage of the elliptical path is covered by each of the belts that you discovered?

Answer: Students will count the total letters along the full orbit as 21 from a to u.

Outer Belt covers the 5 letters J, k, l, m and n which is about 5/ 21 or 100x5/21 = **24%** of the full orbit path perimeter.
The
Middle Belt covers about 4 letters f, g and q, r for a percentage of 100x4/21 =**19%**
and the
Inner Belt covers the 6 letters a, b, c, d and t, u or 100 x 6/21 = **29%** of the elliptical area.



On May 10, 2013, the sun experienced what's called an annular eclipse. This happens when the moon moves directly in front of the sun, but doesn't completely cover the face of the sun. This leaves a thin, fiery ring, called the annulus, visible around the outside. This eclipse was only visible from the South Pacific, Australia, Papua New Guinea, the Solomon Islands and the Gilbert Islands.

The picture shown above was taken by the Hinode X-ray satellite on Jan. 4, 2011 of a previous annular solar eclipse as seen from space. The brilliant corona of the sun can be seen as it glows in X-ray light. The moon appears completely black in front of the sun's surface.

Problem 1 – About two solar eclipses can be seen somewhere on Earth each year. The ones in 2013 occur on May 10 and November 3. In 2014 a pair will occur on April 29 and October 23. The two eclipses in 2015 occur on May 20 and September 13. The one on October 23, 2014 will be visible from North America. On average, about how many days separate the 6 solar eclipses?

Problem 2 – Solar eclipses happen because the size of the Moon from Earth's surface appears the same size as the Sun's disk. If the Sun is located 400 times farther from Earth than the Moon, and the Moon has a diameter of 3,500 kilometers, what is the diameter of the Sun? (Hint: Use a simple proportion)

Annular Eclipse on May 10

May 10, 2013

http://www.nasa.gov/topics/solarsystem/features/2013-annular.html

Problem 1 – About two solar eclipses can be seen somewhere on Earth each year. The ones in 2013 occur on May 10 and November 3. In 2014 a pair will occur on April 29 and October 23. The two eclipses in 2015 occur on May 20 and September 13. The one on October 23, 2014 will be visible from North America. On average, about how many days separate the 6 solar eclipses?

Answer: 178 days, 178 days, 178 days, 210 days, 107 days. **Average = 170 days.**

Problem 2 – Solar eclipses happen because the size of the Moon from Earth's surface appears the same size as the Sun's disk. If the Sun is located 400 times farther from Earth than the Moon, and the Moon has a diameter of 3,500 kilometers, what is the diameter of the Sun? (Hint: Use a simple proportion)

Answer:

$$\frac{\text{Solar Diameter}}{\text{Lunar Diameter}} = \frac{400}{1}$$

so Solar Diameter = 400 x Lunar Diameter = **1,400,000 km**

The InSight Lander will arrive at Mars on September 20, 2016 according to Earth Time, but when will it arrive according to Mars Time?

One Earth Day is exactly 24 hours long, so that the time between two High Noons is exactly 24 hours. But Mars rotates a bit more slowly and by Earth units, one Mars Day (called a Sol) is 24 hours and 40 minutes long.

This photo was taken by the NASA Phoenix Lander on Sol 90, which is Earth date August 25, 2008

The Curiosity Rover can only safely move during the Martian daytime. This is when scientists on Earth can use TV cameras to watch where the rover is traveling. The Martian day is 40 minutes longer then the Earth day. That means scientists have to move their work schedule forward by 40 minutes each Earth day to keep up with sunrise and sunset on Mars. The rover landed on August 5, 2012 at about 10:30 pm Pacific Daylight Time (PDT), which was defined as Sol 0 for this mission. All of the navigation is performed by Navigation Engineers working at the Jet Propulsion Laboratory in Pasadena, California.

Problem 1 - What time and date will it be on Earth on Sol 5?

Problem 2 - Suppose that sunrise at the Curiosity lander happened on February 16, 2013 at 5:20 pm Pacific Standard Time (Sol 190 at 6:00 am Local Mars Time). A Navigation Engineer begins his work shift exactly at that moment. When will his shift have to start after 5 Sols have passed on Mars?

Problem 3 – How many Sols have to elapse before his work shift once again starts at the same Earth time of 5:20 pm PST?

Mars Sunrise and Sunset Tables
http://www.curiosityrover.com/sundata/

Problem 1 - What time and date will it be on Earth on Sol 5?

Answer: This will be 5 x (1 day and 40 minutes) added to August 5 at 10:30 pm PDT.
5 days and 200 minutes = 5 days 3 hours and 20 minutes after August 5 at 10:30 pm, or **August 11 at 1:50 am PDT**.

Problem 2 - Suppose that sunrise at the Curiosity lander happened on February 16, 2013 at 5:20 pm Pacific Standard Time (Sol 190 at 6:00 am Local Mars Time). A Navigation Engineer begins his work shift exactly at that moment. When will his shift have to start after 5 Sols have passed on Mars?

Answer: By his clock, each sunrise will happen 40 minutes later in Earth time, so after 5 Sols have passed, sunrise will happen 5 x 40 minutes = 200 minutes later or 3 hours and 20 minutes added to 5:20 pm PST, which becomes **8:40 pm PST on February 21**.

Problem 3 – How many Sols have to elapse before his work shift once again starts at the same Earth time of 5:20 pm PST?

Answer: One day on Earth is 24 hours long and 1 Sol on Mars is 24 hours 40 minutes long, so we have to wait until the extra 40 minutes after each Sol adds up to exactly 24 hours. Every 3 Sols equals 120 minutes or 2 hours .The progression look like this

Sols	3	6	9	12	15	18…etc
Hours	2	4	6	8	10	12 … etc

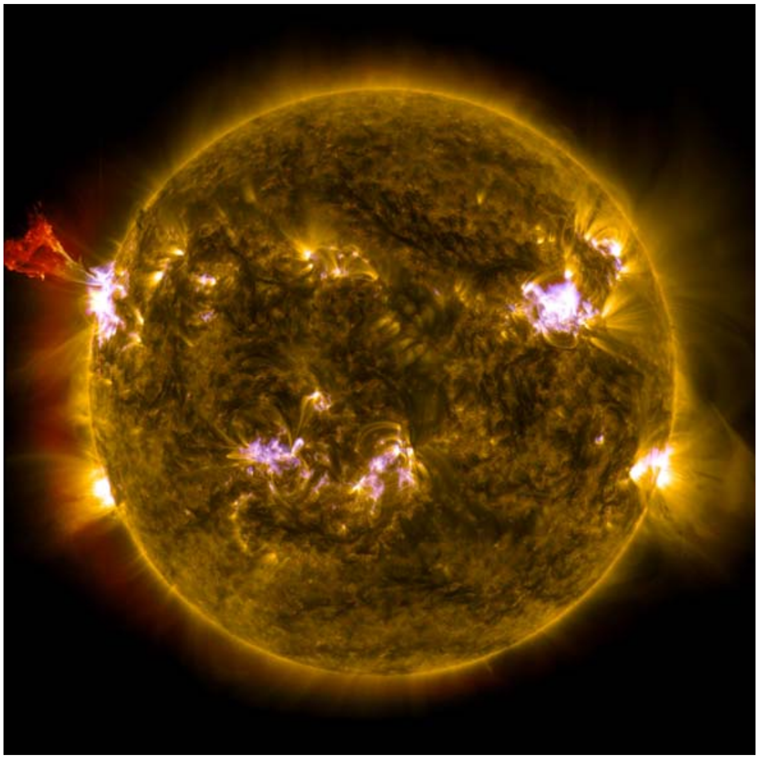
So if 18 Sols equals 12 hours, then 36 Sols equals 24 hours.

The answer is **36 Sols**.

Note to Teacher: 1 Earth day equals 24x60= 1440 minutes so 1 Sol = 1 day plus 40m/ 1440m = 1.028 Earth Days.

We can then convert **36 Sols** to 36 x 1.028 = **37 Earth Days**.

So In the problem above, if the Navigation Engineer starts on February 16 at 5:20 pm for his first Sunrise shift, after 37 days he will start his next Sunrise shift at the same time on March 25, 2013.



This image was obtained by the Solar Dynamics Observatory on May 3, 2013. It shows a solar flare on the edge of the sun.

There are many other active areas on the solar surface near sunspots which have a white color in this image.

The year 2013 is called the Sunspot Maximum because in this year, the sun produces more sunspots and solar storms than any other time in its 11-year cycle.

The year 2009 was called Sunspot Minimum with the fewest number of sunspots counted during each month. For each month in 2009, the average number of sunspots counted was 2, 1, 1, 1, 3, 3, 4, 0, 4, 5, 4, 11. Although sunspots have only been counted for the first 4 months of 2013, the average monthly numbers are 63, 39, 58, 72.

Problem 1 - What is the average number of sunspots counted in 2009 rounded to the nearest integer?

Problem 2 – What is the average number of sunspots counted in 2013 rounded to the nearest integer?

Problem 3 – Suppose that after adding the sunspot counts for May 2013 that the new average became 60. What was the number of sunspots counted in May rounded to the nearest integer?

Problem 4 – In July 2013, suppose that the number of sunspots counted was 5 more than the average for May. The June counts were 6 more than the counts for July, and the May counts were 5 more than the average for January, February, March and April. What were the monthly sunspot counts for May, June and July, and what was the average sunspot number for January-July?

Sun Emits Mid-Level Flare, May 3, 2013
http://www.nasa.gov/mission_pages/sunearth/news/News050313-flare.html

Additional Sunspot Data: http://www.ips.gov.au/Solar/1/6

Problem 1 - What is the average number of sunspots counted in 2009 rounded to the nearest integer?

Answer: (2+1+1+1+3+3+4+0+4+5+4+11)/12 = 39/12 = 3.25 or **3**

Problem 2 – What is the average number of sunspots counted in 2013 rounded to the nearest integer?

Answer= (63+39+58+72)/4 = **58**

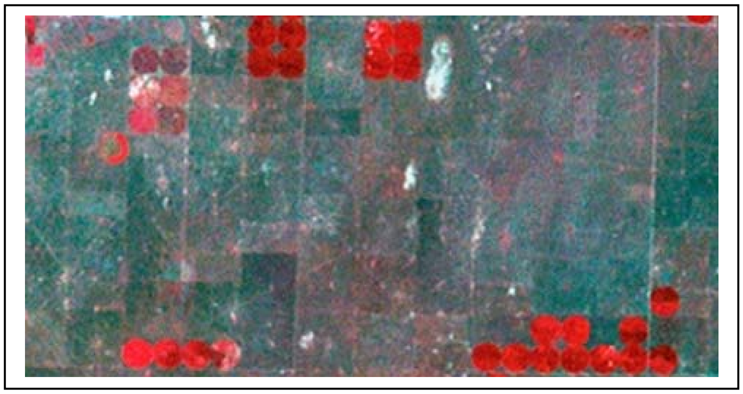
Problem 3 – Suppose that after adding the sunspot counts for May 2013 that the new average became 60. What was the number of sunspots counted in May rounded to the nearest integer?

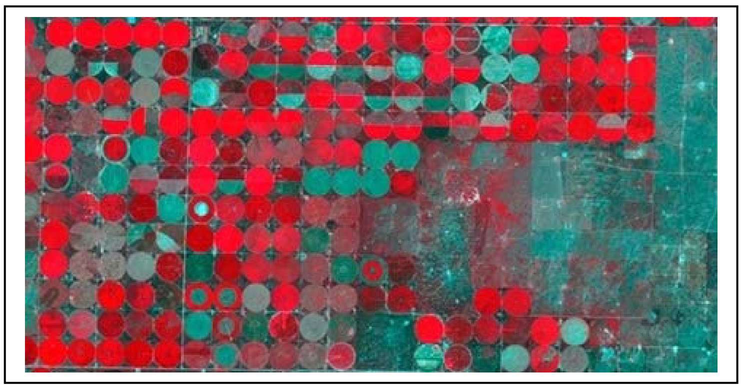
Answer: (63+39+58+72 + X)/5 = 60 then (232+X) = 5 x 60 and X = **68**.

Problem 4 – In July 2013, suppose that the number of sunspots counted was 5 more than the average for May. The June counts were 6 more than the counts for July, and the May counts were 5 more than the average for January, February, March and April. What were the monthly sunspot counts for May, June and July, and what was the average sunspot number for January-July?

Answer: In Problem 2, the average for the first four months was 58. The May counts were 5 more than this average or 58+5 = 63. The July counts were 5 more than the average for May or 63+5 = 68. The June counts were 6 more than the July counts or 68+6 = 74. So we have
January………63
February……..39
March…………58
April…………..72
May……………63
June……………74
July……………..68

The average for the first seven months is then (63+39+58+72+63+74+68)/7 = 62.4 or rounded to the nearest integer we get **62 sunspots**.





The top image obtained by Landsat in 1972 shows arid farm land near Garden City, Kansas. The dimensions of the area are 2 miles wide (East-West) by 1 mile tall (North-south). The bottom image was taken of the same area in 2011.

Center-pivot 'sprinklers' use water pumped from the sub-surface Ogallala aquifer to irrigate circular crop areas 800 meters in diameter.

Because the Ogallala aquifer recharges from new rainwater slowly, some of the water used to irrigate these fields is actually water that's been trapped underground since the last Ice Age. Even with the rise of water-conserving center-pivot irrigation and other efforts to conserve, this aquifer is slowly going dry.

Problem 1 – Farmers measure crop areas in acres. If the radius of one crop circle is 400 meters, and 1 acre = 4047 meters^2, what is the area of a single irrigation circle in the Landsat images? (Use π = 3.14).

Problem 2 – How much additional acreage was irrigated in 2011 compared to 1972?

Problem 3 – Typical water application from center-pivot systems is about 8 gallons/minute per acre for corn, and suppose the irrigation is conducted for 3-hours each day for a normal 120-day growing season. How many extra gallons of water are being drawn out of the Ogallala Aquifer to irrigate the increased acreage of corn in this region of Kansas between 1972 and 2011?

Problem 1 – Farmers measure crop areas in acres. If the radius of one crop circle is 400 meters, and 1 acre = 4047 $meters^2$, what is the area of a single irrigation circle in the Landsat images? (Use $\pi = 3.14$).

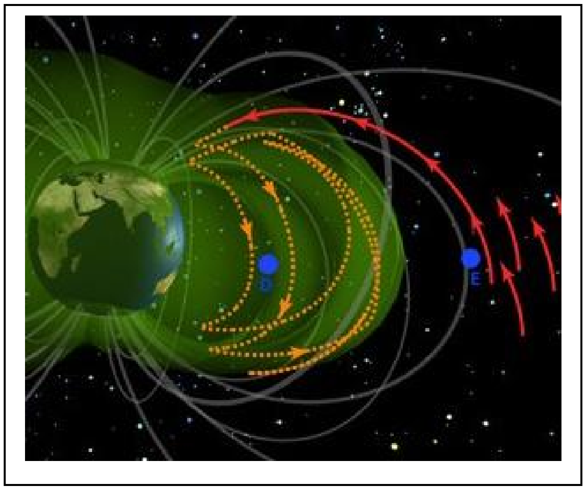
Answer: $3.14 (400)^2 = 502{,}560\ m^2$. Then $502{,}560\ m^2 \times (1\ acre/4047\ m^2)$ = **124 acres**.

Problem 2 – How much additional acreage was irrigated in 2011 compared to 1972?

Answer: Count the number of circles in each image and take the difference to get the increased number of irrigated areas. In 1972 there were 30 in 2011 there were about 189 full circles. Students may obtain slightly different numbers depending on how they count. In this example, the difference is 189-30 = 159. The total acreage added is now 159 areas x (124 acres/1 area) = **19,716 acres** added to the irrigation load of this region.

Problem 3 – Typical water application from center-pivot systems is about 8 gallons/minute per acre for corn, and suppose the irrigation is conducted for 3-hours each day for a normal 120-day growing season. How many extra gallons of water are being drawn out of the Ogallala Aquifer to irrigate the increased acreage of corn in this region of Kansas?

Answer: 8 gallons/min x (60minutes/1 hour) x (3 hours/1 day) x 120 days x 19,716 acres = **3.4 billion gallons** of water each growing season.



Amateur radio operators have been hearing this sound for decades, especially at 'dawn'. It is an eerie sound, like a chorus of birds chirping, so it was called Dawn Chorus.

This sound cannot be heard with ordinary ears even though it is in the right frequency range. Because it is a radio wave, you need a radio receiver to hear it.

Space physicists have tried to understand what produces this electromagnetic 'sound wave', but whatever is producing it is occurring somewhere in the Van Allen belts high above Earth.

During its 60-day checkout phase, the twin Van Allen Probes satellites captured chorus waves close to where they are being produced in the Van Allen belts. As the satellites continue to take more data, scientists hope to be able to triangulate the location of these waves to their place of origin. This will provide scientists with a HUGE clue about what is causing them in the first place.

Let's have a look at how they will 'triangulate' the chorus position in space using simple graphing techniques, a compass and the Pythagorean Theorem!

Problem 1 – Suppose the two spacecraft are located at points P1 (+4.0, +2.0) for VAP-A and P2 (+5.0, -1.0) for VAP-B on a coordinate grid where Earth is at the center and each unit on the coordinate axis is an interval of 6,400 kilometers. (Note 1 unit = radius of Earth). Graph this data on a coordinate grid which has an X-domain from [-5.0, +5.0] and a y-range from [-5.0, +5.0].

Problem 2 – If 1 unit on the graph equals the radius of Earth, what is the domain and range of the graph in kilometers?

Problem 3 - What is the separation between the Van Allen Probes spacecraft in kilometers? (Hint: Use either the 2-point distance formula or a ruler!).

Problem 4 – From the location of VAP-A, the distance to the chorus source was found to be 12,800 km. From VAP-B it was 19,200 km. What point inside the orbits of the spacecraft is consistent with these measurements?

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Problem 2 – If 1 unit on the graph equals the radius of Earth, what is the domain and range of the graph in kilometers?

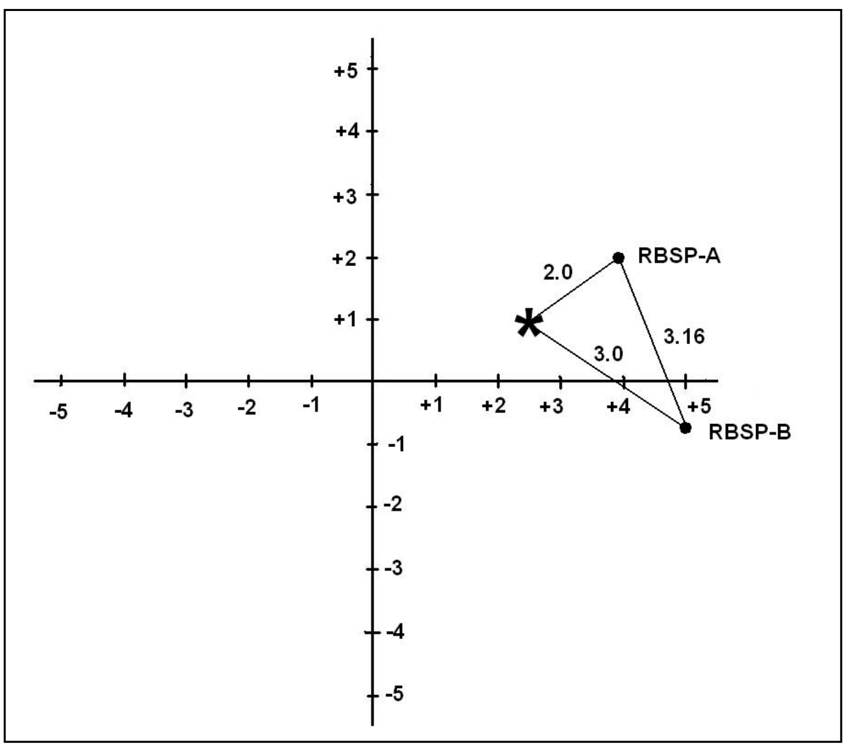
Answer: 5 units = 5 x 6400km = 32,000km.
X domain [-32,000 km, + 32,000 km], Y range [-32,000 km, +32,000 km].

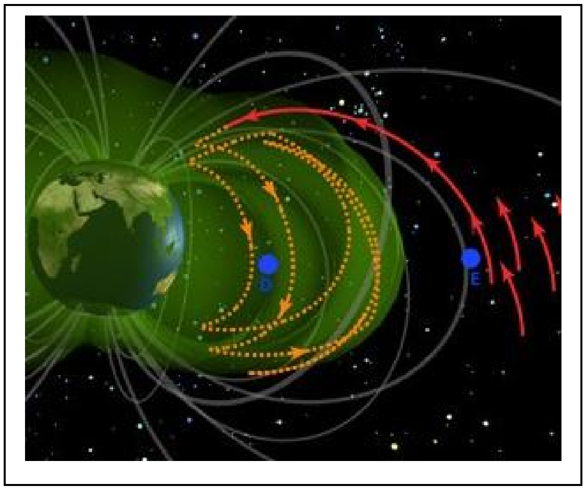
Problem 3 - What is the separation between the Van Allen Probes spacecraft in kilometers? (Hint: Use either the 2-point distance formula or a ruler!).

Answer: $d^2 = (5.0-4.0)^2 + (-1.0-(-2.0)^2$, $d^2 = 10.0$ so d= 3.16 units. In kilometers this is 3.16 x 6400 = **20,224 kilometers**.

Problem 4 – From the location of VAP-A, the distance to the chorus source was found to be 12,800 km. From VAP-B it was 19,200 km. What point inside the orbits of the spacecraft is consistent with these measurements?

Answer: 12,800 km/ 6,400km = 2.0 units. 19,200 km/6400 km = 3.0 units. Pt (+2.5, +1.0) or **(+16000 km, +6400 km)**





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Let's have a look at how they will 'triangulate' the chorus position in space using simple graphing techniques, a protractor and the Pythagorean Theorem!

Problem 1 – Suppose the two spacecraft are located at points P1 (+4.0, +2.0) for VAP-A and P2 (+5.0, -1.0) for VAP-B on a coordinate grid where Earth is at the center and each unit on the coordinate axis is an interval of 6,400 kilometers. (Note 1 unit = radius of Earth). Graph this data on a coordinate grid which has an X-domain from [-5.0, +5.0] and a y-range from [-5.0, +5.0].

Problem 2 – The RBSP-A spacecraft detects the Chorus signal coming from a direction angle of 198^{o}. RBSP-B detects the same signal coming from a direction angle of 135^{o}. Draw two lines at these angles from the locations of VAP-A and B to locate the source of the Chorus signal.

Problem 3 - What is the coordinate of the intersection point of these two lines, and the location of the Chorus signal?

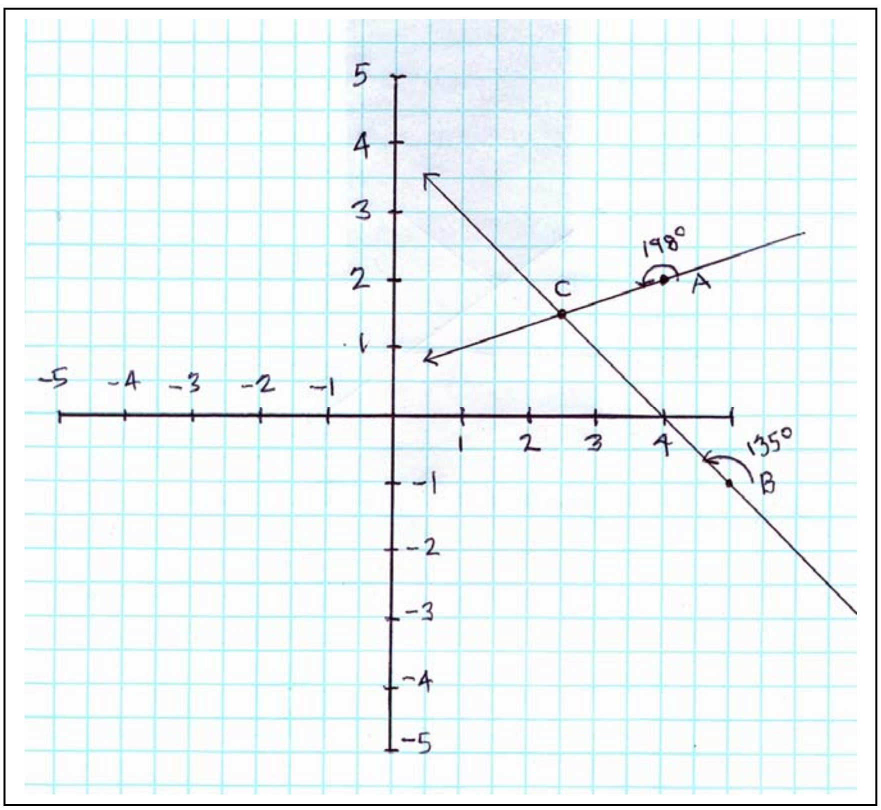
Problem 4 – From the location of VAP-A, and assuming that 1-unit equals 6,400 kilometers, to the nearest hundred kilometers, about how far from the spacecraft is the source of the Chorus signal?

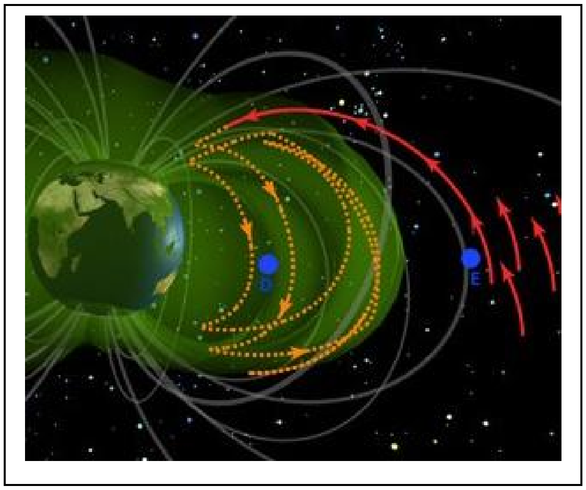
Problem 1 – Suppose the two spacecraft are located at points P1 (+4.0, +2.0) for VAP-A and P2 (+5.0, -1.0) for VAP-B on a coordinate grid where Earth is at the center and each unit on the coordinate axis is an interval of 6,400 kilometers. (Note 1 unit = radius of Earth). Graph this data on a coordinate grid which has an X-domain from [-5.0, +5.0] and a y-range from [-5.0, +5.0]. Answer: **See figure below**

Problem 2 – The VAP-A spacecraft detects the Chorus signal coming from a direction angle of 198^{o}. RBSP-B detects the same signal coming from a direction angle of 135^{o}. Draw two lines at these angles from the locations of VAP-A and B to locate the source of the Chorus signal. Answer: Students place the protractor centered on each spacecraft point, with the bottom edge parallel to the horizontal X-axis. They measure the two degree angles and draw a line through each point.

Problem 3 - What is the coordinate of the intersection point of these two lines, and the location of the Chorus signal? Answer: The intersection point is at **C:(+2.5, +1.5)**

Problem 4 – From the location of VAP-A, and assuming that 1-unit equals 6,400 kilometers, to the nearest hundred kilometers, about how far from the spacecraft is the source of the Chorus signal? Answer: Students can use a ruler and from this scaled drawing determine that the length of the chord from VAP-A to Point C is about 1.6 units or 1.6 x 6400 km = **10,200 kilometers.** Students may also use the distance formula $d^2 = (4.0-2.5)^2 + (2.0-1.5)^2$ so d = 1.6 units and so d = 10,200 km.





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Let's have a look at how they will 'triangulate' the chorus position in space using simple linear equation techniques and the 2-point distance formula !

Problem 1 – Suppose the two spacecraft are located at points P1 (+4.0, +2.0) for VAP-A and P2 (+5.0, -1.0) for VAP-B on a coordinate grid where Earth is at the center, and each unit on the coordinate axis is an interval of 6,400 kilometers. (Note 1 unit = radius of Earth). Graph this data on a coordinate grid which has an X-domain from [-5.0, +5.0] and a y-range from [-5.0, +5.0].

Problem 2 – The VAP-A spacecraft detects the Chorus signal coming from a direction along the line given by the formula $3y = x + 2$.

VAP-B detects the same signal coming from a direction somewhere along the line given by the formula $y = -x + 4$.

What are the coordinates of the intersection point of these two direction lines?

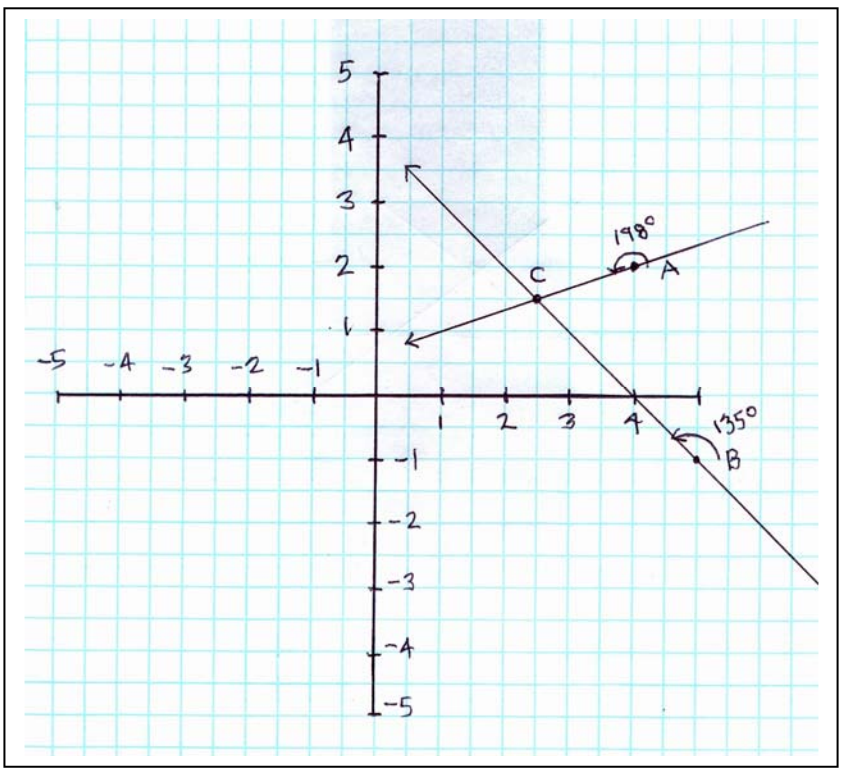
Problem 3 – From the location of VAP-A, and assuming that 1-unit equals 6,400 kilometers, to the nearest hundred kilometers, about how far from the spacecraft is the source of the Chorus signal?

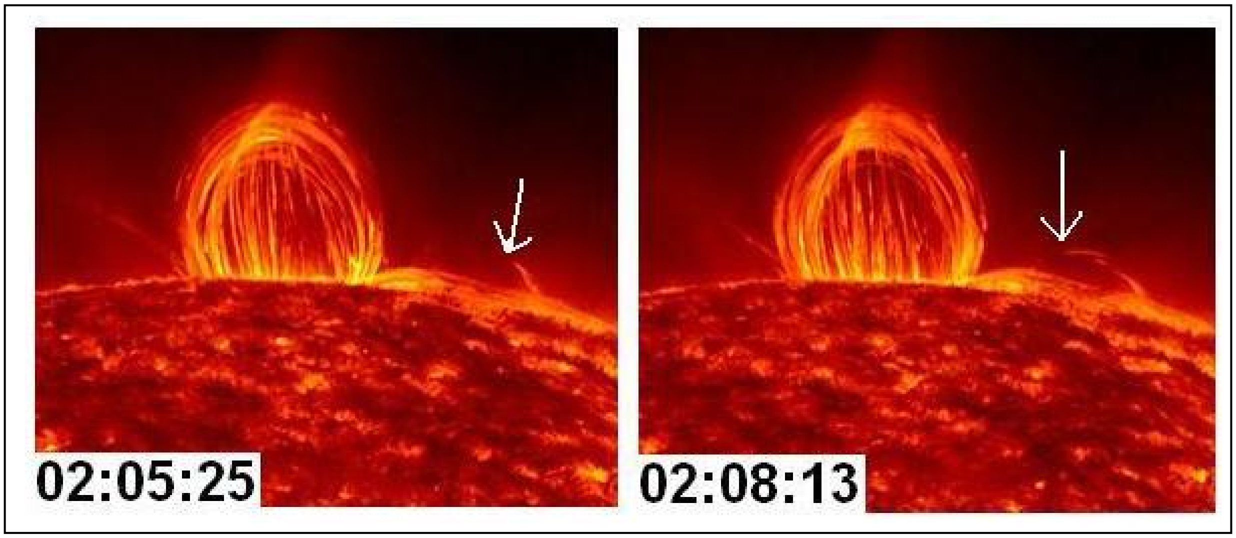
Problem 1 – Suppose the two spacecraft are located at points P1 (+4.0, +2.0) for VAP-A and P2 (+5.0, -1.0) for VAP-B on a coordinate grid where Earth is at the center, and each unit on the coordinate axis is an interval of 6,400 kilometers. (Note 1 unit = radius of Earth). Graph this data on a coordinate grid which has an X-domain from [-5.0, +5.0] and a y-range from [-5.0, +5.0]. Answer: **See figure below**

Problem 2 – The VAP-A spacecraft detects the Chorus signal coming from a direction along the line given by the formula 3y = x + 2. VAP-B detects the same signal coming from a direction somewhere along the line given by the formula y= -x + 4. What are the coordinates of the intersection point of these two direction lines?

Answer: the intersection point is a solution to both equations simultaneously. Students may use the substitution method, 3(-x+4) = x + 2 so -3x + 12 = x + 2 and so 4x = 10 and x = 2.5, then y = -(2.5)+4 = +1.5 so **C: (+2.5, +1.5)**.

Problem 3 – From the location of VAP-A, and assuming that 1-unit equals 6,400 kilometers, to the nearest hundred kilometers, about how far from the spacecraft is the source of the Chorus signal? Answer: Students may use the distance formula $d^2 = (4.0-2.5)^2 + (2.0-1.5)^2$ so d = 1.6 units and so **d = 10,200 km**.





On February 20, 2013, NASA's Solar Dynamics Observatory released a stunning video of a dome-shaped coronal rain event. The footage in this video covers the time between 12:30 a.m. EDT to 10:00 p.m. EDT on July 19, 2012. The width of each image corresponds to 360,000 kilometers near the sun!

These two still images were taken from the SDO video at the time codes indicated above, which are given in terms of playtime beginning at 2 minutes, 5.25 seconds (left image) and ending at 2 minutes, 8.13 seconds (right image). SDO collected one frame every 12 seconds, and the movie plays at 30 frames per second, so each second in this video corresponds to six minutes of real time.

The arrow points to a streamer of plasma ejected from the solar surface, and traveling in an arc from right to left in the 2-dimensional plane of the photograph. Because we don't know how fast the streamer is moving along the line-of-sight in the 3rd dimension, our speed estimate below will be a bit slower than the actual speed of the streamer.

Problem 1 - Astronomers would like to know how fast the heated gas in the streamer is traveling. From the clues in the images and the information in the essay, how fast was the gas traveling in kilometers/sec?

Problem 2 - The Space Shuttle traveled at a speed of 29,000 km/h in its orbit around Earth. Could the astronauts in the Space Shuttle have outraced the gas in the streamer?

Problem 3 - How big would the Earth be at the scale of these images if its diameter is 12,700 kilometers?

February 20, 2013.
NASA's SDO Shows A Little Rain On the Sun
http://www.nasa.gov/mission_pages/sdo/news/coronal-rain.html

Problem 1 - Astronomers would like to know how fast the heated gas in the streamer is traveling. From the clues in the images and the information in the essay, how fast was the gas traveling in kilometers/sec?

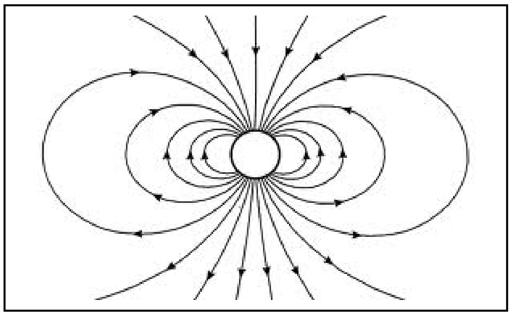
Answer: The time between the still images is 8.13seconds – 5.25 seconds = 2.88 seconds in 'movie time' but the essay says that 1 second in movie time equals 6 minutes (360 seconds) in real time, so the time interval between the images is 2.88 x 360 = 1037 seconds. To get the distance traveled in kilometers, students may use a millimeter ruler to measure the width of the image, which is 360000 kilometers. Typical measurements should get 76 millimeters, so the scale of the image is 360000/76 = 4700 km/mm. Carefully comparing the distance traveled between the two arrows will get a measurement of about 8 millimeters, so the physical distance is 4700 x 8 = 37,600 km. The speed of this streamer is then 37,600 km/1037 seconds **= 36.3 km/s**.

Problem 2 - The Space Shuttle traveled at a speed of 29,000 km/h in its orbit around Earth. Could the astronauts in the Space Shuttle have outraced the gas in the streamer?

Answer: We need to convert the shuttle speed to km/s. 29,000 km/h x (1 h/3600 sec) = 8.0 km/s. This is much slower than the streamer speed of 36.3 km/s so the shuttle **could not outrun the streamer**.

Problem 3 - How big would the Earth be at the scale of these images if its diameter is 12,700 kilometers?

Answer: The image scale is 4700 km/mm, and from the proportion 12700/4700 = X/1 mm we get X = **2.7 millimeters in diameter**.



Magnets have a north and a south pole. If you make this magnet small enough so that it looks like a point, all you will see are the looping lines of force mapped out by iron fillings or by using a compass.

Physicists call these patterns of lines, magnetic lines of force, and they can describe them mathematically!

Problem 1 - Create a standard Cartesian 'X-Y' graph with all four quadrants shown. Select a domain [-5.0, + 5.0] and a range [-2.0, +2.0] and include tic marks every 0.1 along each axis.

Problem 2 - Plot the following points in the order given and connect them with a smooth curve.

X	Y
+0.0	+0.0
+0.1	+0.3
+0.6	+1.1
+1.8	+1.8
+3.2	+1.9
+4.1	+1.6
+4.5	+1.2
+4.9	+0.6
+5.0	+0.0

Problem 2 - Reflect the curve you drew into Quadrant 4, then reflect the curve in Quadrant 1 and 4 into Quadrants 2 and 3 to complete a single magnetic line of force for a magnet located at the origin!

Problem 3 - Add two additional lines of force to your picture by re-scaling the figure you drew so that the X-Y coordinates are now A) 1/4 as large and B) 1.5 times larger.

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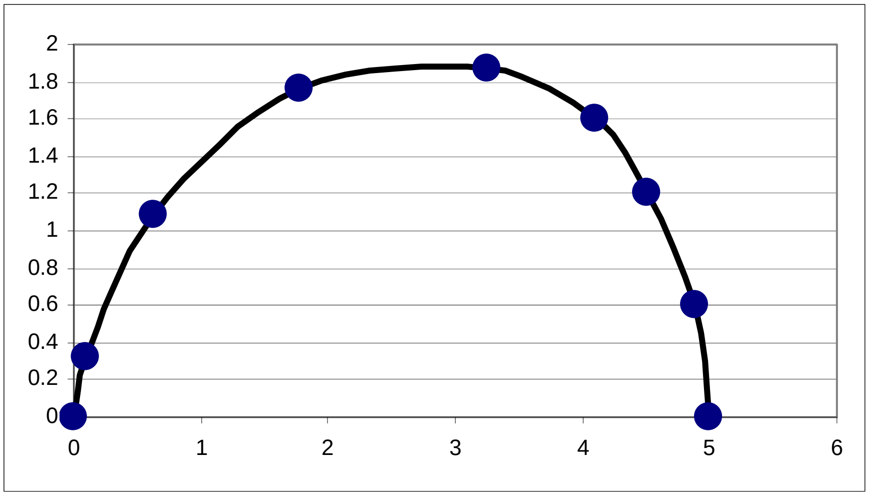

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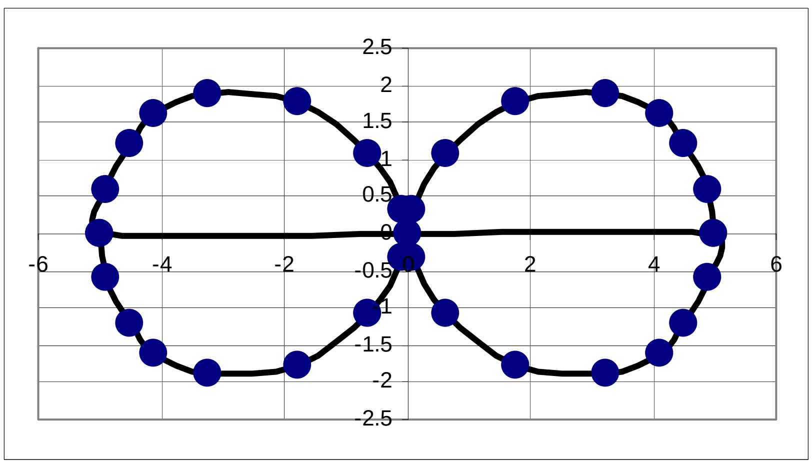
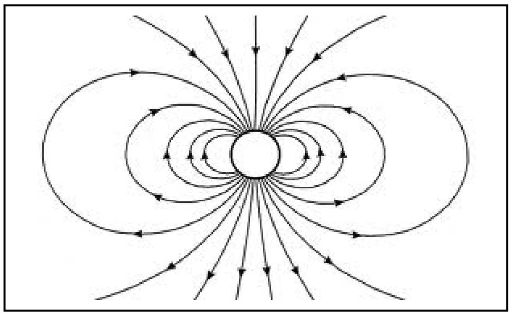


Problem 3 - Add two additional lines of force to your picture by re-scaling (dilating or contracting) the figure you drew so that the X-Y coordinates are now A) 1/4 as large (contraction) and B) 1.5 times larger (dilation).



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Problem 1 - Plot the following points in the order given and connect them with a smooth curve. You have just drawn a mathematical model of a portion of a magnetic line of force in the First Quadrant.

X	Y
+0.0	+0.0
+0.1	+0.3
+0.6	+1.1
+1.8	+1.8
+3.2	+1.9
+4.1	+1.6
+4.5	+1.2
+4.9	+0.6
+5.0	+0.0

Problem 2 - From the x and y coordinates given, find the coordinates of the 8 midpoints between each of the segments you plotted in Problem 1 by finding the average coordinate of each pair of points.

Problem 3 - At each of the midpoints to the segments, draw a line that represents the tangent of the curve at the midpoint. Place an arrow head mark so that the tangent 'arrows' are pointed in a counter-clockwise direction.

Problem 4 - Reflect your diagram in the First Quadrant into quadrants 2, 3 and 4 to complete the magnetic field line drawing!

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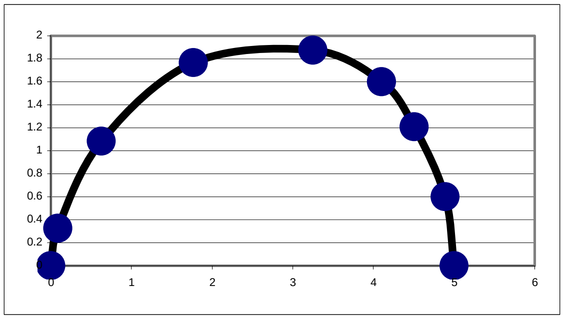
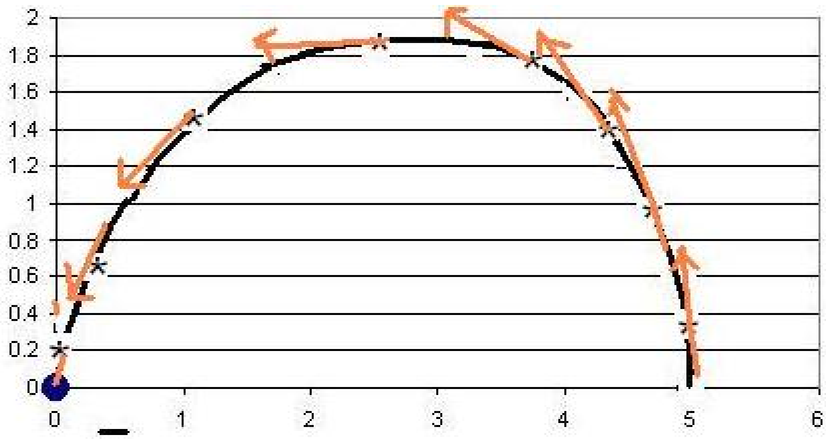


Problem 2 - From the x and y coordinates given, find the coordinates of the 8 midpoints between each of the segments you plotted in Problem 1 by finding the average coordinate of each pair of points.

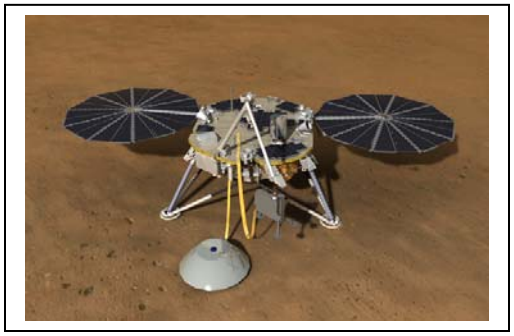
Answer: Example: x1 = (+0.1 + 0.0)/2 = +0.05, y1 = (+0.3 + 0.0)/2 = +0.15 so (+0.05, +0.15) is the first midpoint.

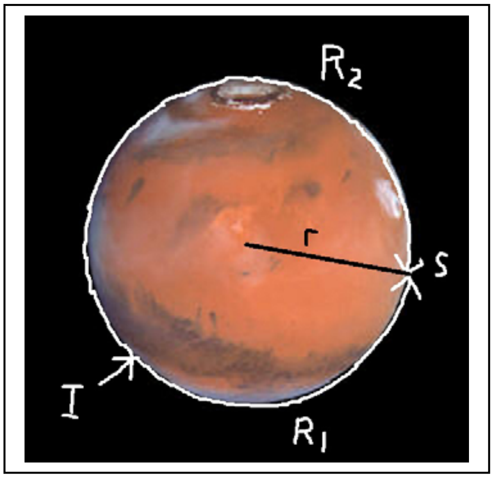
Problem 3 - At each of the midpoints to the segments, draw a line that represents the tangent of the curve at the midpoint. Place an arrow head mark so that the tangent 'arrows' are pointed in a counter-clockwise direction. Answer: See below.



Problem 4 - Reflect your diagram in the First Quadrant into quadrants 2, 3 and 4 to complete the magnetic field line drawing!

Answer: Compare with the drawing in the Introduction to this problem set.





NASA's new mission to Mars called InSight will be launched in March, 2016. It will land on September 20, 2016 in a region of Mars located near the equator and deploy a seismographic station to study the interior of Mars.

Each time a large meteor strikes the surface of Mars, one seismic wave will travel to the InSight station along a clockwise path around mars, and a second seismic wave will travel in the opposite direction to the station.

InSight will measure the arrival times of the two waves, called R1 and R2. From this timing data and the 5 km/s speed of the seismic wave along the martian surface, InSight will calculate where the impact occurred. The radius of Mars is r=3,397 kilometer.

In the following problems, use π = 3.1416, and round all answers to the nearest kilometer and second.

Problem 1 – Suppose that the impact occurred at Point I on the above figure, and the time between the arrival of the R1 and R2 waves was exactly 1423 seconds. How far did the R1 and R2 waves travel to get to the InSight station?

Problem 2 – For a large enough impact, InSight scientists expect that after the R1 and R2 waves are detected by the seismometer, that the waves will continue to 'orbit' the surface of Mars and return once again as a second pair of weaker seismic signals called R3 and R4, followed later on by a third pair of even-weaker signals called R5 and R6. For the example in Problem 1, what are the arrival times of all 6 seismic signals if R1 was detected at the clock time of 13:00:00 local time at the lander site?

Problem 3 - Because of a glitch in the recording of the seismic data, InSight scientists were able to detect the R3 and R6 seismic waves, which arrived at 15:25:30 and 17:00:00 Local Mars Time. How far from the Lander did the impact occur, and when would the arrival times for all 6 seismic waves have occurred in the data?

Problem 1 – Suppose that the impact occurred at Point I on the above figure, and the time between the arrival of the R1 and R2 waves was exactly 1423 seconds. How far did the R1 and R2 waves travel to get to the InSight station?

Answer: To travel once around the circumference of Mars, the wave has to travel $2 \pi r = 2 (3.1416) (3397 \text{ km}) = 21344$ km, so the round trip time is T = 21344 km / (5 km/s) = 4269 seconds. We know that T2 – T1 = 1423 seconds, so the R2 wave had to travel the same distance as the R1 wave plus an additional 1423 seconds. Since 1423 seconds = 1/3 of the full circumference time of 4269 seconds, that means that R1 traveled 1423 seconds from the impact site, I, and R2 traveled 2 x 1423 = 2846 seconds from the impact site. The distance to the impact site using the R1 wave data is 1423 sec x 5 km/sec = **7,115 kilometers**

Problem 2 – For a large enough impact, InSight scientists expect that after the R1 and R2 waves are detected by the seismometer, that the waves will continue to 'orbit' the surface of Mars and return once again as a second pair of weaker seismic signals called R3 and R4, followed later on by a third pair of even-weaker signals called R5 and R6. For the example in Problem 1, what are the arrival times of all 6 seismic signals if R1 was detected at the clock time of 13:00:00 local time at the lander site?

R1 = **13:00:00**
R2 = 13:00:00 + 1423 seconds = 13:00:00 + 23m 43s = **13:23:43**
R3 = 13:00:00 + 4269 seconds = 13:00:00 + 1h 11m 9s = **14:11:09**
R4 = 13:23:43 + 4269 seconds = 13:23:43 + 1h 11m 9s = **14:34:52**
R5 = 14:11:09 + 4269 seconds = 14:11:09 + 1h 11m 9s = **15:22:18**
R6 = 14:34:52 + 4269 seconds = 14:34:52 + 1h 11m 9s = **15:46:01**

Problem 3 - Because of a glitch in the recording of the seismic data, InSight scientists were able to detect the R3 and R6 seismic waves, which arrived at 15:25:30 and 17:00:00 Local Mars Time. How far from the Lander did the impact occur, and when would the arrival times for all 6 seismic waves have occurred in the data?

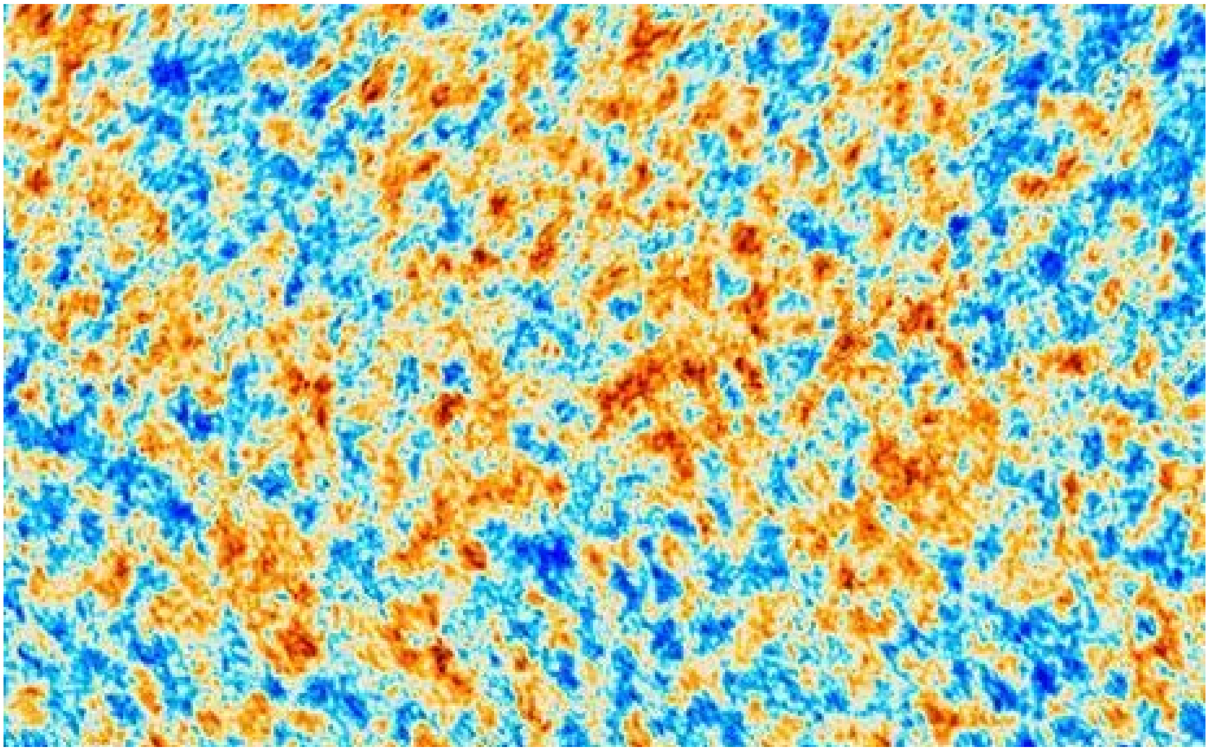
Answer: R3 is the R1 wave which has orbited Mars one additional time so that R3-R1 = 4269 seconds. R6 is the R2 wave which has orbited Mars two additional times so that R6-R2 = 2(4269 seconds). The R5 wave would have arrived one full orbit (4269 seconds) after the R3 wave, so the time intervals are as follows:

R1 = 15:25:30 – 4269 seconds = 15:25:30 – 1h 11m 9s = **14:14:21**
R2 = 17:00:00 – 8538 seconds = 17:00:00 – 2h 22m 18s = **14:37:42**
R3 = **15:25:30**
R4 = 17:00:00 – 4269 seconds = 17:00:00 – 1h 11m 9s = **15:48:51**
R5 = 15:25:30 + 4269 seconds = 15:25:30 + 1h 11m 9s = **16:36:39**
R6 = **17:00:00**

Time interval = R2-R1 = 23:21 = 1401 seconds.

R1 + R2 = 4269 seconds
R1 – R2 = 1401 seconds. Then adding the two equations we get 2R1 = 5670 so R1 = 2835 seconds. Traveling at 5 km/sec, the R1 wave originated **14,175 km from the landing site.**

Planck Mission Sees Ancient Universe Clearly



The Planck Mission, like the COBE and WMAP missions before it, has created this even-clearer image of the cosmic fireball radiation (cosmic background radiation). This image shows the minute temperature differences in the cosmic fireball radiation at a time about 370,000 years after the Big Bang. The differences are caused by the lumpiness of matter that was present by this time." At this time, the average temperature of matter was about 3,000^{o} Celsius. The image spans a portion of the sky 45^{o} wide.

Problem 1 – How many degrees wide are the smallest red flecks in the image?

Problem 2 – The full moon is 0.5 degrees wide. How large are the red flecks that you measured in Problem 1 in terms of the full moon diameter?

Problem 3 - At the distance that this 'surface' is located from Earth, the scale is 62 parsecs/arcsecond. If 1 degree = 3600 arcseconds, how many parsecs wide is the smallest clump?

Problem 4 – The Milky Way has a diameter of about 35,000 parsecs. How large are the features in the Planck image compared to the Milky Way?

Problem 1 – How many degrees wide are the smallest red flecks in the image?

Answer: ½ mm = about 0.14 degrees

Problem 2 – The full moon is 0.5 degrees wide. How large are the red flecks that you measured in Problem 1 in terms of the full moon diameter?

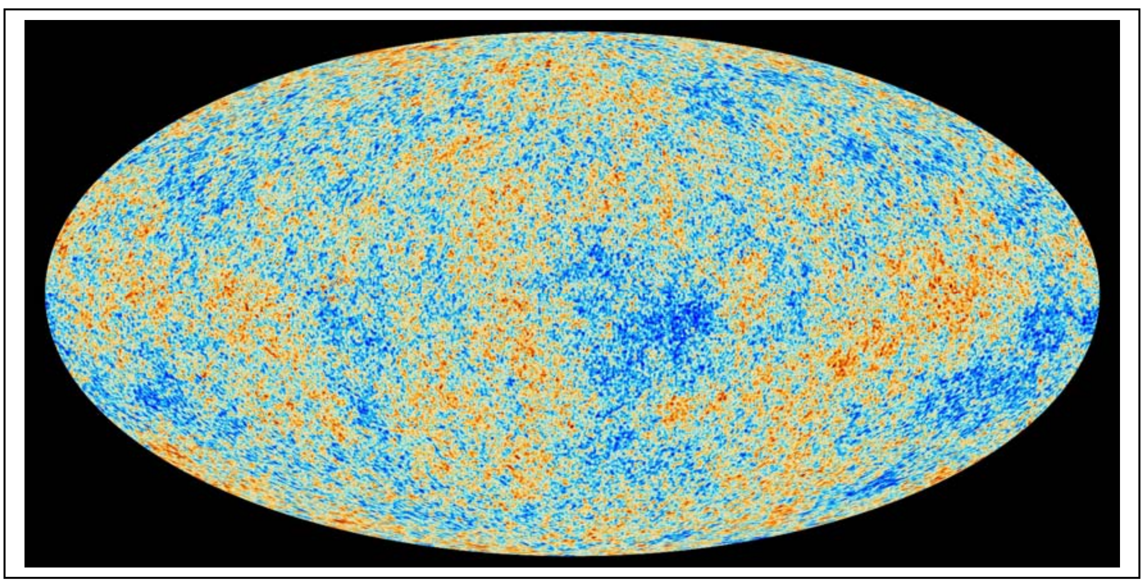
Answer: 0.5/0.14 = about 1/3 the diameter of the full moon.

Problem 3 - At the distance that this 'surface' is located from Earth, the scale is 62 parsecs/arcsecond. If 1 degree = 3600 arcseconds, how many parsecs wide is the smallest clump?

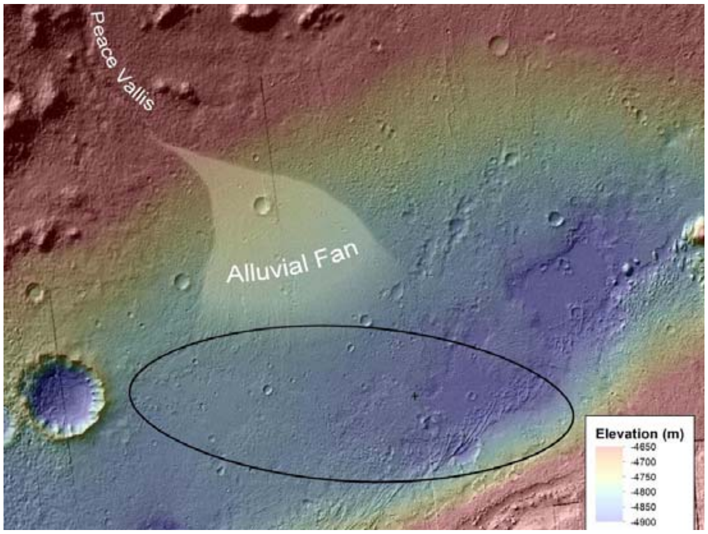
Answer: 0.14 degrees x 3600 x 62 parsecs = 31,200 parsecs.

Problem 4 – The Milky Way has a diameter of about 35,000 parsecs. How large are the features in the Planck image compared to the Milky Way?

Answer: The Milky Way is about the same size as one of the smaller spots in the image!



Curiosity Discovers Ancient Mars River!



The Curiosity Rover discovered rounded pebbles near its original landing site marked with the 'X' in the figure. The figure also shows the elevation changes in this area. Here is what the pebbles looked like! The white bar is 1 cm long.

Geologists studying the pebbles and the landscape believe that the water flow that moved and rounded the pebbles was at least ankle deep and perhaps waist deep. As on Earth, the pebbles were carried by fast moving water and over time became rounded by the constant scraping and bouncing. How fast was the water moving?

Calculating the stream gradient:

Problem 1 – The landing ellipse is 18 km wide. To the nearest kilometer, how far is the Curiosity 'X' from the apex of the alluvial fan near Peace Valllia?

Problem 2 – What is the change in elevation, h, between the alluvial fan vertex and the 'X'.?

Problem 3 – The stream gradient is defined as the elevation difference divided by the distance traveled. What is the stream gradient, SG, in units of meters/meters, for the water which left Peace Vallis and flowed down the alluvial fan?

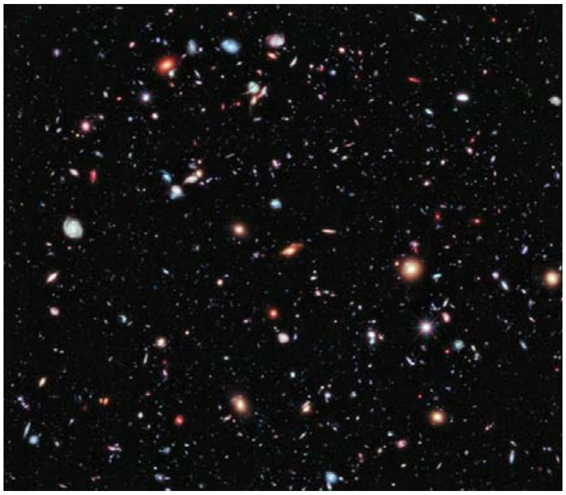
Calculating the stream gradient:

Problem 1 – The landing ellipse is 18 km wide. To the nearest kilometer, how far is the Curiosity 'X' from the apex of the alluvial fan near Peace Vallia? Answer: **About 15 km**.

Problem 2 – What is the change in elevation, h, between the alluvial fan vertex and the 'X'.? Answer: -4650 m – (-4900 m) so h = **250 meters.**

Problem 3 – The stream gradient is defined as the elevation difference divided by the distance traveled. What is the stream gradient, SG, in units of meters/meters, for the water which left Peace Vallis and flowed down the alluvial fan? 250 meters / 15 km = **17 meters/kilometer or SG=0.017 meters/meter.**

The Hubble eXtreme Deep Field – Counting Galaxies



The eXtreme Deep Field (XDF) image was created by astronomers by combining thousands of images of the same spot in the sky. When the data from 2,000 images were carefully combined, astronomers could identify 5,500 galaxies in the combined image.

Some of these galaxies, which appear as the large round spots in the photograph, are nearby galaxies. But in between these are the faint spots of light from the more distant galaxies. Some of these were formed only 500 million years after the Big Bang!

A simple study of this image can tell us a lot about our universe!

Problem 1 - Astronomers use angular measure when referring to locations and areas in the sky. In ordinary angle measure 1 degree can be subdivided into 60 arcminutes. 1 arc minute can be subdivided into 60 arcseconds. How many arcseconds are there in one degree of angular measure?

Problem 2 – The dimensions of the XDF patch are approximately 2.3 arcminutes wide and 2.0 arcminutes tall. What are the dimensions of this patch of the sky in degrees?

Problem 3 - What is the area of this patch of sky in square degrees?

Problem 4 – The full surface area of the entire sky can be found using the formula for the surface area of a sphere, where the 'radius' is equal to 1.0 radians. A radian is an angular measure equal to $180/\pi = 57.296$ degrees. What is the surface area of the entire sky in square degrees?

Problem 5 – How many of the XDF sky patches would cover the entire sky?

Problem 6 - If astronomers counted 5,500 galaxies in the XDF, about how many galaxies would you estimate across the entire sky?

Problem 7 - Astronomers studying the galaxies in the XDF have found about 10% are seen as they were less than 5 billion years ago, 30% are seen as they were between 5 billion and 9 billion years ago, and 60% are being seen as they were more than 9 billion years ago. Across the entire sky, how many galaxies might there be that are more than 9 billion years?

Problem 1 - 1 degree x (60 arcmin/1 deg) x (60 arcses/1 arcmin) = **3600 arcseconds.**

Problem 2 – 2.3 arcmin x (1 deg/60 arcmin) = **0.038 degrees wide** and
2.0 arcmin x (1 deg /60 arcmin) = **0.033 degrees tall**.

Problem 3 - 0.038 degrees x 0.033 degrees = **0.00125 square degrees**.

Problem 4 – $4 \pi (57.296)^2$ = **41,253 square degrees.**

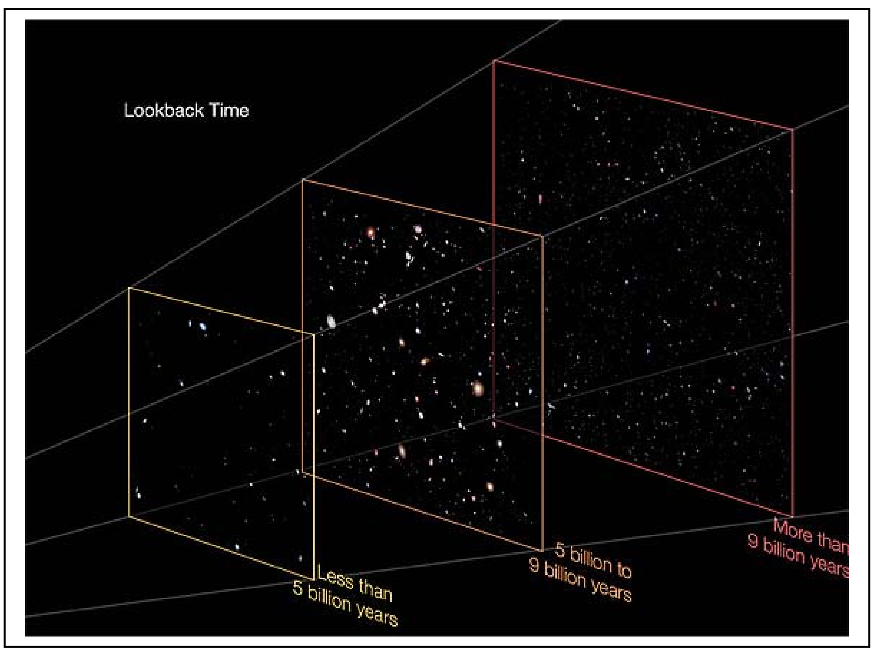
Problem 5 – How many of the XDF sky patches would cover the entire sky?
41,253 / 0.00125 = **33,000,000 patches**. (for 2 significant figure accuracy)

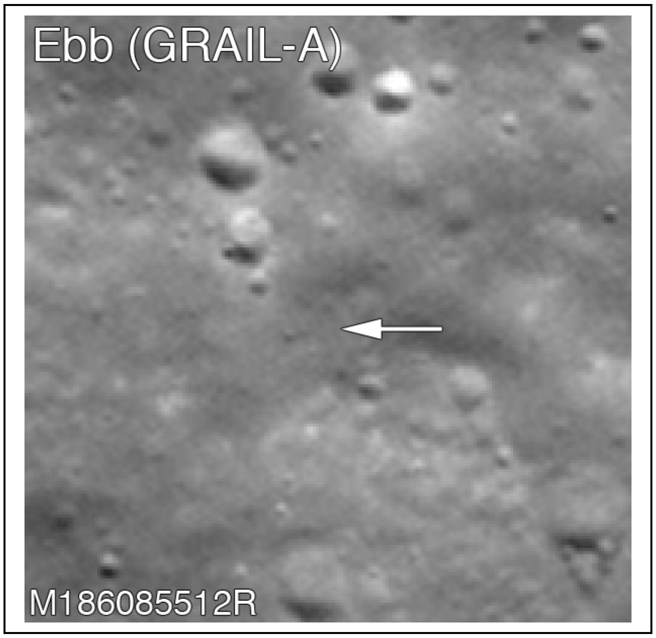
Problem 6 - If astronomers counted 5,500 galaxies in the XDF, about how many galaxies would you estimate across the entire sky? N = 5,500 galaxies/patch x (33000000 patches) = **182 billion galaxies**! With 2 significant figure accuracy this is 180 billion galaxies.

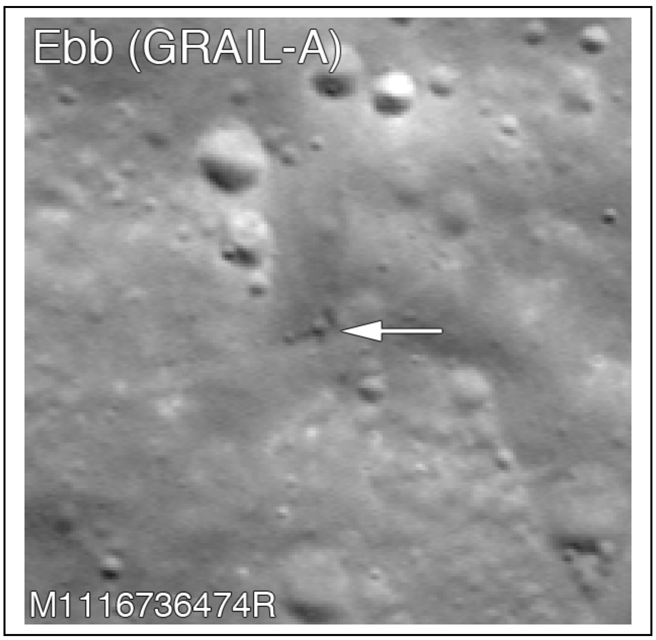
Problem 7 - Astronomers studying the galaxies in the XDF have found about 10% are seen as they were less than 5 billion years ago, 30% are seen as they were between 5 billion and 9 billion years ago, and 60% are being seen as they were more than 9 billion years ago. Across the entire sky, how many galaxies might there be that are more than 9 billion years?

60% of the total would be galaxies seen as they were more than 9 billion years ago, so 0.60 x 182 billion = **109 billion galaxies**. With 2 significant figure accuracy this is 110 billion.

Note: Since the Big Bang occurred 13.7 billion years ago, these galaxies are 13.7 - 9 or less than 4.7 billion years old!







On December 17, 2012 each of the twin Grail spacecraft ended their lunar mapping missions by crashing into the lunar surface. The two images to the left were taken by the Lunar Reconnaissance Orbiter as it flew over the impact site for the Grail-A spacecraft.

The width of each image is 213 meters.

At the moment of impact, each Grail spacecraft was traveling at a speed of 6,070 km/h and carried a mass of 200 kg.

Problem 1 – What was the diameter, in meters and feet, of the crater left behind when Ebb impacted the surface?

Problem 2 – The diameter of the Grail spacecraft was about 1-meter. How many times larger was the crater then the spacecraft?

Problem 3 – For the largest crater in the image, how large would the meteorite have to be to make the crater assuming it has the same density as Grail?

Problem 4 – Assume that the crater was a cylinder with a depth ¼ its diameter. If the density of the lunar soil is 2700 kg/m^3, how many kilograms of lunar soil were excavated by the impact?

Problem 5 – What is the ratio of the excavated mass to the spacecraft mass?

NASA's LRO Sees GRAIL's Explosive Farewell
http://www.nasa.gov/mission_pages/LRO/news/grail-results.html

Problem 1 – What was the diameter, in meters and feet, of the crater left behind when Ebb impacted the surface?

Answer: Use a millimeter ruler to measure the width of the image. You should get about 80 mm .The scale is then 213 meters/80 mm = 2.7 meters/mm. The crater is about 1mm across so this is just **2.7 meters**. Since 3 feet = 1 meter, this is about **8 feet across.**

Problem 2 – The diameter of the Grail spacecraft was about 1-meter. How many times larger was the crater then the spacecraft?

Answer: About **2.7 times larger**.

Problem 3 – For the largest crater in the image, how large would the meteorite have to be to make the crater?

Answer: The largest crater is about 9 mm across or 24 meters. Using Grail, we get 24 meters/2.7 = **8.9 meters across**.

Problem 4 – Assume that the crater was a cylinder with a depth ¼ its diameter. If the density of the lunar soil is 2700 kg/m^3, how many kilograms of lunar soil were excavated by the impact?

Answer: Volume = $\pi R^2 h$ so V = 3.14 x $(2.7/2)^2$ x (2.7/4) = 3.8 meters^3.
Mass = density x volume so Mass = 2700 x 3.8 = **10,000 kilograms**.

Problem 5 – What is the ratio of the excavated mass to the spacecraft mass?

Answer: 10,000 kilograms/200 kg = **50**.

So the amount of excavated mass is 50 times the mass of the impacting spacecraft.

On March 1, 2013, Space Exploration Technologies (SpaceX) launched the Dragon Supply Capsule on a Falcon 9 booster. SpaceX 2 is the second commercial resupply mission to the International Space Station.

Dragon delivered about 1,268 pounds (575 kilograms) of supplies to support continuing space station research experiments and will return with about 2,668 pounds (1,210 kilograms) of science samples from human research, biology and biotechnology studies, physical science investigations, and education activities.

According to the launch video narration, while the first stage engines were operating before Main Engine Cut-Off (called MECO), the rocket position and speed were given by the values in the table below:

Time Min:Sec	Altitude (km)	Down Range (km)	Speed (km/s)
2:20	30	23	1.0
2:45	51	59	1.8

Problem 1 - At what average speed was the altitude changing over this time interval?

Problem 2 - At what average speed was the down-range distance changing over this time interval?

Problem 3 - At what rate was the rocket accelerating over this time interval?

Problem 4 - What was the distance from the launch pad to the rocket at each time?

Problem 5 - At what average speed was the distance increasing over this time interval?

Problem 1 - At what average speed was the altitude changing over this time interval?

Answer: Speed = distance/time. Time = 2:45-2:20 = 25 seconds. Distance = 51-30 = 21 km, so speed = 21 km/25 sec = **0.84 km/sec**.

Problem 2 - At what average speed was the down-range distance changing over this time interval?

Answer: Time = 2:45-2:20 = 25 seconds. Distance = 59-23 = 36 km, so speed = 36 km/25 sec = **1.4 km/sec.**

Problem 3 - At what rate was the rocket accelerating over this time interval?

Answer: Acceleration = speed change/time, so
Acceleration = (1.4 km/s - 0.84km/s) / 25 sec = **0.022 km/sec/sec**.

Problem 4 - What was the distance from the launch pad to the rocket at each time?

Answer: Use the Pythagorean Theorem
At 2:20 $d = (30^2 + 23^2)^{1/2}$ = **38 km**
At 2:45 $d = (51^2 + 59^2)^{1/2}$ = **78 km**

Problem 5 - At what average speed was the distance increasing over this time interval?

Answer: From the distance calculation, speed = (78 - 38)/25 sec = **1.6 km/s**.

IC-443 supernova remnant located 5,000 light years from Earth. The purple color shows the location of the gamma rays seen by Fermi. The supernova remnant has many filaments of gas in which the cosmic rays are boosted in speed.

Cosmic rays are fast-moving particles that travel through space at nearly the speed of light, though they are not electromagnetic radiation at ll. Instead, they are typically electrons and the nuclei of atoms such as hydrogen and helium. Since their discovery in the 1940s, astronomers have speculated that they are created when stars explode as supernovae. The tremendous energy of the explosion ejects matter from the outer layers of the star into space. As this matter spreads out into individual atoms and nuclei, they become the particles we see in cosmic rays.

NASA's Fermi Gamma Ray Observatory has studied the gamma rays that comes from the remains of two nearby supernovae called IC-443 and W44, and has confirmed that the expanding matter does produce cosmic rays.

In space, the average density of cosmic rays is about 0.005 cosmic rays/meter^3.

Problem 1 – The Milky Way is a disk shaped system of stars with a radius of 50,000 light years and a thickness of about 3,000 light years. One light year is equal to 9.5x10^{15} meters. What is the volume of the Milky Way galaxy in cubic meters?

Problem 2 - About what is the total number of cosmic rays in the Milky Way galaxy?

Problem 3 – Suppose a single supernova can eject 2.0x10^{30} kilograms of matter into space. If the mass of a single proton is 1.6x10^{-27} kg, how many hydrogen nuclei does this represent?

Problem 4 – About how many supernova would be required to 'fill up' the Milky Way with cosmic rays if all of the ejected mass in hydrogen atoms were converted into cosmic rays?

Problem 1 - The Milky Way is a disk shaped system of stars with a radius of 50,000 light years and a thickness of about 3,000 light years. One light years is equal to 9.5x1015 meters. About what is the volume of the Milky Way in cubic meters?

Answer: The volume of the galaxy is $V = \pi R^2 H = 3.141 \times (50000 \times 9.5 \times 10^{15})^2 (3000 \times 9.5 \times 10^{15})$ = **2.0×10^{61} meters^3**.

Problem 2 - About what is the total number of cosmic rays in the Milky Way galaxy?

Answer: $N = 0.005 \times 2.0 \times 10^{61} = 1.0 \times 10^{59}$ cosmic rays.

Problem 3 – A single supernova can eject 2.0×10^{30} kilograms of matter into space. If the mass of a single proton is 1.6×10^{-27} kg, how many hydrogen nuclei does this represent?

Answer: 2.0×10^{30} kg x (1 hydrogen atom/1.6×10^{-27} kg) = **1.3×10^{57} hydrogen atoms**.

Problem 4 – About how many supernova would be required to 'fill up' the Milky Way with cosmic rays if all of the ejected mass in hydrogen atoms were converted into cosmic rays?

Answer: 1.0×10^{59} cosmic rays x (1 Supernova/1.3×10^{57} hydrogen atoms) = **77 supernova**.

Note: These are only estimates that fit the simple origins model we used. In fact, the actual amount of matter converted into cosmic rays per supernova explosion is far less than what was used in Problem 3. The present population of cosmic rays has been built up over billions of years as millions of stars have become supernova. Each supernova adds its share to the Milky Way's cosmic ray 'atmosphere'. Over time, many of these cosmic rays actually leave the Milky Way galaxy entirely. The Milky Way is not a closed vessel that accumulates cosmic rays but a leaky bag.

On February 14, 2013 a 10,000 ton meteor about 17-meters in diameter entered Earth's atmosphere over Russia traveling at 40,000 mph (18 km/s). It detonated in the air over the town of Chelyabinsk and the explosion caused major damage to the town injuring 1,000 people. The people were hurt by flying glass when the windows of over 3000 buildings blew out over an area of about 1000 km^2. Unlike the famous Tunguska Event of 1908 which blew down 80 million trees and was not 'discovered' for many decades afterwards, the Chelyabinsk Meteor was extensively videoed by hundreds of dash cams and cell phones as it happened.

Studies of thousands of meteor sightings by scientists can now tell us just how often asteroids of 4-meters or larger enter Earth's atmosphere. About two of these events happens each year over the entire surface area of Earth, which is 500 million km^2!

Problem 1 – The surface area of Earth is consists of 72% oceans and 28% land. Of the land area, only 3% is inhabited. How many years would you have to wait to hear about one of these large meteor events in the News?

Problem 2 – Fireballs are very bright meteors that streak across the sky. They are caused by pieces of meteors that can be 500 grams or more in mass. Astronomers estimate that 50,000 of these 'Bolides' can be seen every year over the entire surface area of Earth. From an inhabited spot on Earth, about how many Bolides should you be able to see in your lifetime if you paid attention to the sky if you could see any bolide entering over an area about 100 km^2?

Problem 3 - The kinetic energy in Joules of a large meteor is given by $KE = 1/2mV^2$ where m is its mass in kilograms and V is its speed in meters/sec. One ton of TNT explodes with an energy of 4.2×10^9 Joules. How many tons of TNT did the Chelyabinsk Meteor yield as it exploded if 1 ton = 1000 kg?

Problem 1 – The surface area of Earth is consists of 72% oceans and 28% land. Of the land area, only 3% is inhabited. How many years would you have to wait to hear about one of these large meteor events in the News?

Answer: The inhabited area of Earth is 3% of 28% of the total surface area or just 0.8% of Earth's total surface area. This is 1/125 of the full area. If you had one event per year, you would have to wait 125 years for the next one. If you had 2 events per year, you would have to wait half this time or about **62 years**. So, in a typical 70-year human lifetime, you will hear about one or two of these major impact events in the News!

Problem 2 – Fireballs are very bright meteors that streak across the sky. They are caused by pieces of meteors that can be 500 grams or more in mass. Astronomers estimate that 50,000 of these 'Bolides' can be seen every year over the entire surface area of Earth. From an inhabited spot on Earth, about how many Bolides should you be able to see in your lifetime if you paid attention to the sky if you could see any bolide entering over an area about 100 km^2?

Answer: The 50,000 bolides arrive somewhere over Earth each year. The chance that that this area is over an inhabited region of Earth is 0.8% or 1/125. So 1/125 of the bolides arrive over an inhabited area which is 50000/125 = 400 each year. For you to personally see the event, it has to happen within 100 km^2 of where you are standing. The inhabited area of Earth has an area of 1/125 x 500 million km^2 = 4 million km^2. Your 100 km^2 is only 1/40000 of this area. So you will see 400 bolides x 1/40000 = 1/100 bolides each year, or will **have to wait about 100 years to see just one**! If you watched the sky every night for your entire life, you might see one of these events!

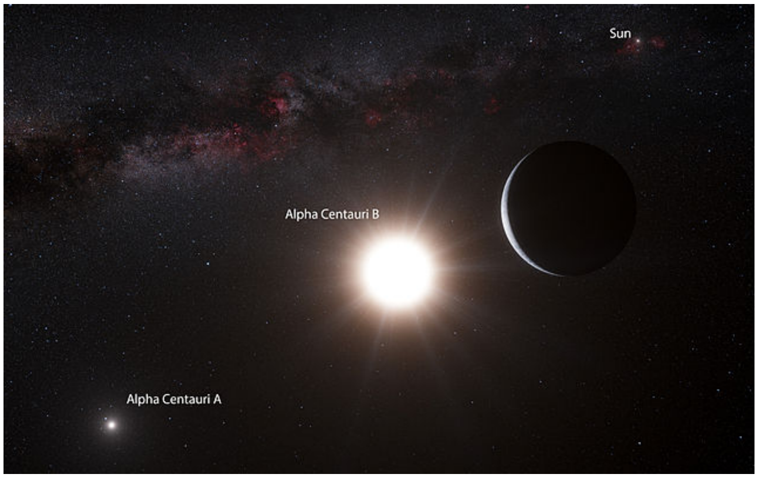
BUT, because of our world-wide news and internet coverage, you
could hear about any bolide that flashed over the inhabited area of Earth or 400 bolides each year! We only hear about a few of these because most of them are too unimpressive to get the attention of the news system. After the Chelyabinsk Meteor event on February 14, 2013, there were many announcements of fireballs or bolides over Los Angeles, San Francisco and other cities as this news topic became popular for a few weeks.

Problem 3 - The kinetic energy in Joules of a large meteor is given by KE = $1/2mV^2$ where m is its mass in kilograms and V is its speed in meters/sec. One ton of TNT explodes with an energy of 4.2×10^9 Joules. How many tons of TNT did the Chelyabinsk Meteor yield as it exploded?

Answer: The mass was 10,000 tons or 10,000 tons x 1000 kg/1 ton = 10 million kg.
The speed was 18 km/s x 1000 m/km = 18,000 m/s, so the energy was

$KE = ½ (1.0\times10^7) \times (1.8\times10^4)^2 = 1.62\times10^{15}$ Joules.

This is equal to 1.62×10^{15} Joules x (1 ton/4.2×10^9Joules) = **386,000 tons of TNT** or about 10 times the energy of a small atom bomb. This is similar to the estimates found in the news reports of this event, and explains why it did so much damage!



Artist rendition courtesy ESO/L. Calçada/N. Risinger (skysurvey.org)

Alpha Centauri is a binary star system located 4.37 light years from our sun. The two stars, A and B, are both sun-like stars, but they are older than our sun by about 1.5 billion years.

Astronomers have used the European Space Agency's 3.6 meter telescope at La Silla in Chile to detect the tell-tail motion of Alpha Centauri B caused by an earth-sized planet in close orbit around this star.

The planet, called Alpha Centauri Bb, orbits at a distance of only six million kilometers from its parent star – closer than Mercury is to the sun. The planet is bathed in unbearable heat, and has a surface temperature of 1,200 Celsius (2,200 F or 1,500 Kelvin). This is hot enough that its surface must be mostly molten lava. Its tight orbit means a year passes in only 3.2 Earth days.

The astronomers made hundreds of measurements of the speed of the Alpha Cen B star to search for a periodic change it its speed through space. They found a change (amplitude) of about 50 cm/sec that increased and decreased with a precise period, which would only be expected from an orbiting object.

This discovery is still being confirmed through independent observations by other astronomers.

We can check the numbers in the information box ourselves. Here are a few of the measurements made of the star's speed:

Time (hours)	Speed (cm/sec)	Time (hours)	Speed (cm/sec)
6	170	48	50
10	150	56	70
21	110	71	130
33	60	83	170

Problem 1 – Graph the speed data. Draw a smooth curve through the data (which need not go through all the points) and estimate the period (in days) of the speed curve to get the orbit period of the proposed planet.

Problem 2 – Kepler's Third Law can be used to relate the period of the planet's orbit (T in years) to its distance from its star (D in Astronomical Units) using the formula

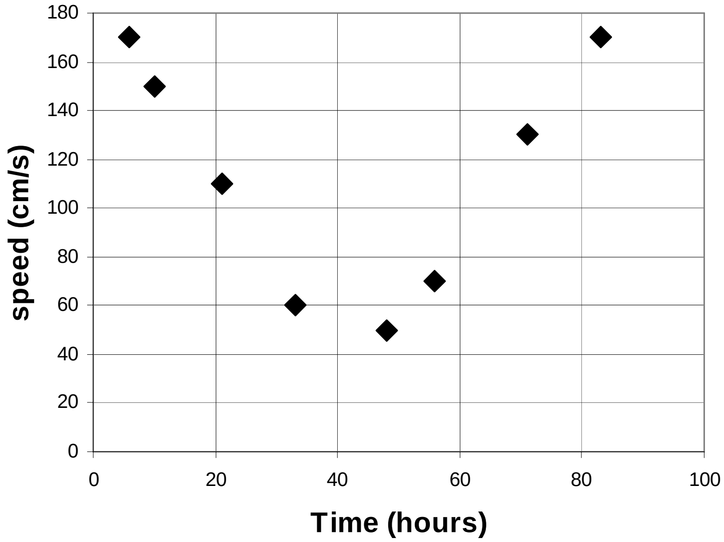
$$T^2 = D^3$$

where 1 Astronomical Unit equals the distance from Earth to our sun (150 million km). Using your estimated planet period, what is the orbit distance of the new planet from Centauri B in A) Astronomical Units? B) kilometers?

Problem 3 – What is the temperature T (in kelvins) of the new planet if its average temperature at a distance of D Astronomical Units is given by the formula:

$$T = \frac{310}{\sqrt{D}}$$

Problem 1 – Graph the speed data. Draw a smooth curve through the data and estimate the period of the speed curve to get the orbit period of the proposed planet. Answer should be about 77 hours or **3.2 days.**



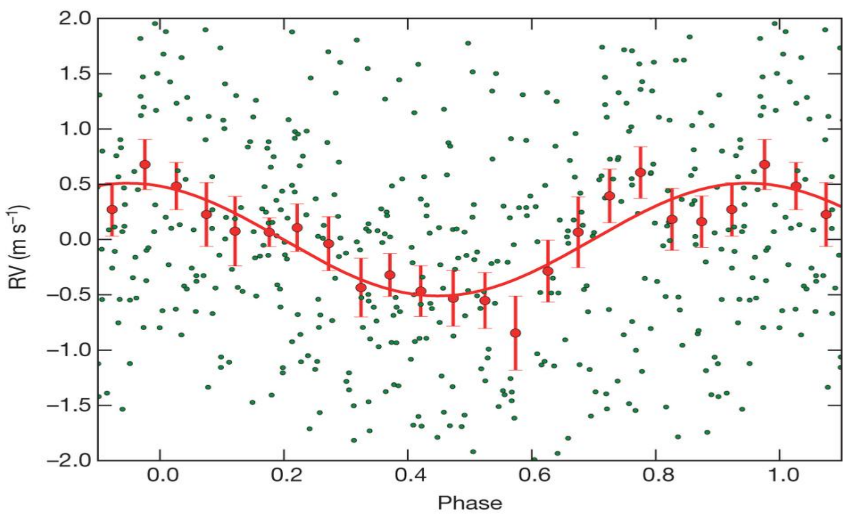
Problem 2 – Kepler's Third Law can be used to relate the period of the planet's orbit (T in years) to its distance from its star (D in Astronomical Units) using the formula

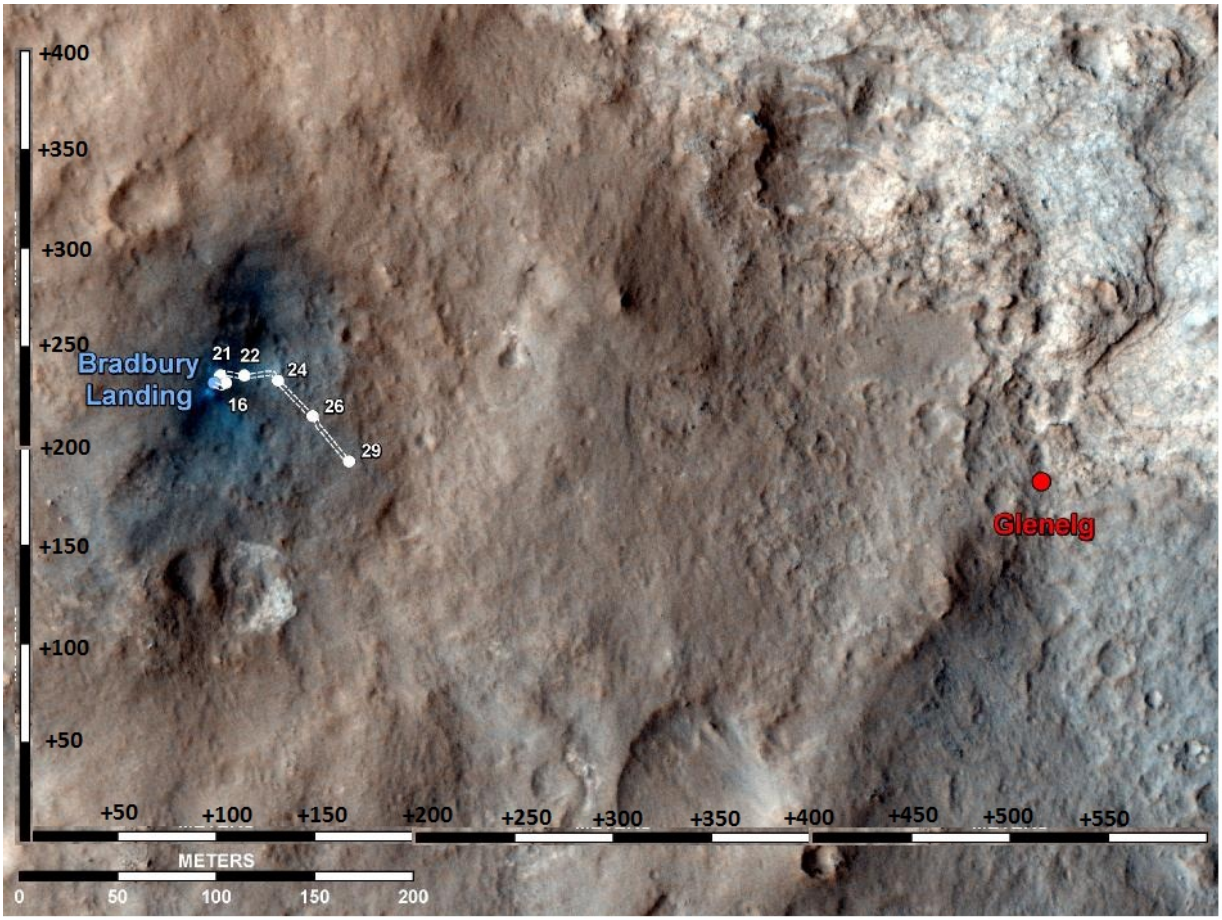
$$T^2 = D^3$$

where 1 Astronomical Unit equals the distance from the earth to our sun (150 million km). Using your estimated planet period, what is the orbit distance from Centauiri B in A) Astronomical Units? B) kilometers? Answer: T = 3.2 days or in terms of earth-years T = 3.2/365 = 0.00877 years. Then $D^3 = (0.00877)^2$, $D^3 = 7.69\times10^{-5}$ $D = (7.69\times10^{-5})^{1/3}$ **D = 0.043 Astronomical Units**. B) In kilometers, this is 0.043 AU x (150 million km/1 AU) = **6.4 million kilometers**.

Problem 3 – What is the temperature T (in kelvins) of the planet if its average temperature at a distance of D Astronomical Units. Answer: $T = 310 / (0.043)^{1/2}$ so T = **1,500 kelvins**.

The actual graph of the data published by the astronomers is shown below:





The Curiosity Rover on Mars landed at Bradbury Station on Day 0 (Called Sol 0) and is headed for an important geological site called Glenelg. This map shows the location of the Rover until Sol 29. Also shown on the map is a coordinate grid marked in intervals of 50-meters. Bradbury Station is located at approximately (+100, +230). The table below gives the location of Curiosity for the period from Sol 29 to Sol 56. Students should use the distance formula to determine interval lengths: $d^2 = (x2-x1)^2 + (y2-y1)^2$ but they may also use millimeter rulers and the image scale to determine the distances between the points.

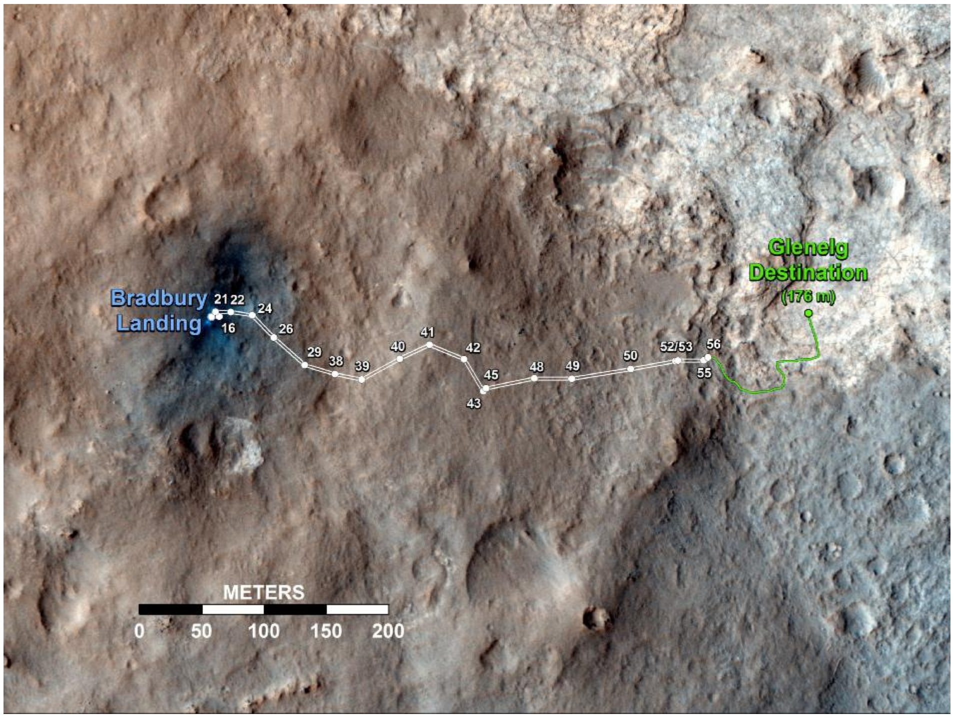
Day	X	Y	Day	X	Y
39	+210	+180	48	+360	+175
41	+270	+210	49	+390	+180
42	+300	+200	52	+470	+200
45	+315	+165	56	+500	+205

Problem 1 – Graph the additional points and connect them with line segments to show Curiosity's path across the martian landscape.

Problem 2 – During which segment was Curiosity traveling the fastest?

Problem 3 – During which segment was Curiosity traveling the slowest?

Problem 4 – What has been the average speed of Curiosity between Sol 39 and Sol 56?



Problem 1 – Graph the additional points and connect them with line segments to show Curiosity's path across the martian landscape. Answer: **See actual course above**.

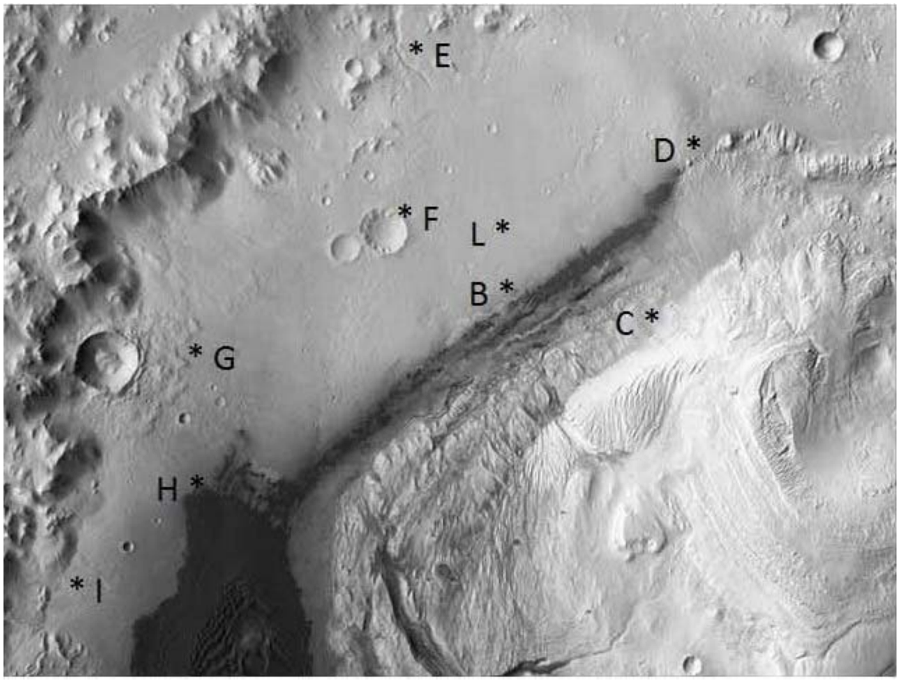
Day	X	Y	Segment Time (days)	Segment Distance (m)	Segment Speed (m/d)
39	+210	+180			
41	+270	+210	2	67	34
42	+300	+200	1	32	32
45	+315	+165	3	38	13
48	+360	+175	3	46	15
49	+390	+180	1	30	30
52	+470	+200	3	82	27
56	+500	+205	4	30	8

Example: Day 45 - Day 42 = 3 days. $D^2 = (315-300)^2 + (165-200)^2 = 1450$ so d = 38 meters and speed = 38meters/3days = 13 meters/day.

Problem 2 – During which segment was Curiosity traveling the fastest? **Between Sol 41 and Sol 42 at a speed of 34 meters per day.**

Problem 3 – During which segment was Curiosity traveling the slowest? **Between Sol 52 and Sol 56.**

Problem 4 – What has been the average speed of Curiosity between Sol 39 and Sol 56?
Total segment distance traveled = (67+32+38+46+30+82+30)=325 meters in 17 days
So **average speed = 19 meters/day.**



The table below gives the coordinates for the locations visited by the Curiosity Rover shown in the figure above. The X and Y coordinate units are in kilometers. Although Curiosity is free to travel between most points in the map, Point C is at a much higher elevation than the other points, and a steep cliff wall exists between Point B and C and runs diagonally to the lower left.

Label	Name	(X,Y)	Label	Name	(X,Y)
L	Landing Area	(45,40)	F	Crater Wall	(38,43)
B	Layered Wall	(50,35)	G	Mudslide	(17,30)
C	Alluvial Fan	(60,32)	H	Dark Sands	(17,19)
D	Summit Access	(65,50)	I	Mystery Valley	(5,10)
E	River Bed	(37,58)			

Problem 1 – Curiosity can travel a top speed of 300 meters/hr. When it landed, it was instructed to move as quickly as possible to Point B in case the mission malfunctioned. What is the distance between Point L and Point B, and how long did it take Curiosity to get there?

Problem 2 – To the nearest kilometer, what is the distance directly from Point D to Point I as, and how long would it take Curiosity to make this trip without stopping to do any scientific research?

Problem 3 – One possible path Curiosity might take connects all of the points in the sequence L-B-D-C-D-E-F-B-G-H-I. To the nearest kilometer, what is the total distance traveled, and to the nearest tenth, how many days would this journey take?

Problem 4 – Can you think of a different trip, and include a 3-day stay at each point along the way?

Problem 1 – Curiosity can travel a top speed of 300 meters/hr. When it landed, it was instructed to move as quickly as possible to Point B in case the mission malfunctioned. What is the distance between Point L and Point B, and how long did it take Curiosity to get there?

Answer: L(45,40) and B(50,35). Using the Pythagorean distance formula, $D = ((50-45)^2 + (35-40)^2)^{1/2}$ = **7 kilometers**. The time taken is just T = 7000 meters/300 m/h = **23 hours.**

Problem 2 – To the nearest kilometer, what is the distance directly from Point D to Point I as, and how long would it take Curiosity to make this trip without stopping to do any scientific research?

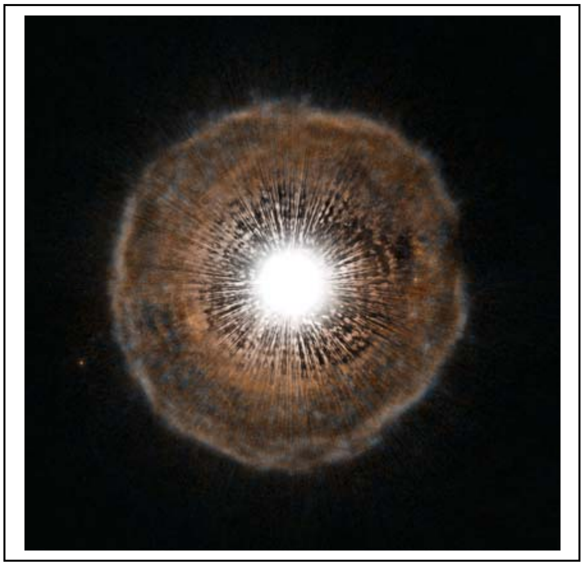
Answer: Point D(65,50) and Point I(5,10) than d = ((5-65)2 + (10-50)2)1/2 = **72 kilometers**. This takes T = 72000 meters/300 m/h = 240 hours or **10 days**.

Problem 3 – One possible path Curiosity might take connects all of the points in the sequence L-B-D-C-D-E-F-B-G-H-I. To the nearest kilometer, what is the total distance traveled, and to the nearest tenth, how many days would this journey take?

Answer: Using the distance formula between consecutive point pairs we get:
D(LB) = 7 km
D(BD) = 21 km
D(DC) = 19 km
D(CD) = 19 km
D(DE)= 29 km
D(EF) = 15 km
D(FB) = 14 km
D(BG)= 33 km
D(GH)=11 km
D(HI) = 15 km Total distance = 183 km, time = 183,000/300 = 610 hrs = **25.4 days**.

Problem 4 – Can you think of a different trip, and include a 3-day stay at each point along the way?

Answer: Students may take the points in any interesting order. One strategy is to do the most interesting points first to make sure that the mission science goals are achieved. Be cafreful of the big cliff between Points B and C!



This dramatic image taken by the Hubble Space Telescope reveals details in the shell of gas ejected by the star U Camelopardalis thousands of years ago.

Located 1,400 light years from our sun, the shell is expanding at a speed of about 25 km/sec, and its outer edge is about 500 billion km from the central star, whose image has been greatly over exposed making it seem huge in this image!

Although it looks impressive, the amount of mass in this shell is actually quite small. It is only about 1/10 the mass of our own planet Earth!

Problem 1 - The radius of the shell is 500 billion kilometers, and the estimated speed is about 25 km/sec. How many years did it take for the shell to get this big if 1 year = 31 million seconds?

Problem 2 – The mass of Earth is 6.0×10^{24} kg. The mass of a hydrogen atom is 1.6×10^{-27} kg. If the entire mass of the shell were evenly spread out in a sphere with the shell's radius, how many hydrogen atoms would you expect to find in a cubic meter of this shell to the nearest 10,000 atoms?

Problem 1 - The radius of the shell is 500 billion kilometers, and the estimated speed is about 25 km/sec. How many years did it take for the shell to get this big if 1 year = 31 million seconds?

Answer: Time = distance/speed
= 500 billion km / 25 km/sec
= 20 billion seconds

Since 1 year = 31 million seconds, Time = 20 billion/31 million = **645 years**.

Problem 2 – The mass of Earth is 6.0×10^{24} kg. The mass of a hydrogen atom is 1.6×10^{-27} kg. If the entire mass of the shell were evenly spread out in a sphere with the shell's radius, how many hydrogen atoms would you expect to find in a cubic meter of this shell to the nearest 10 million atoms?

Answer: Convert all lengths to meters: 5×10^{11} km x (1000 m/1km) = 5.0×10^{14} meters.

Volume = $4/3\ \pi\ R^3$ so V = $1.333 \times 3.141 \times (5.0 \times 10^{14})^3 = 5.2 \times 10^{44}$ meters^3.

Mass of shell is 1/10 Earth = $0.1 \times 6.0 \times 10^{24}$kg = 6.0×10^{23} kg.

Density = mass/volume
= 6.0×10^{23} kg/5.2×10^{44} m^3
= 1.1×10^{-21} kg/m^3

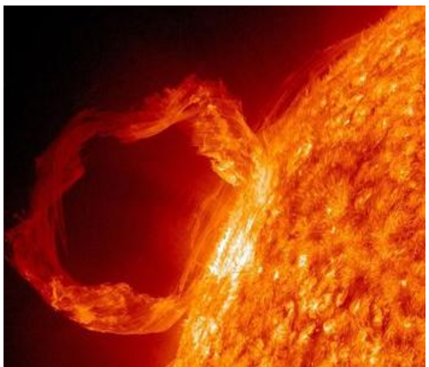
Since 1 hydrogen atom = 2.0×10^{-27} kg, the density corresponds to

N = 1.1×10^{-21} kg x (1 atom/2.0×10^{-27} kg) = 576,923 atoms/meter^3

Rounded to the nearest 10,000 atoms we get **580,000 atoms**.

Mass = 9.5×10^{31} kg
Volume = 5.0×10^{47} m^3

Mass = 5.1×10^{18} kg
Volume = 4.2×10^{18} m^3



Mass = 1.2×10^{25} kg
Volume = 6.0×10^{28} m^3

Mass = 2.0×10^{39} kg
Volume = 1.6×10^{60} m^3

In addition to mass and volume, density is the next most important feature of matter that we can easily determine. Density is just the mass of an object divided by its volume. Although density is usually given in terms of the unit kilograms/meter^3, in astronomy we prefer to use the number of particles per cubic meter. It is easy to imagine how a cubic meter might contain 1000 atoms, but the equivalent density of 1.6×10^{-24} kg/m^3 seems mysterious and doesn't provide much of a clue for how to think about it physically!

Problem 1 – Complete the table below by calculating the density of each astronomical object in terms of atoms per cubic meter.

Name	Volume (m^3)	Mass (kg)	Density (atoms/m^3)
Atmosphere of Earth	4.2×10^{18}	5.1×10^{18}	
Red supergiant star	2.3×10^{33}	4.0×10^{31}	
Surface of our Sun	6.0×10^{28}	1.2×10^{25}	
Atmosphere of Moon	1.9×10^{15}	1.0×10^{4}	
Solar Corona	9.0×10^{26}	8.9×10^{13}	
Interstellar Cloud	5.0×10^{47}	9.5×10^{31}	
Orion Nebula	6.2×10^{51}	1.5×10^{33}	
Solar Wind	1.4×10^{34}	4.5×10^{14}	
Milky Way galaxy	1.6×10^{60}	2.0×10^{39}	
Van Allen radiation belts	1.3×10^{23}	1.1×10^{-2}	

Problem 1 – Complete the table below by calculating the density of each astronomical object in terms of atoms per cubic meter.

Answer: example Solar Corona:

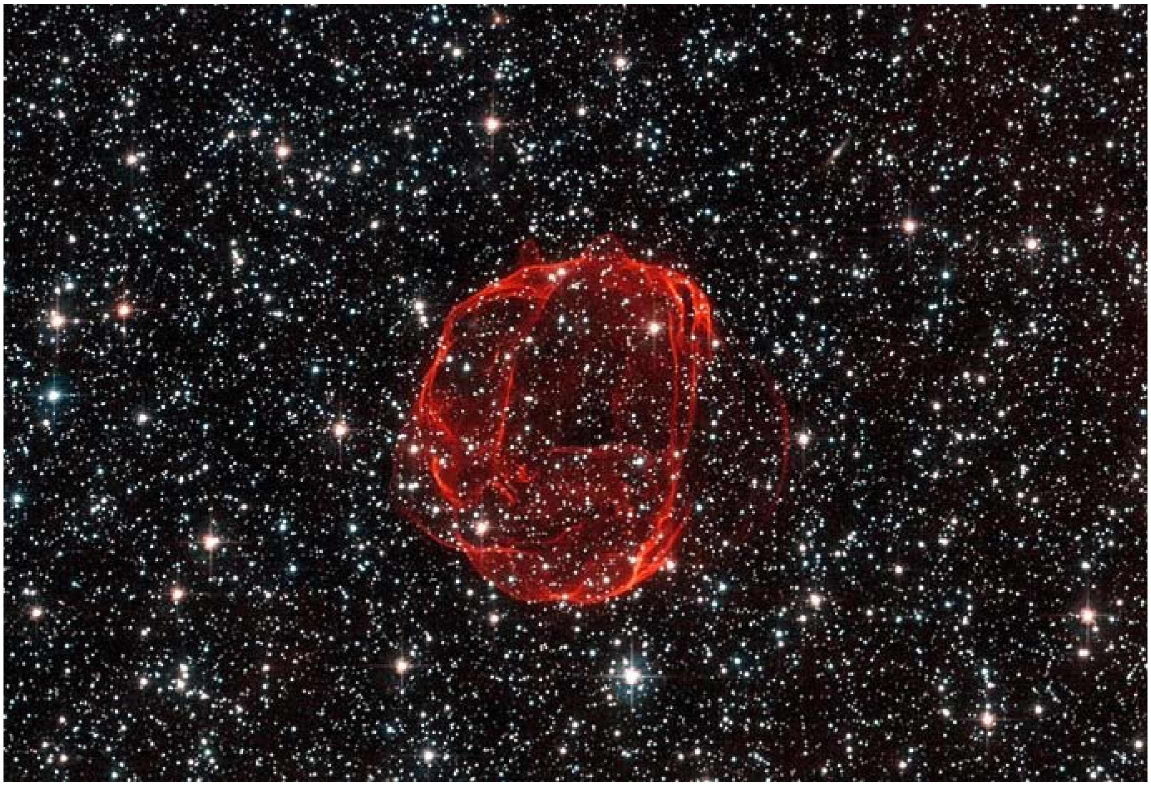
$D = M/V = 8.9\text{x}10^{13}$ kg/$9.0\text{x}10^{26}$ m^3 = $9.9\text{x}10^{-14}$ kg/m^3

$N = 9.9\text{x}10^{-14} / 1.6\text{x}10^{-27} = 6.2\text{x}10^{13}$ atoms/m^3

Name	Volume (m^3)	Mass (kg)	Density (atoms/m^3)
Atmosphere of Earth	$4.2\text{x}10^{18}$	$5.1\text{x}10^{18}$	$7.5\text{x}10^{26}$
Red supergiant star	$2.3\text{x}10^{33}$	$4.0\text{x}10^{31}$	$1.1\text{x}10^{25}$
Surface of our Sun	$6.0\text{x}10^{28}$	$1.2\text{x}10^{25}$	$1.3\text{x}10^{23}$
Atmosphere of Moon	$1.9\text{x}10^{15}$	$1.0\text{x}10^{4}$	$3.3\text{x}10^{15}$
Solar Corona	$9.0\text{x}10^{26}$	$8.9\text{x}10^{13}$	$6.2\text{x}10^{13}$
Interstellar Cloud	$5.0\text{x}10^{47}$	$9.5\text{x}10^{31}$	$1.2\text{x}10^{11}$
Orion Nebula	$6.2\text{x}10^{51}$	$1.5\text{x}10^{33}$	$1.5\text{x}10^{8}$
Solar Wind	$1.4\text{x}10^{34}$	$4.5\text{x}10^{14}$	$2.0\text{x}10^{7}$
Milky Way galaxy	$1.6\text{x}10^{60}$	$2.0\text{x}10^{39}$	$8.1\text{x}10^{4}$
Van Allen radiation belts	$1.3\text{x}10^{23}$	$1.1\text{x}10^{-2}$	53

Students can also be asked to compare how many times more dense is Object A than Object B? Example: Atmosphere of Moon / Milky Way galaxy = $3.3\text{x}10^{15}/8.1\text{x}10^{4}$ = 41 billion times!

A Distant Supernova Remnant Discovered



These delicate wisps of gas make up an object known as supernova remnant SNR 0519. The thin, blood-red shells are actually the remnants from when an unstable star exploded violently as a supernova around 600 years ago. SNR 0519 is located over 150,000 light-years from Earth in the southern constellation of Dorado, a constellation that also contains most of our neighboring galaxy, which is called the Large Magellanic Cloud. One light year equals about 10 trillion kilometers.

Problem 1 – The diameter of this supernova remnant shell is about 24 light years. The distance from our sun to the nearest star Alpha Centauri is 4.3 light years. If the sun were placed at the center of the supernova shell, about where would Alpha Centauri be at the same scale?

Problem 2 – The star that produced this shell exploded 600 years ago. If there are about 30 million seconds in one year, how fast was the shell traveling in kilometers/second expressed A) as a simplified fraction? B) As a decimal number?

http://www.nasa.gov/mission_pages/hubble/science/snr-0519.html

Hubble Sees the Remains of a Star Gone Supernova

May 3, 2013

Problem 1 – The diameter of this supernova remnant shell is about 24 light years. The distance from our sun to the nearest star Alpha Centauri is 4.3 light years. If the sun were placed at the center of the supernova shell, about where would Alpha Centauri be at the same scale?

Answer: **About 1/3 of the way from the center to the edge of the shell**.

Problem 2 – The star that produced this shell exploded 600 years ago. If there are about 30 million seconds in one year, how fast was the shell traveling in kilometers/second expressed A) as a simplified fraction? B) As a decimal number?

Answer: The center of the shell is 12 light years from the edge, so the distance traveled in 600 years is 12 x 10 trillion km, or 120 trillion kilometers.

Since 600 years equals 600x30 million seconds = 18 billion seconds,

the shell travels 120 trillion km / (18 billion seconds)
= 120,000 billion/18 billion
= 120,000/18

A) As a simplified fraction $\frac{\mathbf{20000}}{\mathbf{3}}$ **kilometers per second**

B) As a decimal: 120000/18 = **6,666 kilometers per second**.

This speed is about 100 times faster than the Space Station orbiting Earth.

An image from an instrument aboard NASA's Landsat Data Continuity Mission or LDCM satellite may look like a typical black-and-white image of a dramatic landscape, but it tells a story of temperature. The dark waters of the Salton Sea are shown in the semi-circle on the left-hand edge of the image. Crops create a checkerboard pattern stretching south to the Mexican border.

The size of this image is 26 km wide and 17 km tall. Each green square represents a planted crop measuring 160 meters on a side and an area of about 6 acres.

Problem 1 - What percentage of the total area of this image is occupied by planted crops?

Problem 2 – What percentage of all the farmed areas actually have growing crops?

Problem 3 – The annual rain fall is about 3 inches per year (0.076 meters/yr). If one gallon of water has a volume of 0.0038 $meters^3$, how many gallons of water fall on the planted crop area each year?

New NASA Satellite Takes the Salton Sea's Temperature
April 22,2013
http://www.nasa.gov/mission_pages/landsat/news/salton-sea.html

Problem 1 - What percentage of the total area of this image is occupied by planted crops?

Answer: The total area of this image is 26 km x 17 km = 442 km^2.
Students should count the number of green squares to tally the number of planted areas. A typical number would be about 50, so the total planted area is 50 x 0.16 km x 0.16 km = 1.3 km^2. The percentage of the total area is then 100% x 1.3/442 = **0.3 %.**

Problem 2 – What percentage of all the farmed areas actually have growing crops?

Answer: This is a bit more difficult because students have to count all of the square patches that they can see in the image, not just the green ones. A typical answer would be about 100 patches, so the total number of green + brown patches is about 150, and so the percentage of the planted areas is 100% x 50/150 = **33% or 1/3**.

Problem 3 – The annual rain fall is about 3 inches per year (0.076 meters/yr). If one gallon of water has a volume of 0.0038 $meters^3$, how many gallons of water fall on the planted crop area each year?

Answer:

From Problem 1, the total planted area is 1.3 km^2 or $1.3x10^6$ $meters^2$. If the rain covers a depth of 0.076 meters each year, the rain volume is just $1.3x10^6$ x 0.076 = 98800 cubic meters. This equals 98800 $meters^3$ x (1 gallon/0.0038 m^3) = **26 million gallons each year.**

Giant Gas Cloud in System NGC 6240

Scientists have used Chandra to make a detailed study of an enormous cloud of hot gas enveloping two large, colliding galaxies. This unusually large reservoir of gas contains as much mass as 10 billion Suns, spans about 300,000 light years, and radiates at a temperature of more than 7 million degrees.

This giant gas cloud, which scientists call a "halo," is located in the system called NGC 6240. Astronomers have long known that NGC 6240 is the site of the merger of two large spiral galaxies similar in size to our own Milky Way. Each galaxy contains a supermassive black hole at its center. The black holes are spiraling toward one another, and may eventually merge to form a larger black hole.

Problem 1 - If 1 light year equals 9.5×10^{15} meters, and the cloud is in the shape of a sphere with a diameter of 300,000 light years, what is the volume of this cloud in cubic meters? ($\pi = 3.141$)

Problem 2 - The mass of the sun is 2.0×10^{30} kilograms. What is the density of this cloud in kilograms/m^3?

Problem 3 - If a single hydrogen atom has a mass of 1.7×10^{-27} kilograms, how many hydrogen atoms per cubic meter does the gas density represent?

Giant Gas Cloud in System NGC 6240
April 30, 2013
http://www.nasa.gov/mission_pages/chandra/multimedia/ngc6240.html

Problem 1 - If 1 light year equals 9.5 x 10^{15} meters, and the cloud is in the shape of a sphere with a diameter of 300,000 light years, what is the volume of this cloud in cubic meters? (π = 3.141)

Answer: V = 4/3 π R^3 so

V = 1.333 (3.141) (150,000 x 9.5x1015 meters)^3

= **1.2 x 10^{64} meters^3**

Problem 2 - The mass of the sun is 2.0 x 10^{30} kilograms. What is the density of this cloud in kilograms/m^3?

Answer: Mass of cloud = 10 billion suns = 10^{10} x 2.0x10^{30} kg = 2.0x10^{40} kg.

Density = mass/volume

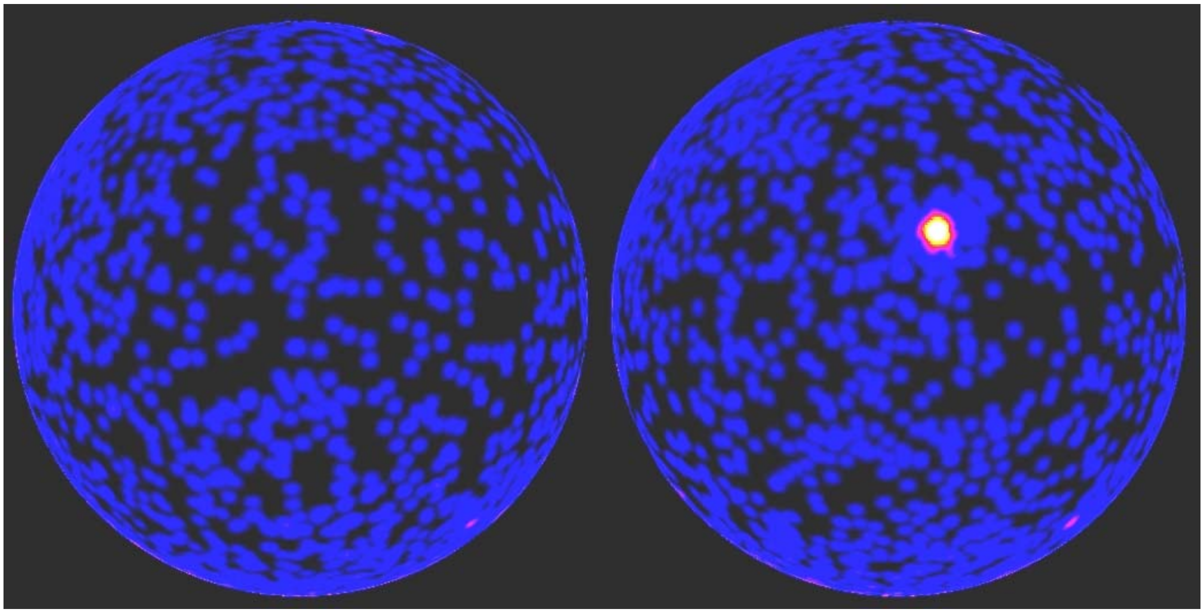
= 2.0x10^{40} kg / 1.2x10^{64} m^3

= **1.7x10^{-24} kg/m^3**

Problem 3 - If a single hydrogen atom has a mass of 1.7x10^{-27} kilograms, how many hydrogen atoms per cubic meter does the gas density represent?

Answer: Density = 1.7x10^{-24} kg/m^3 x (1 atom/1.7x10^{-27} kg)

= **1000 atoms/meter^3**



In April, 2013, a record-setting blast of gamma rays from a dying star in a distant galaxy wowed astronomers around the world. The eruption, which was classified as a gamma-ray burst, or GRB, and designated GRB 130427A, produced the highest-energy light ever detected from such an event. Fermi's Large Area Telescope (LAT) detected the gamma-ray emission from the burst, which lasted for hours, and it remained detectable for the better part of a day, setting a new record for the longest gamma-ray emission from a GRB. The event occurred 3.6 billion light years from Earth in the direction of the constellation Leo. The estimated energy flux from this event at the location of Earth was about F = 2.0×10^{-8} watts/meter^2.

Problem 1 – Although instruments at Earth can measure how many watts of energy were detected over an area of 1 square meter, called the radiant flux, they would really like to know how much energy in watts, was released by the source of the event. Suppose that the same flux of energy flowed through every square meter of surface, of a sphere whose radius equaled the distance to the source of 3.6 billion light years (1 light year = 9.5×10^{15} meters). What was the total power emitted by the source into space in A) watts, B) solar units where 1 solar unit = 4.0×10^{26} watts)?

Problem 2 - Instead of the energy emitted across the entire sky like our sun does, astronomers think that the energy from a gamma-ray burst is emitted in two narrow beams that come from the core of the star as it becomes a supernova. If the area of each beam on the sky is only 1 square degree, and the full area of the sky is 42,000 square degrees, what will be the estimated total power of the source in watts and in solar units?

NASA's Fermi, Swift See 'Shockingly Bright' Burst
May 3, 2013
http://www.nasa.gov/topics/universe/features/shocking-burst.html

Problem 1 – Although instruments at Earth can measure how many watts of energy were detected over an area of 1 square meter, called the radiant flux, they would really like to know how much energy in watts, was released by the source of the event. Suppose that the same flux of energy flowed through every square meter of surface, of a sphere whose radius equaled the distance to the source of 3.6 billion light years (1 light year = 9.5×10^{15} meters). What was the total power emitted by the source into space in A) watts, B) solar units where 1 solar unit = 4.0×10^{26} watts)?

Answer: The surface area of a sphere with a radius of 3.6 billion light years is
R = 3.6 billion ly x (9.5×10^{15} meters/ly) = 3.4×10^{25} meters.

Then S = 4 (3.141) ($3.4 \times 10^{25})^2$ = 1.45×10^{52} meters^2.

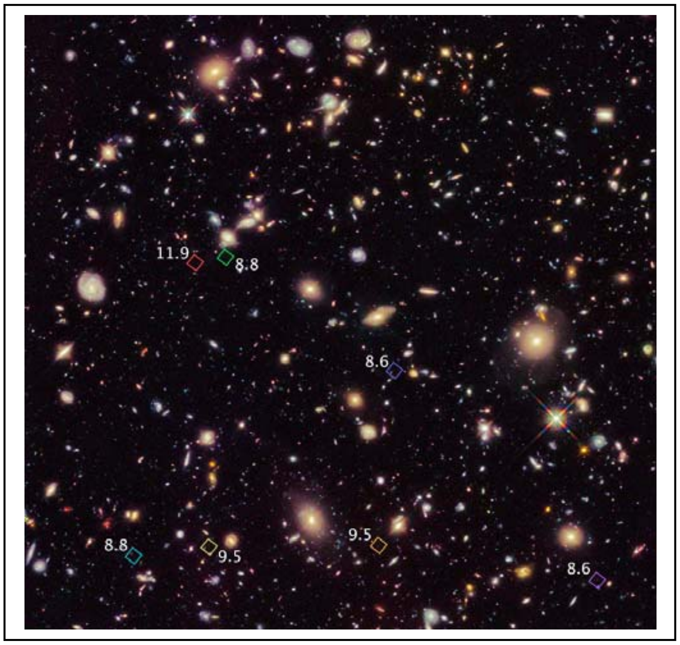
A) The total power = 2.0×10^{-8} watts/m^2 x 1.45×10^{52} m^2 = **2.9×10^{44} watts**.

B) 2.9×10^{44} watts x (1 solar unit / 4.0×10^{26} watts) = **7.3×10^{17} solar units**.

Note: If the source radiation was emitted over the entire surface of the sphere, it would equal the energy produced each second by 730 thousand trillion stars like our sun!

Problem 2 - Instead of the energy emitted across the entire sky like our sun does, astronomers think that the energy from a gamma-ray burst is emitted in two narrow beams that come from the core of the star as it becomes a supernova. If the area of each beam on the sky is only 1 square degree, and the full area of the sky is 42,000 square degrees, what will be the estimated total power of the source in watts and in solar units?

Answer: Each beam only emits energy into 1/42,000 of the full sky, so the area on the spherical surface for each beam is only 1/42000 x 1.45×10^{52} meters^2 or 3.5×10^{47} meters^2. For both beams the total area is 2 x 3.5×10^{47} meters^2 or 7.0×10^{47} meters^2. The total power from the gamma-ray source is then 2.0×10^{-8} watts/meter^2 x 7.0×10^{47} meters^2 = **1.4×10^{40} watts**. This equals the power from **35 trillion suns**!



Using the Ultra Deep Field Image created by NASA's Hubble Space Telescope, astronomers have uncovered a previously unseen population of seven primitive galaxies that formed more than 13 billion years ago.

The age of the universe is 13.7 billion years, so we are seeing these galaxies as they were when the universe was only 4 percent of its present age. This is like an 80 year old human seeing a picture of themselves when they were only 3 years old!

Among the predictions of Big Bang cosmology is a mathematical relationship T(Z) between the redshift of a galaxy denoted by the variable Z, and the time since the light was emitted by that galaxy, T, so that it can arrive at Earth today, some 13.7 billion years after the Big Bang occurred. During this time, the space within the universe has expanded, and Z indicates the amount of stretching of space that has occurred between the time when the light was emitted and the current era.

Using the exact solution for T(Z) from Big Bang theory, an astronomer wants to create a faster way to compute T(Z) that he can use on his hand calculator. He derives an approximation to the function T(Z) for Z between 2 and 15 given by

$$T(Z) = 0.000154z^5 - 0.0072Z^4 + 0.1301Z^3 - 1.143Z^2 + 5.0141Z + 3.7677$$

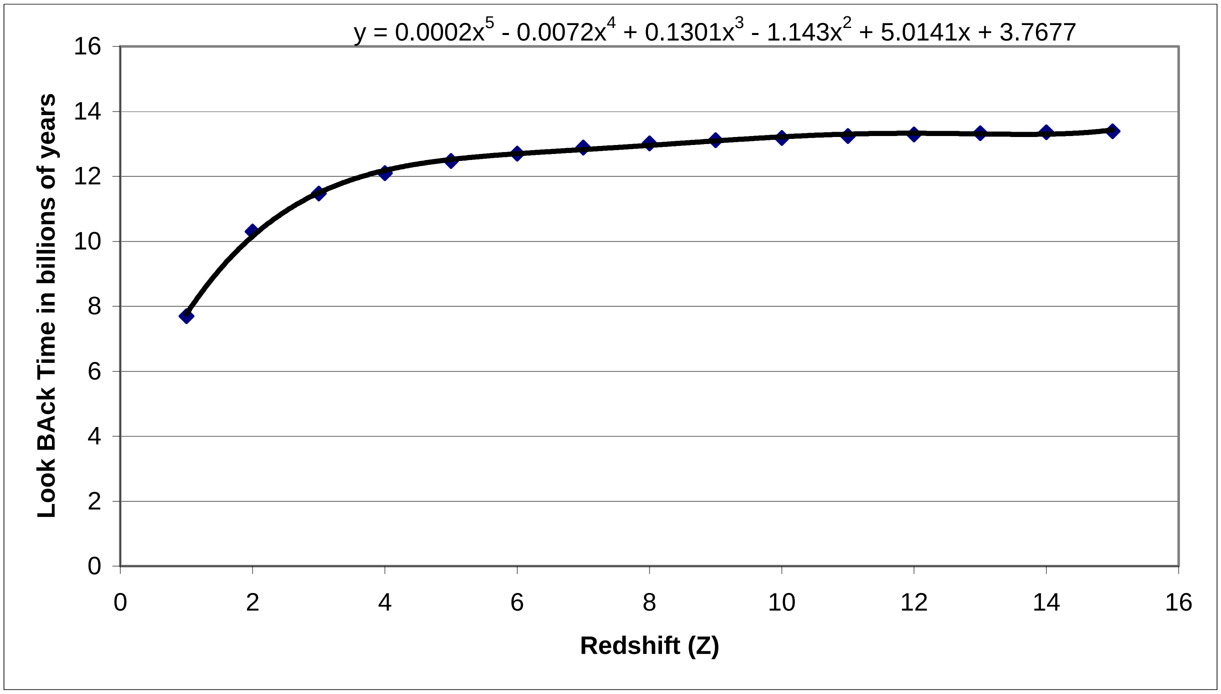
Problem 1 - Graph this function over the redshift range Z:[2.0, 15.0].

Problem 2 - Most of the galaxy redshift measurements the astronomer will make will be made over the much smaller range from 9.0 to 12.0. What is the slope of T(Z) over the range Z:[9.0, 12.0]?

Problem 3 - What is the linear equation that matches the redshift formula over Z:[9.0, 12.0]?

Problem 4 - If the astronomer uses the 'linear approximation' to T(Z) over Z:[9.0, 12.0] by what percentage will his estimates for the look-back time T differ from the original equation over the range Z:[5.0 ,15.0]?

Problem 1 - Graph this function over the redshift range Z:[2.0, 15.0].



Problem 2 - Most of the galaxy redshift measurements will be made over the much smaller range from 9.0 to 12.0. What is the slope of T(Z) over the range Z:[9.0, 12.0]?

Answer: change in Z = 12-9 = 3.0, T(12) = 13.18 billion years T(9)= 13.00 billion years, Slope M = 13.18-13.00/3.0 = 0.06.

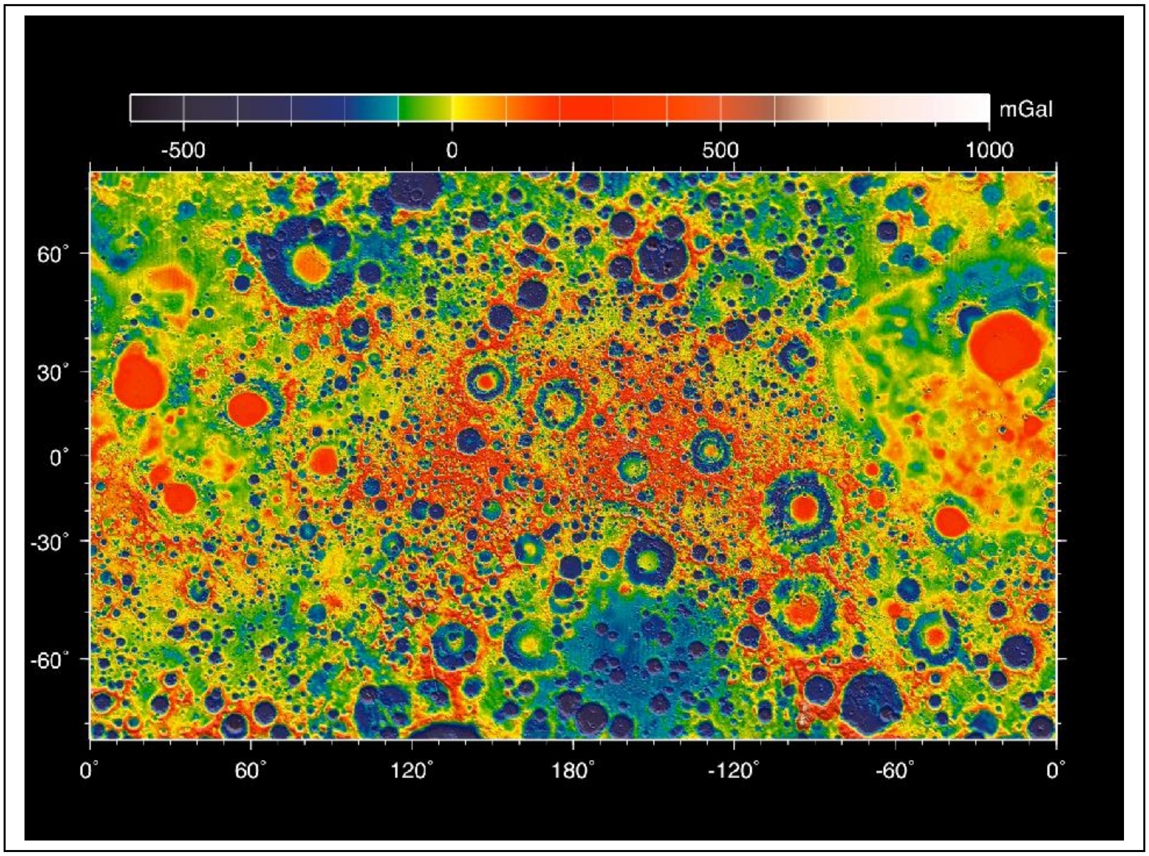
Problem 3 - What is the linear equation that matches the redshift formula over Z:[9.0, 12.0]?

Answer: T = mZ + b, T(9) = 13.00 and T(12) = 13.18. Use the two-point formula: y-y1 - (x-x1) (y2-y1)/(x2-x1) then T - 13.00 = (z-9) (13.18-13.00)/(12-9) so T = 13.00 +(0.06(z-9) and so **T(Z) = 12.46 + 0.06Z**.

Problem 4 - If the astronomer uses the new 'linear approximation' to T(Z) over Z:[9.0, 12.0] by what percentage will his estimates for the look-back time T differ from the original equation over the range Z:[5.0 ,15.0]?

Z	Actual	Predicted	Difference	Percent
5	12.47	12.76	- 0.29	2.3%
15	13.39	13.36	+0.03	0.2%

So using the new, faster to compute, linear approximation only gives answers that differ by no more than 3% from the original, slower to compute, fifth-order polynomial approximation for T(Z).



During 2012, NASA's twin Grail satellites orbited the moon at altitudes of only 30 km. As they traveled, minute changes in their speeds tracked from Earth revealed changes in the gravitational field of the moon. These changes could be mapped, and revealed density changes in the lunar surface below them. In this way, scientists could look hundreds of kilometers beneath the lunar surface and explore how the surface was formed billions of years ago! On Earth, the acceleration of gravity is 9,807 cm/sec^2. The normal acceleration of gravity on the average lunar surface is 1620 cm/sec^2, but in the blue regions of the map this is as low as 1520 cm/sec^2, and in the red regions it is as high as 1920 cm/sec^2. A pendulum clock has a swinging period, T in seconds, given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum in centimeters, and g is the acceleration of gravity in cm/sec^2.

Problem 1 - A lunar colony in a lunar 'blue' area has a Blue Clock with a pendulum length L = 100 cm. What is the swing period? (use π = 3.141)

Problem 2 - A lunar colony in a lunar 'red' area has an identical Red Clock. What is the swing period? (use π = 3.141)

Problem 3 - After how many swings on the Blue Clock will the clocks differ in time by 1 hour?

Problem 4 - If both clocks were synchronized to 1:00:00 am local time, what will the time on the Blue Clock and the Red Clock be when the two colony clocks are off by 1 hour relative to each other?

Problem 1 - A lunar colony in a lunar 'blue' area has a Blue Clock with a pendumum length L = 100 cm. What is the swing period?

Answer: $T = 2 (3.141) (100/1520)^{1/2}$ = **1.61 seconds**.

Problem 2 - A lunar colony in a lunar 'red' area has an identical Red Clock. What is the swing period?

Answer; $T = 2 (3.141) (100/1920)^{1/2}$ = **1.43 seconds**.

Problem 3 - After how many swings on the Blue Clock will the clocks differ in time by 1 hour?

Answer: Each swing on the slower Blue Clock pendulum is behind the faster Red Clock by 1.61 - 1.43 = 0.18 seconds. We want this difference to be 3600 seconds in 1 hour, which will take N = 3600/0.18 = **20,000 swings** on the Blue Clock.

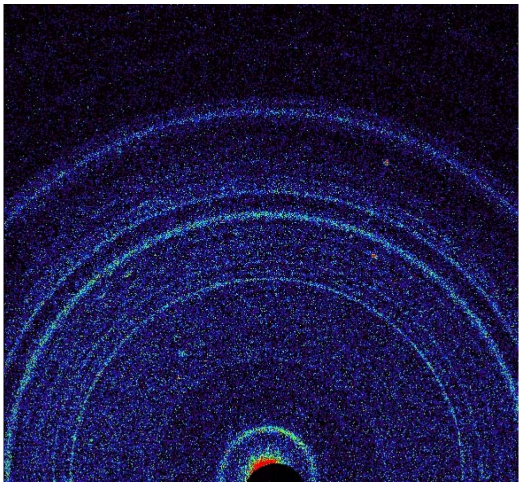
Problem 4 - If both clocks were synchronized to 1:00:00 am local time, what will the time on the Blue Clock and the Red Clock be when the two colony clocks are off by 1 hour relative to each other?

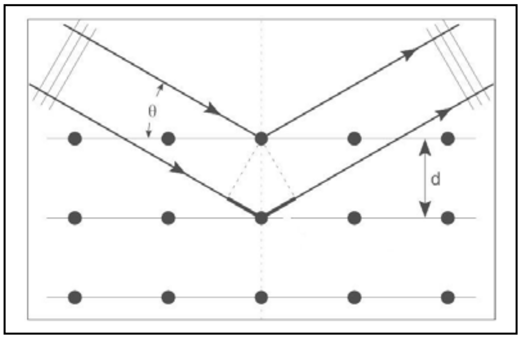
Answer: On the Blue Clock, 20,000 swings have to pass, each taking 1.61 seconds for a total time of 32,200 seconds or 8 hours, 56 minutes, 40 seconds. So the time on the Blue Clock will read **09:56:40 am local time**.

On the Red Clock, because after 20,000 swings it is exactly 1 hour behind the Blue Clock, its time will read 08:56:40 am local time. Another 'long way' to see this is that we still need 20,000 swings to add up to a 1 hour time difference, but on the Red Clock each swing is only 1.43 seconds long and so this takes 28,600 seconds or 7 hours, 56 minutes, 40 seconds. The time on the Red Clock will be **08:56:40 am local time**.

This is why colonists will NOT be using pendulum clocks on the moon!!

Note: Devices that act like pendulum clocks were once used by prospectors on Earth to search for oil and other valuable materials below ground before the advent of more accurate magnetometer-based technology. Minute changes in the pendulum period indicate changes in the density of rock below ground and these can be used to identify high-gravity, density regions (like iron ore) or low-gravity regions (like caverns). Another way to measure minute gravity changes is by the shape of a satellite orbit, or by the subtle changes in speed between two satellites on the same orbit. Lunar scientists used this orbit method with the two Grail spacecraft only 200 kilometers apart.





The Curiosity Rover recently used a technique called X-ray Diffraction Crystallography to determine the identity of compounds found in a rock sample on the surface of Mars. The image to the left shows what this data looks like. The exact radii of these rings, and the locations of spots along these rings, serve as a fingerprint of the shape of the mineral compound in space. We all know how human fingerprints work, and even 'DNA' fingerprinting is commonly mentioned in TV programs like NCIS or CSI. But how does this technique work?

The figure to the left shows a beam of light striking the surface of a crystal with 15 atoms arranged into three parallel planes. The light strikes the atoms and is 'diffracted' into a new direction defined by the angle θ.

If two beams of light are out-of-phase by 90 degrees, when they are added together, the crests of one wave interfere with the troughs of the other wave and you end up with no light. If they are in-phase, they will add together, you get the light intensified and you also get a ring of light!

The diagram shows the added distance that the lower ray gains by being diffracted through the angle θ.

Problem 1 – From the information in the diagram, what is the extra distance, s, traveled by the x-ray light in a crystal lattice where the planes are separated by a distance d?

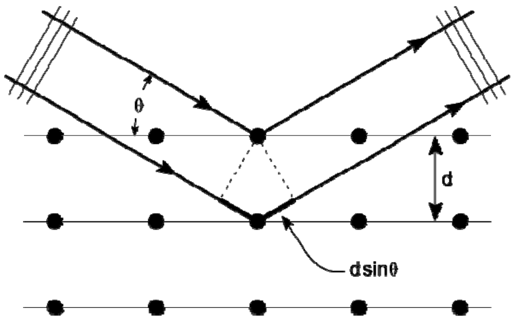
Problem 2 - If the light struck the crystal exactly face-on ($\theta=90^o$) how much extra distance would the second beam travel compared to the first beam that was reflected only from the top surface?

Problem 3 – If the wavelength of the x-ray light is L, what is the relationship between L and s so that the wave crests exactly match up?

Problem 4 – Suppose that in the Curiosity data, a diffraction ring is detected at an angle of incidence $\theta=2^o$. The Curiosity instrument uses X-rays with an energy of 6.929 keV, which have a wavelength of 1.79×10^{-10} meters. What is the separation, d, of the crystal planes in the mineral sample?

Problem 1 – From the information in the diagram, what is the extra distance, L, traveled by the x-ray light in a crystal lattice where the planes are separated by a distance d?

Answer: From the diagram below, **s = 2d sin θ**



Problem 2 - If the light struck the crystal exactly face-on ($\theta=90^{O}$) how much extra distance would the second beam travel compared to the first beam that was reflected from the top surface?

Answer: **s = 2d**

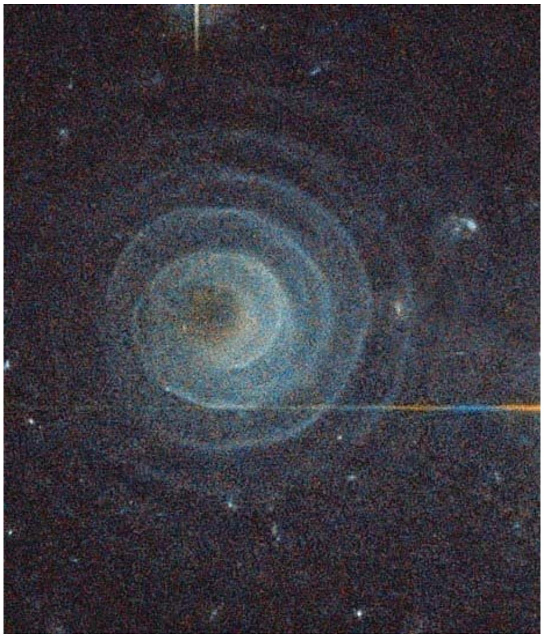
Problem 3 – If the wavelength of the x-ray light is L, what is the relationship between L and s so that the wave crests exactly match up?

Answer: The wavelength is L and for the waves to exactly match up, the two waves can either be shifted by s=0, or by **one full wavelength** s = L. So the first non-zero condition is that L = 2 d sinθ and so

$$\mathbf{d = \frac{L}{2\sin\theta}}$$

Problem 4 – Suppose that in the Curiosity data, a diffraction ring is detected at an angle of incidence $\theta=2^{O}$. The Curiosity instrument uses x-rays with an energy of 6.929 keV, which have a wavelength of 1.79×10^{-10} meters. What is the separation, d, of the crystal planes in the mineral sample?

Answer: d = 1.79×10^{-10} meters/($2\sin 2^{O}$) so d = **2.56×10^{-9} meters**.



Courtesy Advanced Camera/ HST (2010)

This remarkable picture captures the formation of an unusual spiral nebula around the star LL Pegasi located 3,000 light years from the sun in the constellation of Pegasus (the Winged Horse). The spiral is made the same way that an 'arc' of water is produced by a revolving water sprinkler on your lawn.

The picture shows a thin spiral pattern winding around the star, which is itself hidden behind a cloud of thick dust at the center. The spiral is thought to arise because LL Pegasi is a binary system, with the star that is losing material and a companion star orbiting each other. The spacing between layers in the spiral is equal to the 800-year orbit period of the binary.

Problem 1 – If the expansion speed is 50,000 km/hour, how far does the gas travel every year?

Problem 2 - Draw a line from the center of the spiral to the edge of the picture in any direction. How far apart will the successive 'shells' of gas be along this axis?

Problem 3 - Find a direction where the number of shells is a maximum. How many years did it take for the gas to reach the farthest shell you can see?

Problem 4 - Suppose that the brightness of each shell decreases according to the function $B = 128\ N^{-3/2}$ where N is the shell number. If the sky has a brightness of B=2 in these units, what is the maximum number of shells that you could count in a picture?

Problem 5 - About how long ago was the last detectable shell in Problem 4 emitted by the binary system?

Problem 1 – If the expansion speed is 50,000 km/hour, how far does the gas travel every year?

Answer: 50,000 km/hour x (24 hours / 1 day) x (365 days / 1 year) = **438 million km**.

Problem 2 - Draw a line from the center of the spiral to the edge of the picture in any direction. How far apart will the successive 'shells' of gas be along this axis?

Answer; the gas travels 438 million km/yr x 800 yrs = **350 billion kilometers**.

Problem 3 - Find a direction where the number of shells is a maximum. How many years did it take for the gas to reach the farthest shell you can see?

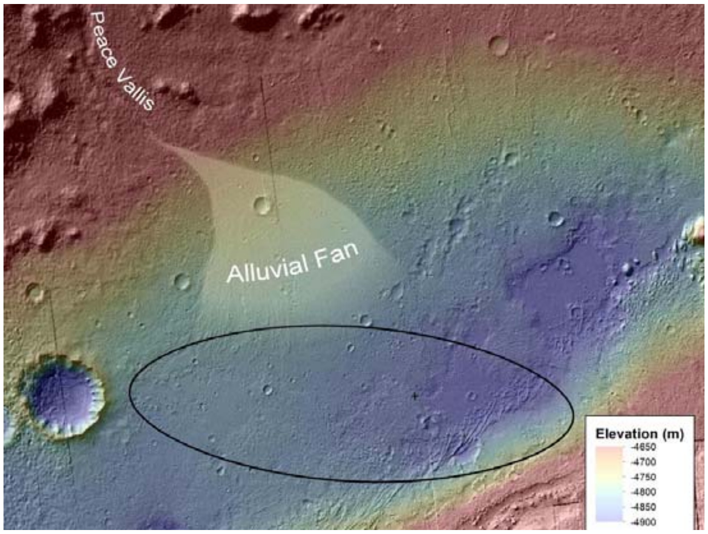
Answer; If students count 6 shells, then 6 x 800 years = **4800 years**.

Problem 4 - Suppose that the brightness of each shell decreases according to the function $B = 128\ N^{-3/2}$ where N is the shell number. If the sky has a brightness of B=2 in these units, what is the maximum number of shells that you could count in a picture?

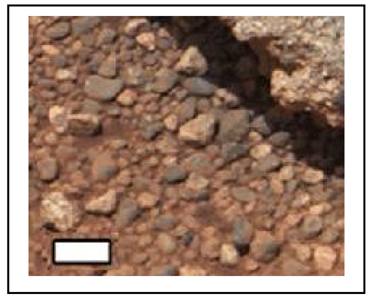
Answer $N^{3/2} = 128/2$ so $N^{3/2} = 2^6$ and $N = (2^6)^{2/3}$ so $\mathbf{N = 16}$

Problem 5 - About how long ago was the last detectable shell in Problem 4 emitted by the binary system?

Answer: 800 years x (16) = **12,800 years ago**!



The Curiosity Rover discovered rounded pebbles near its original landing site marked with the 'X' in the figure. The figure also shows the elevation changes in this area. Here is what the pebbles looked like! The white bar is 1 cm long.



Geologists studying the pebbles and the landscape believe that the water flow that moved and rounded the pebbles was at least ankle deep and perhaps waist deep. As on Earth, the pebbles were carried by fast moving water and over time became rounded by the constant scraping and bouncing. How fast was the water moving?

Calculating the stream gradient:

Problem 1 – The landing ellipse is 18 km wide. To the nearest kilometer, how far is the Curiosity 'X' from the apex of the alluvial fan near Peace Vallis?

Problem 2 – What is the change in elevation, h, between the alluvial fan vertex and the 'X'.?

Problem 3 – The stream gradient is defined as the elevation difference divided by the distance traveled. What is the stream gradient, SG, in units of meters/meters, for the water which left Peace Vallis and flowed down the alluvial fan?

ADVANCED) Calculating the stream flow speed:

Problem 4 – On Mars, the stream flow can be approximated by $V = (2gh)^{1/2} \sin(\theta)$ where $\tan(\theta) = SG$, $g = 3.8$ meters/sec^2 is the acceleration of gravity on Mars, and h is the difference in elevation of the top and bottom of the stream. About how fast was the water flowing past the Curiosity landing area to create the pebbles?

Calculating the stream gradient:

Problem 1 – The landing ellipse is 18 km wide. To the nearest kilometer, how far is the Curiosity 'X' from the apex of the alluvial fan near Peace Vallia? Answer: **About 15 km**.

Problem 2 – What is the change in elevation, h, between the alluvial fan vertex and the 'X'.? Answer: -4650 m – (-4900 m) so h = **250 meters.**

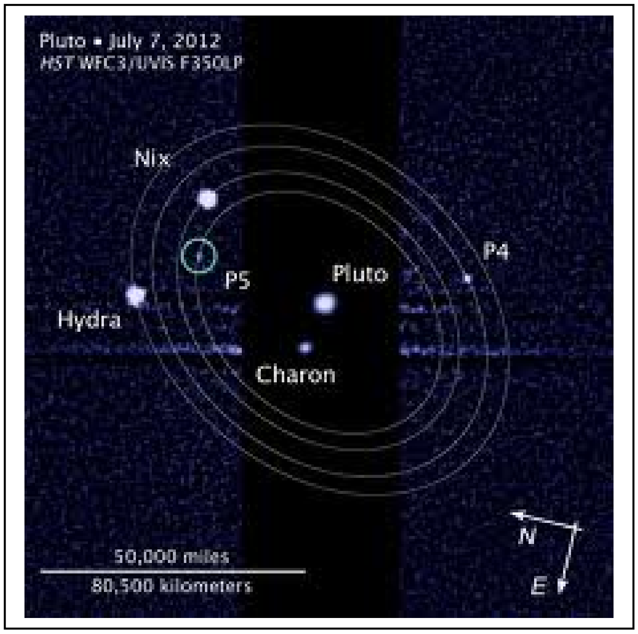
Problem 3 – The stream gradient is defined as the elevation difference divided by the distance traveled. What is the stream gradient, SG, in units of meters/meters, for the water which left Peace Vallis and flowed down the alluvial fan? 250 meters / 15 km = **17 meters/kilometer or SG=0.017 meters/meter.**

Calculating the stream flow speed:

Problem 4 – On Mars, the stream flow can be approximated by $V = (2gh)^{1/2} \sin(\theta)$ where $\tan(\theta)$ = SG, and g = 3.8 meters/sec^2 is the acceleration of gravity on Mars. About how fast was the water flowing past the Curiosity landing area to create the pebbles?

Answer: h = 650 meters, SG=0.017, g = 3.8 meters/sec^2 . Then θ = 1.0 degrees.
$V = (2 \times 3.8 \times 650)^{1/2} \sin(1.0)$
V = 1.2 meters/sec.

This is about as fast as a human walking very slowly (about 0.3 miles/hr or 4.3 km/hr).



The most distant well-known object in our solar system, Pluto, is an irresistible object for Hubble Space Telescope investigations.

In July, 2012, Hubble scientists released an image of Pluto showing a new moon called P5. It is irregularly shaped and about 15 km in diameter, and probably made from ice.

Its average orbit radius is 47,000 km and appears to lie in the same orbit plane as the other four moons, and takes about 20 days to make one orbit.

The table below gives the details for the four new moons of Pluto as of 2012.

Name	Discovery	Diameter	Distance	Period
		(km)	(km)	(hours)
Pluto V	2012	10 to 25	42,000	485
Nix	2005	46 to 137	48,700	598
Pluto IV	2011	13 to 34	59,000	770
Hydra	2005	61 to 167	65,000	917

Problem 1 – Compute for each moon the cube of the distance,D^3, and the square of the period, P^2. Calculate the value $R = D^3/P^2$ for each moon. What do you notice about the values for R? What is the average value for R using the data from the five moons?

Problem 2 – Suppose future observations discover a new moon, P6, orbiting at a distance of 35,000 km from Pluto. What would you predict as the orbit period for this satellite?

Problem 3 - The mass of a body can be determined from Kepler's Third Law, which you verified in Problem 1. By using the formula $M = 6.9\times10^{10}$ R, where R is in units of $km^3/days^2$, what is the mass of Pluto in kilograms?

Name	Discovery	Diameter	Distance	Period	R
		(km)	(km)	(hours)	
Pluto V	2012	10 to 25	42,000	485	3.15×10^8
Nix	2005	46 to 137	48,700	598	3.23×10^8
Pluto IV	2011	13 to 34	59,000	770	3.46×10^8
Hydra	2005	61 to 167	65,000	917	3.27×10^8

Problem 1 – Compute for each moon the cube of the distance, D^3, and the square of the period, P^2. Calculate the value $R = D^3/P^2$ for each moon. What do you notice about the values for R? What is the average value for R using the data from the five moons?

Answer: The values for R are very similar to each other even though there is a large range in orbit distances and periods**. The average value is 3.28×10^8 km^3/hours^2**

Problem 2 – Suppose future observations discover a new moon, P6, orbiting at a distance of 35,000 km from Pluto. What would you predict as the orbit period for this satellite in days?

Answer: $P^2 = (35000)^3/3.28 \times 10^8 = 130{,}716$ so
P = 361 hours or **15.1 days**.

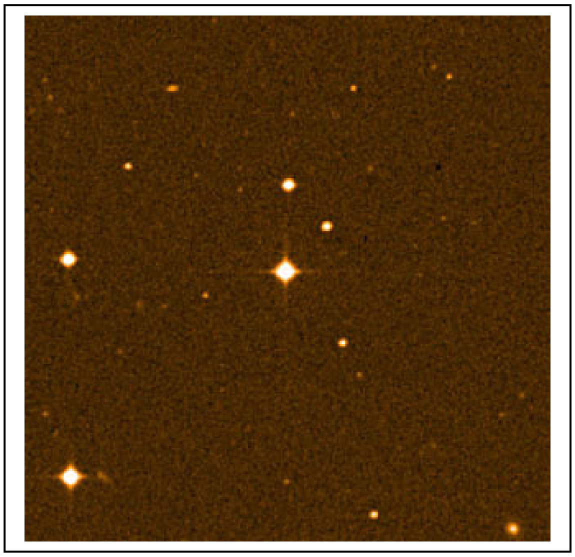
Problem 3 - The mass of a body can be determined from Kepler's Third Law, which you verified in Problem 1. By using the formula $M = 6.9 \times 10^{10}$ R, where R is in units of km^3/days^2, what is the mass of Pluto in kilograms?

Answer: First convert R into the correct units:

$R = 3.28 \times 10^8$ km^3/hours^2 x (24 hours/1 day)^2 = 1.88×10^{11} km^3/day^2

Then $M = 6.9 \times 10^{10} \times 1.88 \times 10^{11}$ so M = **1.3×10^{22} kg**.

Note, the mass of Earth is 6.0×10^{24} kg, so Pluto has a mass equal to about
1.3×10^{22} kg/6.0×10^{24} = 0.002 Earths!



Cayrel's Star (See ESO photo) is a faint star located 13,000 light years away in the constellation Cetus the Whale. In 2001 a group of astronomers led by Roger Cayrel made the first direct measurement of the age of this star by using the radioactive decay of Uranium-238, which decays to lead with a half-life of 4.47 billion years.

A detailed spectroscopic study revealed numerous, clear lines of uranium in the star's spectrum. From this they could determine the abundances of the elements, and use their ratios to figure out an age for this star.

The measurements showed that the uranium abundances are about 1/7 that of our own sun. The age our sun is known to be 4.6 billion years.

Problem 1 – The half-life formula states that the initial abundance of a radioactive element, N_o, with a half-life of $t_{1/2}$ is related to its current abundance N(t) by

$$N(T) = N_0 \, e^{-0.69(T / t_{1/2})}$$

In the case of our sun, what fraction of the Uranium-238 isotope remains in the sun today given an age of 4.6 billion years?

Problem 2 – If the abundance of Uranium-238 in Cayrel's Star is 1/7 of our sun, what is the age of Cayrel's Star?

Problem 3 – Astronomers were able to detect the element Thorium-232 in the atmosphere of Cayrel's Star. It has a half-life of 14.1 billion years. If the amount of thorium-232 in our sun today is 4.4×10^{19} kg, how much was there in the sun when it was first formed?

Problem 1 – The half-life formula states that the initial abundance of a radioactive element, No, with a half-life of t1/2 is related to its current abundance N(t) by

In the case of our sun, what fraction of the Uranium-238 isotope remains in the sun today given an age of 4.6 billion years?

Answer:
$N(4.6)=n_o\ e^{(-0.69 \times 4.6/4.7)}$ so $N/n_0 = 0.49$ and so there is **half as many** atoms of uranium-238 today as there was when the sun was formed.

Problem 2 – If the abundance of Uranium-238 in Cayrel's Star is 1/7 of our sun, what is the age of Cayrel's Star?

Answer:

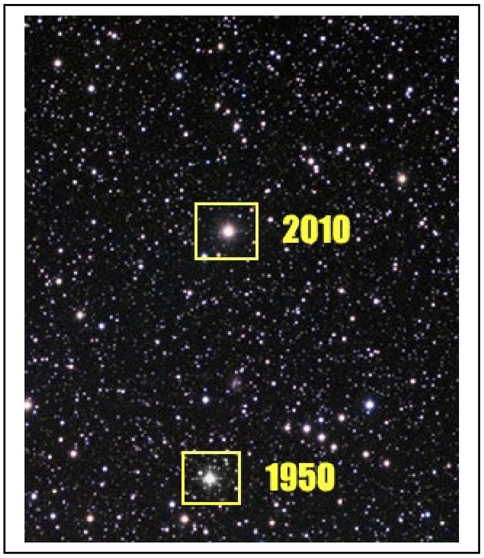
$1/7 = e^{(-0.69\ T/4.47)}$ solving for T we get $Ln(1/7)= -0.69(T/4.47)$ then **T= 12.6 billion years**.

Problem 3 – Astronomers were able to detect the element Thorium-232 in the atmosphere of Cayrel's Star. It has a half-life of 14.1 billion years. If the amount of thorium-232 in our sun today is 4.4×10^{19} kg, how much was there in the sun when it was first formed?

Answer: The decay fraction is just $e^{(-0.69 \times 4.6/14.1)} = 0.80$. So the initial amount was N(4.6)/0.8 = no and so n_o = **5.5×10^{19} kg.**

Note: The abundance of Thorium-232 in our sun today is estimated to be 0.0335 parts per million of silicon atoms, which are 650 ppm of the mass of the sun, which in turn has a total mass of 2.0×10^{30} kg. So,

Amount of thorium = 2.0×10^{30} kg x 650 x 10^{-6} (silicon/Msun) x 0.0335 $\times10^{-6}$ (thorium/silicon) = 4.4×10^{19} kg.



Stars travel along different orbits through the Milky Way. Near our sun, stars are going mostly in the same direction, but from time to time they pass close together. Barnard's Star is currently in the constellation Ophiuchus, but travels across the sky so quickly that it traverses the diameter of the full moon every 180 years.

With the sun at the center of a Cartesian coordinate grid, Barnard's Star can be represented as a point located at (2.0, 5.6) where the units are in light years. But because it is moving through space at a speed of 143 km/sec, its future position relative to the sun changes quickly in time.

The parametric equations for the X and Y location of Barnard's Star can be approximated as follows:

$$X(T) = 2.0 + 0.09\,T \qquad Y(T) = 5.67 - 0.25\,T$$

where T is in thousands of years from the present time, and all units are in light years.

Problem 1 – What is the distance to Barnard's Star at the present time?

Problem 2 – By using the differential calculus, what is the equation of the line y = Mx + B that is represented by the parametric functions?

Problem 3 – By using the Pythagorean distance formula and using the differential calculus to find the minimum distance, at what time, T will Barnard's Star be closest to the sun along this trajectory?

Problem 1 – What is the distance to Barnard's Star at the present time?

Answer: $d = (2.0^{2} + 5.67^{2})^{1/2} = 6.0$ light years.

Problem 2 – What is the equation of the line Y = Mx + B that is represented by the parametric functions?

$$M = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$ so M = -0.25/0.09 and so M = -2.8

Since a point on this line is at (2.0, 5.6) we have y – 5.6 = M(x-2.0)
Then y = 5.6 -2.8x +5.6
Therefore **y = 11.2 – 2.8x**

Problem 3 – At what time, T, will Barnard's Star be closest to the sun along this trajectory?

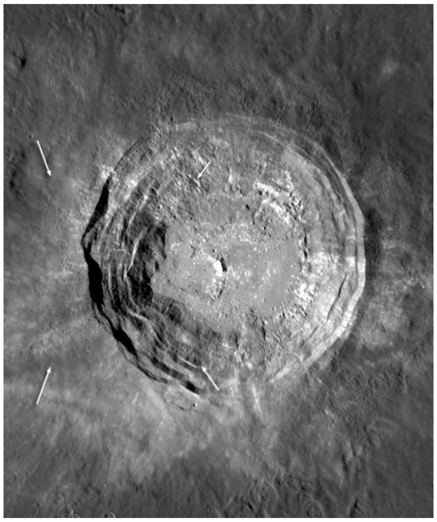
$D(T)^2 = (2.0 + 0.09T)^2 + (5.67 – 0.25T)^2$

Take the derivative to get $2D\frac{dD}{dt} = 2(2.0 + 0.09T)(0.09) + 2(5.67-0.25T)(-0.25)$

This simplifies to $D\frac{dD}{dt} = 0.18 + 0.0081T - 1.42 + 0.0625T = -1.24 + 0.0706T$

Set this equal to zero to get 0 = -1.24 + 0.0706T then **T = 17.6**.

So in about 18,000 years from now, Barnard's Star will be at its closest to the sun. Its distance at this time will be $d^2 = (2.0+0.09*17.6)^2 + (5.67 – 0.25*17.6)^2 = 14.5$ so d= **3.8 light years**.



Lunar craters have been excavated by asteroid impacts for billions of years. This has caused major remodeling of the lunar surface as the material that once filled the crater is ejected. Some of this returns to the lunar surface hundreds of kilometers from the impact site.

An example of a lunar crater is shown in the image to the left taken of the crater Aristarchus by the NASA Lunar Reconnaissance Orbiter (LRO).

Astronomers create models of the rock that was displaced that try to match the overall shape of the crater. The shape of a crater can reveal information about the density of the rock, and even the way that it was layered below the impact area.

One such mathematical model was created for a 17-kilometer crater located at lunar coordinates 38.4 °North, and 194.9 °West. For this particular crater, its depth, D, at a distance of x from its center can be approximated by the following 4^{th}-order polynomial:

$$\mathbf{D = 0.0001x^4 - 0.0055x^3 + 0.0729x^2 - 0.2252x - 0.493}$$

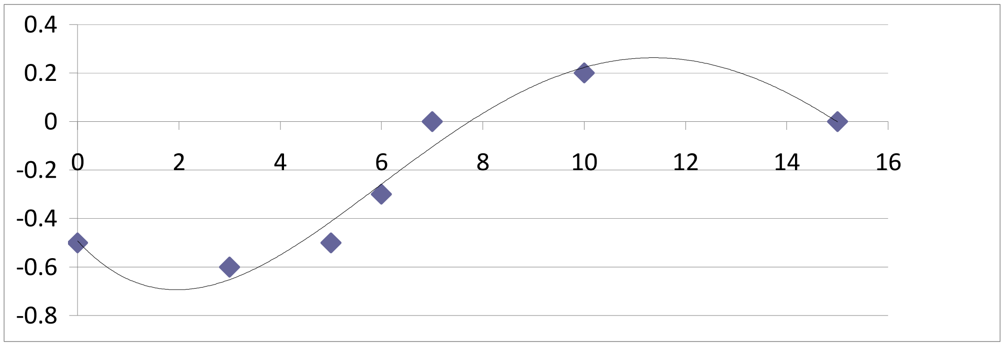
where D and x are in kilometers. The crater is symmetric around the axis x=0.

Problem 1 – Graph this function in the interval (0, +15.0) for which it provides a suitable model.

Problem 2 - To 2 significant figures, what is the approximate excavated volume of this symmetric crater using the method of inscribed and circumscribed disks?

Problem 3 – To 2 significant figures, what is the volume of this crater bounded by the function h(R) and the line y = +0.2 km, and rotated about the y-axis? (Use π = 3.141)

Problem 1 – Graph this function in the interval (0, +15.0) for which it provides a suitable model.



Problem 2 - To 2 significant figures, what is the approximate excavated volume of the symmetric crater using the method of inscribed and circumscribed disks? Answer; The volume of a disk is V = π R^{2} h. For the crater, the inscribed disk has a radius of 5 kilometers and a height of 0.7 km, so its volume is Vi = 3.141 $(5)^{2}$ (0.7) = 55 km^{3}. The circumscribed disk has a radius of 10 km and a height of 0.8 km, so Vc = 3.141 $(10)^{2}$ (0.8) = 251 km^{3}. The estimated volume is then the average of these two or V = (Vc + Vi)/2 = 153 km^{3}. To 2 Significant Figure, the correct answer would be V = 150 km^{3}.

Problem 3 – To 2 significant figures, what is the volume of this crater bounded by the function h(R) and the line y = +0.2 km, and rotated about the y-axis? (Use π = 3.141) Answer: Using the method of shells, the volume differential for this problem is dV = 2π x [0.2 - h(x)] dx

The definite integral to evaluate is then $V = \int_0^{10} 2\pi x[0.2 - h(x)]dx$ Then :

$$V = 2\pi(0.2)\int_0^{10} x dx - 2\pi \int_0^{10} 0.0001x^5 - 0.0055x^4 + 0.0729x^3 - 0.2252x^2 - 0.493x \, dx$$

$$V = 2\pi(0.2)\frac{x^2}{2}\Big]_0^{10} - 2\pi\left(0.0001\frac{x^6}{6} - 0.0055\frac{x^5}{5} + 0.0729\frac{x^4}{4} - 0.2252\frac{x^3}{3} - 0.493\frac{x^2}{2}\right)_0^{10}$$

$$V = 2\pi(0.2)\frac{10^2}{2} - 2\pi\left(0.0001\frac{10^6}{6} - 0.0055\frac{10^5}{5} + 0.0729\frac{10^4}{4} - 0.2252\frac{10^3}{3} - 0.493\frac{10^2}{2}\right)$$

V = 2(3.141) [10 - 16.7 + 110 - 182.3 + 75.1 + 24.7]   V = 6.24 [20.8]

V = 129.8 km^{3} To 2 significant figures this becomes **V = 130 km^{3}**. So to check that Vi < V < Vo we have 55 km^{3} < 130 km^{3} < 251 km^{3}.

As they are forming, planets and raindrops grow by accreting matter (water or asteroids) at their surface.

The basic shape of a planet or a raindrop is that of a sphere. As the sphere increases in size, there is more surface area for matter to be accreted onto it, and so the growth rate increases.

In the following series of problems, we are going to follow a step-by-step logical process that will result in a simple mathematical model for predicting how rapidly a planet or a raindrop forms.

Problem 1 - The differential equation for the growth of the mass of a body by accretion is given by Equation 1 and the mass of the body is given by Equation 2

Equation 1) $\frac{dM}{dt} = 4\pi\ \rho\ V\ R(t)^2$ Equation 2) $M(t) = \frac{4}{3}\pi\ D\ R(t)^3$

where R is the radius of the body at time t, V is the speed of the infalling material, ρ is the density of the infalling material, and D is the density of the body.

Solve Equation 2 for R(t), substitute this into Equation 1 and simplify.

Problem 2 - Integrate your answer to Problem 1 to derive the formula for M(t).

Raindrop Condensation - A typical raindrop might form so that its final mass is about 0.0001 kilograms and D = 1000 kg/m^3, under atmospheric conditions where ρ = 1 kg/m^3 and V = 1 m/sec. How long would it take such a raindrop to condense?

Planet Accretion - A typical rocky planet might form so that its final mass is about that of Earth or 5.9x10^{24} kg, and D = 3000 kg/m^3, under conditions where ρ = 0.000001 kg/m^3 and V = 1 km/sec. How long would it take such a planet to accrete using this approximate mathematical model?

Figure from 'Planetary science: Building a planet in record time' by Alan Brandon, Nature 473,460–461(26 May 2011)

Answer 1:

$$R(t) = \left(\frac{3M(t)}{4\pi D}\right)^{\frac{1}{3}} \quad \text{then} \quad \frac{dM}{dt} = 4\pi\ \mathrm{V}\rho\left(\frac{3M}{4\pi D}\right)^{\frac{2}{3}} \quad \text{so} \quad \frac{dM}{dt} = 4\pi V\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}}\ \mathrm{M}^{\frac{2}{3}}$$

Answer 2: First re-arrange the terms to form the integrands:

$$\frac{dM}{M^{\frac{2}{3}}} = 4\pi\mathrm{V}\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}} dt \quad \text{Integrate both sides:} \quad 3M^{\frac{1}{3}} = 4\pi V\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}} t$$

Now solve for M(t) to get the answer: $M(t) = \left(\frac{4\pi V\rho}{3}\right)^3 \left(\frac{3}{4\pi D}\right)^2 t^3$

Raindrop Condensation - A typical raindrop might form so that its final mass is about 0.0001 kilograms and **D** = 1000 kg/m^3, under atmospheric conditions where ρ = 1 kg/m^3 and **V** = 2.0 m/sec (about 5 miles per hour). How long would it take such a raindrop to condense using this approximate mathematical model?

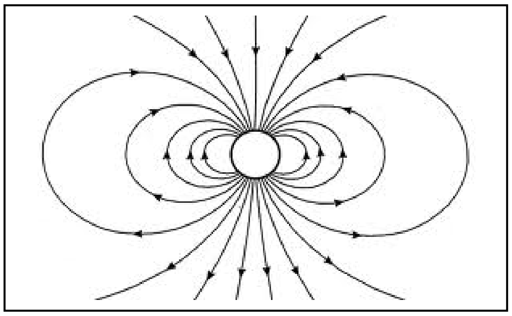
$$0.0001 = \left(\frac{4\pi(2.0)(1.0)}{3}\right)^3 \left(\frac{3}{4\pi(1000)}\right)^2 t^3 \quad \text{so} \quad t^3 = 2.98$$

and so it takes about **1.4 seconds**.

Planet Accretion - A typical rocky planet might form so that its final mass is about that of Earth or 5.9x10^{24} kg, and **D** = 3000 kg/m^3, under conditions where ρ = 0.000001 kg/m^3 and **V** = 1 km/sec. How long would it take such a planet to accrete using this approximate mathematical model?

$$5.9x10^{24} = \left(\frac{4\pi(1000)(0.000001)}{3}\right)^3 \left(\frac{3}{4\pi(3000)}\right)^2 t^3 \quad \text{so} \quad t^3 = 1.27\text{x}10^{40}$$

and so it takes about 2.3x10^{13} seconds or **750,000 years**.



Mathematically, every point in space near a magnet can be represented by a vector, B. Because the field exists in 3-dimensional space, it has three 'components'. The equations for the coordinates of B in 2-dimensions looks like this:

$$B_r = -\frac{2M\sin\theta}{r^3} \qquad B_\theta = \frac{M\cos\theta}{r^3}$$

It is convenient to graph a magnetic field on a 2-dimensional piece of paper to show its shape. The lines that are drawn are called 'magnetic field lines', and if you placed a compass at a particular point on the field line, the direction of the line points to 'north' or 'south'.

The slope of the magnetic field at any point (R,θ) is defined by $\frac{B_\theta}{B_r}$.

From calculus, in a polar coordinate system, the slope of a line is defined by $\frac{rd\theta}{dr}$.

Problem 1 – What is the differential equation that relates $\frac{B_\theta}{B_r}$ to $\frac{rd\theta}{dr}$?

Problem 2 – Integrate your answer to Problem 1 to find the polar coordinate equation of a magnetic field line.

Problem 1 -

$$\frac{rd\theta}{dr} = \frac{M\cos\theta}{-2M\sin\theta} \quad \text{so} \quad \frac{rd\theta}{dr} = -\frac{\cos\theta}{2\sin\theta}$$

Problem 2 -

Rearrange the terms into two integrands: $$\frac{dr}{r} = -\frac{2\sin\theta}{\cos\theta}d\theta$$

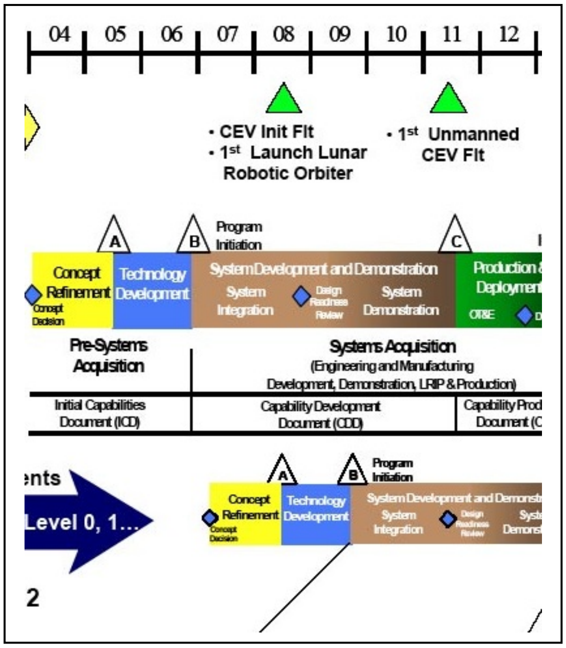
The integrals become $$\int\frac{dr}{r} = -2\int\frac{\sin\theta}{\cos\theta}d\theta$$

These are both logarithmic integrals that yield the solution:

$$ln(r) \ + \ \text{C} \ = 2 \ ln(\cos\theta) \ + \ \text{C}$$

r_0 is the distance from the center of the magnetic field to the point where the field line crosses the equatorial plane of the magnet at θ=0. Each field line is specified by a unique crossing point distance. In other words, the constants of integration are specified by the condition that r = r_0 for θ = 0, which then gives us the final form of the equation:

$$r = r_0\cos^2\theta$$



NASA's new mission to Mars called InSight will be launched in March, 2016. It will land in a region of Mars located near the equator and deploy a seismographic station to study the interior of Mars.

From the time a mission is imagined to the time it actually launches, many events have to be scheduled so that everything is built, tested, and delivered in time for the launch of the mission. The date and time that a mission launches is very carefully determined and called a Launch Window. Sometimes it is only few hours long on a specific date that is many years in the future. The InSight mission must land on Mars on September 20, 2016.

To make sure that all of the thousands of components to a spacecraft are built, tested and delivered on time, Mission Planners develop very detailed schedules that show all of the components, their state of construction, and when the many critical tests and 'milestones' have to be reached to keep a mission on time for launch.

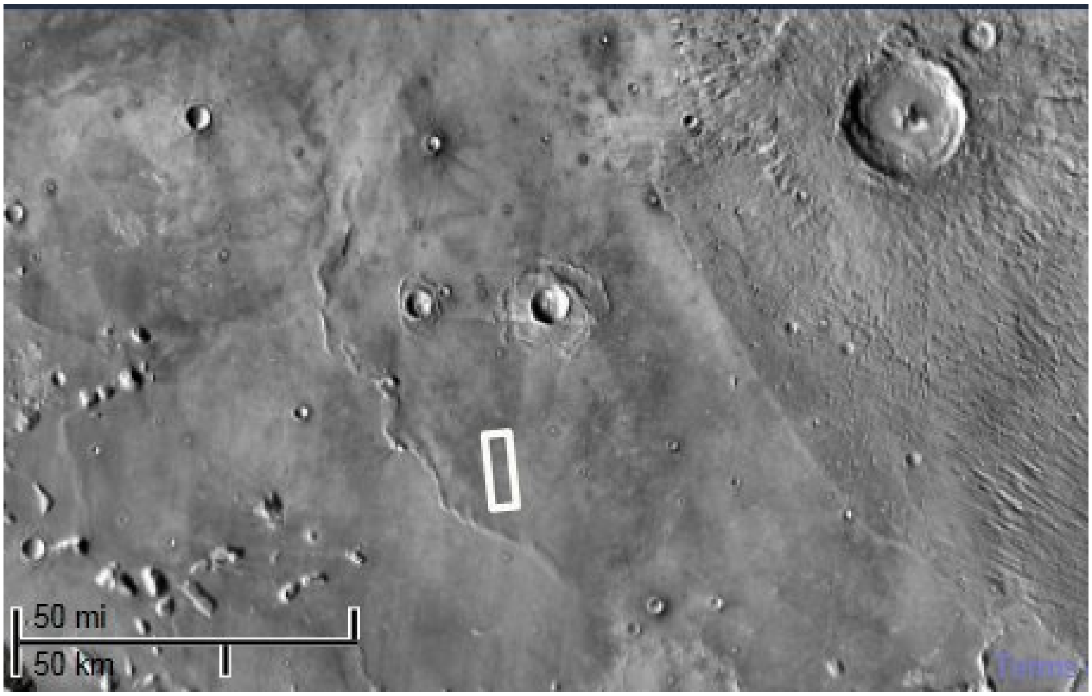
Believe it or not, you do this kind of planning yourself and probably do not even know it!

Imagine that you are taking a vacation to visit a family member, and that you are flying on a jet plane. Your parents have already booked your reservations for August 5 and the plane will leave the gate at exactly 11:35 AM. Allow exactly 2:00 to travel from your home, park your car, pass through Airport Security and walk to the gate. You have to arrive at the gate 45 minutes before the flight leaves to check your baggage and to board the flight.

Problem – Create a timeline for each person in your family that includes waking up on August 5, packing bags and loading them in the family car, eating breakfast, taking showers and other bathroom preparations. Oh…suppose, also, that you only have one bathroom to share, and that no two family members took exactly the same amount of time to do each of their tasks!

Problem – Students timelines will vary. Students should organize each person's activities as a row on the table, and time progressing from left to right along the columns of the table as shown in the example below. The times denote the start of each event.

	7:15	7:30	7:45	8:15	8:30	9:20	9:34	9:35	11:35
Person 1	Wake up	Get dressed	Eat breakfast	Use bathroom	Pack bags	Load car	Fasten seatbelt	Car Leaves Home	Flight Leaves
Person 2	Sleeping	Wake up	Use bathroom	Get dressed	Pack bags	Load car	Fasten seatbelt	Car Leaves Home	Flight Leaves



MRO image**: ESP_027702_1815** Title: Future Landing Site for InSight mission, ellipse E11 Interactive Image: http://www.uahirise.org/hiwish/view/69980
The center of the rectangle is at Latitude +1.617°N and Longitude 138.684° East.

Problem 1 – How big, in meters, are the smallest features you can see, and what are they?

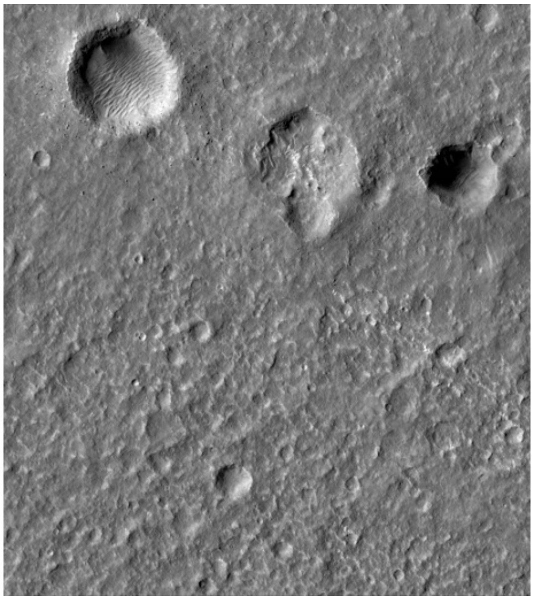
Problem 2 – What clues in this image suggest that, instead of solid rock, the martian ground in this area might actually be much looser?

Answer Key

Problem 1 – The scale of the image is 5okm/28mm = 1.8 km/mm and the smallest crater you can easily see is about 0.5mm across, so the smallest feature is about 900 meters across.

Problem 2 – The two craters near the center of the image have skirts of ejecta that look like mud flowing downhill. The wavy ridge to their left also looks like the same kind of flow, so the ground is not solid rock, but a mixture of materials that can flow like mud when struck.

Comparing the InSight Landing Area to a City Block!



The image on the left was taken by the Mars Reconnaissance Orbiters High-Resolution camera. It shows a possible landing area for the InSight mission. The image to the right is a satellite view from GOOGLE Earth of a neighborhood somewhere in the United States. Both images have a width of 400 meters.

Problem 1 – How wide are the streets in this neighborhood in meters and feet?

Problem 2 – What is the diameter of the crater towards the bottom edge of the image in meters and feet?

Problem 3 – What is the diameter in meters and feet of the smallest crater you can see in the image?

Problem 4 - Find the tennis court in the neighborhood. Which crater is about the same size as a tennis court?

Problem 1 – How wide are the streets in this neighborhood in meters and feet?

Answer: When reproduced wit a standard printer for ‘8 1/2x11’ paper, the scale of the image is 400 meters = 70 millimeters or 5.7 meters/mm. The street in the center of the image is about 3 mm wide or 17 meters, which is about 51 feet.

Problem 2 – What is the diameter of the crater towards the bottom edge of the image in meters and feet?

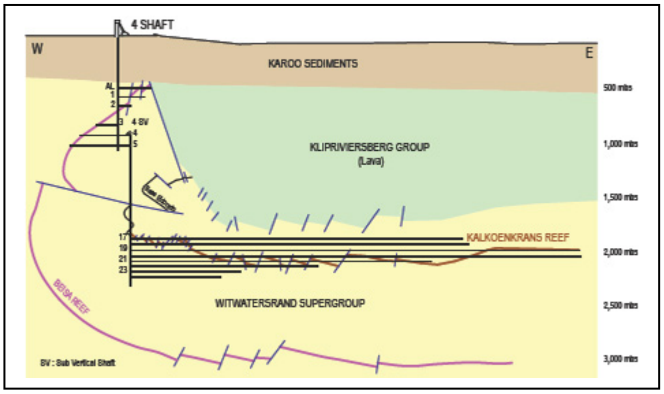
Answer: The crater is about 5 mm wide or 28 meters in diameter or 84 feet wide.

Problem 3 – What is the diameter in meters and feet of the smallest crater you can see in the image?

Answer: Students selections will differ, but 0.3 millimeters equals 1.7 meters or 5 feet is a good estimate. Some ambitious students may see features smaller than this.

Problem 4 - Find the tennis court in the neighborhood. Which crater is about the same size as a tennis court?

Answer: The tennis court is located in the upper left corner of the image as a greenish rectangle. It is about 150 feet long. The large crater near the top right edge is a close match to the size of the tennis court.



As miners on Earth dig deeper mines, they have noticed that the temperature of rocks gets higher. This tells scientists that the interior of Earth is much hotter than its surface. Exactly how much hotter depends on how deep into Earth you travel.

The Beatrix Mine is located in the extreme southern Witwatersrand Basin in South Africa. The basin, created by an ancient asteroid impact over 3 billion years ago, is the world's largest gold deposit. To get to the richest veins of gold, miners have to dig 2,200 meters below ground level.

A measure of how fast the temperature increases for a given depth is called the geothermal gradient. Gradient is a mathematical term that just means how fast one number (temperature in degrees) changes as you move a certain distance (depth in meters). The geothermal gradient for the Beatrix Gold Mine is about +29^{o}C/km. This means that for every kilometer of depth, the temperature of the rock increases by 29^{o} Celsius (+52^{o}F).

Problem 1 – On a summer day at the mine's surface, the temperature is about 90^{o}F (32^{o}C). What temperature in Celsius and Fahrenheit will a miner experience at the bottom of Shaft 4 at a depth of 2,200 meters?

Problem 2 – A geologist measures the geothermal gradient of +0.02^{o}C/meter at one location in Africa, and +29^{o}C/km at another location in Texas. Which location has the fastest temperature change with depth?

Problem 3 – You want to dig a basement floor to your home so that the lowest floor is always at a temperature of 60^{o}F. It is known that at a depth of 3 meters, the ground is always at a temperature of 13^{o}C (55^{o}F) year-round because the soil above acts as an insulator. How deep must you excavate to reach the proper depth if you are in a volcanically-active area where the geothermal gradient is +35^{o}C/km?

Problem 4 - One of the deepest laboratories in the world is in the small town of Soudan, Minnesota. It is a physics research facility and is at a depth of 690 meters (2,263 feet) below the surface. It is operated by the University of Minnesota. If the geothermal gradient is +25^{o}C/km, how warm are the walls of this laboratory if the average surface temperature is +55^{o}F (+13^{o}C)?

The temperature gradient in the Beatrix Mine is given by T. C. Onstott, D. P. Moser, M. F. DeFlaun, L. M. Pratt, and B. Sherwood Lollar, Abstr. 101st Gen. Meet. Am. Soc. Microbiol., p. 515, 2001 in their study of bacteria living in the crust at high temperatures. (http://www.ncbi.nlm.nih.gov/pmc/articles/PMC93369

"The geothermal gradients were found to vary among the four mines and appeared to be correlated with the amounts of water moving through fractures or along dyke contacts. At East Driefontein only a few slowly flowing boreholes were noted at the time of this study and the geothermal gradient is 9°C km−1, whereas at West Driefontein, Kloof, and ***Beatrix fissure*** *water emanating from boreholes is common and the geothermal gradients are 13, 16, and* ***29°C km−1****, respectively"*

Problem 1 – On a summer day at the mine's surface, the temperature is about 90°F (32°C). What temperature in Celsius and Fahrenheit will a miner experience at the bottom of Shaft 4 at a depth of 2,200 meters?

Answer: The gradient is +29°C/km so the increase in temperature will be +29°C/km x (2.2km) = +64°C, and so adding this to the surface temperature you get +32°C +64°C = +96°C. In terms of Fahrenheit we have (+96°C x 9/5) + 32°F =+205° F. That means you can use the mined rocks to boil water, and that makes them very dangerous to the miners.

Problem 2 – A geologist measures the geothermal gradient of +0.02°C/meter at one location in Africa, and +29°C/km at another location in Texas. Which location has the fastest temperature change with depth?

Answer: 1 km = 1000 meters so we can convert to the same units: +0.02°C/meter x (1000 meters/1km) = +20°C/km. The location in Africa has a faster increase of temperature with depth than Texas.

Problem 3 – You want to dig a basement floor to your home so that the lowest floor is always at a temperature of 60°F (16°C). It is known that at a depth of 3 meters, the ground is always at a temperature of 13°C (55°F) year-round because the soil above acts as an insulator. How deep must you excavate to reach the proper depth if you are in a volcanically-active area where the geothermal gradient is +35°C/km?

Answer: First you need to excavate to 3 meters depth (about 9 feet) where you start with a temperature of 13°C. You need an additional 3°C of warming. The geothermal gradient is +35°C/km or +0.35°C/meter. To get an additional 3°C of heating you need to dig an additional +3°C/(+0.35°C/m) = 8.6 meters for a total depth of 11.6 meters (or 35 feet).

Problem 4 - One of the deepest laboratories in the world is in the small town of Soudan, Minnesota. It is a physics research facility and is at a depth of 690 meters (2,263 feet) below the surface. It is operated by the University of Minnesota. If the geothermal gradient is +25°C/km, how warm are the walls of this laboratory if the average surface temperature is +55°F (+13°C)?

Answer: T = +13°C + 0.69km x (+25°C/km) = **+30°C (or +86°F)**

Transferring digital data from place to place takes time. Like water flowing into a lake, the faster it flows the more rapidly the lake fills up and overflows. With computer data, we have a similar problem. You have probably had to do this yourself many times. Each time you copy your playlist from your PC to your portable music player, you will have to wait a certain length of time. The transfer rate is fixed, so the more songs you want to transfer the longer you have to wait. Here's how this works!

Problem 1 - Suppose you want to transfer 1000 songs from your PC collection to your music Player. Each 4-minute song takes up 4 megabytes on the PC, and the cable link from your computer to your Player can handle a transfer rate of 3 million bytes/second. How many minutes does it take to transfer all your songs to the Player?

Imagine a lake fed by one large slow-moving river that brings water to it, and a second small, fast-moving river that takes water from the lake. If the rates at which the water enters and leaves the lake are not in step, the lake's water level will overflow. The InSight lander has a similar problem. It is gathering data at one rate, but transmitting it to Earth at another rate. We don't want to lose any of the data, so the data has to be stored in a memory device called a buffer.

InSight has two instruments that generate constant streams of digital data. The SEIS seismometer produces 48 megabytes/hr and the HP3 produces 2 megabytes/hr. This data is stored in a 500 megabyte buffer. Every 2 hours, the data in the buffer is transmitted to Earth at a rate of 4 megabytes/sec.

Problem 2 - How long will it take to fill up the buffer with data?

Problem 3 - How long will be required to transmit the buffer data to Earth during each 2-hour transmission cycle?

Problem 4 – The receiver on Earth can be scheduled to contact the Lander as often as once every 2 hours. How large a buffer would you need so that you could gather as much data as 4 megabytes/sec over 2 hours? How long does it take the instruments to gather this much data?

Problem 1 - You want to transfer 1000 songs from your PC collection to your music Player. Each 4-minute song takes up 4 megabytes, and the cable link from your computer to your Player can handle a transfer rate of 3 million bytes/second. How many minutes does it take to transfer all your songs?

Answer: 4000 megabytes x (1 second/ 3 megabytes) = 1333 seconds or about **22 minutes**.

The InSight lander has two instruments that generate constant streams of digital data. The SEIS seismometer produces 48 megabytes/hr and the HP3 produces 2 megabytes/hr which is stored in a 500 megabyte digital memory called a buffer. Every 2 hours, the data in the buffer is transmitted to Earth at a rate of 4 megabytes/sec.

Problem 2 - How long will it take to fill up the buffer with data?

Answer: The data enters the buffer at 50 megabytes/hr and the buffer contains 500 megabytes, so it can store data for 500 MBytes/(50Mbytes/hr) = **10 hours**.

Problem 3 - How long will be required to transmit the buffer data to Earth during each 2-hour transmission cycle?

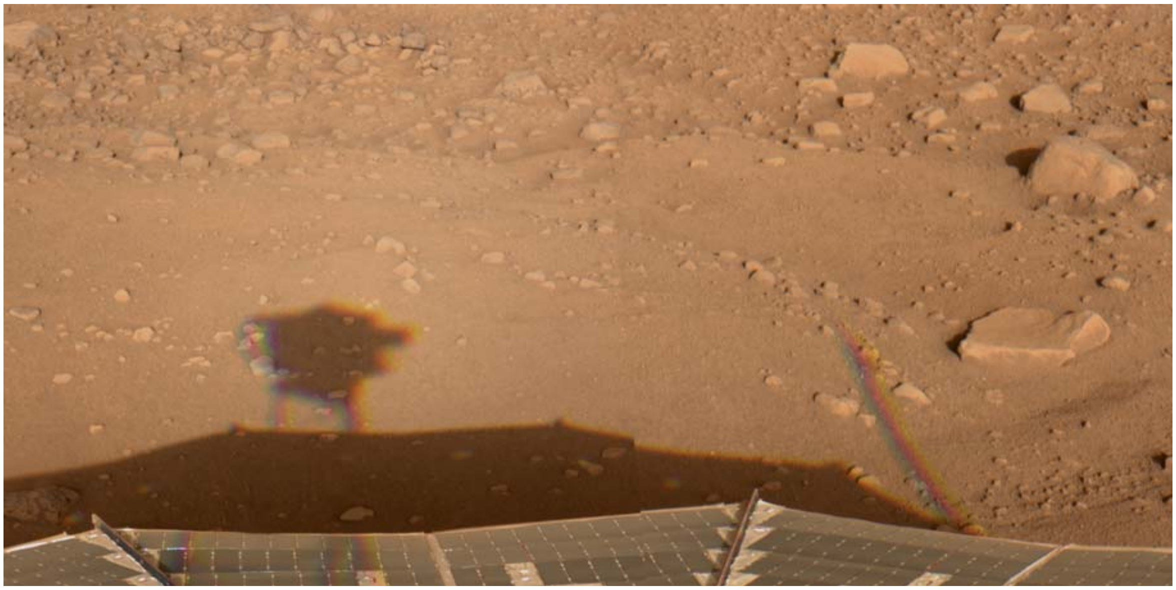
Answer: In 2 hours at a data rate of 50 megabytes/hr you have 100 megabytes stored in the buffer. At a transmission rate of 4 megabytes/sec it takes 100 Mbytes/(4 Mbytes/sec) = **25 seconds** to transmit the100 megabytes from the buffer to Earth.

Problem 4 – The receiver on Earth can be scheduled to contact the Lander as often as once every 2 hours. How large a buffer would you need so that you could gather as much data as 4 megabytes/sec over 2 hours? How long does it take the instruments to gather this much data?

Answer: 2 hours equals 2 x 3600 = 7200 seconds. At a transmission rate of 4 Mbytes/sec, this equals 7200 sec x 4 Mbytes/sec = 28,800 Megabytes or **28.8 Gigabytes.**

The instruments gather 50 Megabytes/hour, so it would take them 28,800 Megabytes/(50 MB/hr) = 576 hours or 24 days to gather this much data.

What this says is that if you had a buffer this large (28.8 gigabytes) it could store 24 days of data from InSight and only take 2 hours to transmit to Earth. The danger of waiting so long to transmit data (every 24 days) is that something could happen to the lander and you would lose all this data! That's why scientists try to download their data as often as possible.



The InSight lander will arrive at Mars on September 20, 2016. One of its first jobs will be to photograph a three square meter area in front of the lander where the Seismic Experiment for Interior Structure (SEIS) and the Heat Flow and Physical Properties Package (HP^3) will be deposited with a mechanical arm. The photo above was taken by the successful NASA Phoenix Lander in 2008 using a similar camera system.

The InSight camera system called IDC is a digital camera with a one megapixel square format (1024x1024 pixels). Each pixel can see a square area of the surface that is 0.82 milliradians on each side.

Problem 1 - If one radian is 57.296 degrees, how many arcseconds across is the angular resolution of one pixel?

Problem 2 - What is the width of the camera's field of view in degrees: A) along one side of the image? B) Along the diagonal of the image?

Problem 3 - At a height of one meter from the ground, what is the ground resolution of the camera in millimeters/pixel?

Problem 4 - If the area to survey is 1.6 meters x 2.4 meters, and each image must overlap 50% of the previous image, how many images have to be taken by the camera to survey the entire instrument area?

Problem 1 - If one radian is 57.296 degrees, how many arcseconds across is the angular resolution of one pixel?

Answer: 0.82 milliradians x (1 radian/1000 milli) x (57.296 degrees/1 radian) x (3600 seconds/1 degree) = **169 arcseconds/pixel.**

Problem 2 - What is the width of the camera's field of view in degrees: A) along one side of the image? B) Along the diagonal of the image?

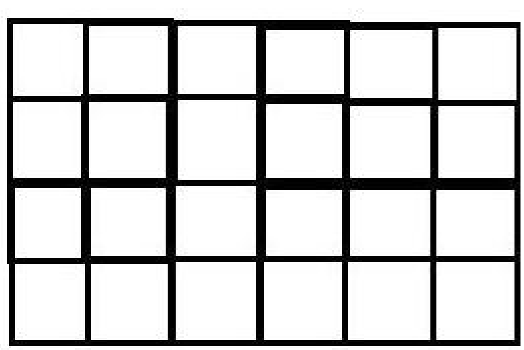
Answer: A) Width = 1024 pixels x 169 arcseconds/pixel x (1 degree/3600 seconds) = **48.1 degrees**. B) The image is a square, so the hypotenuse is 1.414 x side, and so the diagonal length is 1.414 x 48.1 degrees = **68 degrees**.

Problem 3 - At a height of one meter from the ground, what is the ground resolution of the camera in millimeters/pixel?

Answer: Because the angle is much less than one degree, we use the 'skinny triangle' proportion which says that $x/L = \theta$ / 1radian and so since 169 arcseconds = 0.00082 radians we have x = I meter x 0.00082 so x = **0.82 millimeters per pixel**.
Note: From trigonometry, $\tan(\theta) = x/L$ and for small values of θ, θ = 1 radian x (x/L) which is called the 'skinny triangle' approximation.

Problem 4 - If the area to survey is 1.6 meters x 2.4 meters, and each image must overlap 50% of the previous image, how many images have to be taken by the camera to survey the entire instrument area?

Answer: Each square image has a side length of 1024 pixels x 0.00082 meters or 0.82 meters. The surface has a width of 1.6 meters and a length of 2.4 meters. If you tiled this field with camera images, you would need a tiling of 2 x 3 images or 6 images with <u>no overlap</u>. For a 50% overlap, the figure below shows how this would work out.



It would take three images vertically and 5 images horizontally for a total of **15 images**.

The InSight Lander will arrive at Mars on September 20, 2016 according to Earth Time, but when will it arrive according to Mars Time?

One Earth Day is exactly 24 hours long, so that the time between two High Noons is exactly 24 hours. But Mars rotates a bit more slowly and by Earth units, one Mars Day is 24 hours and 40 minutes long.

Since the first Viking Lander touched down on Mars in July 20, 1976, NASA scientists use the convention that the first day of operations of a Lander is called Sol 0, and each Mars solar day, called a Sol is 24 hours and 40 minutes long.

This image was taken by the Phoenix lander at sunrise on Sol 86 which corresponded to Earth date August 21, 2008.

Problem 1 - Draw two clock faces for John and Johanna. They live in the same town, but Johanna's clock runs 40 minutes later that John's clock. John says that it is 12:00 Noon. Draw the hour and minute hands for each clock that show when 12:00 Noon happens on Johanna's clock according to John's clock.

Problem 2 - After 10 days, Johanna's clock says that it is 12:00 Noon. What does John's clock say?

Problem 3 - Suppose Johanna is a colonist on Mars and John is back on Earth. On a particular date, time and place on Mars where Johanna was living, it was 12:00 Noon at exactly the same time as it was on Earth where John was living. When John calls Johanna the next Earth day he says that he is having lunch because it is 12:00 Noon by his clock. What time does Johanna's clock read?

Problem 4 - After how many Mars days will both clocks once again show that it is 12:00 Noon?

Problem 1 - Draw two clock faces for John and Johanna. They live in the same town, but Johanna's clock runs 40 minutes behind John's clock. John says that it is 12:00 Noon. Draw the hour and minute hands for each clock that show when 12:00 Noon happens on Johanna's clock according to John's clock.

Answer: John's clock shows the hour and minute hands both on 12, but Johanna's clock has the hands indicating 11:20 am.

Problem 2 - After 10 days, Johanna's clock says that it is 12:00 Noon. What does John's clock say?

Answer: Johanna's clock is 40 minutes behind Johns clock so if John's clock says 12:00 Noon, Johanna's clock says it is **11:20 AM**. It is the same 40 minutes as in Problem 1 because both locations use Earth's 24-hour day.

Problem 3 - Suppose Johanna is a colonist on Mars and John is back on Earth. On a particular date, time and place on Mars where Johanna was living, it was 12:00 Noon at exactly the same time as it was 12:00 Noon on Earth where John was living. When John calls Johanna two Earth days later he says that he is having lunch because it is 12:00 Noon by his clock. What time does Johanna's clock read?

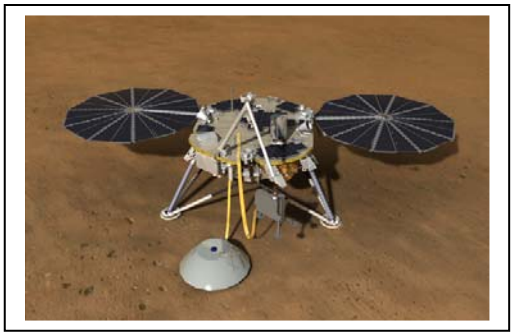
Answer: Because Mars rotates once every 24 hours and 40 minutes, the slower Mars clock will fall behind by 40 minutes after the first Earth day and an additional 40 minutes after the second Earth day, so on Johanna's clock, John will be having Noon at 12:00 - 80 minutes or **10:40 AM**.

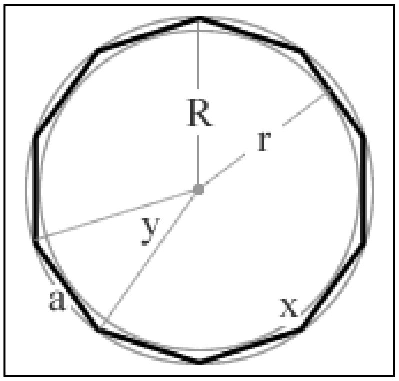
Problem 4 - After how many Mars days will both clocks once again show that it is 12:00 Noon?

Answer: For every Earth day, Mars falls behind by 40 minutes. We want N x 40 minutes to add up to one full Mars day of 24h 40 minutes. Solving for N we get:

N = 24h 40m/ 40 m = 24.6666/0.6666 = **37 Mars days**.

Scientists work on Mars Time because Landers use solar panels and so most activity occurs during the Martian daytime. Sunrise and sunset on Mars follow the 24h 40m length of the martian day, not the shorter 24h 00m Earth solar day. This means that by the normal earth clock, martian sunrise occurs 40 minutes earlier each Earth day.





NASA's new mission to Mars called InSight will be launched in March, 2016. It will land on September 20, 2016 in a region of Mars located near the equator and deploy a seismographic station to study the interior of Mars.

To provide the electricity it needs, the lander will deploy two solar panels, each shaped like a regular, 10-sided polygon called a decagon.

In a regular decagon, the lengths of each of the 10 sides, a, are equal. For the two InSight lander solar panels:

a = 0.62 meters,
r = 0.95 meters,
R = 1.0 meters.

Problem 1 – What is the measure of the interior angle, y for a regular decagon?

Problem 2 – An isosceles triangle is formed by the base a and side length R. What is the length, r, in terms of a and R?

Problem 3 – What is the area of the isosceles triangle in Problem 2?

Problem 4 – What is the area of the regular decagon in terms of a and r?

Problem 5 - Calculate the area of one InSight solar panel in $meter^2$.

Problem 6 - What is the estimated area of one solar panel by using the inscribed circle with a radius of r and the circumscribed circle with a radius R?

Problem 7 – To two significant figures, if the solar panels produce 75 $watts/m^2$ of electricity at the distance of Mars from the sun, what is the total power produced by the two solar panels using either area method?

Problem 1 – What is the measure of the interior angle, y for a regular decagon?

Answer: $y = 360/10 =$ **36^{o}**.

Problem 2 – An isosceles triangle is formed by the base a and side length R. What is the length, r, in terms of a and R?

Answer: The segment with the length, r, is called the apothem and is the perpendicular bisector of the side with the length a, so from the Pythagorean Theorem we get **$r = (R^2 - (a/2)^2)^{1/2}$**.

Note for the InSight dimensions: $0.95 = (1 - 0.096)^{1/2}$

Problem 3 – What is the area of the isosceles triangle in Problem 2?

Answer: $A = 2 \times ½ (a/2) \times r$ so **$A = ar/2$**

For the InSight solar panel: $A = 0.62 \times 0.95/2 = 0.29\ m^2$.

Problem 4 – What is the area of the regular decagon in terms of a and r?

Answer: $A = 10 \times (ar/2)$ so **$A = 5ar$**.

Problem 5 - Calculate the area of one InSight solar panel in meter^2.

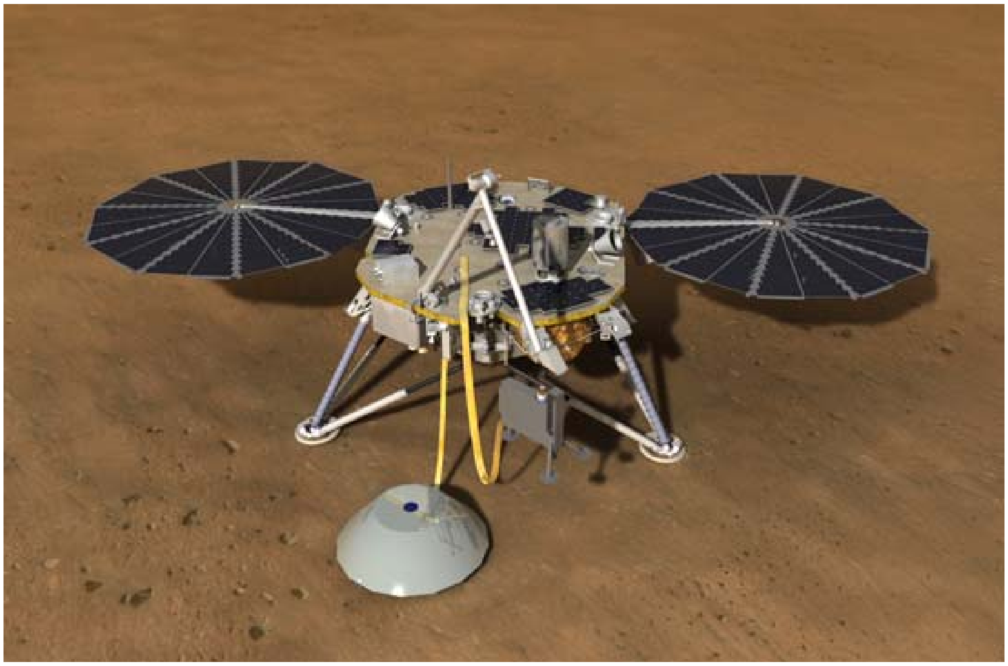
Answer: For the Insight solar panel, $A = 5 (0.62)(0.95) =$ **$2.95\ m^2$**.

Problem 6 - What is the estimated area of one solar panel by using the inscribed circle with a radius of r and the circumscribed circle with a radius R?

Answer: Take the average areas of the inscribed and circumscribed circles to get $A = 0.5 \pi (R^2 + r^2)$. For Insight, $A = 0.5 \times 3.141 \times (1 + 0.90) =$ **$2.98\ m^2$**.

Problem 7 – To two significant figures, if the solar panels produce 75 watts/m^2 of electricity at Mars, what is the total power produced by the two solar panels using either area method?

Answer: To 2 SF, the areas are both 3.0 m^2, so $P = 2$ panels $\times$ 75 w/m^2 $\times$ 3.0 m^2 = **450 watts**.



NASA's new mission to Mars called InSight will be launched in March, 2016. It will land in a region of Mars located near the equator and deploy a seismographic station and a heat probe to study the interior of Mars.

Once the InSight lander touches down on September 20, 2016, during the next 60 days it will slowly and carefully deploy the seismometer station called SEIS (the conical package in the image above) and the heat flow experiment called HP^3 (the rectangular instrument package). Both are attached to the lander by yellow cables that carry instrument power and data. A single maneuverable arm will do this work, but it cannot reach closer than 1.5 meter to the Lander, or farther than 2.5 meters from the lander. It can swing left to right, sweeping out an arc of 90 degrees.

Problem 1 – What is the total surface area on Mars near the lander where the maneuverable instrument arm can place its two payloads? (Use π = 3.141)

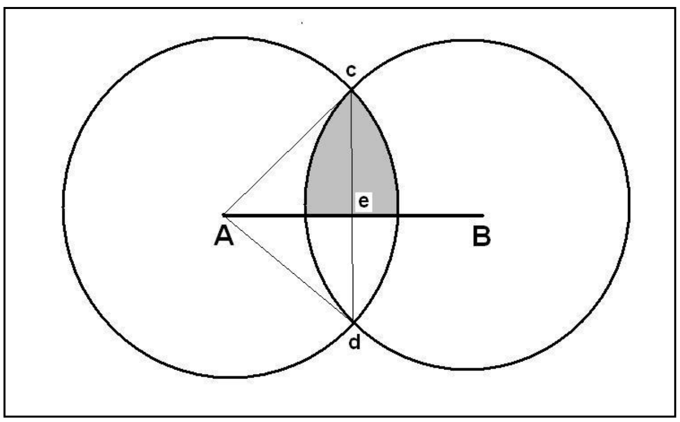
Problem 2 – Suppose the Lander had two identical arms, A and B, whose pivot points were separated by 2.0 meters on the Lander's body. If the arms could reach a maximum of 1.0 meters from their pivot points, what is the area on the martian surface that is common to both arms but above the line joining their pivot points? (Note: For safety reasons, only this common area would be where instruments might be placed so that one or the other arms always have access to them.) Use any method to determine the area.

Problem 1 – What is the total surface area on Mars near the lander where the maneuverable instrument arm can place its two payloads? (Use π = 3.141)

Answer: The area will be ¼ the area of a circular ring with inner radius of 1.5 meter and outer radius of 2.5 meters. This area is just ¼ the area of the larger circle $A = \pi (2.5)^2 = 19.6\ m^2$ minus the area of the inner circular 'hole' $A = \pi (1.5)^2 = 7.1\ m^2$ which leaves an area of ¼ (12.5) **= 3.1 m^2.**

Problem 2 – Suppose the Lander had two identical arms, A and B, whose pivot points were separated by $2/2^{1/2}$ = 1.41 meters on the Lander's body. If the arms could reach a maximum of 1.0 meters from their pivot points, what is the area on the martian surface that is common to both arms but above the line joining their pivot points? (Note: For safety reasons, only this common area would be where instruments might be placed so that one or the other arms always have access to them.) Use any method to determine the area.

Answer: It always helps to draw a diagram of the area in question given the information in the problem. This visual information helps you formulate a mathematical approach to the problem and can suggest solutions.



Method 1 – Students can reproduce the figure on gridded graph paper and count the number of whole squares in the overlap region.

Method 2 – Students can use Method 1 but add into their sum an estimate for the total area in the fractional squares.

Method 3 – An exact solution to this problem requires a bit of geometry.
The distance Ae = Be = $1/2^{1/2}$ and the radius of the circle is 1.0, so the triangle **cAe** is a 45-45-90 right triangle with side lengths $R/2^{1/2}$ and an area A=1/2base X height = $1/4\ R^2$ where R=1 for InSight.
The full arc **cAd** is 90^o, so its area is ¼ of the circle whose radius is R=1 meter so Aarc = $\pi/4R^2$.
But we only want the area above the line **AB** so Aarc = $\pi/8R^2$.
The shaded area to the right of the line **ce** has an area A = Aarc – Atriangle = $\pi/8\ R^2 - ¼\ R^2$.
A similar solution by symmetry works for the shaded area to the left of the line **ce**, so the total area of the shaded region is $A = 2 \times (\pi/8\ R^2 - 1/4R^2)$ or $A = R^2/4\ (\pi - 2)$
For the specific case of R=1 meter, $A = (\pi-2)/4$ = **0.28 meters^2**. This should be close to the answers determined by Method 1 and 2.

Artist rendition of impact on Mars courtesy William Hartman (Planetary Science Institute)

A more useful earthquake severity scale is the Moment Magnitude scale represented by the logarithmic number Mw. Instead of giving the amount of ground displacement, it is directly related to the amount of seismic energy released by the earthquake. On this scale, Mw = 5.5 represents $E=2.0\times10^{17}$ Joules of energy released.

The InSight seismometer experiment (called SEIS) will be able to detect marsquakes with magnitudes from M=3.5 to 6. The relationship between E and M_W is given by the formula:

$$M_W = 2/3 \text{Log} E - 6.0$$

Problem 1 - What is the seismic energy range for the InSight SEIS instrument in Joules? How many megatons of TNT does this range equal if 1 megaTon TNT = 4.2×10^{15} Joules?

Problem 2 – The energy, E, of an earthquake (or marsquake) can be estimated from the formula E = R x A x d where A is the area of the fault slippage, d is the amount of slippage by the fault and R is the rigidity of the crust. Typically, R = 3.3×10^{10} Newton/m^2 and A and d are given in square-meters and meters respectively and E is in Joules of energy. A) What is the energy of an earthquake for which the area is 2000 km^2 and the slippage was 2 meter? B) What is the Moment Magnitude for this event?

Problem 3 - Most of the marsquakes detected by InSight will be due to meteors of various masses impacting the surface of Mars. Each impact will deliver an amount of energy equal to its kinetic energy or $E = \frac{1}{2} mV^2$ where E is in Joules if v is in meters/sec and m is in kilograms. Typical impact speeds will be about 15 km/sec. What is the formula that relates the Moment Magnitude to the mass of the meteorite in kilograms? What is the range of InSight sensitivites in terms of the mass range of the meteors in metric tons?

Problem 1 - What is the seismic energy range for the InSIght SEIS instrument in Joules? How many megatons of TNT does this range equal if 1 megaTon TNT = 4.2x1015 Joules?

Answer: $3.5 < M_w < 6.0$ then solving for E in the above equation we get

$E = 10^{(1.5M_w + 9.0)}$

and so for $M_w = 3.5$ we have **E = 1.8x10^{14} Joules** and for M_w=6.0 we have **E = 1.0x10^{18} Joules**. This range of energy corresponds to **0.4 mTons < E < 238 mTons**.

Problem 2 – The energy, E, of an earthquake (or marsquake) can be estimated from the formula E = R x A x d where A is the area of the fault slippage, d is the amount of slippage by the fault and R is the rigidity of the crust. Typically, R = 3.3x10^{10} Newton/m^2 and A and d are given in square-meters and meters respectively and E is in Joules of energy. A) What is the energy of an earthquake for which the area is 2000 km^2 and the slippage was 2 meter? B) What is the Moment Magnitude for this event?

Answer: A) E = 3.3x10^{10} x 2000 x 2 = **1.3x10^{14} Joules**.
B) Mw = 2/3 logE – 6 so **Mw = 3.4**.

Problem 3 - Most of the marsquakes detected by InSight will be due to meteors of various masses impacting the surface of Mars. Each impact will deliver an amount of energy equal to its kinetic energy or E = ½ mV^2 where E is in Joules if v is in meters/sec and m is in kilograms. Typical impact speeds will be about 15 km/sec. What is the formula that relates the Moment Magnitude to the mass of the meteorite in kilograms? What is the range of InSight sensitivites in terms of the mass range of the meteors in metric tons?

Answer: For v = 15 km/sec we have E = 1.125x10^{8}m. We have Mw = 2/3LogE – 6 and substituting for E we get Mw = 2/3 log(1.125x10^{8}m) – 6 ; and so the formula is

Mw = 2/3Log(mass) – 0.633.

For Mw = 3.5 we have m = 1.58x10^{6} kg or 1580 metric tons.
For Mw = 6.0 we have m = 8.9x10^{9} kg or 8.9 megatons.
So **1580 tons < m < 8.9 megatons** is the mass range.

Event	Magnitude R	Tons of TNT
Hand grenade	0.2	0.00003
1 stick dynamite	1.2	0.0012
Chernobyl	3.9	9.5
2010 Quebec	5.0	480
2011 Washington	5.8	15,000
2010 Haiti	7.0	480,000
1906 San Francisco	8.0	15 million
1883 Krakatoa	8.8	200 million
1964 Anchorage	9.2	950 million
Chicxulub Impact	12.6	100 trillion

On Earth, the severity of an earthquake is measured by the amount of ground movement that it produces. The Richter Scale has been in use for many years and is an example of a logarithmic scale.

Logarithmic scales are linear scales in 'x' such as 1.0, 2.0, 3.0 etc, but they represent magnitude changes of 10, 100 and 1000 etc. Because natural phenomena span such a large range in energy, logarithmic scales are often used to represent them.

Problem 1 – The common earthquake Richter Scale is a measure of how much ground movement a local earthquake produces. For example, an R=5.0 earthquake produces 10 times more ground movement than an R=4.0 earthquake. This scale is calibrated so that an R=0 earthquake at a distance of 100 km produces a ground change of 1 micron (10^{-6} meters), which is measured by a seismometer. In 2011, the Washington DC area was struck by an R=6.0 earthquake. About how much ground movement was produced in Washington DC, about 100 km from the epicenter?

Problem 2 – One of the largest modern earthquakes occurred in Anchorage Alaska in 1964 and was measured as R=9.2. How much ground motion occurred 100 km from the epicenter of this quake?

Problem 3 – The detonation of three tons of TNT produces an energy similar to an R=3.5 earthquake. If the energy of an earthquake is proportional to $10^{1.5R}$, how many tons of TNT is the equivalent energy for the Krakatoa Explosion in 1883 which was recorded as R=8.8?

Problem 1 – The common earthquake Richter Scale is a measure of how much ground movement a local earthquake produces. An R=5.0 earthquake produces 10 times more ground movement than an R=4.0 earthquake. This scale is calibrated so that an R=0 earthquake at a distance of 100 km produces a vertical ground change of 1 micron (10^{-6} meters). In 2011, the Washington DC area was struck by an R=5.8 earthquake. About how much ground movement was produced near Washington DC, which was about 100 km from the epicenter?

Answer: 1 micron x $10^{(5.8)}$ = **0.6 meters**.

Problem 2 – One of the largest modern earthquakes occurred in Anchorage Alaska in 1964 and was measured as R=9.2. How much ground motion occurred 100 km from the epicenter of this quake?

Answer: 1 micron x $10^{(9.2)}$ = **1.6 kilometers**.
Note: Actual vertical displacements were only about 11 meters in some locations.

Problem 3 – The detonation of three tons of TNT produces an energy similar to an R=3.5 earthquake. If the energy of an earthquake is proportional to $10^{1.5R}$, how many tons of TNT is the equivalent energy for the Krakatoa Explosion in 1883 which was recorded as R=8.8?

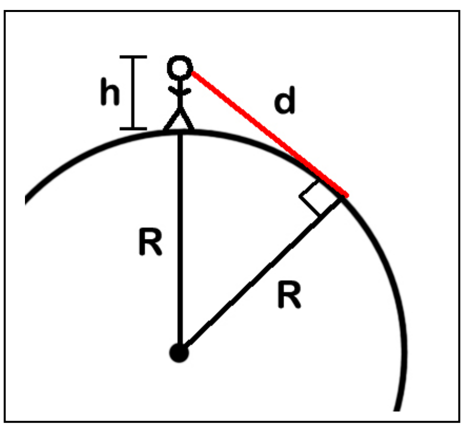
Answer: 8.8-3.5 = 5.3 then E = 3 tons x $10^{1.5(5.3)}$ = **267 megatons**.

Imagine you and your friend standing on the surface of a perfectly flat planet. Your friend starts walking away from you, and you see her size get smaller and smaller until at a distance of 100 kilometers you can't even see her at all.

Now imagine the same experiment on a spherical planet. As many sea-farers discovered 1000 years ago, because Earth is curved, you will see the ships hull disappear from the bottom upwards, then the last thing that vanishes is the top of the main mast.

The image above comes from Johannes de Sacrobosco's Tractatus de Sphaera (*On the Sphere of the World*) written in 1230 AD. It showcases the knowledge that the appearance of ships on the horizon testified to a curved earth. A bit of simple geometry, and some help from the Pythagorean Theorem, will let you calculate the distance to the horizon on Mars as viewed from the InSight Lander!

Problem 1 – Use the Pythagorean Theorem to solve for the distance, d, in terms of h and R.



Problem 2 – R is the radius of Mars, which is 3,378 kilometers. If h is the height of an observer in meters, write a simplified equation for d when h << R. What is the distance to the martian horizon as viewed from the IDS camera located 1 meters above the ground?

Problem 3 - Mars has no ionosphere, so radio signals cannot be 'bounced' around Mars to distant locations. Instead, tall 'cell towers' have to be used. If the cell tower is 100 meters tall, how many cell towers will you need to cover the entire surface of Mars?

Problem 1 – Use the Pythagorean Theorem to solve for the distance, d, in terms of h and R.

Answer: $d^2 = (R+h)^2 - R^2$

So $d^2 = 2Rh + h^2$

And so $d = (2Rh + h^2)^{1/2}$

Problem 2 – R is the radius of Mars, which is 3,378 kilometers. If h is the height of an observer in meters, write a simplified equation for d when h << R. What is the distance to the martian horizon as viewed from the IDS camera located 1 meters above the ground?

Answer: For the formula to work, R and h must be in the same units of meters (or kilometers!). When h is much less then R, the quantity h^2 is always much, much smaller than 2Rh, so the formula simplifies to $d = (2Rh)^{1/2}$.

For Mars, h = 1 meter and R = 3378000 meters and so **d = 2599 meters or about 2.6 kilometers.**

Problem 3 - Mars has no ionosphere, so radio signals cannot be 'bounced' around Mars to distant locations. Instead, tall 'cell towers' have to be used. If the cell tower is 100 meters tall, how many cell towers will you need to cover the entire surface of Mars?

Answer: h = 100 meters or 0.1 km, so the horizon distance for one cell tower has a radius of $d = (2 \times 3378 \text{ km} \times 0.1 \text{ km})^{1/2}$ = 26 kilometers. The area of this reception circle is $A = \pi R^2 = 2123 \text{ km}^2$. The total surface area of Mars is $A = 4 \times \pi \times (3378)^2 = 1.43\times10^8 \text{ km}^2$.

Then dividing the surface area of Mars by the cell tower reception area we get $N = 1.43\times10^8/2123$ = **67,530 cell towers**.

As of 2013, there are over 200,000 cell towers in the Unites States alone!

Once astronomers have measured the diameter and mass of a planet, they can determine the average density of the planet by dividing its mass by its volume. This is a valuable 'first look' into the interior of a planet because if the average density is close to 1000 kg/m^3, then most of the planet consists of light materials and gas or even water and ice like Saturn and Uranus. If the value is large and near 4000 kg/m^3, then the planet may consist mostly of rocky materials like Mercury and Earth.

Problem 1 - The mass of Mars is known to be 6.39×10^{23} kilograms, and the outer radius of the planet is 3400 kilometers. What is the average density of Mars in kilograms/meter^3? What would you estimate as the composition of the martian interior if ice has a density of 917 kg/m^3, granite has a density of 2700 kg/m^3 and iron ore has a density of 7000 kg/m^3?

Problem 2 – The interior of mars can be represented by three main geologic regions: The core is a spherical region with a radius of about 1800 km; the mantle is a spherical shell with an outer radius of 3300 km, and the crust is a 100 km spherical shell located above the mantle. The crust of Mars has been sampled by several NASA landers including Viking, Spirit, Opportunity, Phoenix and Curiosity. The density of the surface rocks appears to be about 2000 kg/m^3. If models of the core of Mars suggest a density of 6400 kg/m^3, what is the average density of the rocks in the martian mantle zone to two significant figures?

Problem 1 - The mass of Mars is known to be 6.39×10^{23} kilograms, and the outer radius of the planet is 3400 kilometers. What is the average density of Mars in kilograms/meter^3? What would you estimate as the composition of the martian interior if ice has a density of 917 kg/m^3, granite has a density of 2700 kg/m^3 and iron ore has a density of 7000 kg/m^3?

Answer: The volume of mars as a sphere is given by $V = 4/3\ \pi\ R^3$ so
$V = 1.333 \times 3.141 \times (3400000)^3 = 1.65\times10^{20}\ m^3$, then the density is just
$D = 6.39\times10^{23}\ kg/1.65\times10^{20}\ m^3 =$ **3872 kg/m^3**. This is between the density of granite and iron, but closer to granite, so on average there is probably very little iron in the interior of Mars.

Problem 2 – The interior of mars can be represented by three main geologic regions: The core is a spherical region with a radius of about 1800 km; the mantle is a spherical shell with an outer radius of 3300 km, and the crust is a 100 km spherical shell located above the mantle. The crust of Mars has been sampled by several NASA landers including Viking, Spirit, Opportunity, Phoenix and Curiosity. The density of the surface rocks appears to be about 2000 kg/m^3. If models of the core of Mars suggest a density of 6400 kg/m^3, what is the average density of the rocks in the martian mantle zone to two significant figures?

Answer: We know:

The total mass of mars is 6.39×10^{23} kg.
The radius of the core is 1800 km.
The inner and outer radius of the mantle shell as 1800 km and 3300 km.
The inner and outer radius of the crust shell as 3300 km and 3400 km.
The density of the core as 6400 kg/m^3
The density of the crust as 2000 kg/m^3.

So we subtract from the mass of Mars the mass of the core and the crust to get the mass of the mantle. From the mantle shell volume we can then determine it density:

$Mcore = 6400 \times 4/3\ \pi\ (1800000)^3 = 1.56\times10^{23}$ kg.
$Mcrust = 2000 \times 4/3\pi\ (3400000^3 – 3300000^3) = 2.82\times10^{22}$ kg
$Mmantle = 6.39\times10^{23}\ kg – 1.56\times10^{23}\ kg – 2.82\times10^{22}\ kg = 4.55\times10^{23}$ kg.
$Volume(mantle) = 4/3\pi\ (3300000^3 – 1800000^3) = 1.26\times10^{20}\ m^3$.

So Density = $4.55\times10^{23}\ kg/1.26\times10^{20} m^3 =$ **3600 kg/m^3**.

Astronomers can determine the mass of a planet by using Kepler's Third Law, which is written in algebraic form as follows:

$$P^2 = \frac{4\pi^2}{G M} a^3$$

where G is Newton's constant of gravity equal to $6.67 \times 10^{-11}\ m^3/kg\ sec^2$, M is the mass of the planet in kilograms, a is the average orbit radius in meters and P is the orbit period in seconds.

Here are some problems that let you work with this very important formula in astronomy.

Problem 1 – The martian moon Phobos was observed from Earth through telescopes to have an orbit radius of a=9380 km and a period of P=0.32 days, what is the mass of Mars?

Problem 2 – The martian moon Deimos has an orbit period of P=1.26 days. What is its orbit radius from the center of Mars in kilometers to 3 significant figures?

Problem 3 – On March 10, 2006, the Mars Reconnaissance Orbiter was originally placed into a very elliptical orbit with a period of P=35.5 hours and an average radius of a=25,889 km. What is the mass of Mars based on this orbit? Its final orbit was achieved in September 2006 with a circular radius of a=3700 km. What is the final orbit period of MRO in the new circular orbit?

Problem 1 – The martian moon Phobos was observed from Earth through telescopes to have an orbit radius of 9380 km and a period of 0.32 days, what is the mass of Mars?

Answer: First solve the algebraic equation for M to get M = $(4\pi^2/G)(a^3/p^2)$. Then use a = 9380 km x 1000m/km = 9.38×10^6 m, and p = 0.32days x 24h/d x 3600s/hr = 27648 seconds, to get

$M = [4 \times (3.141)2/6.67\times10^{-11}]\ (9.38\times10^6)^3/(27648)^2 =$ **6.39×10^{23} kg**.

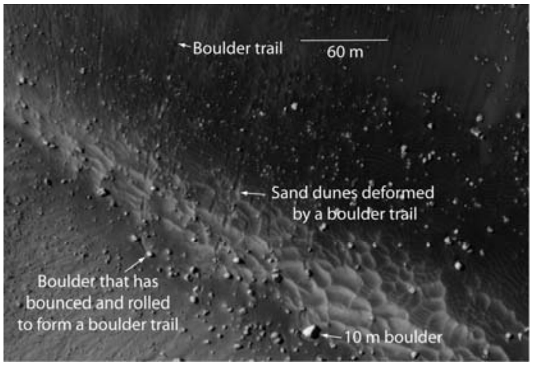
Problem 2 – The martian moon Diemos has an orbit period of P=1.26 days. What is its orbit radius from the center of Mars in kilometers to 3 significant figures?

Answer: In this case we know M and P and need to solve for a:

$a^3 = GMP^2/4\pi^2$ so P = 1.26d x 24h/d x 3600 s/h = 108864 seconds, and so $a^3 = (6.67\times10^{-11})(6.39\times10^{23})(108864)^2/(4 \times 3.141^2) = 1.27\times10^{22}$ so a = 23352 kilometers and so a = **23,400 kilometers**.

Problem 3 – On March 10, 2006, the Mars Reconnaissance Orbiter was originally placed into a very elliptical orbit with a period of P=35.5 hours and an average radius of a=25,889 km. What is the mass of Mars based on this orbit? Its final orbit was achieved in September 2006 with a circular radius of a=3700 km. What is the final orbit period in minutes of MRO in the new circular orbit?

Answer: Solving for P we get $P^2 = (4\pi^2/GM)\ a^3$, and so for a = 3,700,000 meters and M = 6.39×10^{23} kg we get $P^2 = 4.69\times10^7$ and so P = 6848 seconds or **114 minutes**.



The InSight seismometer (SEIS) will measure vibrations caused by distant meteor impacts and 'marsquakes'. Scientists measure these impacts in three ways: By their energy in Joules, by their frequency in impacts per year, and by their magnitude on a Richter-like 'M' scale.

It is estimated that M=5.6 impacts deliver about 2×10^{17} Joules of energy, and occur about once each year. This amount of impact energy is equal to 48 million tons (48 megatons) of TNT!

The image above was taken by the Mars Reconnaissance Orbiter and shows boulders dislodged by marsquakes near the region known as Cerberus Fossae. Tracks made by the rolling boulders can be easily seen in the sand dunes.

Problem 1 – On Mars, scientists have predicted that there will be 100 events per year with M=3.5, 10 events per year with M=4.5 and 1 event per year with M=5.5. If an M=4.5 quake is 10 times more violent than an M=3.5 quake, write a mathematical formula that gives the impact rate, R, as a function of the quake severity, M.

Problem 2 – An impact with M=5.0 is strong enough that if it occurred closer than 100 km from the InSight seismometer, it would overpower the sensors and not be recorded accurately; a condition called data saturation. How long would scientists have to wait for such an impact to occur? (The radius of Mars is 3,376 km).

Problem 3 – For what magnitude of marsquake, M, would you expect to detect one event within 100km of the InSight seismometer?

Problem 4 – If an M=5.6 delivers 2.0×10^{17} Joules of energy, and M=4.6 delivers 30x less energy, about how many tons of TNT will the earthquake described in Problem 3 deliver?

Problem 1 – On Mars, scientists have predicted that there will be 100 events per year with M=3.5, 10 events per year with M=4.5 and 1 event per year with M=5.5. If an M=4.5 quake is 10 times more violent than an M=3.5 quake, write a mathematical formula that gives the impact rate, R, as a function of the quake severity, M.

Answer: $R(M) = 100 \times 10^{(3.5-M)}$

Problem 2 – An impact with M=5.0 is strong enough that if it occurred closer than 100 km from the InSight seismometer, it would overpower the sensors and not be recorded. How long would scientists have to wait for such an impact to occur? (The radius of Mars is 3,376 km).

Answer: For M=5.5, the rate is once per year over the entire planet. The surface area of Mars is $4 \times 3.141 \times (3376)^2 = 1.4 \times 10^8$ km^2, and the area of the zone near the seismometer is $3.141 \times (100\text{km})^2 = 3.1 \times 10^4$ km^2. The ratio of the areas is about 1/4500, so the rate of impacts inside the InSight zone is 1/4500 per year or an average of 4500 years between impacts.

Problem 3 – For what magnitude of marsquake, M, would you expect to detect one event within 100km of the InSight seismometer?

Answer: The ratio of the martian surface area to the area inside 100km is 1/4500. We need 4500 events per year over all of Mars in order to get 1 event inside our detection zone. From our rate function:

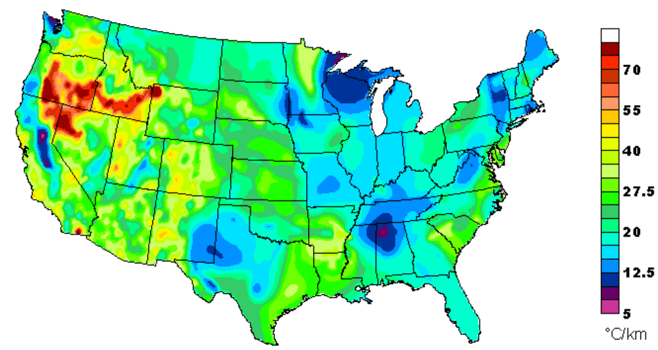
4500 = 100 x 10(3.5-M)

Solving for M we get 45 = 10(3.5-M)
Taking Log10 on both sides we get Log10(45) = 3.5-M
Then 1.6 = 3.5-M
And so M = 3.5-1.6 **M = 1.9**

Problem 4 – If an M=5.6 delivers 2.0×10^{17} Joules of energy, and M=4.6 delivers 30x less energy, about how many tons of TNT will the earthquake described in Problem 3 deliver?

Answer: 2×10^{17} Joules equals 48 megatons, so we use that unit directly.
The difference between M=5.6 and M=1.9 is 3.7. Rounding this to 4.0, we have $30^{4.0}$ = 810,000 times less energy, so 48,000,000/810,000 = 59 tons of TNT.

Comparing the Heat Output of Mars and Earth



Temperature gradient map of the United States. Courtesy Southern Methodist University, Geothermal Lab, (http://smu.edu/geothermal/heatflow/heatflow.htm)

The amount of heat that a planet produces at its surface gives us clues about its interior.

For Earth, geologists have measured the heat flow at its surface to be about 80 milliWatts/m^2. For Mars the value is about 20 milliWatts/m^2.

Earth's interior is producing almost 4 times more heat flow than Mars, suggesting that the interior of Earth is significantly hotter.

When these measurements are combined with a knowledge of the properties of the crust of Earth and Mars, we can estimate what the value of the thermal gradient is near the surface of each planet! The basic formula is

$$F = K\frac{DT}{Z}$$

where F = heat flux in watts/m^2, K = heat diffusion coefficient of the crust, z = thickness of the crust in meters and DT = temperature difference in Celsius.

Problem 1 - For Earth, the crust is granite with K=2.0 watts/meter ^{o}C. For Mars, the crust is loosely packed rock with K = 0.08 Watts/meter ^{o}C. What are the temperature gradients near the surface of each planet in ^{o}C/meter?

Problem 2 - For a given value of F, what happens to the temperature gradient as the material becomes a better insulator (K smaller)? Explain what this means in words.

Problem 3 - The radius of Earth is 6378 km and Mars is 3389 km. What is the total heat power emitted by Earth and Mars in teraWatts? (1 TW = 1 trillion watts).

Problem 4 - Two planets, A and B, have the same diameter. If $F_B = 1/4F_A$ and K_B=8 K_A, which planet has the largest crustal temperature gradient?

Problem 1 - For Earth, the crust is granite with K=2.0 watts/meter ^oC. For Mars, the crust is loosely packed rock with K = 0.08 Watts/meter ^oC. What are the temperature gradients near the surface of each planet in ^oC/meter?

Answer: Mars: 0.020 watts/m^2 = (0.08 watts/meter ^oC) x DR/Z so
DR/Z = 0.25 ^oC/meter.

Earth: 0.080 watts/m^2 = (4.0 watts/meter ^oC) x DR/Z so
DR/Z = 0.02 ^oC/meter.

Problem 2 - For a given value of F, what happens to the temperature gradient as the material becomes a better insulator (K smaller)? Explain what this means in words.

Answer: As K becomes smaller, the temperature gradient becomes larger. That means that for every meter you travel through the material, the temperature difference decreases by a large value. In other words, most of the warmth remains close to the hottest side of the material.

Problem 3 - The radius of Earth is 6378 km and Mars is 3389 km. What is the total heat power emitted by Earth and Mars in teraWatts? (1 TW = 1 trillion watts).

Answer:
Mars surface area = 4 π $(3378000)^2$ = 1.4×10^{14} m^2
Power = 0.020 watts/m^2 x 1.4×10^{14} m^2 = 2.9×10^{12} watts = **2.9 teraWatts**.

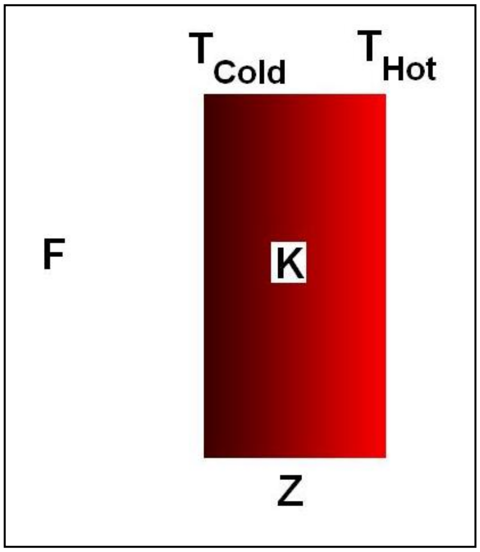
Earth surface area = 4 π $(6378000)^2$ = 5.1×10^{14} m^2
Power=0.080 watts/m^2 x 5.1×10^{14} m^2 = 4.1×10^{13} watts = **41 teraWatts**.

Problem 4 - Two planets, A and B, have the same diameter. If $F_B = 1/4F_A$ and $K_B=8 K_A$, which planet has the largest crustal temperature gradient?

Answer: For Planet B: $(DT/Z)_B = F_B/K_B$ so by substitution:

$$(DT/Z)_B = 1/4F_A / 8K_A$$
$$= 1/16\ (F_A/K_A)$$
$$= 1/16\ (DT/Z)_A$$

So Planet B has a temperature gradient 1/16 of Planet A. **Planet A has the largest**.



Every time you put on clothing to go outside in the winter you are experimenting with heat flow and insulation. Your body is at a temperature of 98.6^{o}F (37^{o}C) and emits about 100 watts of heat. By putting on a layer of clothing, you are preventing this 100 watts from quickly leaking out into the cold air, and this keeps you nice and warm.

The type (K value) of the insulation you wear (cotton versus down) will determine how rapidly this 100 watts leaks out, and how cold you will feel.

Another factor involved in keeping warm is the temperature difference given by DT= (T_{hot} - T_{cold}) between your body and the surrounding air. If there is a big temperature difference, the heat will flow more rapidly and you will cool off more quickly. Also, a thin insulation (Z small) will make you feel cooler than a thick insulation (Z large) and allow more heat to escape more quickly. Scientists can create a formula that follows all of these relationships.

Problem 1 - Convert the following statement into a mathematical formula using P = heat flux in watts/m^2, K = heat diffusion coefficient of the insulation, z = thickness of insulator in meters and DT = temperature difference in Celsius:

The amount of heat flux in watts per square meter is proportional to the temperature difference in degrees Celsius and inversely proportional to the thickness of the insulator in meters. The constant of proportionality is given by K.

Problem 2 - A cook is using an aluminum pot (K=250) and a stainless steal pot (K=16) to boil water. If the pots have a thickness of 2 millimeters and the temperature difference between the hot plate and the inside of the pot is 200^{o}C, which pot will boil the water the fastest?

Problem 3 - On the surface of Mars, the heat flux is measured to be 20 milliWatts/m^2. The temperature gradient DT/Z = 5^{o}C/20 meters. What is the heat diffusion coefficient K for the martian soil?

Problem 1 - Convert this statement into a mathematical formula using P = heat flux in watts/m^2, K = heat diffusion coefficient of insulation, z = thickness of insulator in meters and DT = temperature difference in Celsius: *The amount of heat power per square meter is proportional to the temperature difference and inversely proportional to the thickness of the insulator. The constant of proportionality is given by K*.

Answer: $$F = K\frac{DT}{Z}$$

Problem 2 - A cook is using an aluminum pot (K=250) and a stainless steal pot (K=16) to boil water. If the pots have a thickness of 2 millimeters and the temperature difference between the hot plate and the inside of the pot is 200^{o}C, which pot will boil water the fastest?

Answer: Aluminum: F = 250 x (200/0.002) = 25,000,000 watts/m^2
Stainless Steal: F = 16 x (200/.002) = 1,600,000 watts m^2

There is 16 times more heat entering the aluminum pot so water will boil faster in an aluminum pot that a stainless steel pot.

Problem 3 - On the surface of Mars, the heat flux is measured to be 20 milliWatts/m^2. The temperature gradient DT/Z = 5^{o}C/20 meters. What is the heat diffusion coefficient K for the martian soil?

Answer: 20 miliWatts/m^2 = 0.020 Watts/m^2. From the formula we have

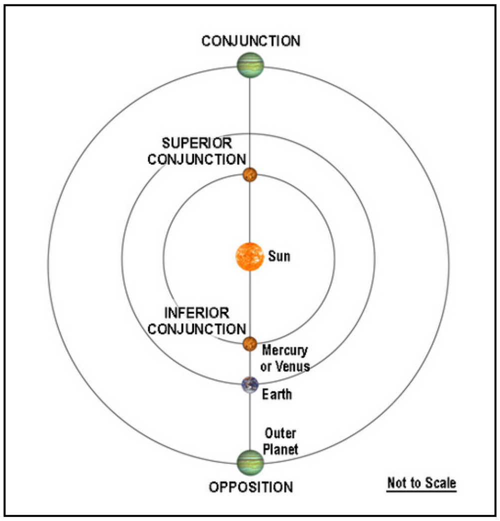
0.020 watts/m^2 = K x (5^{o}C/20meters) so

K = 0.020 x 20 meters/5^{o}C and so

K = 0.08 watts/meter ^{o}C

Note: Solid granite has a value of K = 3.0
Volcanic pumice rock has K = 0.5
Lunar soil has K between 0.1 and 0.01

So the martian surface is similar as an insulator to lunar soil which has been produced by numerous asteroid bombardments and has lots of space between the soil particles.



The InSight Lander needs to be in constant communication with Earth every day the data it gathers can be sent back to Earth. As viewed from the surface of Mars, Earth never gets very far from the sun. Over the course of about 780 days, Earth travels from its farthest westward position in the sky (morning star) of 46^o W, to its farthest eastward position (evening star) of 41^o E and back in about 780 days.

When the Earth-Sun angle is near zero as viewed from Mars, Earth can either be between Mars and the sun (called Inferior Conjunction) or Earth can be on the opposite side of the sun as viewed from Mars (called Superior Conjunction).

When Mars and Earth are in inferior conjunction, Earth can receive signals from the Lander, but the Lander will have to broadcast its data almost directly at the sun, which is a hazard for the transmitter. When Earth and Mars are in superior conjunction, Neither InSight nor the Earth radio communication system can transmit or receive data with Earth behind the sun.

Problem 1 – The following table gives the Earth-Sun angle viewed from Mars during the time InSight is operating on the martian surface. During this time, inferior conjunction occurred on May 22, 2016 and July 26, 2018, with superior conjunction on July 27, 2017. When will the next inferior conjunction occur after July 26, 2018?

Month	Angle	Month	Angle	Month	Angle	Month	Angle
9/16	+46	3/17	+24	9/17	-10	3/18	-40
10/16	+45	4/17	+18	10/17	-17	4/18	-41
11/16	+42	5/17	+13	11/17	-23	5/18	-37
12/16	+38	6/17	+7	12/17	-28	6/18	-27
1/17	+34	7/17	+1	1/18	-33	7/18	-7
2/17	+29	8/17	-5	2/18	-38		

Angular data obtained from *Eyes on the Solar System* (February 28, 2013).

Problem 2 – InSight will end its operations after one full martian year (687 days). If it lands on September 20, 2016, during which months of operation will Earth be within 10 degrees of the sun at conjunction, and unable to communicate with Earth?

Problem 3 - Graph the data in the table. During which months will the Earth-Mars angle A) be changing the most rapidly? B) the slowest?

Problem 1 – When will the next superior conjunction occur after July 26, 2018?

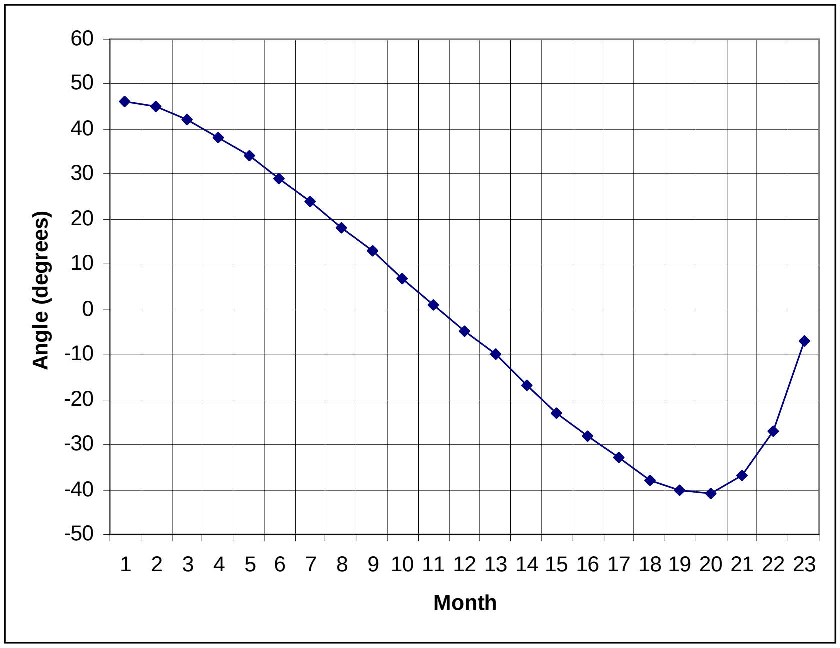
Answer: Each pair of superior and inferior conjunctions happen in cycles. The time for one cycle can be found by the time between the dates of the two listed inferior conjunctions on 5/22/2016 and 7/26/2018. Students can find the number of days between these dates manually (hard), or they can use an online calculator (fun!) like http://www.timeanddate.com/date/duration.html to get 796 days. To find the next superior conjunction, add 796 days to July 27, 2017 (eg. http://www.timeanddate.com/date/dateadd.html) to get **May 4, 2019**.

Problem 2 – InSight will end its operations after one full martian year (687 days). If it lands on September 20, 2016, during which months of operation will Earth be within 10 degrees of the sun at conjunction, and unable to communicate with Earth?

Answer: Conjunction occurs on July 27, 2017. From the table, Earth is within 10 degrees of the sun during the months of **June 2017 to August 2017** so it will be difficult to directly transmit or receive data during this time.

Problem 3 - Graph the data in the table. During which months will the Earth-Mars angle A) be changing the most rapidly? B) the slowest?

Answer: Students may use Microsoft Excel. X-axis may be the month number.
A) June-July, 2018.
B) During September-November, 2016 and February-April, 2018. Slow changes correspond to a nearly flat 'slope' while fast changes correspond to the steepest slope.



Estimating the Mass of a Martian Dust Devil!

This image shows a Martian dust devil tearing across the surface of Mars in the region called Amazonis Planitia. The image was obtained by the Mars Reconnaissance Orbiter on February 16, 2012. The dust devil rises up more than a half mile high (1000 meters) and is about 100 feet (30 meters) wide. Although rare in the equatorial region where NASA's InSight lander will be located, the movement of so much mass near the sensitive seismometer will disturb its measurements. So, how much mass is in a dust devil like the one above?

Problem 1 – The average martian dust devil observed by the Mars Pathfinder Rover has a diameter of about 200 meters. If its height is 1000 meters and is cylindrically shaped, what is its total volume in cubic meters? (Use $\pi = 3.14$)

Problem 2 – If the average dust density is about 0.0000035 kg/m^3, how many kilograms of dust would be present in an average dust devil to the nearest kilogram? How much is this in pounds to the nearest pound? (1 kg = 2.2 pounds)

Sources:

http://aoss-research.engin.umich.edu/planetaryenvironmentresearchlaboratory/docs/Ferri.et.al.JGR03.pdf

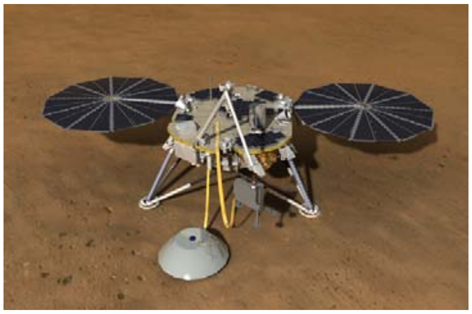
Problem 1 – The average martian dust devil observed by the Mars Pathfinder Rover has a diameter of about 200 meters. If its height is 1000 meters and is cylindrically shaped, what is its total volume in cubic meters?

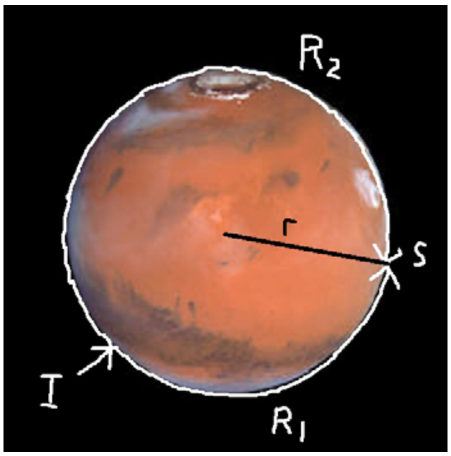
Answer: $V = \pi R^2 h$ so $V = (3.14)\ (100m)^2\ (1000m)$ = **3.1×10^7 meters^3**.

Problem 2 – If the average dust density is about 0.0000035 kg/m^3, how many kilograms of dust would be present in an average dust devil, to the nearest kilogram? How much is this in pounds, to the nearest pound? (1 kg = 2.2 pounds)

Answer: Mass = density x volume, so $M = 3.5 \times 10^{-6} \times 3.1 \times 10^7$ = **109 kilograms**.

In pounds this is M = 2.2 x 109 = **240 pounds**.





NASA's new mission to Mars called InSight will be launched in March, 2016. It will land on September 20, 2016 in a region of Mars located near the equator and deploy a seismographic station to study the interior of Mars.

Each time a large meteor strikes the surface of Mars, one seismic wave will travel to the InSight station along a clockwise path around mars, and a second seismic wave will travel in the opposite direction to the station.

InSight will measure the arrival times of the two waves, called R1 and R2. From this timing data and the 5 km/s speed of the seismic wave along the martian surface, InSight will calculate where the impact occurred. The radius of Mars is r=3,397 kilometer.

In the following problems, use π = 3.1416, and round all answers to the nearest kilometer and second.

Problem 1 – Suppose that the impact occurred at Point I on the above figure, and the time between the arrival of the R1 and R2 waves was exactly 1423 seconds. How far did the R1 and R2 waves travel to get to the InSight station?

Problem 2 – For a large enough impact, InSight scientists expect that after the R1 and R2 waves are detected by the seismometer, that the waves will continue to 'orbit' the surface of Mars and return once again as a second pair of weaker seismic signals called R3 and R4, followed later on by a third pair of even-weaker signals called R5 and R6. For the example in Problem 1, what are the arrival times of all 6 seismic signals if R1 was detected at the clock time of 13:00:00 local time at the lander site?

Problem 3 - Because of a glitch in the recording of the seismic data, InSight scientists were able to detect the R3 and R6 seismic waves, which arrived at 15:25:30 and 17:00:00 Local Mars Time. How far from the Lander did the impact occur, and when would the arrival times for all 6 seismic waves have occurred in the data?

Problem 1 – Suppose that the impact occurred at Point I on the above figure, and the time between the arrival of the R1 and R2 waves was exactly 1423 seconds. How far did the R1 and R2 waves travel to get to the InSight station?

Answer: To travel once around the circumference of Mars, the wave has to travel $2 \pi r = 2 (3.1416) (3397 \text{ km}) = 21344$ km, so the round trip time is T = 21344 km / (5 km/s) = 4269 seconds. We know that T2 – T1 = 1423 seconds, so the R2 wave had to travel the same distance as the R1 wave plus an additional 1423 seconds. Since 1423 seconds = 1/3 of the full circumference time of 4269 seconds, that means that R1 traveled 1423 seconds from the impact site, I, and R2 traveled 2 x 1423 = 2846 seconds from the impact site. The distance to the impact site using the R1 wave data is 1423 sec x 5 km/sec = **7,115 kilometers**

Problem 2 – For a large enough impact, InSight scientists expect that after the R1 and R2 waves are detected by the seismometer, that the waves will continue to 'orbit' the surface of Mars and return once again as a second pair of weaker seismic signals called R3 and R4, followed later on by a third pair of even-weaker signals called R5 and R6. For the example in Problem 1, what are the arrival times of all 6 seismic signals if R1 was detected at the clock time of 13:00:00 local time at the lander site?

R1 = **13:00:00**
R2 = 13:00:00 + 1423 seconds = 13:00:00 + 23m 43s = **13:23:43**
R3 = 13:00:00 + 4269 seconds = 13:00:00 + 1h 11m 9s = **14:11:09**
R4 = 13:23:43 + 4269 seconds = 13:23:43 + 1h 11m 9s = **14:34:52**
R5 = 14:11:09 + 4269 seconds = 14:11:09 + 1h 11m 9s = **15:22:18**
R6 = 14:34:52 + 4269 seconds = 14:34:52 + 1h 11m 9s = **15:46:01**

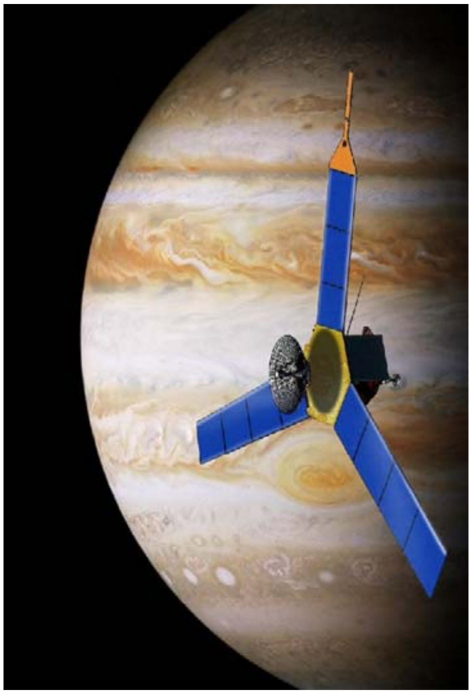
Problem 3 - Because of a glitch in the recording of the seismic data, InSight scientists were able to detect the R3 and R6 seismic waves, which arrived at 15:25:30 and 17:00:00 Local Mars Time. How far from the Lander did the impact occur, and when would the arrival times for all 6 seismic waves have occurred in the data?

Answer: R3 is the R1 wave which has orbited Mars one additional time so that R3-R1 = 4269 seconds. R6 is the R2 wave which has orbited Mars two additional times so that R6-R2 = 2(4269 seconds). The R5 wave would have arrived one full orbit (4269 seconds) after the R3 wave, so the time intervals are as follows:

R1 = 15:25:30 – 4269 seconds = 15:25:30 – 1h 11m 9s = **14:14:21**
R2 = 17:00:00 – 8538 seconds = 17:00:00 – 2h 22m 18s = **14:37:42**
R3 = **15:25:30**
R4 = 17:00:00 – 4269 seconds = 17:00:00 – 1h 11m 9s = **15:48:51**
R5 = 15:25:30 + 4269 seconds = 15:25:30 + 1h 11m 9s = **16:36:39**
R6 = **17:00:00**

Time interval = R2-R1 = 23:21 = 1401 seconds.

R1 + R2 = 4269 seconds
R1 – R2 = 1401 seconds. Then adding the two equations we get 2R1 = 5670 so R1 = 2835 seconds. Traveling at 5 km/sec, the R1 wave originated **14,175 km from the landing site.**



On August 5, 2011 NASA launched the $700 million Juno spacecraft atop an Atlas V551 rocket from Cape Canaveral. Its mission is to reach Jupiter in 2016, go into orbit, and study its enormous radiation belts and atmosphere.

To provide the nearly 500 watts of electricity to power its many instruments, Juno will use three solar panels that will convert sunlight directly into electricity.

Solar panels, consisting of hundreds of individual solar cells, are one of the most reliable energy technologies of the Space Age. They have been used on satellites and spacecraft since the late-1950s. Because of the damaging radiation from Jupiter, Juno's solar cells are much more durable than the solar cells you can buy at your local hardware store.

Problem 1 – The spacecraft has three rectangular solar panels. Two of the panels have dimensions 2.7meters x 8.9 meters, and the third panel has a size of 2.1 meters x 8.9 meters. What is the total area of the solar panels in A) square-meters?, B) Square-centimeters?

Problem 2 – Near Earth's orbit, at a distance of 150 million km from the sun, a square-centimeter of solar cells generates 0.019 watts of electrical power. To the nearest watt, what is the total electrical power that these panels will generate near Earth's orbit?

Problem 3 – The planet Jupiter is located 5.2 times farther from the sun than Earth. Use the Inverse-square Law to A) calculate the percentage of sunlight reaching Jupiter compared to Earth's orbit and B) The amount of electrical power that the solar panels will generate at the orbit of Jupiter.

Problem 1 – The spacecraft has three rectangular solar panels. Two of the panels have dimensions 2.7meters x 8.9 meters, and the third panel has a size of 2.1 meters x 8.9 meters. What is the total area of the solar panels in A) square-meters?, B) Square-centimeters?

A) Area = 2 (2.7m x 8.9) + (2.1m x 8.9m) = **66.8 meters^2**

B) 66.8 meters 2 x (100 cm / 1 meter) x (100 cm / 1 meter) = **668,000 cm^2**

Problem 2 – Near Earth's orbit, at a distance of 150 million km from the sun, a square-centimeter of solar cells generates 0.019 watts of electrical power. To the nearest watt, what is the total electrical power that these panels will generate near Earth's orbit?

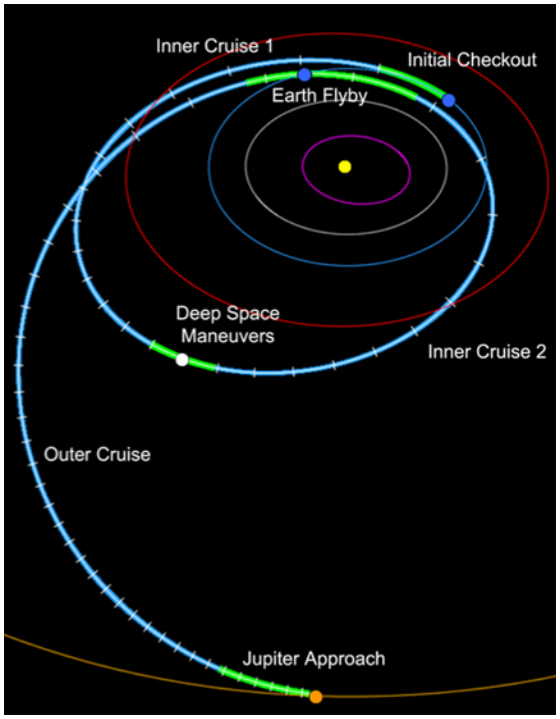
P = area x rate
= 668,000 cm^2 x 0.019 watts/cm^2
= **12,692 watts**.

Problem 3 – The planet Jupiter is located 5.2 times farther from the sun than Earth. Use the Inverse-square Law to A) calculate the percentage of sunlight reaching Jupiter compared to Earth's orbit and B) The amount of electrical power that the solar panels will generate at the orbit of Jupiter.

Answer: A) The amount of sunlight is $(1.0 / 5.2)^2$ = 1/27.0 of the sunlight at Earth's orbit. The percentage reaching Jupiter is then about 100%/27.0 = **3.7%.**

B) Because only 1/27 of the sunlight arriving at Earths distance reaches Jupiter, the solar panels will be illuminated by sunlight that is 1/27 as bright, so the amount of electricity the panls generate will be 1/27 what they did near Earth.
P = 12,692 watts/27.0 = **470 watts**.

Investigating Juno's Elliptical Transfer Orbit



The Juno spacecraft was initially placed in an elliptical orbit near Earth soon after its launch on August 5, 2011. The orbit was elliptical and designed so that a Deep Space Maneuver in August 2012 would send the spacecraft into a flyby of Earth in 2013. This encounter with Earth would boost the spacecraft's speed and place it into an elliptical transfer orbit that would intersect Jupiter's orbit in 2016.

This added speed would not require extra fuel by the spacecraft making it a free resource that keeps the cost of the mission small. These kinds of 'billiard shot' gravitational assists are commonly used by NASA to place spacecraft in trajectories to the outer solar system.

An approximate equation for the transfer orbit is given by the formula: $5.15x^2 + 9.61y^2 = 49.49$. The units for x and y are given in terms of Astronomical Units where 1 AU = 150 million kilometers, which is the average orbit distance of Earth from the Sun.

Problem 1 - What is the equation of the orbit written in Standard Form for an ellipse?

Problem 2 – What is the semimajor axis length in AU?

Problem 3 – What is the semiminor axis length in AU?

Problem 4 – What is the distance between the focus of the ellipse and the center of the ellipse, defined by c?

Problem 5 - What is the eccentricity, e, of the orbit?

Problem 6 – What are the spacecraft's aphelion and perihelion distances?

Problem 7 – Kepler's Third Law states that the period, P, of a body in its orbit is given by $P = a^{3/2}$ where a is the semimajor axis distance in AU, and the period is given in years.If Juno spends ½ of its orbit to get to Jupiter after October, 2013 about when will it arrive at Jupiter?

Problem 1 - What is the equation of the orbit written in Standard Form for an ellipse?
Answer:
$5.15x^2 + 9.61y^2 = 49.49$ Divide both sides by 49.49 to get

$$\frac{x^2}{9.61} + \frac{y^2}{5.15} = 1$$

Problem 2 – What is the semimajor axis length in AU?
Answer: For an ellipse written in standard form: $x^2/a^2 + y^2/b^2 = 1$
Comparing with the equation from Problem 1 we get that the longest axis of the ellipse is along the x axis so the semimajor axis is $a^2 = 9.61$ so **a = 3.1 AU**

Problem 3 – What is the semiminor axis length in AU?
Answer: The semiminor axis is along the y axis so $b^2 = 85.15$ and **b = 2.3 AU**

Problem 4 – What is the distance between the focus of the ellipse and the center of the ellipse, defined by c?
Answer: $c = (a^2 – b^2)^{1/2}$. With a= 3.1 and b=2.3 we have **c = 2.1**.

Problem 5 - What is the eccentricity, e, of the orbit?
Answer: The eccentricity e = c/a so e = 2.1/3.1 and so **e = 0.68**

Problem 6 – What are the spacecraft's aphelion and perihelion distances?
Answer: The closest distance to the focus along the orbit is given by a – c so the perihelion distance is 3.1 – 2.1 = **1.00 AU**. The farthest distance is a + c = 3.1 + 2.1 = **5.2 AU**. *Note the perihelion distance is at Earth's orbit and the aphelion distance is at Jupiter's orbit.*

Problem 7 – Kepler's Third Law states that the period, P, of a body in its orbit is given by $P = a^{3/2}$ where a is the semimajor axis distance in AU, and the period is given in years. If Juno spends ½ of its orbit to get to Jupiter after October, 2013 about when will it arrive at Jupiter?

Answer: Since a = 3.1 we have $P = 3.1^{3/2}$ = 5.5 years for a full orbit. For Juno it spands 5.5/2 = 2.8 years in the elliptical transfer orbit, so it arrives at Jupiter in October 2013 + 2.8 years = October, 2013 + 2 years 8 months = **June, 2016**.

This sequence of images was taken of the launch of the Juno spacecraft on August 5, 2011 from Cape Canaveral. The images were taken, from left to right, at T+21, T+23 and T+25 seconds after launch, which occurred at 12:25:00 pm EDT. The original video can be found on *YouTube*. The distance from the base of the Atlas-Centaur rocket to its top is 45 meters (148 feet). As the video was produced, the camera zoomed-out between the T+21 image and the T+23 image. Both the T+23 and T+25 images were taken at exactly the same zoom scale.

Problem 1 - From the information given, find the speed of the rocket in meters/sec and kilometers/hr between A) 21 and 23 seconds after launch and B) 23 to 25 seconds after launch.

Problem 2 - What is the average acceleration of the rocket in meters/sec^2 between 21 and 25 seconds after launch?

Problem 3 - At the average acceleration of this rocket, about when will it be traveling faster then the speed of sound (Mach 1) which is 340 meters/sec?

Problem 1 - From the information given, find the speed of the rocket in meters/sec and kilometers/hr between A) 21 and 23 seconds after launch and B) 23 to 25 seconds after launch.
Answer: Students will need to determine the scale of each image by using a millimeter ruler to measure the length of the rocket body, which is known to be 45 meters. When printed using a regular laser printer, the lengths of the rockets are about 21) 5.5mm 23) 4.0 mm and 25) 3.0 mm
The image scales are therefore 8.2 meters/mm, 11.3 meters/mm and 15 meters/mm

To measure speed, all we need to do is measure the height of the bottom of the rocket vertically from a well-defined point near the bottom of the image away from the exhaust cloud. The horizontal band of water just below the exhaust plume provides a good reference. Using the millimeter ruler we get 21) 35 mm 23) 36 mm and 25) 43 mm Converting this in to meters using the three scales we get 21) 287 meters 23) 407 meters and 25) 645 meters

Speed: 21 to 23 seconds; s1 = (407-287) / 2 sec so s1 = **60 meters/sec**
23 to 25 seconds: s2 = (645 - 407)/ 2 sec so s2 = **119 meters/sec**

In km/h we get s1 = **216 km/hour** and s2 = **428 km/hr**.

Students estimates will vary depending on the method and measuring accuracy used.

Problem 2 - What is the average acceleration of the rocket in meters/sec^2 between 21 and 25 seconds after launch?

Answer: acceleration = difference in speed/difference in time so
Acc = (119 meters/sec - 60 meters/sec) / (4 seconds)
= **15 meters/sec^2**

Problem 3 - At the average acceleration of this rocket, about when will it be traveling faster then the speed of sound (Mach 1) which is 340 meters/sec?

Answer: speed = initial speed + acceleration x time

340 = 119 + 15 x T

So **T = 15 seconds after the initial speed** of 119 m/s was reached. This occurs about T = 25 sec + 15 sec = **40 seconds after launch**.

According to actual flight information, Mach 1 was reached a bit later at T+ 51 sec.

The Juno spacecraft was launched on August 5, 2011 on a 5 year journey to Jupiter. This image was taken 120 seconds after launch and shows one of the solid rocket boosters being jettisoned. The camera is on the Atlas booster and looks down on the engines and the distant arc of a cloudy Earth. Scenes from the launch can be found on *YouTube*, and show a dazzling launch from multiple viewing locations on Earth and in space.

During the launch, and the boost to orbit, the altitude of the rocket changes continuously as the engines provide thrust, eventually lifting the entire payload into orbit at a planned altitude of 420 kilometers (261 miles). A short table of the rocket's altitude and times is provided below.

Elapsed Time (seconds)	Altitude (miles)	Altitude (kilometers)
160	46	
192	60	
268	81	
274	83	
315	95	
319	97	
339	112	

Problem 1 - American engineers use English units for all measurements including the details of the rocket launch where 1 mile = 1.6 kilometers. Use this information to complete the table above in metric units rounded to the nearest kilometer.

Problem 2 - From the tabular data, graph the altitude of the rocket in time.

Problem 3 - From the data, find the function A(t) that predicts the altitude of the rocket at future times. The function will be of the form

$$A(t) = a + b\ln(t)$$

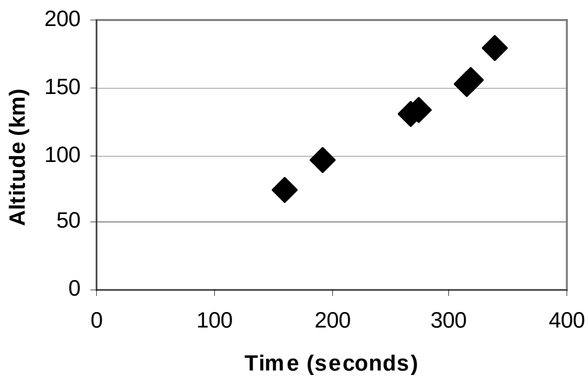
Find the constants a and b for A(t) in kilometers and A(t) in miles.

Problem 4 - How many seconds after launch will it take for the rocket to reach orbit altitude at 420 kilometers?

Problem 1 - American engineers use English units for all measurements including the details of the rocket launch where 1 mile =1.6 kilometers. Use this information to complete the table above in metric units rounded to the nearest kilometer.

Elapsed Time (seconds)	Altitude (miles)	**Altitude (kilometers)**
160	46	**74**
192	60	**96**
268	81	**130**
274	83	**133**
315	95	**152**
319	97	**155**
339	112	**179**

Problem 2 - From the tabular data, graph the altitude of the rocket in time.



Problem 3 - From the data, find the function A(t) that predicts the altitude of the rocket at future times. The function will be of the form

$$A(t) = a + b\ln(t)$$ Find the constants a and b.

Answer: Choose (160,74) and (319,155) then
74 = a + 5.1b and 155 = a + 5.8b
Solve by substitution: a = 74 - 5.1b then 155 =(74-5.1b) + 5.8b
So 81 = 0.7b and b = 116. Then a = -518
So with A(t) in kilometers, and t in seconds, we have:

$$A(t) = -518 + 116\ln(t)$$

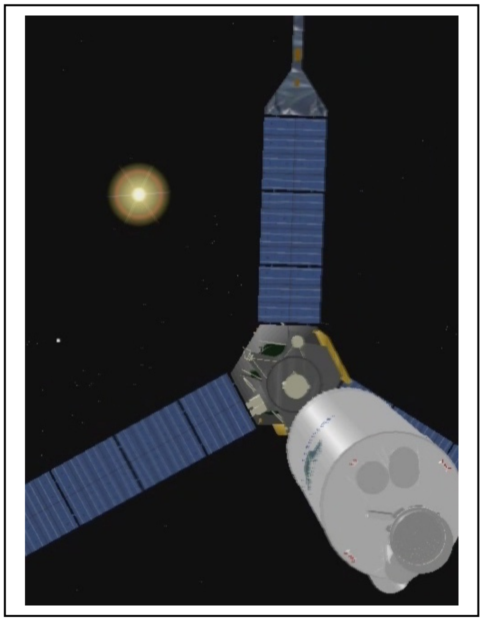
In miles this becomes $$A(t) = -324 + 73\ln(t)$$

Problem 4 - How many seconds after launch will it take for the rocket to reach orbit altitude at 420 kilometers?

Answer: $420 = -518 + 116\ln(t)$ so t = **3,248 seconds or about 54 minutes**.

Note: The actual time is about 3,229 seconds.

Solar Energy and the Distance to Juno from the Sun



As the Juno spacecraft travels to Jupiter, it gets farther from the sun every day. Because the spacecraft generates its electrical power using solar cells, as the sun gets farther away, the amount of power constantly diminishes. At Earth, the solar panels generate about 12,700 watts. Because the spacecraft's trajectory is a portion of an ellipse, the formula for its distance, r, from the sun, located at one of the ellipse's foci, is given by the formula:

$$r(\theta) = \frac{a(1-e^2)}{1-e\cos\theta}$$

where r is in Astronomical Units, a is the semi-major axis length and e is the orbit eccentricity.

The specific equation for the Juno spacecraft can be approximately represented by a = 3.0 AU and e = 2/3, where all distances are given in units of the Astronomical Unit. The Astronomical Unit is a measure of the distance between Earth and the sun; a physical distance of 150 million km.

Problem 1 – What is the simplified form for R given the initial parameters, a and e, for the Juno spacecraft?

Problem 2 – If the amount of solar energy falling on the Juno solar panels is determined by the inverse-square law, and the amount of solar energy generated by the solar panels at r = 1.0 AU is exactly 12,690 watts, what is the formula for the solar panel power at any distance defined by the function P(r)?

Problem 3 – For what angle, θ, will the spacecraft be able to generate only ¼ of the electrical power it did when it left Earth orbit?

Problem 4 – What will the spacecraft power be when it reaches Jupiter at its maximum distance from the sun?

Problem 1 – What is the simplified form for R given the initial parameters, a and e, for the Juno spacecraft?

$$r(\theta) = \frac{5}{3 - 2\cos\theta}$$

Problem 2 – If the amount of solar energy falling on the Juno solar panels is determined by the inverse-square Law, and the amount of solar energy generated by the solar panels at r = 1.0 AU is 12,000 watts, what is the formula for the solar panel power at any distance defined by the function P(r)?

$$P(r) = 480(3 - 2\cos\theta)^2$$

Problem 3 – For what angle, θ in degrees, will the spacecraft be able to generate only ¼ of the electrical power it did when it left Earth orbit?

P = 12000/4 = 3000 watts
$3000 = 480\ (3\text{-}2\cos\theta)^2$ so $2.5 = 3 - 2\cos\theta$ and $\cos\theta = 0.25$ so **θ= 76 degrees**

Problem 4 – What will the spacecraft power be when it reaches Jupiter at its maximum distance from the sun?

Answer: The maximum value for r occurs at $\theta = 0$, so **P = 480 watts**.

The Speed of the Juno Spacecraft

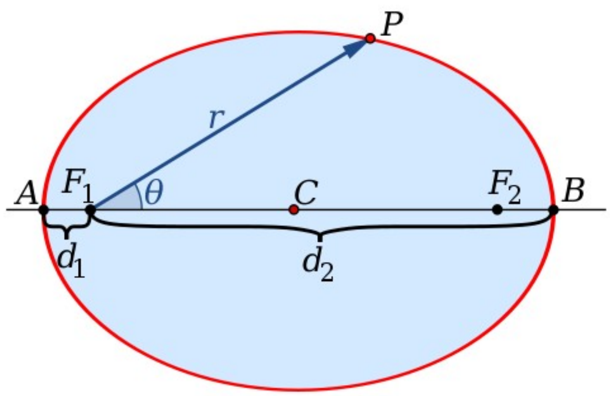


Diagram defining r and q. Spacecraft at Point P. Sun at focus F1. Jupiter at Point B (Image courtesy Wikipedia entry for 'Ellipse')

The speed of the Juno spacecraft in its elliptical orbit around the sun is given by two equations. The first one specifies the spacecraft location in its orbit given by

$$r(\theta) = \frac{5}{3 - 2\cos\theta}$$

The second equation is the speed approximately given for the Juno spacecraft by

$$s = 40\sqrt{\left(\frac{2}{r} - \frac{1}{3}\right)}$$

where S is in kilometers/sec, r is Juno's distance from the sun in AUs, and θ is the orbital angle.

Problem 1 – Combining the two equations, and evaluating the constant to the nearest km/s, what is the function specifying the spacecraft speed defined solely in terms of the angle parameter s(θ) ?

Problem 2 - About how far from the sun will Juno be in its orbit for A) θ=0? B) θ=180?

Problem 3 - About what will Juno's orbital speed be when it arrives at Jupiter?

Problem 4 - For what value of θ will A) the spacecraft speed be about 25 km/s, and B) how far will it be from the sun at that time?

Problem 1 – Combining the two equations, and evaluating the constant to the nearest km/s, what is the function specifying the spacecraft speed defined solely in terms of the angle parameter s(θ) ?

$$s = 40\sqrt{\frac{2}{(5/(3-2\cos\theta))} - \frac{1}{3}}$$

$$s = 40\sqrt{\frac{6(3-2\cos\theta)-5}{15}}$$

$$s = \frac{8\sqrt{15}}{3}\sqrt{13-12\cos\theta}$$

$$s = 10\sqrt{13-12\cos\theta}$$

Note: $8(15)^{1/2}/3 = 10.33$ which to the nearest km/sec becomes 10

Problem 2 - About how far from the sun will Juno be in its orbit for A) θ=0? B) θ=180?

Answer: A) **5 Astronomical Units**. B) **1 Astronomical Unit**.

Problem 3 - About what will Juno's orbital speed be when it arrives at Jupiter?

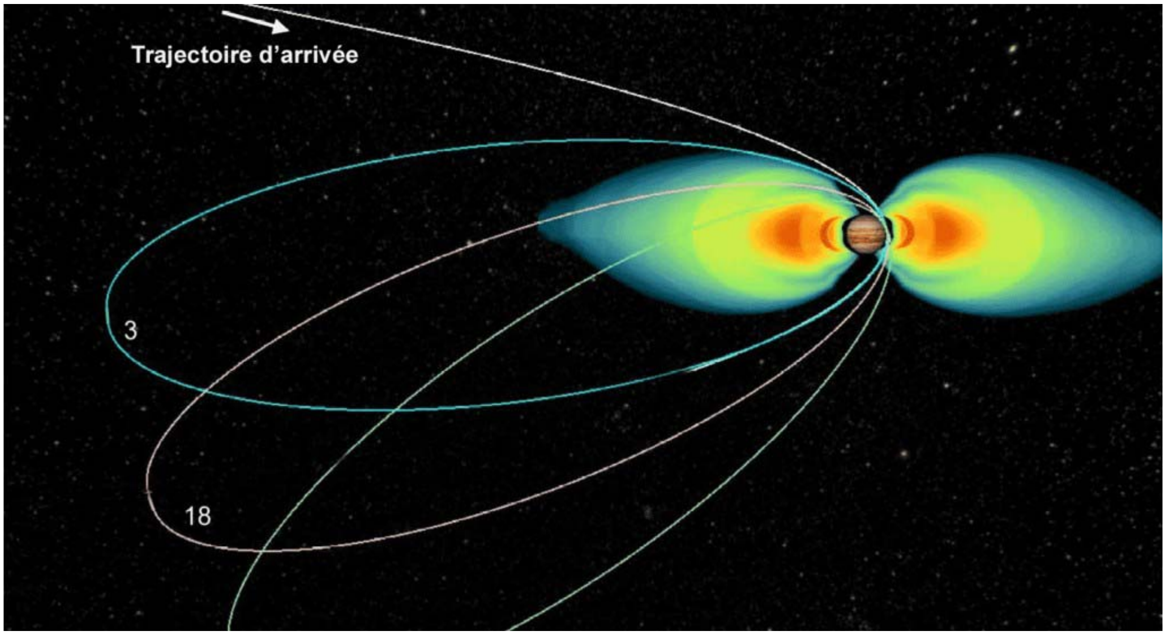
Answer: $s = 40\sqrt{\left(\frac{2}{5} - \frac{1}{3}\right)}$ so s = **10 km/sec**.

Problem 4 - For what value of θ A) will the spacecraft speed be about 25 km/s, and B) how far will it be from the sun at that time?

Answer: A)

$25 = 40\sqrt{\left(\frac{2}{r} - \frac{1}{3}\right)}$ so solving for r we get r = **2.76 Astronomical Units**.

B) $2.76 = \frac{5}{3-2\cos\theta}$ so cos θ = 0.594 and so θ **= 54 degrees**.



The Juno spacecraft, launched August 5, 2011, will arrive at Jupiter in early-July, 2016. Upon arrival, it will be placed in an elliptical polar orbit, where it will remain until the end of the mission after 33 orbits. The spacecraft will then dive into Jupiter's atmosphere and burn-up.

Problem 1 - For convenience, distances around Jupiter are given in multiples of Jupiter's radius, which is 71,600 km. This unit is called RJ, so that, for example, a distance of 3.0 RJ corresponds to 3 x (71,600 km) or 214,800 km. If the formula for Juno's elliptical orbit is given by $\mathbf{38x^2 + 400y^2 = 15,200}$ where x and y are in units of RJ, what is the equation for Juno's orbit written in Standard Form for an ellipse?

Problem 2 - The periJovian distance, Rp, is the closest distance from the orbit to the center of Jupiter, while the apoJovian distance, Ra, is the farthest distance. Use the following properties of an ellipse to determine, Rp, Ra and the eccentricity, e, of Juno's orbit, where f is the shortest distance from the focus to the ellipse and

$b = a\,(1-e^2)^{1/2}$ $\quad Ra = a + f \quad Rp = a-f \quad f = ae$

Problem 3 - According to Kepler's Third Law, the period, P, in days for a body in an orbit around Jupiter is related to its semi-major axis, a, in RJ given by

$$a^3 = 66.5P^2$$

A) To the nearest day, what is the orbit period of Juno in days?
B) How long will the spacecraft remain in orbit before atmospheric entry?

Problem 1 - For convenience, distances around Jupiter are given in multiples of Jupiter's radius, which is 71,600 km. This unit is called RJ, so that, for example, a distance of 3.0 RJ corresponds to 3 x (71,600 km) or 214,800 km. If the formula for Juno's elliptical orbit is given by **$38x^2 + 400y^2 = 15{,}200$** where x and y are in units of RJ, what is the equation for Juno's orbit written in Standard Form for an ellipse?

Answer: Divide both sides by 15,200 and simplify to the Standard Form:

$\left(\frac{38}{15200}\right)x^2 + \left(\frac{400}{15200}\right)y^2 = 1$ so in Standard Form: $\frac{x^2}{20^2} + \frac{y^2}{6.2^2} = 1$

The semi-major axis is a = 20 RJ and is along the x-axis. The semi-minor axis is b = 6.2 RJ and is along the y-axis.

Problem 2 - The periJovian distance, Rp, is the closest distance from the orbit to the center of Jupiter, while the apoJovian distance, Ra, is the farthest distance. Use the following properties of an ellipse to determine, Rp, Ra and the eccentricity, e, of Juno's orbit, where f is the shortest distance from the focus to the ellipse and

$$b = a\,(1-e^2)^{1/2} \qquad Ra = a + f \qquad Rp = a - f \qquad f = ae$$

Answer: From the first equation, a and b are known so solve for e to get **e = 0.95**. Use the fourth equation to solve for f to get f = 19.0 RJ. Then use the second and third equations to find the periJovium and apoJovium as **Ra=1.0 RJ** and **Rp=39 RJ**.

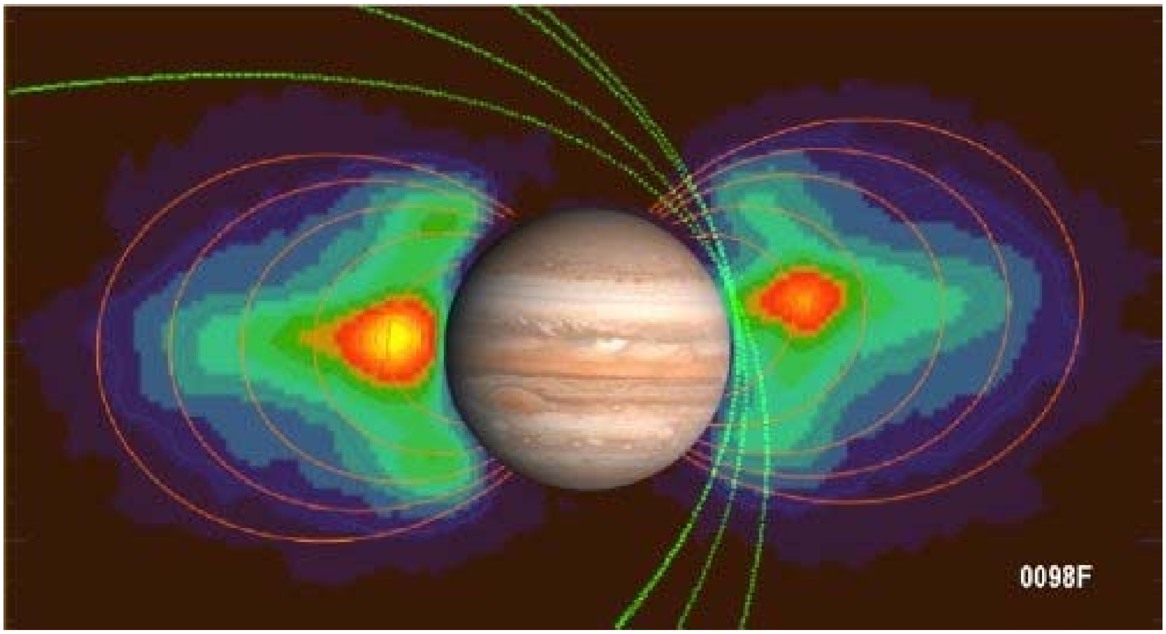
Problem 3 - According to Kepler's Third Law, the period, P, in days for a body in an orbit around Jupiter is related to its semi-major axis, a, in RJ given by

$$a^3 = 66.5P^2$$

A) To the nearest day, what is the orbit period of Juno in days?
B) How long will the spacecraft remain in orbit before atmospheric entry?

Answer: A) Since a = 20 RJ, so solving for P we get P = 10.96 days or **11 days**.
B) The information states that after 33 orbits it will be directed to enter the Jovian atmosphere to burn up. The mission lifetime is then T = 33 x 11 days = **330 days.**

Note: Due to the intense 'van Allen' radiation belts around Jupiter, the spacecrafts electrical systems will not survive intact for more than a year so no useful data will be returned for a prolonged stay. Meanwhile. Atmospheric entry will allow the instruments to measure the Jovian atmosphere before they start to fail.



In the above NASA artist rendition, we see the inner portions of the elliptical orbits of the Juno spacecraft as it passes close to Jupiter and its intense radiation belts. The orbit was designed to avoid as much of the radiation belts as possible so that the spacecraft could survive long enough to carry-out its science objectives.

As the spacecraft travels along its orbit, high-energy protons and electrons penetrate the spacecraft and degrade its electrical systems and computer systems. A simple, and very approximate, mathematical model for the number of these particles encountered by every square centimeter of the satellite surface area, along the orbit is given by

$$N = 26,000 \sin^2\left(\frac{\pi T}{11}\right) \text{ particles per day}$$

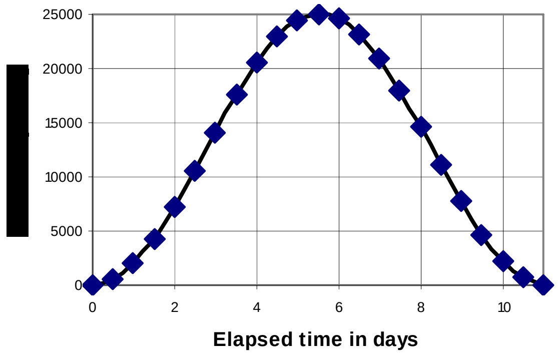
where T is the elapsed time in days.

Problem 1 - If the orbital period of the spacecraft is 11.0 days, graph this function for one complete orbit.

Problem 2 - What is the approximate total number of particles encountered by the spacecraft in one complete orbit?

Problem 3 - If the JunoCam camera has an unshielded imaging array that has 1 million pixels, and if each radiation particle destroys one pixel, A) how many pixels are lost by the camera each orbit? B) About how many orbits will be required to destroy all of JunoCam's imaging pixels?

Problem 1 - If the orbital period of the spacecraft is 11.0 days, graph this function for one complete orbit.



Problem 2 - What is the approximate total number of particles encountered by the spacecraft in one complete orbit?

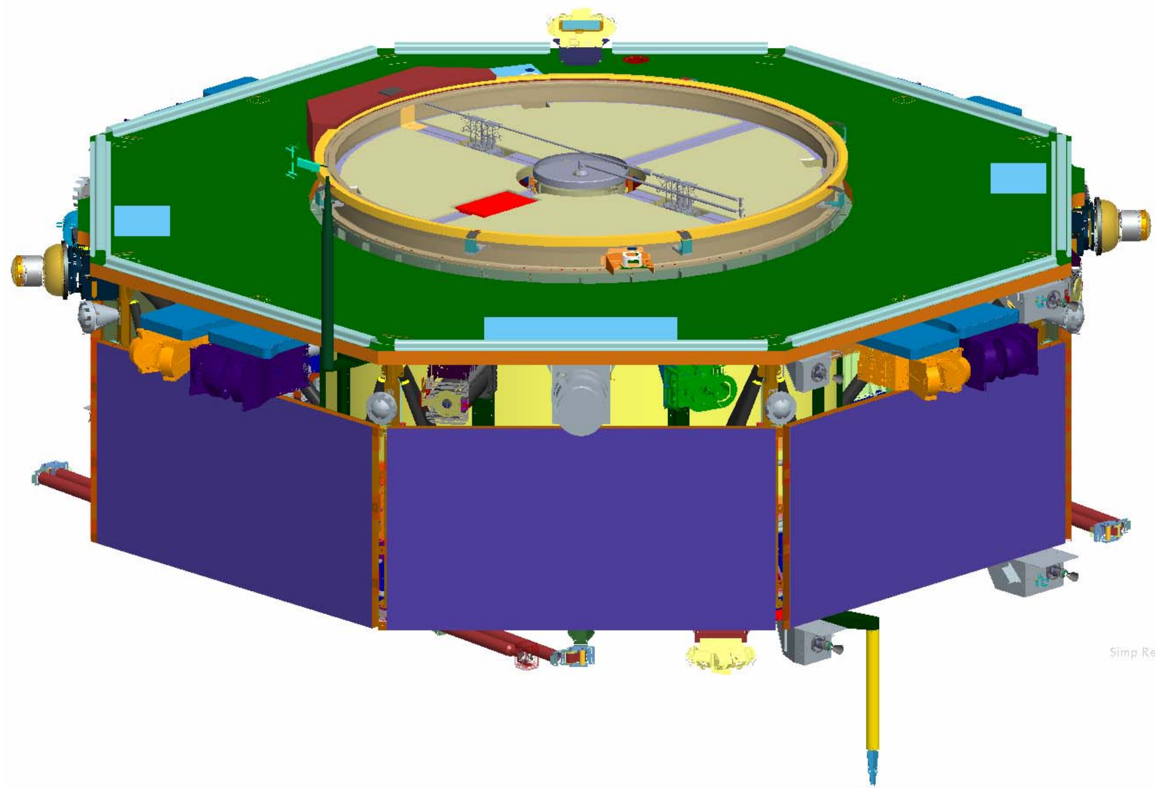
Answer: Since the function gives the number of particles per day, the area under this curve gives the total number of particles since the area is: (particles/day) x day = particles. Students can approximate the area by using the inscribed rectangles and compare their answers using rectangles with smaller bases. Using 1 rectangle with a base of 8 days and an average height of N = 15,000 you get **N = 120,000 particles**. Another method is to take the average between the inscribed rectangle and the circumscribed rectangle. The dimensions of the circumscribed rectangle is width= 11 days, height = 25,000 so A = 275,000 particles. The average is then N = (275,000 + 120,000)/2 = **197,500 particles**.

The exact answer using integral calculus (or a LOT of very small rectangles!) is N = 137,500 particles.

Problem 3 - If the JunoCam camera has an unshielded imaging array that has 1 million pixels, and if each radiation particle destroys one pixel, A) how many pixels are lost by the camera each orbit? B) About how many orbits will be required to destroy all of JunoCam's imaging pixels?

Answer: Students answers will differ depending on their answer to Problem 2, but the answer using the three values for N are 1 million/120000 = 8 orbits; 1 million/197500 = 5 orbits, and the exact answer would be 1 million/137500 = 7 orbits. **Answers between 5 to 8 orbits are acceptable.**

So the JunoCam will not last as long as the entire mission of 33 orbits!



Each of the Magnetosphere Multiscale (MMS) satellites is in the shape of an octagonal prism. The faces of the satellite are partially covered with solar cells that will generate the electricity to operate the satellite and its many experiment modules.

Problem 1 - If the distance between opposite faces of the satellite is D = 3.150 meters, and the height of each solar panel is h = 0.680 meters, what is the width of one rectangular solar panel if w = 0.414 D?

Problem 2 - What is the surface area of one rectangular solar panel in the satellite?

Problem 3 If the solar cells generate 0.03 watts per square centimeter of surface area, how much power will be generated by a single face of the satellite?

Answer Key

Problem 1 - If the distance between opposite faces of the satellite is D = 3.150 meters, and the height of each solar panel is h = 0.680 meters, what is the width of one rectangular solar panel if w = 0.414 D?

Answer: The problem says that D = 3.150 meters, so w = 0.414 (3.150) = **1.30 meters**.

Advanced students who know trigonometry can determine the width from the following geometry:

The relevant right-triangle has two sides that measure D/2 and w/2 with an angle of 45/2 = 22.5 . Then from trigonometry, tan(45/2) = w/D. Since tan(22.5) = 0.414, we have w = 0.414 (D), and so **w = 1.30 meters**.

Problem 2 - The area of each solar panel is just A = (1.30 meters) x (0.680 meters) so

Area = 0.88 meter^2.

Problem 3 - If the solar cells generate 0.03 watts per square centimeter of surface area, how much power will be generated by a single face of the satellite?

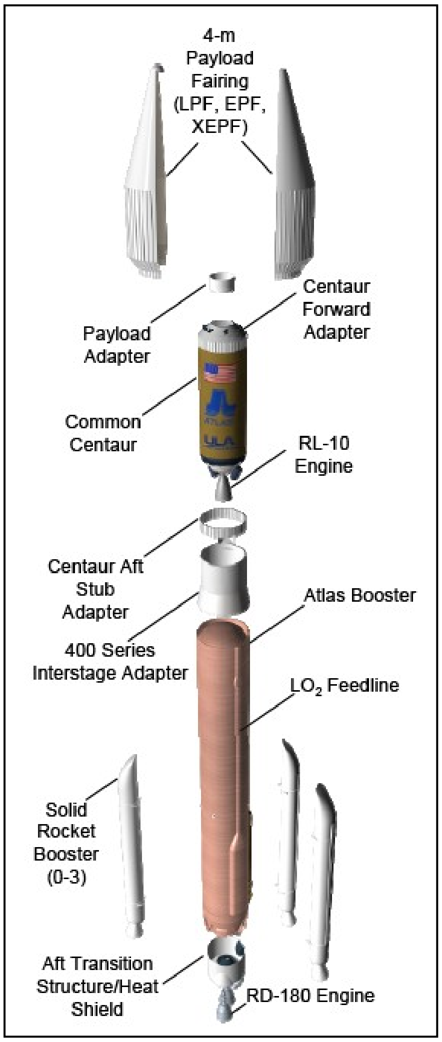
Answer: A single face of the satellite has a solar panel with an area of 0.88 meters^2 . First convert this into square centimeters

A = 0.88 meter^2 x (100cm / 1 m) x (100 cm/1 m)

= 8800 cm^2

Then multiply by the solar power area factor of 0.03 watts/cm^2 to get

Power = 264 watts.



The Magnetosphere Multiscale (MMS) satellite constellation will be launched into orbit in 2014 atop an Atlas V421XEPF rocket. There are dozens of different configurations of Atlas rockets depending on the mass and orbit destination of the payload being launched. One of these that is similar to the MMS rocket is shown in the figure to the left. The Atlas V421 consists of single Atlas rocket booster with two strap-on solid rocket boosters, and a second-stage Centaur rocket booster with the MMS satellite stack attached above the Centaur.

Atlas: Height = 32.46 meters
Diameter = 3.81 meters
Mass = 399,700 kg

Centaur: Height = 12.68 meters
Diameter = 3.05 meters
Mass = 23,100 kg

Payload: Height = 13.81 meters
Diameter = 4.2 meters
Mass = 7,487 kg

Problem 1 - What is the total length of the MMS launch vehicle in A) meters? B) feet? C) inches? (Note: 1 meter = 3.281 feet)

Problem 2 - To the nearest tenth of a percent, what percentage of the launch vehicle height is occupied by the payload?

Problem 3 - To the nearest tenth of a percent, if each satellite has a mass of 1,250 kg, what percentage of the entire rocket mass is occupied by the four satellites?

Answer Key

Atlas: Height = 32.46 meters
Diameter = 3.81 meters Mass = 399,700 kg

Centaur: Height = 12.68 meters
Diameter = 3.05 meters Mass = 23,100 kg

Payload: Height = 13.81 meters
Diameter = 4.2 meters Mass = 7,487 kg

Problem 1 - What is the total length of the MMS launch vehicle in A) meters? B) feet? C) Inches?

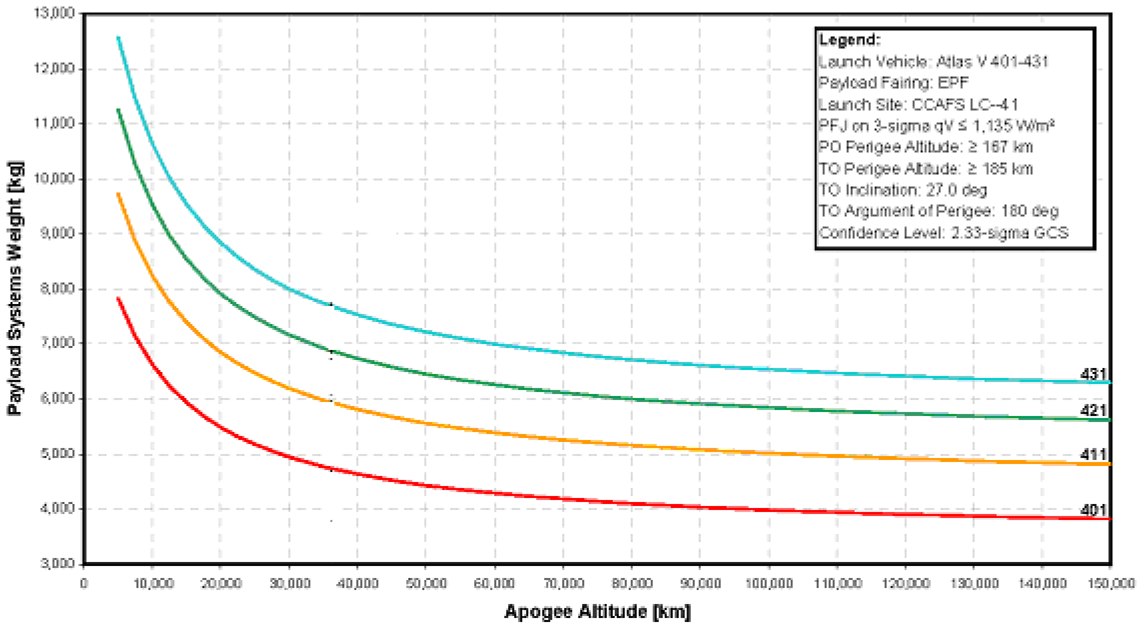
Answer: A) H = 32.46 meters + 12.68 meters + 13.81 meters = **58.95 meters**.
B) H = 58.95 meters x (3.281 feet/meters) = **193.41 feet**.
C) H = 193.41 feet x (12 inches / 1 foot) = **2320.92 inches**.

Problem 2 - To the nearest tenth of a percent, what percentage of the launch vehicle height is occupied by the payload?

Answer: Total length = 58.95 meters. Payload = 13.81 meters so
P = 100% (13.81/58.95) = **23.4%**

Problem 3 - To the nearest tenth of a percent, if each satellite has a mass of 1,250 kg, what percentage of the entire rocket mass is occupied by the four satellites?

Answer: The combined masses for the Atlas booster, the Centaur upper stage and the payload is M = 399,700 kg + 23,100 kg + 2,487 kg + 4(1,250 kg) = 430,287 kg. The mass in the satellites is m = 4(1,250 kg) = 5000 kg, so the percentage is just
P = 100% (5000/430287) = **1.2%**.



This figure, obtained from the Boeing Corporation '*Atlas V Users Guide: 2010*' shows the maximum payload mass that can be launched into a range of orbits with apogees from 6,378 km to 150,000 km. The apogee distance is the maximum distance from the center of Earth that the orbit will take the payload. The specific models of Atlas rocket are given on the right-hand edge and include the Atlas V401, V411, V421 and V431. For example, the Atlas V401 can lift any payload with a mass of less than 4,000 kg into an orbit with an apogee of no more than 100,000 km.

Problem 1 – For a particular Atlas V model, what does the curve for that model tell you about payload mass and maximum altitude?

Problem 2 – Suppose a scientist wanted to place a 6,500 kg research satellite into orbit with an apogee of 70,000 km. What is the best choice of launch vehicle to satisfy this requirement?

Problem 1 – For a particular Atlas V model, what does the curve for that model tell you about payload mass and maximum altitude?

Answer: The curves all decline in payload mass as the orbit apogee increases. This means that there is a trade-off between the mass you can place into orbit and the maximum apogee for that mass. The less mass your payload has, the greater is the apogee of the orbit that you can reach. This is the same concept as it is easier for you to throw a 3 ounce tennis ball to a higher altitude than a 2-pound lead weight! The same relationship works for rockets.

Problem 2 – Suppose a scientist wanted to place a 6,500 kg research satellite into orbit with an apogee of 70,000 km. What is the best choice of launch vehicle to satisfy this requirement?

Answer: We find ’70,000 km’ on the horizontal axis and draw a vertical line. Next we draw a horizontal line from ‘6,500 kg’ on the vertical axis until it intercepts our vertical line at ’70,000 km’. The intersection point lies just above the curve for the V 421, meaning that it is not powerful enough for the task, but it lies below the curve for the **Atlas V431** launch vehicle, so this vehicle has the capacity to launch this payload to the indicated apogee.

Exploring the Atlas-V Launch Pad at Cape Canaveral

The Magnetosphere Multiscale (MMS) mission will be launched from Pad 41 at the Cape Canaveral US Air Force Launch Facility in Florida in 2014. The image above shows a spectacular satellite view of this complex. The short, horizontal line to the lower left indicates a length of 100 meters.

Problem 1 – With the help of a millimeter ruler, what is the scale of this image in meters per millimeter?

Problem 2 – The circular road is centered on the location of the launch pad for the Atlas V rocket. What is the circumference of this road to the nearest meter?

Problem 3 – At a comfortable walking pace of 1.5 meter/sec, to the nearest minute, how long would it take you to walk around this perimeter road?

Problem 4 – There are four towers surrounding the launch pad, and their shadows can be seen pointing to the upper left. If a tower is 73 meters tall, create a scale model showing the horizontal shadow length and the vertical tower height, and determine the angle of the sun at the time the image was made.

Answer Key

Problem 1 – With the help of a millimeter ruler, what is the scale of this image in meters per millimeter?

Answer: For ordinary reproduction scales, students should measure the '100 meter' bar to be 12 mm long, so the scale of the image is 100 meters/12 mm = **8.3 meters/mm.**

Problem 2 – The circular road is centered on the location of the launch gantry for the Atlas-V rocket. What is the circumference of this road to the nearest meter?

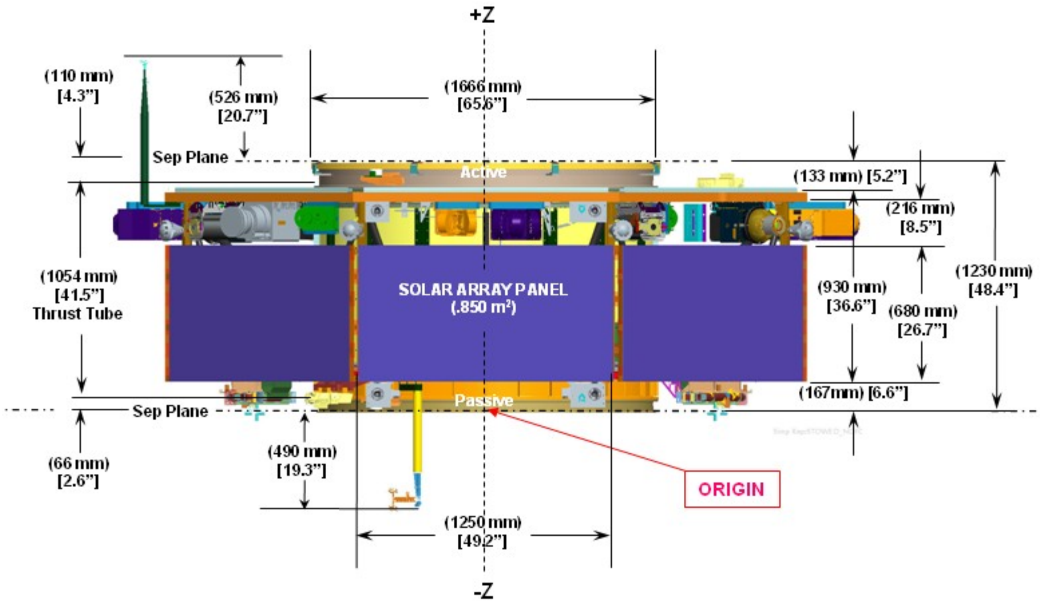
Answer: The diameter of the road is 41 mm or 340 meters. The circumference is C = 3.141 x 340 meters = **1068 meters**.

Problem 3 – At a comfortable walking pace of 1.5 meter/sec, to the nearest minute, how long would it take you to walk around this perimeter road?

Answer: T = distance/speed so
T = 1068 meters/1.5 = 712 seconds or **12 minutes**.

Problem 4 – There are four towers surrounding the launch gantry, and their shadows can be seen pointing to the upper left. If a tower is 73 meters tall, create a scale model showing the horizontal shadow length and the vertical tower height, and determine the angle of the sun at the time the image was made.

Answer: The shadow of one of the towers measures about 17 mm or 141 meters. The two sides of the right-triangle are therefore 73 meters and 141 meters. Students may draw a scaled model of this triangle, then use a protractor to measure the sun elevation angle opposite the '73 meter' segment. Or they may use $\tan(\theta) = 73$ meters/141 meters and so θ **= 27 degrees**.



In 2014, NASA will launch four octagonal satellites in the Magnetosphere Multiscale (MMS) constellation into space to measure Earth's magnetic field. The satellites will be stacked vertically in the nose-cone of the rocket. Once in space, they will be released one at a time into their specific orbits.

The diagram above shows the dimensions of the satellite in side-view projection. The satellites will be stacked vertically along the vertical 'Z-axis' shown in the figure. The two vertical antennas for each satellite will be stowed inside the satellite and will not protrude above the faces of the satellite. Each satellite consists of a main octagonal body, with two attachment flanges labeled 'Passive' and 'Active' in the diagram above. The Active flange will clamp onto the bottom Passive flange of the previous satellite when the satellites are stacked in the nose-cone. When the Passive and Active flanges are connected, the total height of the combined coupling will only be 167 millimeters.

Problem 1 – Using the measurements labeled in the figure above, when the four satellites are stacked, what will be the total height of the satellite stack in the nose-cone in A) millimeters? B) meters?

Problem 2 - To fit inside the rocket, the maximum diameter of the stacked satellites must be 3.478 meters. From your answer to Problem 1B, to three significant figures, what is the total cylindrical volume of the stacked satellites in cubic meters?

Problem 1 - Using the measurements labeled in the figure above, when the four satellites are stacked, what will be the total height of the satellite stack in the nose-cone in A) millimeters? B) meters??

Answer: A) From the measurements in the diagram, each satellite body has a height of 133 mm + 216mm + 680 mm or 1029 millimeters.

The stack will include a top 'Active' flange from Satellite 4, and a bottom 'Passive' flange from Satellite 1. The Active flange has a height of 133 millimeters. The Passive flange has a height of 167 millimeters.

There will also be 3 internal coupled flanges with a height of 3 x 167 mm = 501 millimeters.

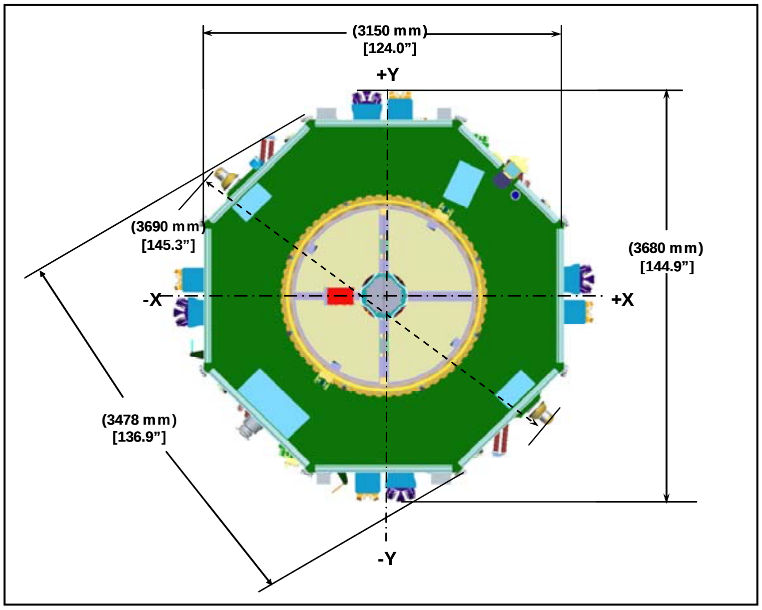
So the total satellite stack height is then
H = 4(1029) + 133 + 167 + 501
H = 4917 mm.

B) Since 1000 millimeters = 1 meter, **H = 4.917 meters**.

Problem 2 - The maximum diameter of the stacked satellites will be 3.478 meters. From your answer to Problem 1B, to three significant figures, what is the total cylindrical volume of the stacked satellites in cubic meters?

Answer: $V = \pi R^2 H$, so
$V = 3.141 (3.478/2)^2 (4.917)$,
and so **V = 46.7 meters^3**.

The Volume and Surface Area of an Octagonal MMS Satellite



The Magnetosphere Multiscale (MMS) constellation consists of 4 identical satellites that will be launched into orbit in 2014 to investigate Earth's magnetic field in space. Each satellite is an octagonal prism with a face-to-face diameter of 3.15 meters and a height of 0.896 meters. A central, cylindrical hole has been cut out of each satellite to accommodate the steering rockets and fuel tanks. This cylindrical hole has a diameter of 1.66 meters.

Problem 1 - What is the formula for the area of an octagon with a diameter of D meters, if the area of one of the 16 inscribed right triangles is given by the formula $A = 0.104D^2$?

Problem 2 - To three significant figures, what is the total surface area, including side faces, of a single MMS satellite? (Hint: don't forget the cylindrical hole!)

Problem 1 - What is the formula for the surface area of an octagon with a diameter of D meters if the area of one of the 16 inscribed right triangles is given by the formula $A = 0.104D^2$?

Answer: Note: Draw an octagon with the stated dimensions. Reduce the octagonal area to the areas of 16 right-triangles with sides w and D/2. Using trigonometry, w/2 = D/2 tan(45/2) so w = 0.414 D, and then the area of a single triangle is A = 1/2 (D/2) x (0.414D) or $A = 0.104 D^2$.

The total area of a single octagonal face is then $A = 16 \times (0.104) D^2$, or $\mathbf{A = 1.664D^2}$.

Problem 2 - To three significant figures, what is the total surface area, including side faces, of a single MMS satellite?

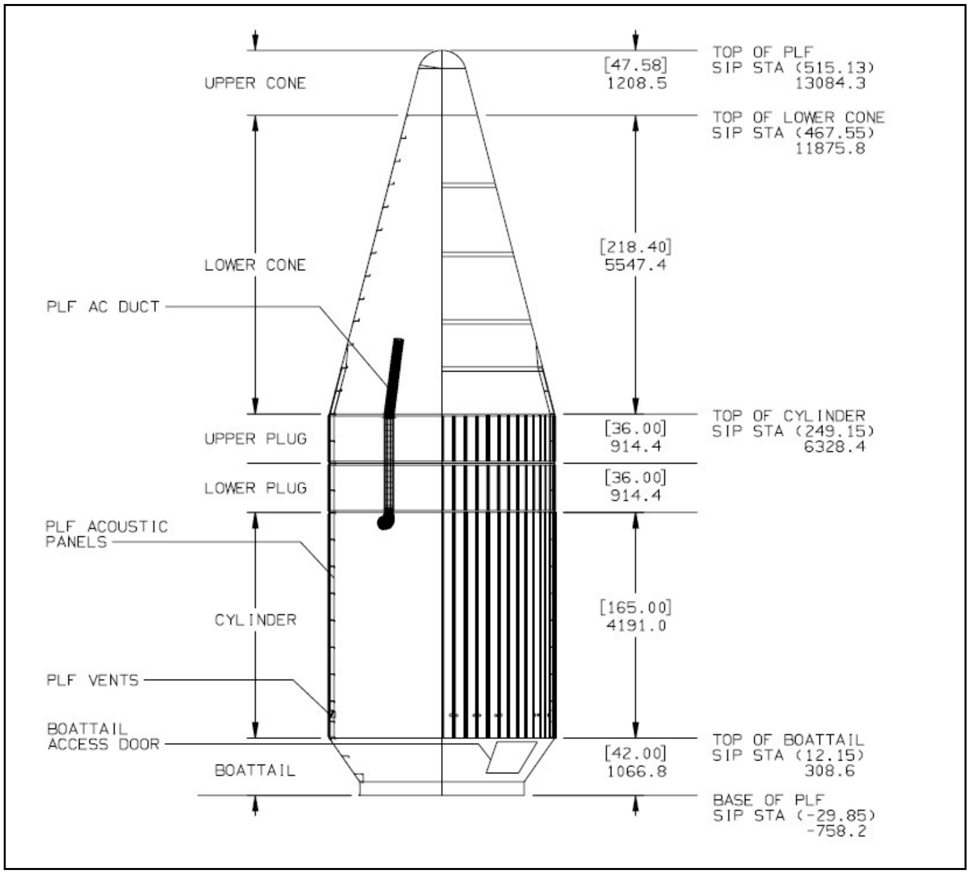
Answer: The surfaces consist of 2 octagons, and 8 rectangular side panels. The total area is then $A = 2 (1.664D^2) + 8 (h)(w)$. For D = 3.15 meters, h = 0.896 meters and w = 0.414(3.15) = 1.30 meters, we have a total area of

$A = 2(1.664)(3.15)^2 + 8(0.896)(1.30) = 42.34$ meters^2. This is for a solid, regular octagonal cylinder. However, for an MMS satellite, a cylindrical hole has been removed. This means that for the top and bottom faces, a circular area of $A = 2 \times \pi (1.66/2)^2 = 4.33$ meters^2 has been removed, and a surface area for the inside cylindrical hole has been ADDED. This surface area is just $A = 2 \pi r h$ or A = 2 (3.14) (1.66/2)(0.896) = 4.67 meters^2 .So the total area is just

A = 42.34 meters^2 - 4.33 meters^2 + 4.67 meters^2
A = 42.68 meters^2 ,which to three SF is just **A = 42.7 meters^2**.

Problem 3 - To three significant figures, what is the volume of a single MMS satellite?

Answer: The surface area of a single satellite hexagonal face is $A = 1.664D^2$ and the volume is just $V = 1.664D^2 h$ .With a central cylindrical volume subtracted with $V_c = \pi r^2 h$, then $V = h (1.664D^2 - \pi r^2)$. For a single MMS satellite
$V = (0.896)(1.664(3.15)^2 - 3.141(1.66/2)^2)$
V = 14.79 - 1.94
V = 12.85 meters^3. so to three SF we have **V = 12.9 meters^3**.



The four satellites of the Magnetosphere Multiscale (MMS) mission will be stacked vertically inside a nose-cone payload shroud, which will be jettisoned when the payload reaches orbit. The diagram above shows the dimensions of the payload. The bracketed numbers are in inches. The numbers below the brackets are the corresponding dimensions in millimeters. The diameter of the cylindrical section is 4.2 meters. The cylindrical payload section is topped by a conical 'nose-cone' section, which in turn comes to an end in a hemispherical cap with a diameter of 910 millimeters.

Problem 1 – To 2 significant figures, what is the volume of the cylindrical payload section including the Upper and Lower Plugs; A) in cubic meters? B) in cubic feet? (1 meter = 3.28 feet).

Problem 2 – To 2 significant figures, what is the volume for the hemispherical cap at the top of the nose-cone in A) in cubic meters? B) in cubic feet?

Problem 3 – The formula for the volume of a truncated right circular cone is given by $V = 1/3\ \pi\ (R^2 + rR + r^2)h$, where R is the radius at the base, r is the radius at the truncation, and h is the height of the truncated cone. To 3 significant figures, what is the volume of the conical section in A) in cubic meters? B) in cubic feet? (1 meter = 3.28 feet)

Problem 1 – To 2 significant figures, what is the volume of the cylindrical payload section including the Upper and Lower Plugs A) in cubic meters? B) in cubic centimeters? C) in cubic feet? (1 foot = 30.48 cm

Answer: A) r = 4.2/2 = 2.1 meters, and converting the measurements in millimeters to meters, h = 4.191m + 0.914m + 0.914m = 6.019m,
so $V = \pi r^2 h = 3.141 (2.100)^2 (6.019) =$ **83 meters^3**.

B) $V = 83000000 \text{ cm}^2 \times (1 \text{ foot} / 30.48 \text{ cm})^3 =$ **2,900 feet^3**.

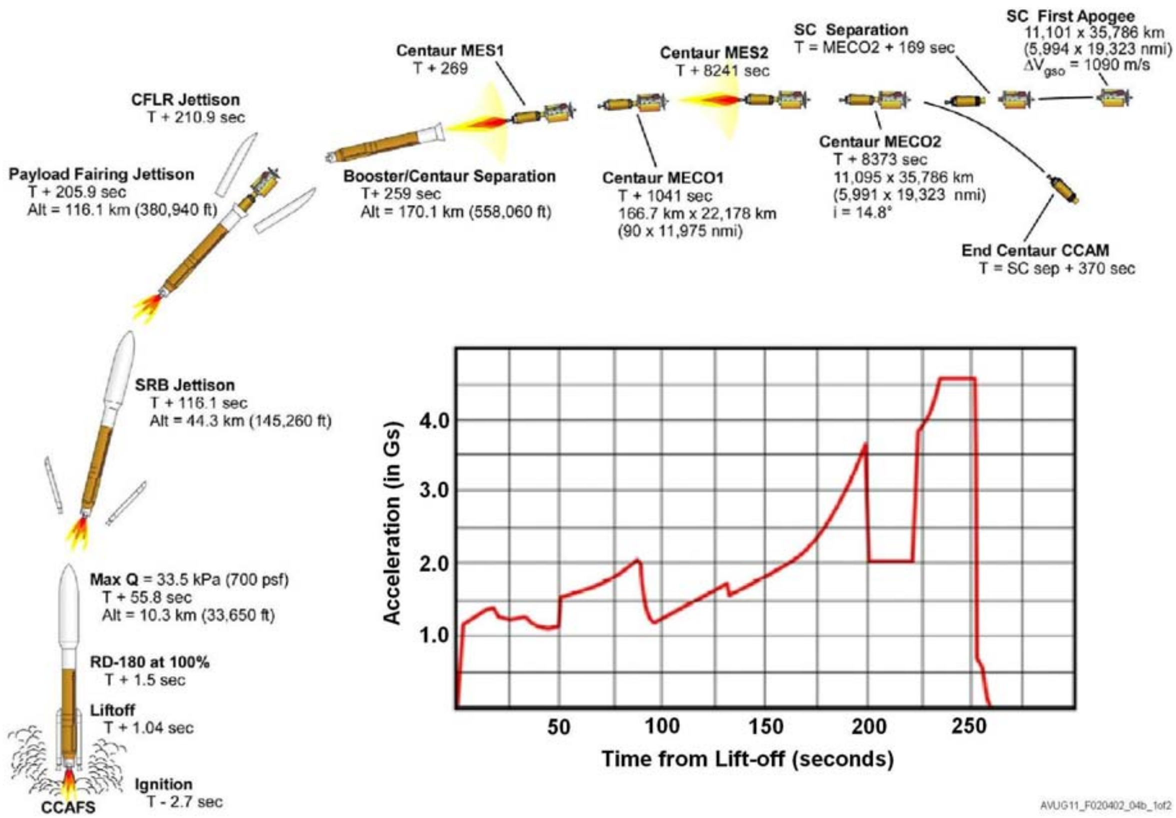
Problem 2 – To 2 significant figures, what is the volume for the hemispherical cap at the top of the nose-cone in A) in cubic meters? B) in cubic centimeters? C) in cubic feet?

Answer: A) $V = 2/3 \pi R^3$ and R = 0.91 m/2 = 0.455 m, so **V = 0.20 meters^3**.
B) $V = 200{,}000 \text{ cm}^3 \times (1 \text{ foot}/30.48 \text{ cm})^3 =$ **7.0 feet^3**

Problem 3 – The formula for the volume of a truncated circular cone is given by $V = 1/3 \pi (R^2 + rR + r^2)h$, where R is the radius at the base, r is the radius at the truncation, and h is the height of the truncated cone. To 3 significant figures, what is the volume of the conical section in A) in cubic meters? B) in cubic feet?

Answer: A) R = 4.2 m/2 = 2.1 meters, and r = 0.910 m/2 = 0.455 m, h = 5.547 m so
$V = 0.333 (3.141)(2.1^2 + (2.1)(0.455) + 0.455^2)(5.547) =$ **32.2 meters^3**.

B) $V = 32.2 \text{ m}^3 \times (3.28 \text{ feet} / 1 \text{ m})^3 = 1136.3 \text{ feet}^3$ to 3 SF this becomes **1140 feet^3**



This diagram, provided by the Boeing *Atlas V Launch Services User's Guide* shows the major events during the launch of an Atlas V521 rocket, which is similar to the rocket planned for the MMS launch in 2014. The figure shows the events in the V521 timeline starting from -2.7 seconds before launch through the Centaur rocket main engine cut off 'MECO2' event at 8,373 seconds after launch. The graph shows the acceleration of the payload during the first 250 seconds after launch. Acceleration is given in Earth gravities, where 1.0 G = 9.8 meters/sec^2.

Problem 1 – To the nearest tenth of a G, what is the acceleration at the time of:

A) Maximum aerodynamic pressure, called Max-Q ?
B) Solid rocket booster (SRB) jettison?
C) Payload fairing jettison?
D) Booster/Centaur separation?

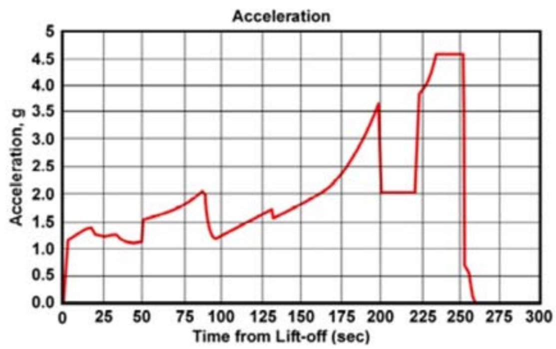
Advanced Math Challenge: The speed of the rocket at a particular time, T, is the area under the acceleration curve (in meters/sec^2) from the time of launch to the time, T. By approximating the areas as combinations of rectangles and triangles, and rounding your final answers to two significant figures, about what is the rocket speed at a time of:

A) T = 50 seconds?
B) T = 100 seconds?
C) T = 200 seconds?
D) T = 250 seconds?

Answer Key

Problem 1 – To the nearest tenth of a G, what is the acceleration at the time of:
Answer:

A) Maximum aerodynamic pressure, called Max-Q? A = 1.6 Gs
B) Solid Rocket Booster (SRB) jettison? A = 1.5 Gs
C) Payload Fairing Jettison? A = 2.0 Gs
D) Booster/Centaur Separation? A = 0.0 Gs (thrust no longer applied)



Advanced Math Challenge – The speed of the rocket at a particular time, T, is the area under the acceleration curve (in meters/sec^2) from the time of launch to the time, T. By approximating the areas as combinations of rectangles and triangles, and rounding your final answers to two significant figures, about what is the rocket speed at a time of:

A) T = 50 seconds?
V = (50 seconds) (1.3Gs) (9.8 m/s^2) = **640 meters/sec**.

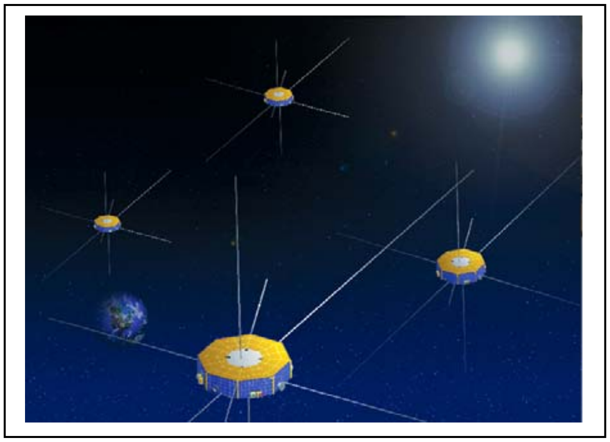
B) T = 100 seconds?
V = 640 meters/sec + (50 sec)(1.5Gs)(9.8m/s^2) + 1/2(50sec)(2.0-1.5)(9.8m/s^2)
= 640 meters/sec + 735 m/sec + 123 m/sec
= **1500 meters/sec**

C) T = 200 seconds?
V = 1500 meters/sec + (200-100)(1.2 Gs)(9.8 m/s^2) + ½ (200-100)(3.6-1.2)(9.8m/s^2)
= 1500 m/sec + 1200 m/sec + 1200 m/sec
= **3900 meters/sec**.

D) T = 250 seconds?
V = 3900 meters/sec + (225-200)(2.0Gs)(9.8 m/s^2) + (250-225)(4.5 Gs)(9.8 m/s^2)
= 3900 m/sec + 490 m/s + 1100 m/s
= **5500 meters/sec**.

Note: Students answers will vary. The biggest challenge is to convert Gs to meters/sec^2 to get physical units in terms of meters and time.

To reach 'orbit', the payload needs a speed of about 7,000 m/sec, which is provided by the ignition of the second stage about 270 seconds after launch. The specific speed needed is determined by the orbit desired. Lower orbits require a smaller final speed (6,000 to 8,000 m/sec) than higher orbits (8,000 to 10,000 m/sec).



In 2014, the four satellites of the Magnetosphere Multiscale (MMS) mission will be launched into orbit atop an Atlas-V421 rocket. These satellites, working together, will attempt to measure dynamic changes in Earth's magnetic field that physicists call magnetic reconnection events. These changes in the magnetic field are responsible for many different phenomena including Earth's polar aurora.

Soon after launch, the satellites will be placed into a Phase-1 elliptical orbit with Earth at one of the foci. The closest distance between the satellite and Earth, called perigee, will be at a distance of 1.2 R_e, where 1.0 R_e equals the radius of Earth of 6,378 km. The farthest distance from Earth, called apogee, occurs at a distance of 12.0 R_e. After a few years, the satellites will be moved into a Phase-2 elliptical orbit with an apogee of 25.0 R_e and a perigee of 1.2 R_e.

The standard form for an ellipse is $$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

where x and y are the coordinates of a point on the ellipse, and the ellipse constants a and b are the semi-major and semi-minor axis dimensions of the ellipse. The relationship between the apogee (A) and perigee (P) distances, and the ellipse constants, a and b, and the eccentricity of the ellipse, e, are as follows:

$$A = a + c \quad P = a - c \qquad e = c/a \qquad b = a(1-e^2)^{1/2}$$

where c is the distance between the foci of the ellipse.

Problem 1 – What are the equations for the semi-major and semi-minor axis, a and b, in terms of only A and P?

Problem 2 – What are the equations of the MMS elliptical orbit in standard form, with all distances given in multiples of R_e for A) Phase-1 and B) Phase-2?

Problem 3 – What are the equations of the MMS elliptical orbit in the form $kx^2 + gy^2 = s$ where k, g and s are numerical constants rounded to integers for A) Phase-1 and B) Phase-2?

The relationship between the apogee (A) and perigee (P) distances, and the elementary properties of ellipses are as follows:

$$A = a + c \quad P = a - c \qquad e = c/a \qquad b = a(1-e^2)^{1/2}$$

Problem 1 – What are the equations for the semi-major and semi-minor axis, a and b, in terms of only A and P?

Answer: Add the equations for A and P to get $A+P = 2a$, and $a = (A+P)/2$
Subtract the equations for A and P to get $A-P = 2c$, and $c= (A-P)/2$
Then by substitution $e = (A-P)/(A+P)$
And so $b = \frac{(A+P)}{2}\sqrt{1-\frac{(A-P)^2}{(A+P)^2}}$ so $b = \frac{1}{2}\sqrt{(A+P)^2-(A-P)^2}$
Expand and simplify:

$b = ½ (A^2+2AP + P^2 – A^2 +2AP – P^2)^{1/2}$

$b = ½ (4AP)^{1/2}$

$\mathbf{b = (AP)^{1/2}}$

Problem 2 –

Answer: A) for A = 12.0 and P=1.2, $a = (13.2/2) = 6.6\ R_e$ $c = (12-1.2)/2 = 5.4\ R_e$
$e = (5.4/6.6) = 0.82$
$b = (12 \times 1.2)^{1/2} = 3.8\ R_e$

then $1 = \frac{x^2}{(6.6)^2} + \frac{y^2}{(3.8)^2}$

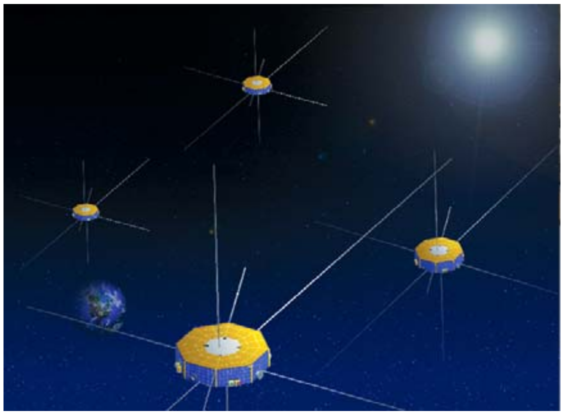
B) for A = 25.0 and P=1.2, $a = (26.2/2) = 13.1\ R_e$ $c = (25-1.2)/2 = 11.9\ R_e$
$e = (11.9/13.1) = 0.91$
$b = (25\times1.2)^{1/2} = 5.5\ R_e$

then $1 = \frac{x^2}{(13.1)^2} + \frac{y^2}{(5.5)^2}$

Problem 3 –

Answer: A) $1 = x^2/(6.6)^2 + y^2/(3.8)^2$ cross-multiply and simplify to get
$(6.6)^2(3.9)^2 = (3.8)^2x^2 + (6.6)^2y^2$
$662.55 = 14.4x^2 + 43.56\ y^2$ and rounded to the nearest integer:
$\mathbf{663 = 14x^2 + 44y^2}$

B) $1 = x^2/(13.1)^2 + y^2/(5.5)^2$ cross-multiply and simplify to get
$(13.1)^2(5.5)^2 = (5.5)^2x^2 + (13.1)^2y^2$
$5191.20 = 30.25x^2 + 171.61\ y^2$ and rounded to the nearest integer:
$\mathbf{5191 = 30x^2 + 172y^2}$



In 2014, the four satellites of the Magnetosphere Multiscale (MMS) mission will be launched into orbit atop an Atlas V421 rocket. These satellites, working together, will attempt to measure dynamic changes in Earth's magnetic field that physicists call magnetic reconnection events. These changes in the magnetic field are responsible for many different phenomena including Earth's polar aurora.

Soon after launch, the satellites will be placed into a Phase-1 elliptical orbit with Earth at one of the foci. After a few years, the satellites will be moved into a different 'Phase-2' elliptical orbit. The closest distance between the satellite and Earth is called the perigee. The farthest distance from Earth is called the apogee. The relationship between the apogee (A) and perigee (P) distances, and the elementary properties of ellipses are as follows:

$$A = a + c \quad P = a - c \qquad e = c/a \qquad b = a(1-e^2)^{1/2}$$

Where a, b and e are the semi-major and semi-minor axis lengths, and e is the eccentricity of the ellipse. The equations describing the Phase-1 and Phase-2 orbits are as shown below, with all distance units given in terms of multiples of 1 Earth radius (1 R_e = 6,378 km):

Phase-1 $663 = 14x^2 + 44y^2$

Phase-2 $5191 = 30x^2 + 172y^2$

Problem 1 – In terms of kilometers, and to two significant figures, what is the semi-major axis distance, a, for the A) Phase-1 orbit? B) Phase-2 orbit?

Problem 2 – According to Kepler's Third Law, for orbits near Earth, the relationship between the semi-major axis distance, a, and the orbital period,T, is given by

$$a^3 = 287\ T^2$$

where a is in units of R_e and T is in days. To the nearest tenth of a day, what are the estimated orbit periods for the MMS satellites in A) Phase-1? B) Phase-2?

The relationship between the apogee (A) and perigee (P) distances, and the elementary properties of ellipses are as follows:

$$A = a + c \quad P = a - c \qquad e = c/a \qquad b = a(1-e^2)^{1/2}$$

The equations describing the Phase-1 and Phase-2 orbits are as shown below, with all distance units given in terms of multiples of 1 Earth radius (1 R_e = 6,378 km):

Phase-1 $663 = 14x^2 + 44y^2$

Phase-2 $5191 = 30x^2 + 172y^2$

Problem 1 – In terms of kilometers, and to two significant figures, what is the semi-major axis distance, a, for the A) Phase-1 orbit? B) Phase-2 orbit?

Answer: A) Writing the Phase-1 equation in standard form:

$$1 = \frac{x^2}{(6.9)^2} + \frac{y^2}{(3.9)^2}$$

then a = 6.9 R_e
= 6.9 x (6,378 km)
= 44,008 km.
= 44,000 km, to 2 SF

B) Writing the Phase-2 equation in standard form:

$$1 = \frac{x^2}{(13.1)^2} + \frac{y^2}{(5.5)^2}$$

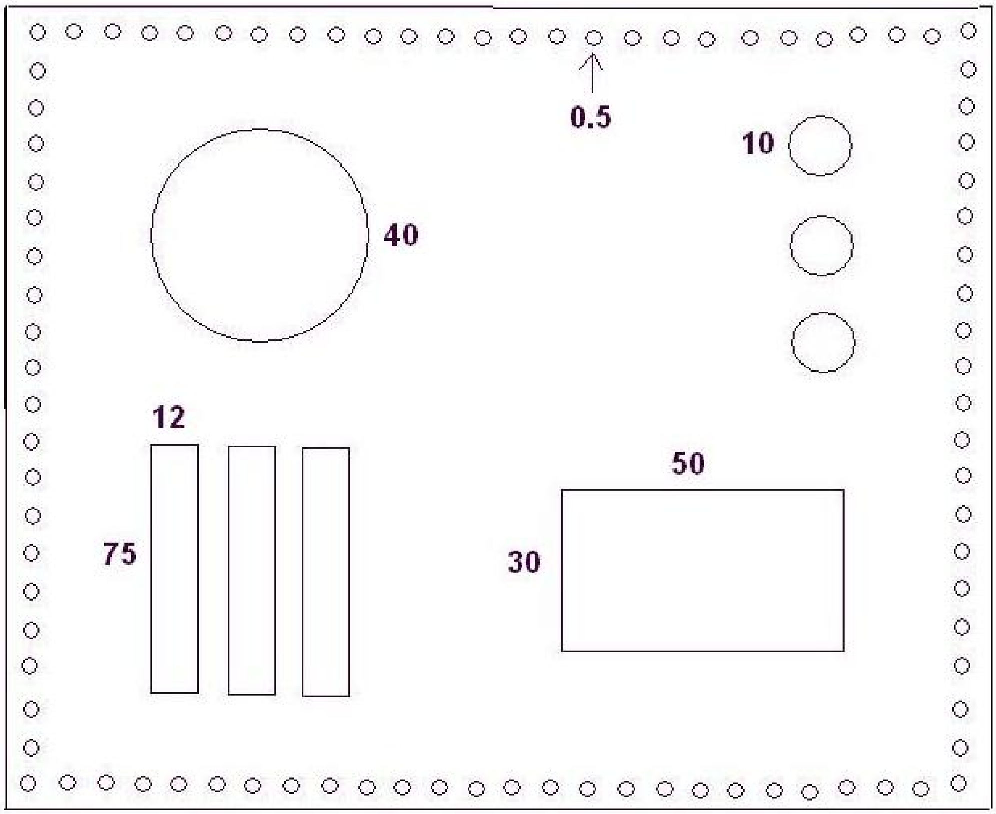
then b = 13.1 R_e
= 13.1 x (6,378 km)
= 83,552 km.
= 84,000 km, to 2 SF

Problem 2 – According to Kepler's Third Law, for orbits near Earth, the relationship between the semi-major axis distance, a, and the orbital period, T, is given by

$$a^3 = 287\ T^2$$

where a is in units of R_e and T is in days. To the nearest tenth of a day, what are the estimated orbit periods for the MMS satellites in A) Phase-1? B) Phase-2?

Answer: A) $(6.9)^3 = 287\ T^2$, so for Phase-1, T = 1.1 days.
B) $(13.1)^3 = 287\ T^2$, so for Phase-2, T = 2.8 days.



This is a diagram of a panel from a spacecraft showing all of the openings. All units are in centimeters. The panel is 190 centimeters wide and 150 centimeters tall. The diameters of each circular opening is also given. The small holes around the circumference are for the screws that fasten the panels together.

Problem 1 - To the nearest square-centimeter, what is the total area of the panel in square-centimeters before the openings were made?

Problem 2 - To the nearest square-centimeter, what is the total area of all of the openings in the panel? (Use $\pi = 3.1415$)

Problem 3 - To the nearest square-centimeter, what is the area of the panel after the openings were made?

Problem 4 - The panel is 1.5 centimeters thick. To the nearest cubic-centimeter, what is the volume of the finished panel?

Problem 5 - To the nearest cubic-centimeter, what is the volume of the material that was removed to make the holes?

Problem 6 - If the density of the aluminum in the panel is 2.7 grams/cm^3, to the nearest tenth of a kilogram, what is the mass of the finished panel?

Problem 7 - To the nearest tenth of a kilogram, how many grams of aluminum were removed to make all of the openings?

Problem 1 - To the nearest square-centimeter, what is the total area of the panel in square-centimeters before the openings were made?

Answer: A = w x h so A = 190 x 150 = **28,500 cm^2**

Problem 2 - To the nearest square-centimeter, what is the total area of all of the openings in the panel?

Answer:

A = (30 x 50) + 3 (12 x 75) + 1 $(3.1415)(40/2)^2$ + 3 x $(3.1415)(10/2)^2$ + 90x$(3.1415)(0.5/2)^2$
A = 1500 + 2700 + 1256.6000 + 235.6125 + 17.6709
A = 5709.8834
A = 5710 cm^2.

Problem 3 - To the nearest square-centimeter, what is the area of the panel after the openings were made?

Answer: 28,500 - 5710 = **22,790 cm^2**

Problem 4 - The panel is 1.5 centimeters thick. To the nearest cubic-centimeter, what is the volume of the finished panel?
Answer: Volume = Area x Thickness
= 22,790 cm^2 x 1.5 cm
= **34,185 cm^3**

Problem 5 - To the nearest cubic-centimeter, what is the volume of the material that was removed to make the holes?

Answer: Volume = 5710 cm^2 x 1.5 cm = **8,565 cm^3**

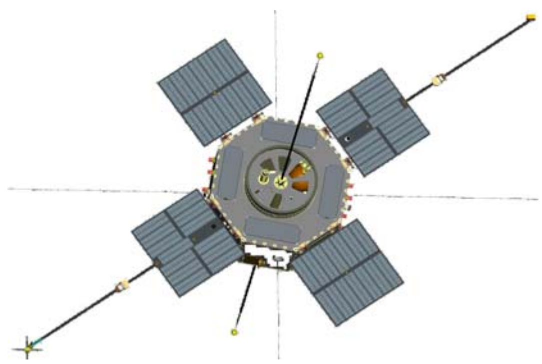
Problem 6 - If the density of the aluminum in the panel is 2.7 grams/cm^3, to the nearest tenth of a kilogram, what is the mass of the finished panel?

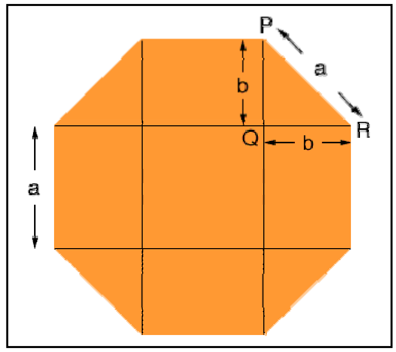
Answer: Mass = Density x volume
= 2.7 grams/cm^3 x 34,185 cm^3
= 92,299.5 grams
= **92.3 kilograms**

Problem 7 - To the nearest tenth of a kilogram, how many grams of aluminum were removed to make all of the openings?

Answer: Mass = 2.7 grams/cm^3 x 8,565 cm^3
= 23,125.5 grams
= **23.1 kilograms**

The Van Allen Probes: Working with Octagons





NASA's Van Allen Probes satellites are in the shape of an octagon with a thickness of 84 centimeters between the front octagonal face and the back octagonal face. An engineer needs to determine the total surface area of this 'octagonal prism' in order to create a mathematical model of how fast the satellite is warming up and cooling down as it orbits Earth.

Problem 1 – The figure on the right shows how the geometric area of an octagon can be broken up into rectangles, squares and triangles. What are the formulas for the areas of each of the squares, rectangles and triangles?

Problem 2 – What is the formula for the total area of the octagonal face in terms of the measurements for a and b?

Problem 3 – What is the formula for the total surface area of the spacecraft if h is the distance between the top and bottom octagonal faces?

Problem 4 - The engineer determines that $a + 2b = 1.7$ meters and $a = 0.7$ meters and $h = 0.84$ meters. What is the total surface area to the nearest tenth of a square meter of A) one octagonal face? B) the entire satellite?

Problem 1 – The figure on the right shows how the geometric area of an octagon can be broken up into rectangles, squares and triangles. What are the formulas for the areas of each of the squares, rectangles and triangles?

Answer: A(square) = $a \times a = a^2$ A(rectangle) = $a \times b$ A(triangle) = $\frac{1}{2}\, b \times b$

Problem 2 – What is the formula for the total area of the octagonal face in terms of the measurements for a and b?

Answer: A = 1 x A(square) + 4 x A(rectangle) + 4 x A(triangle)

$A = a^2 + 4ab + 2b^2$

Problem 3 – What is the formula for the total surface area of the spacecraft if h is the distance between the top and bottom octagonal faces?

Answer: There are two octagonal areas and 8 rectagonal side faces each with an area of a x h, so the total area of the spacecraft is

$A = 2(a^2 + 4ab + 2b^2) + 8ah$

Problem 4 - The engineer determines that a + 2b = 1.7 meters and a = 0.7 meters and h = 0.85 meters. What is the total surface area to the nearest tenth of a square meter of A) one octagonal face? B) the entire satellite?

Answer: a + 2b = 1.7meters and a = 0.7meters so b = 0.5 meters

A) $A = a^2 + 4ab + 2b^2$ so $A = (0.7)^2 + 4(0.7)(0.5) + 2(0.5)^2$ = **2.4 meter^2**.

B) $A = 2(2.4\ m^2) + 8(0.7)(0.85)$ = **9.6 meters^2**

A modern home computer has a hard drive whose capacity is typically about 500 gigabytes. A song that you download typically has a size of about 10 megabytes, and if you have a fast internet connection you can usually download at a rate of about 1 megabytes/sec. At that rate it takes about 10 seconds to download a song, and if your entire hard drive were available to store songs on your playlist, you could accommodate about 50000 songs!

The Van Allen Probes spacecraft instruments will be generating data that has to first be stored onboard the spacecraft, then at a specific time in the orbit, transmitted to Earth before the next round of data has to be stored on top of the old data.

The spacecraft engineers have worked with the scientists to determine how often the scientists want to store their measurements on each satellite. The average rate is 9,000 bytes/second. Each satellite will download the data once every orbit when the satellite is closest to Earth (perigee). The ground station has a busy schedule working with other satellites so each of the two Van Allen Probes spacecraft will only have 10 minutes every orbit to download all of its stored data. The orbit period is 9 hours.

Problem 1 – How many megabytes of data do both of the spacecraft collect after one orbit?

Problem 2 – At what rate in bytes/second will the data have to be downloaded to the ground station?

Problem 3 – The mission is being supported by NASA to last two years during its first operations cycle. How much data will both of the Van Allen Probes spacecraft accumulate during its first 2-year period in :

A) DVD disks (1 DVD = 4 gigabytes)

B) Songs (1 song = 10 megabytes).

Problem 1 – How many megabytes of data do both of the spacecraft collect after one orbit?

Answer: Time = 9 hours x (3600 seconds / 1 hr) = 32,400 seconds.
Total data = 9,000 bytes/sec x 32400 seconds = 291,600,000 bytes or 291.6 megabytes per spacecraft and **583.2 megabytes**/orbit for two spacecraft combined.

Problem 2 – At what rate in bytes/second will the data have to be downloaded to the ground station?

Answer: 583.2 megabytes have to be downloaded within 10 minutes or 600 seconds so the rate will be R = 583,200,000 bytes / 600 seconds = **972,000 bytes/second**.

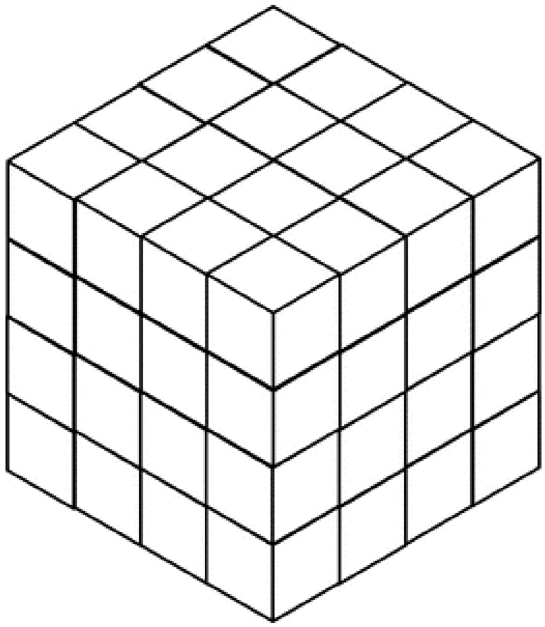
Problem 3 – The mission is being supported by NASA to last two years during its first operations cycle. How much data will both of the Van Allen Probes spacecraft accumulate during its first 2-year period A) In DVD disks (1 DVD = 4 gigabytes) B) Songs (1 song = 10 megabytes).

Answer: A) 1 year = 365 days x 24 hours/day x 3600 seconds/hr = 31,536,000 seconds.
The data rate for two satellites is 18000 bytes/second, so in 2 years the satellites will accumulate 18000 x 31536000 = 567.6 billion bytes or 567.6 gigabytes.

A single DVD stores 4 gigabytes so you will need 567.6/4 = 141.9 or **141 DVDs**.

B) Number of songs = 567,600 megabytes / 10 megabytes = 56,760 or **56,760 song**s.

Exploring Gas Density in Space



Earth's atmosphere does not have a hard edge that says 'this is where space starts'. Instead, the density of the atmosphere gets smaller and smaller…but it never quite becomes zero!

Scientists measure gas density in space in terms of the number pf particles that you would find in any cubic meter of space. This is called the **Number Density, n,** and is measured in particles/m^3. Near the Earth, the gas densities are so large that we have to use scientific notation to write them. For instance, at Earth's surface, the number density of air is n= 2.5×10^{25} molecules/m^3. In the mesosphere at 70 km altitude, it is n= 2.5×10^{20} molecules/m^3.

Imagine the each particle sits at the center of its own cube. The number of these cubes, N, in <u>one cubic meter</u> is just the gas number density: **N = n.** In the figure above n = 64 if the length of each cube edge is 1 meter.

Problem 1 - Suppose you had 64 cubes arranged in a cube with a side length of one meter. How far apart would the centers of each cube be?

Problem 2 – Suppose that the large cube had an edge length of 1 meter and it contained 1 million identical cubes. What would the distance between the cube centers be?

Problem 3 – In the van Allen belts, the average number density is about 900 particles/m^3. What is the average distance between the atoms in the van Allen Belts?

Problem 4 – In the mesosphere, the average number density is about 2.5×10^{20} particles/m^3. What is the average distance between the atoms in the mesosphere in microns, where 1 micron = 10^{-6} meters?

Problem 1 - Suppose you had 64 cubes arranged in a cube with a side length of one meter. How far apart would the centers of each cube be?

Answer: 64 cubes arranged in a cube means that you have 4 cubes along each side so that 4 x 4 x 4 = 64 cubes total. If the length of each side is 1 meter, then the center to center distance for the cubes is just 1 meter/4 = **25 centimeters**.

Problem 2 – Suppose that the large cube had an edge length of 1 meter and it contained 1 million identical cubes. What would the distance between the cube centers be?

Answer: 1 million = 100 x 100 x 100 so there are 100 cubes along each edge, and since each edge measures 1 meter, the separation between the cubes would be 1 meter/100 = 1 centimeter.

Problem 3 – In the van Allen belts, the average number density is about 1000 particles/m^3. What is the average distance between the atoms in the van Allen Belts?

Answer: The number of cubes along each side is (1000)1/3 = 10 so the distance is d = 1 meter/10 = **10 centimeters.**

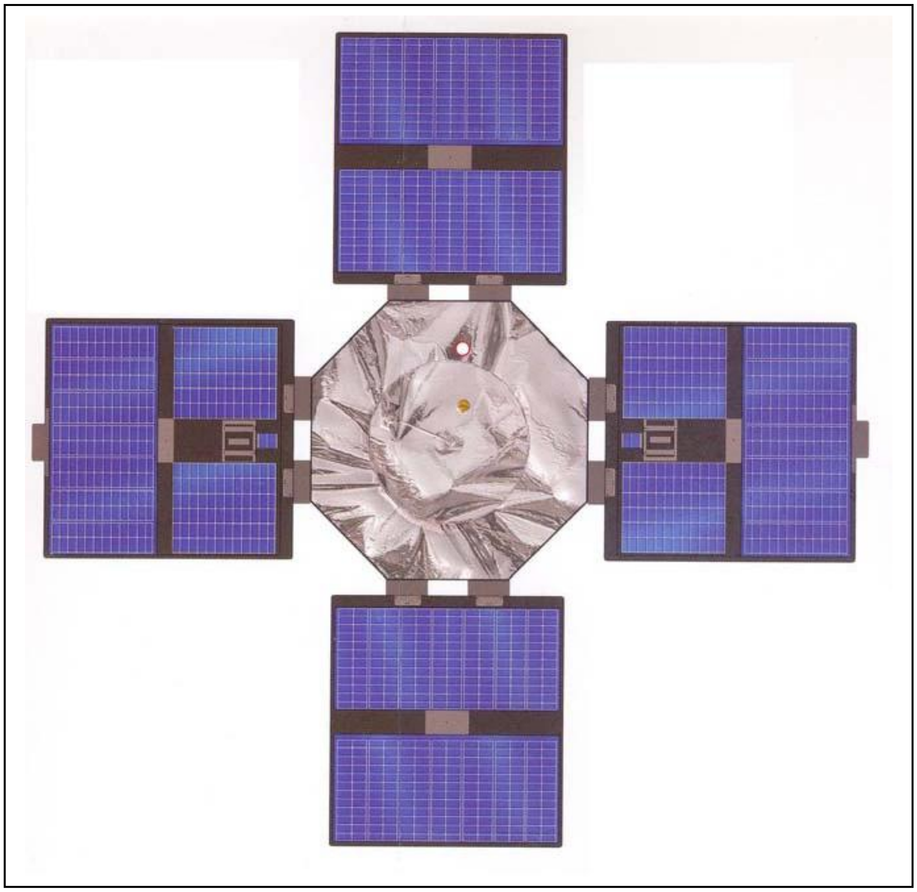
Problem 4 – In the mesosphere, the average number density is about 2.5×10^{20} particles/m^3. What is the average distance between the atoms in the mesosphere in microns, where 1 micron = 10^{-6} meters?

Answer: The number of cubes along each 1-meter side is $(2.5\text{x}1020)^{1/3} = 6.3\times10^{6}$. The average separation between atoms is then 1 meter/$6.3\times10^{6} = 1.6\times10^{-7}$ meters. Since 1 micron equals 10^{-6} meters so the atoms are separated by **0.16 microns**.

Note: The general formula for particle separation is

$$D = \frac{1 \text{ meter}}{n^{1/3}}$$

where n is the number density in particles/m^3



NASA's twin Van Allen Probes spacecraft will be launched in 2012. The figure above shows the octagonal spacecraft body and the location of the surrounding four solar panel 'wings' that provide power to the spacecraft instruments. The small blue rectangles within each of the four solar panels show the location of the solar cells used to power the satellite. As the spacecraft orbits Earth, the four solar panels continuously face the sun to provide constant power.

Problem 1 – Using a millimeter ruler to measure the (silver) octagonal satellite body in the above figure, and the fact that the actual top-to-bottom height of the octagon is 2.0 meters, what is the scale of this figure in centimeters/millimeter?

Problem 2 –What is the total area of the 10 solar cells in square-meters?

Problem 3 – The amount of electrical power generated by a solar panel is 0.0077 watts/cm^2. What is the total power generated by the four solar panels on one Van Allen Probes spacecraft to the nearest hundred watts?

Problem 1 – Using a millimeter ruler to measure the (silver) octagonal satellite body in the above figure, and the fact that the actual top-to-bottom height of the octagon is 2.0 meters, what is the scale of this figure in centimeters/millimeter?

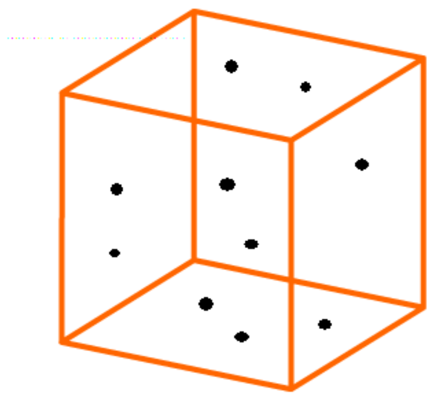
Answer: If you print this problem on a standard '8.5x11' page, the top-to-bottom length is 37 millimeters. This corresponds to 2 meters or 200 cm, so the scale is 200 cm/37mm = **5.4 cm/mm**.

Problem 2 –What is the total area of the 10 solar cells to the nearest tenth of a square-meter?

Answer: The 5 large rectangles have dimensions of 29mm x 13mm, and the 5 small rectangles measure 13mm x 12mm, so their actual dimensions are 157cm x 70 cm, and 70cm x 65cm. The total area is 5(157x70) + 5(70x65) = 77,700 cm^2. Since 1 meter = 100 cm, the area in square-meters is just 77700 cm^2 x (1 m/100 cm)(1m/100cm) = 7.77 $meters^2$, or to the nearest tenth of a square-meter we get **7.8 $meters^2$.**

Problem 3 – The amount of electrical power generated by a solar panel is 0.0077 $watts/cm^2$. What is the total power generated by the four solar panels on one Van Allen Probes spacecraft to the nearest hundred watts?

Answer: In square centimeters, the total area of the solar panels is 78,000 cm^2. The electrical power produced is then P = 0.0077 $watts/cm^2$ x (78000 cm^2) = **600 watts**.



This cube has edges that are 1 meter long. Its volume is 1 cubic meter. There are 10 atoms inside this cube, so its density is **10 atoms per cubic meter**.

The NASA, van Allen Probe spacecraft orbit Earth so high up that there is hardly any air at all. Scientists use the term 'density' to measure how many kilograms or gas there are in each cubic-meter of space, but when the density is too low, a unit like kg/m^3 is not very helpful. That's because instruments often measure individual atoms, and kg/m^3 is just too big a unit! It's like using 'kilometers' to measure the size of a bacterium.

A much more convenient unit is 'atoms/m^3'. This tells scientists immediately just how often their very sensitive instruments will be affected by their environment.

Problem 1 – The density of the van Allen belts is typically about 900 atoms/m^3. How many atoms would you expect to find in a box that measures 15 centimeters on a side?

Problem 2 – The opening to one of the van Allen spacecraft instruments is about 10 cm^2. As the satellite completes one orbit, it travels about 70,000 km. How many atoms will pass through the spacecraft instrument window each orbit?

Problem 3 – How many kilometers would the spacecraft have to travel in order to encounter 9 million atoms?

Problem 1 – The density of the van Allen belts is typically about 900 atoms/m^3. How many atoms would you expect to find in a box that measures 15 centimeters on a side?

Answer: 10 cm = 0.15 meters, so the volume of the box is 0.15 x 0.15 x 0.15 = 0.0034 meters3 .Then the number of atoms is density x volume = 900 x 0.0034 = **3 atoms**.

Problem 2 – The opening to one of the van Allen instruments is about 10 cm^2. As the satellite completes one orbit, it travels about 70,000 km. How many atoms will pass through the spacecraft instrument window each orbit?

Answer: Convert the area into square meters, and the orbit length into meters to get

Area = 10 cm^2 x (1 m/100cm)x(1m/100cm)
= 0.001 m^2,

and 70,000 km x (1000 m/1 km) = 70,000,000 m.

Then volume = area x length to get (0.001 m^2) x (70,000,000 m) = 70,000 m^3. Now multiply this 'swept out' volume by the density to get the number of atoms that passed through the window: 900 atoms/m^3 x 70,000 m^3 = **63 million atoms**.

Problem 3 – How many kilometers would the spacecraft have to travel in order to encounter 9 million atoms?

Answer: The window area is 0.001 m^2 and the density of atoms is 900 atoms/m^3.
You want 9 million atoms, so
9 million = 900 x area x length
9 million = 900 atoms/m^3 x 0.001 m^2 x Length

Length = 9 million / 0.9 = 10,000,000 meters! **This equals 10,000 kilometers**.

Kepler's Third Law says that the cube of the satellite's orbit radius is directly proportional to the square of its orbit period. The proportionality constant, c, depends only on the mass of the planet that the satellite (or moon) is orbiting. For distances measured in meters, periods measured in seconds, and masses measured in kilograms, the proportionality constant for satellites orbiting Earth is just

$$C = 1.7 \times 10^{-12} M$$

Problem 1 – What is the equation described by the paragraph above?

Problem 2 – Solve the equation for M – the mass of Earth.

Problem 3 – For objects near Earth, it is convenient to measure their distances in multiples of Earth's radius so that 1.0 Re = 6,378 kilometers. It is also more convenient to use hours as a measure of orbit period. Re-write your equation so that it gives the mass of Earth in kilograms, in terms of the orbit period in hours, and the distance in multiples of Earth's radius.

Problem 4 – The Van Allen Probes spacecraft will be in orbits with a period of 9 hours, and a distance of 3.4 Re. What would you estimate as the mass of Earth given these spacecraft parameters?

Problem 1 – What is the equation described by the paragraph above?

Answer: After substituting for the constant, C, you get

$$\mathbf{R^3 = 1.7x10^{-12}\ M\ T^2}$$

Problem 2 – Solve the equation for M – the mass of Earth.

$$\mathbf{M = 5.9x10^{11}\ R^3/T^2}$$

Problem 3 – For objects near Earth, it is convenient to measure their distances in multiples of Earth's radius so that 1.0 Re = 6,378 kilometers. It is also more convenient to use hours as a measure of orbit period. Re-write your equation so that it gives the mass of Earth in kilograms, in terms of the orbit period in hours, and the distance in multiples of Earth's radius.

Answer:

$M = 5.9x10^{11}\ (6378000)^3/(3600)^2\ \ R^3/T^2$

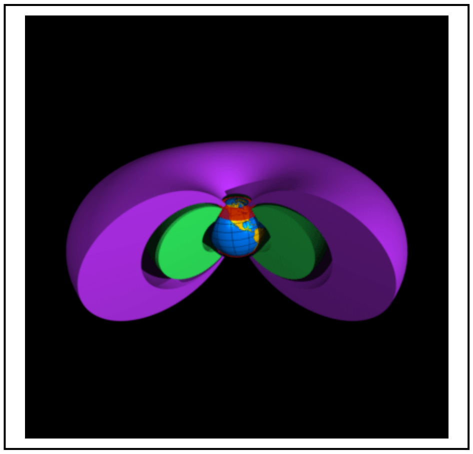
$\mathbf{M = 1.2 \times 10^{25}\ R^3/T^2}$

Problem 4 – The Van Allen Probes spacecraft will be in orbits with a period of 9 hours, and a distance of 3.4 Re. What would you estimate as the mass of Earth given these spacecraft parameters?

Answer : $M = 1.2 \times 10^{25}\ (3.4)^3/\ (9.0)^2$

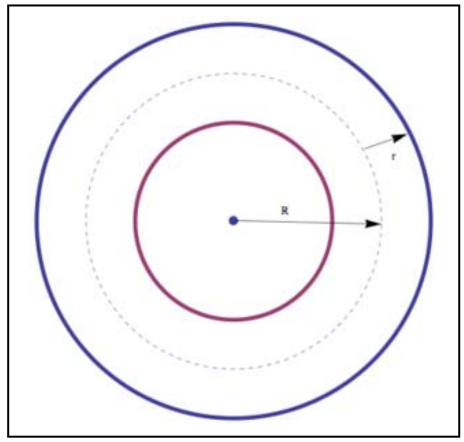
$\mathbf{M = 5.8 \times 10^{24}}$ **kilograms**

The actual value is 5.98×10^{24} kg.



The Van Allen belts were discovered in the late-1950s and resemble two donut-shaped clouds of protons (inner belt) and electrons (outer belt) with Earth at its center.

A donut is an example of a simple mathematical shape called a **torus** that is created by rotating a circle with a radius of r, through a circular path with a radius of R.



In terms of the variables r and R, the formula for the volume of a torus is given by the rather scary-looking formula:

$$V = 2\pi^2 R r^2$$

Problem 1 – What is the circumference of the circle with a radius of R?

Problem 2 – What is the area of a circle with a radius of r?

Problem 3 – If you dragged the area of the circle in Problem 2, along a distance equal to the circumference of the circle in Problem 2, what would be the formula for the volume that you swept out?

Problem 4 – If the Van Allen belts can be approximated by a torus with r = 16,000 km, and R = 26,000 km, to two significant figures, what is the total volume of the Van Allen belts in cubic kilometers?

Problem 5 – To two significant figures, how many spherical Earths can you fit in this volume if r = 6378 km?

Problem 1 – What is the circumference of the circle with a radius of R?

Answer: $C = 2 \pi R$

Problem 2 – What is the area of a circle with a radius of r?

Answer: $A = \pi r^2$

Problem 3 – If you dragged the area of the circle in Problem 2, along a distance equal to the circumference of the circle in Problem 2, what would be the formula for the volume that you swept out?

Answer: Volume = Area x distance

$= (\pi r^2) \times (2 \pi R)$

$\mathbf{= 2 \pi^2 R r^2}$

Problem 4 – If the Van Allen belts can be approximated by a torus with r = 16,000 km, and R = 26,000 km, to two significant figures what is the total volume of the Van Allen belts in cubic kilometers?

Answer:
r = 16000 km x (1000 m/1km) = 16,000,000 meters
R= 26000 km x (1000 m/1km) = 26,000,000 meters

$V = 2 (3.14)^2 (2.6\times10^7) (1.6\times10^7)^2$

$=$ **1.3×10^{23} meters^3**

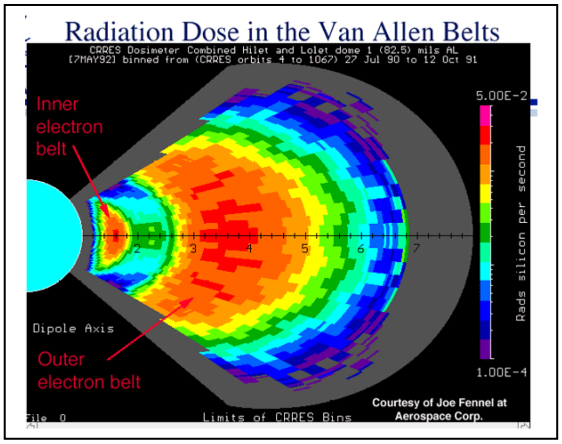
Problem 5 – To two significant figures, how many spherical Earths can you fit in this volume if r = 6378 km?

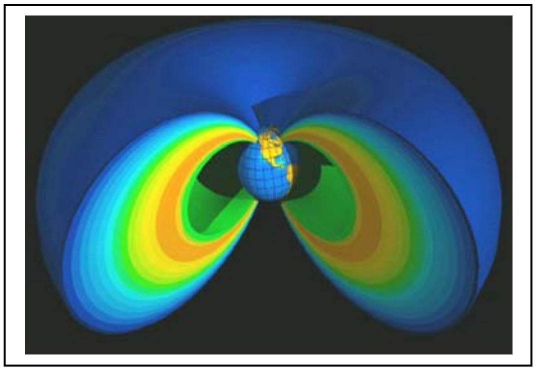
Answer: $V = 4/3 \pi r^3$

$V = 1.33 (3.14) (6.378\times10^6 \text{ m})^3$

$V = 1.1 \times 10^{21}$ meters^3

So 1.3×10^{23} meters^3 / 1.1×10^{21} meters^3 = 118 or **120 Earths!**





Most artistic illustrations of the Van Allen belts make them look almost solid, and the colors chosen make them look especially brilliant in vibrant crimsons and blues. These colors are symbolic and are chosen to represent information about the belts rather than what they actually look like. In fact, if you were standing in the middle of the belts you would not even see them at all!

The Van Allen belts contain trillions of high energy particles that over time can be lethal to an exposed astronaut. They can also damage satellites and spacecraft. But there are very few of these particles in any cubic meter of space. The particles are very small and amount to very little mass at all when added together.

The volume occupied by the Van Allen belts forms a donut-shaped region called a torus, which extends from about 10,000 km to 42,000 km from Earth and equals about 1.3×10^{23} meters^3**.** To find the total mass of the Van Allen belts we use the basic principle that mass = density x volume.

Problem 1 – The average density of electrons and protons in the Van Allen belts is about 100 particles per meter^3. There are about equal numbers of electrons and protons. The protons have a mass of 1.7×10^{-27} kg and electrons have a mass of about 9.1×10^{-31} kg. What are the densities of the electrons and protons in kg/m^3?

Problem 2 – Based on the estimated volume of the Van Allen belts, what is the total mass in A) electrons? B) protons C) combined mass in grams?

Problem 3 – A typical donut has a mass of 33 grams. What is the mass of the Van Allen belts in donuts?

Problem 1 – The average density of electrons and protons in the Van Allen belts is about 100 particles per meter^3. There are about equal numbers of electrons and protons. The protons have a mass of 1.7 x 10^{-27} kg and electrons have a mass of about 9.1 x 10^{-31} kg. What are the densities of the electrons and protons in kg/m^3?

Answer: Density = 50 electrons/m^3 x (9.1 x 10^{-31} kg/electron) = 4.6 x 10^{-29} kg/m^3
Density = 50 protons/m^3 x (1.7 x 10^{-27} kg/electron) = 8.5 x 10^{-26} kg/m^3

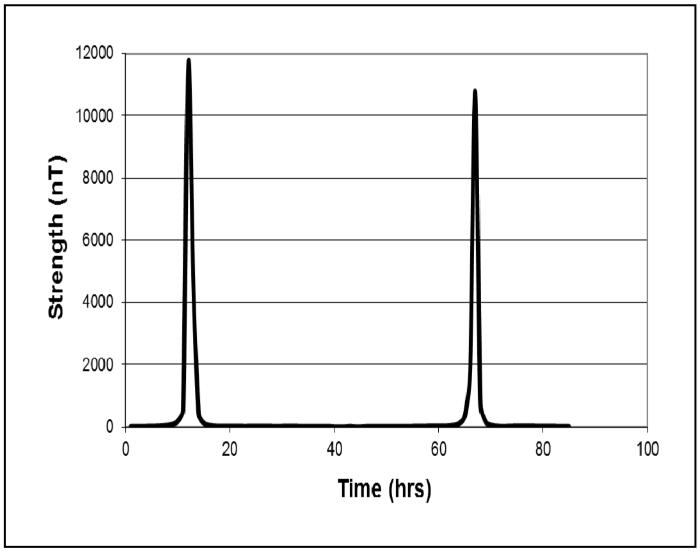
Problem 2 – Based on the estimated volume of the Van Allen belts, what is the total mass in A) electrons? B) protons C) combined mass in grams?

A) M(electrons) = density x volume
= (4.6 x 10^{-29} kg/m^3) (1.3 x 10^{23} meters^3) = 6.0 x 10^{-6} kilograms
B) M(Protons) = density x volume
= (8.5 x 10^{-26} kg/m^3) (1.3 x 10^{23} meters^3) = 1.1 x 10^{-2} kilograms

C) Combined = 0.011 kg x (1000 grams/1kg) = **11 grams!**

Problem 3 – A typical donut has a mass of 33 grams. What is the mass of the Van Allen belts in donuts?

Answer: Our 'donut-shaped' Van Allen belts have **1/3 the mass** of an actual donut!!!



The Cluster satellite constellation consists of 5 satellites orbiting Earth in a close formation. They were designed to measure Earth's magnetic field, and particles in space such as protons and electrons.

This graph shows the strength of Earth's magnetic field measured by the Cluster C1 satellite as it orbited Earth between January 1 and January 6, 2010.

Problem 1 - About what is the highest magnetic field strength measured along the satellite's orbit?

Problem 2 - The satellite's orbit had a perigee (closest point to Earth) of 10,000 km and an apogee (farthest distance from Earth) of 140,000 km. About what was the strength of the magnetic field at A) Perigee? B) Apogee?

Problem 3 - How many hours did it take the satellite to complete one orbit? Explain how you determined this from the graph.

Problem 4 - Below is a table of data taken at specific distances from Earth during the orbit. The strength of the magnetic field is given in units of the nanoTesla (nT). Graph this data. Does the strength decrease as the inverse-square or inverse-cube of the distance?

Point	Distance (km)	Strength (nT)
1	10,000	10,000
2	30,000	370
3	40,000	160
4	50,000	83
5	60,000	44
6	70,000	30
7	140,000	6

Problem 1 - Answer: The two peaks are at 11,700 nT and 10,800 nT so the maximum value occurs for the first peak with **11,700 nT**.

Problem 2 - Answer: A) At perigee, the satellite is closest to Earth so the strength of the magnetic field should be at its highest point along the orbit or 11,700 nT. B) At apogee the spacecraft is farthest from Earth and the strength is lowest, which from the graph is near-zero.

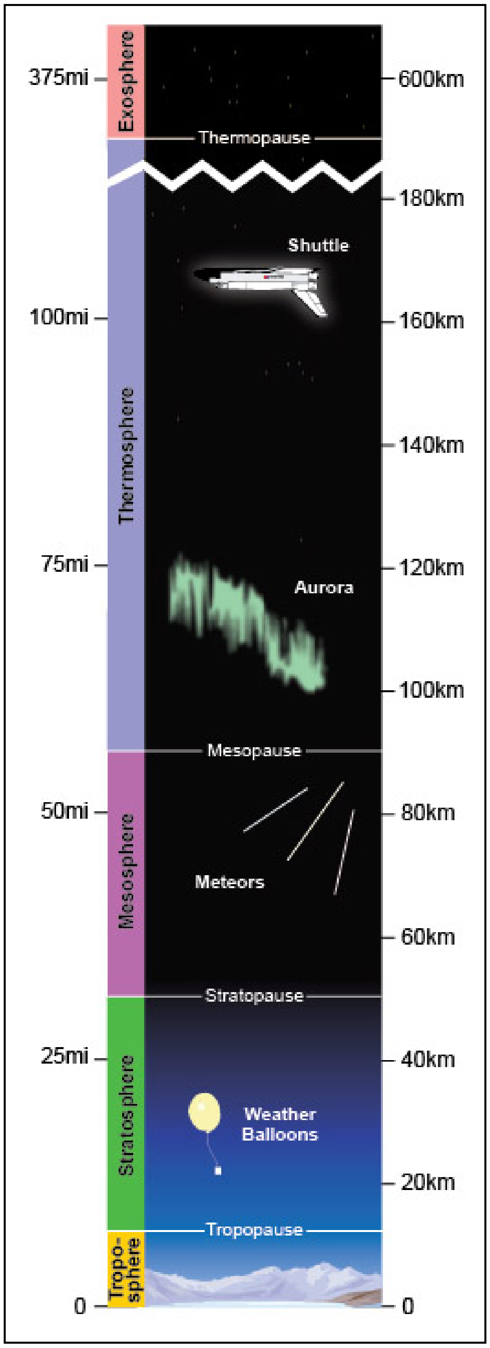
Problem 3 - Answer: The time between perigee of one orbit and perigee of the next orbit is just the time between the maximum measured magnetic strengths of these two consecutive orbits. From the graph, and using a millimeter ruler to get the correct scale, the time between the peaks is about 54 hours.

Problem 4 - Below is a table of data taken at specific distances from Earth during the orbit. The strength of the magnetic field is given in units of the nanoTesla (nT). Graph this data. Does the strength of the magnetic field decrease as the inverse-square or inverse-cube of the distance from Earth?

Answer: Inverse-square: Example of this model would predict between Point 1 and Point 2 that the intensity would drop by $1/(3^2)$ so it would be 1,111 nT
Inverse-cube: it would be $10{,}000/(3^3)$ = 370 nT as shown in the table.

We see that the inverse-cube model fits the data much better than the inverse-square distance law.

Point	Distance (km)	Strength (nT)	**Inverse-square**	**Inverse-cube**
1	10,000	10,000	**10,000**	**10,000**
2	30,000	370	**1,111**	**370**
3	40,000	160	**625**	**156**
4	50,000	83	**400**	**80**
5	60,000	44	**278**	**46**
6	70,000	30	**204**	**29**
7	140,000	6	**51**	**4**



The Van Allen Belt Probes will be exploring a region of space near Earth where the atmosphere of Earth is almost non-existent, but it can still be measured. Scientists use density as a way to show just how much gas there is in a cubic meter of space if you were to collect all of the gas in such a box.

On Earth we often talk about a rock being dense, and measure density in kilograms per cubic meter. For most rocks, their densities are 3000 kg/m^3, so if you had a pick-up truck that could hold 1 cubic meter of rock, it would hold 3000 kilograms of mass or 3 metric tons!

Gas is so dilute that, instead of writing density as kilograms/m^3 we use atoms (or molecules) per cubic meter. This tells us how many particles of gas are in a cubic meter. Because the number of particles is so large, we sometimes have to use scientific notation to write them!

Problem 1 – At sea level, the average density of air molecules (oxygen and nitrogen) is 2.5×10^{25} molecules/m^3. Write this number in: A) decimal form, B) by using 'million', 'billion', 'trillion' etc.

Problem 2 – Imagine a piece of paper 1000 kilometers on a side. How many dots would you have to place in each square that is 1 cm on a side in order to fill up the page with this many dots?

Problem 3 – The mesosphere is one of the highest levels of the atmosphere and at 70 km has a density of 0.00001 kg/m^3. This is 100,000 times lower than the density of the atmosphere at sea level. How many dots would you have in each cell of the paper you used in Problem 2?

Problem 4 - In the van Allen Belts, which is located above the exosphere, the density of particles is about 900 atoms/m^3. If you used the same 1000 km wide piece of paper, how far apart would the atoms of the van Allen Belt be on this scale?

Problem 1 – At sea level, the average density of air molecules (oxygen and nitrogen) is 2.5×10^{25} molecules/m^3. Write this number in: A) decimal form, B) by using 'million', 'billion', 'trillion' etc.

Answer: A) 25,000,000,000,000,000,000,000,000 molecules/m^3.
B) 250 trillion trillion molecules/m^3.

Problem 2 – Imagine a piece of paper 1000 kilometer on a side. How many dots would you have to place in each square that is 1 cm on a side in order to fill up the page with this many dots?

Answer: First we have to find out how many 1 cm^2 squares there are in a 1000 km x 1000 km piece of paper. Since 1 km = 1000 meters and 1 meter = 100 cm, each side of the paper measures 100,000,000 cm, (10^8) so the area of the paper is $(100{,}000{,}000)^2 = 10^{16}$ = 10 thousand trillion cm^2. There are 10 thousand trillion squares on the page, so the number of dots we need to put in each square is 250 trillion trillion / 10 thousand trillion = $2.5 \times 10^{25}/10^{16}$ = 2.5×10^9 or **2,500,000,000 dots!**

Problem 3 – The mesosphere is one of the highest levels of the atmosphere and at 70 km has a density of 0.00001 kg/m^3. This is 100,000 times lower than the density of the atmosphere at sea level. How many dots would you have in each cell of the paper you used in Problem 2?

Answer: If the density is 100,000 lower, then you only need a **total** of $2.5 \times 10^{25}/10^5$ = 2.5×10^{20} dots over the entire sheet. This means that each 1 cm^2 square will only need 100,000 times fewer dots or $2.5 \times 10^9/10^5 = 2.5 \times 10^4$ = **25,000 dots**.

Problem 4 - In the van Allen Belts, which is located above the exosphere, the density of particles is about 900 atoms/m^3. If you used the same 1000 km wide piece of paper, how far apart would the atoms of the van Allen Belt be on this scale?

Answer: You want 900 atoms scattered across a 1000 km x 1000 km piece of paper. Because 30x30 =900, that means that along each side of the paper we would mark off 30 atoms, and complete a grid with this spacing covering the 1000km x 1000km page to mark 900 points. Since 1000 km/30 = 33 1/3 kilometers, the atoms of the van allen belts on this scale would be about **33 kilometers apart!**

Using Proportions to Estimate the Height of a Cloud!


'A lonely cloud and its shadow!'
Courtesy: Henriette (2005)
The Cloud Appreciation Society

Sometimes, if you are lucky, you can see a single cloud and its shadow, perhaps while you were visiting the beach, standing in a meadow, or driving across the desert.

By using a simple proportion and the properties of similar triangles you can use this cloud and its shadow to figure out how high up the cloud is! You need a meter stick, and a bit of help from a friend to do this, though.

Let's see how this works for an example so that you can try this the next time you are at the beach...or the desert!

Problem 1 – Measure the length of your out-stretched arm in centimeters. Now find a cloud near you that has a shadow close by where you are standing. Make sure that the cloud's shadow is directly below the cloud, which will happen around local Noon. Holding the meter stick at arm's length, how many centimeters is it from the base of the cloud down to the ground?

Problem 2 – Draw a scaled model right-triangle ABC, where side AB is the length of your arm in centimeters, and side BC is the vertical distance to the base of the cloud that you measured in Problem 1. Let's suppose that for this problem, AB = 20 inches and AC = 12 inches and that 1 inch = 2.5 cm.

Problem 3 – This next part is a bit tricky. As best you can, estimate how far it is from where you are standing to where the shadow of the cloud begins. You can also note some feature at this location like a tree or a rock formation, or a distant person sitting on a blanket! Count the number of paces it takes to get to this spot. Suppose that for this problem your pace was 2-feet long (0.7 meters) and you completed 3000 paces to get to the spot. How many meters did you travel?

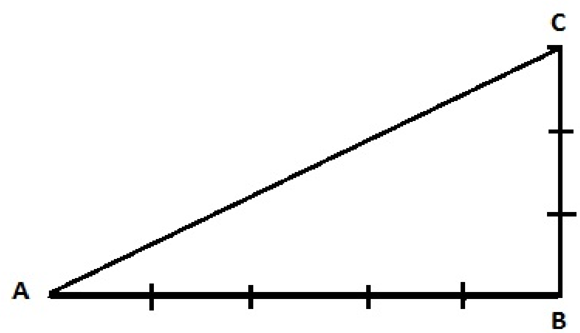
Problem 4 – It is now time to use proportional reasoning. Use the similar triangle you created in Problem 1, with the distance you paced in Problem 3 to determine the actual height of the cloud above the distant point! What would be your answer for the example we used?

Common Core Math Standards:
CCSS.Math.Content.7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Problem 1 – Measure the length of your out-stretched arm in centimeters. Now find a cloud near you that has a shadow close by where you are standing. Holding the meter stick at arm's length, how many centimeters is it from the base of the cloud down to the ground?

Problem 2 – Draw a scaled model right-triangle ABC, where side AB is the length of your arm in centimeters, and side BC is the vertical distance to the base of the cloud that you measured in Problem 1. Let's suppose that for this problem, AB = 20 inches and AC = 12 inches and that 1 inch = 2.5 cm.

Answer: We want all measurements to be in centimeters in order to draw the scaled triangle. AB = 20 inches x (2.5 cm/1 inch) = 50 cm. AC = 12 inches x (2.5 cm/ 1 inch) = 30 cm. The drawing looks like this. Each division is 10 centimeters.



Problem 3 – This next part is a bit tricky. As best you can, estimate how far it is from where you are standing to where the shadow of the cloud begins. You can also note some feature at this location like a tree or a rock formation, or a distant person sitting on a blanket! Count the number of paces it takes to get to this spot. Suppose that for this problem your pace was 2-feet long (0.7 meters) and you completed 3000 paces to get to the spot. How many meters did you travel?

Answer: In this example, 3000 paces x (0.7 meters/1 pace) = **2,100 meters**.

Problem 4 – It is now time to use proportional reasoning. Use the similar triangle you created in Problem 1, with the distance you paced in Problem 3 to determine the actual height of the cloud above the distant point! What would be your answer for the example we used?

Answer: Let X be the actual height of the cloud, then from the similar triangle BC/AB = X/2100 meters, and since BC=30cm and AB=50 cm we have 30/50 = X/2100 and so X = 2100 (30/50) = 1260 meters. **So the cloud is about 1260 meters above the ground!**

'A lonely cloud and its shadow!'
Courtesy: Henriette (2005)
The Cloud Appreciation Society

Sometimes, if you are lucky, you can see a single cloud and its shadow, perhaps while you were visiting the beach, standing in a meadow, or driving across the desert.

By using a simple proportion and the properties of similar triangles you can use this cloud and its shadow to figure out the size of the cloud! You need a meter stick, and a bit of help from a friend to do this, though.

Let's see how this works for an example so that you can try this the next time you are at the beach...or the desert!

Problem 1 – Measure the length of your out-stretched arm in centimeters. Now find a cloud near you that has a shadow close by where you are standing. Holding the meter stick at arm's length, how many centimeters across does the cloud appear to be from where you are standing?

Problem 2 – Draw a scaled model right-triangle ABC, where side AB is the length of your arm in centimeters, and side BC is the width of the cloud that you measured in Problem 1. Let's suppose that for this problem, AB = 20 inches and AC = 12 inches and that 1 inch = 2.5 cm.

Problem 3 – This next part is a bit tricky. As best you can, estimate how far it is from where you are standing to where the cloud is located. For instance, you might see the shadow of the cloud on the ground in the distance. If it is close-by, count the number of paces it takes to get to where the cloud shadow is located. Suppose that for this problem your pace was 2-feet long (0.7 meters) and you completed 3000 paces to get to the spot. How many meters did you travel?

Problem 4 – It is now time to use proportional reasoning. Use the similar triangle you created in Problem 1, with the distance you paced in Problem 3 to determine the actual width of the cloud! What would be your answer for the example we used?

Common Core Math Standards:
CCSS.Math.Content.7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Problem 1 – Measure the length of your out-stretched arm in centimeters. Now find a cloud near you that has a shadow close by where you are standing. Holding the meter stick at arm's length, how many centimeters across does the cloud appear to be from where you are standing?

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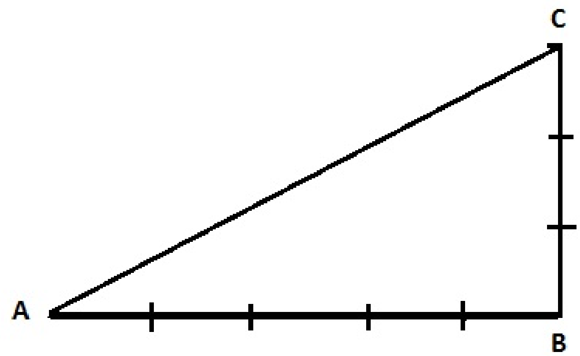
Answer: We want all measurements to be in centimeters in order to draw the scaled triangle. AB = 20 inches x (2.5 cm/1 inch) = 50 cm. AC = 12 inches x (2.5 cm/ 1 inch) = 30 cm. The drawing looks like this. Each division is 10 centimeters.



Problem 3 – This next part is a bit tricky. As best you can, estimate how far it is from where you are standing to where the cloud is located. For instance, you might see the shadow of the cloud on the ground in the distance. If it is close-by, count the number of paces it takes to get to where the cloud shadow is located. Suppose that for this problem your pace was 2-feet long (0.7 meters) and you completed 3000 paces to get to the spot. How many meters did you travel?

Answer: In this example, 3000 paces x (0.7 meters/1 pace) = **2,100 meters**.

Problem 4 – It is now time to use proportional reasoning. Use the similar triangle you created in Problem 1, with the distance you paced in Problem 3 to determine the actual width of the cloud! What would be your answer for the example we used?

Answer: Let X be the actual width of the cloud, then from the similar triangle BC/AB = X/2100 meters, and since BC=30cm and AB=50 cm we have 30/50 = X/2100 and so X = 2100 (30/50) = 1260 meters. **So the cloud is about 1260 meters wide!**

Estimating the Mass of a Cloud!

You look up at the sky one day and see puffy little cumulus clouds hovering over the beach, a meadow, or over your town. Did you ever wonder just how much a cloud might weigh as it drifts by over your head?

Different clouds carry different amounts of water droplets and so they have different densities. Brilliant white cumulus clouds, for example, have densities of 0.3 grams/meter^3.

From the known cloud densities, we can estimate their masses once we know their volumes because Density = Mass/Volume.

Problem 1 – From the definition of density, what are the other two equations you can create that define mass and volume?

Problem 2 – A puffy cumulus cloud looks almost like a sphere. If its diameter is 3.0 kilometers, what is its volume in cubic meters? (use π = 3.14)

Problem 3 – What is the total mass of the cumulus cloud in kilograms and metric tons?

Problem 4 – You spot two clouds in the sky. The cumulus cloud is 1/5 the diameter of the cumulonimbus cloud, and the cumulonimbus cloud has 8 times the density of the cumulus cloud. What is the ratio of the mass of the cumulus cloud to the cumulonimbus cloud if both clouds are spherical in shape?

Grade 7 - Working with Density, mass and volume: Examples: 'California Mathematics Standards' - *Students can calculate the mass of a cylinder given its dimensions and density*. - Utah State Science Standards: I.2.c. "*Calculate the density of various solids and liquids.*"

Grade 8 - Common Core Math Standards:
CCSS.Math.Content.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Problem 1 – From the definition of density, what are the other two equations you can create that define mass and volume?

Answer: Mass = Density x Volume   Volume = Mass/Density

Problem 2 – A puffy cumulus cloud looks almost like a sphere. If its diameter is 3.0 kilometers, what is its volume in cubic meters? (use $\pi = 3.14$)

Answer: $V = 4/3\ \pi\ R^3$ and for D = 3.0 km, we have R = 1500 meters and so $V = 4/3\ \pi\ (1500\text{meters})^3 =$ **1.4×10^{10} meters^3**

Problem 3 – What is the total mass of the cumulus cloud in kilograms and metric tons?

Answer: Mass = Density x Volume so $M = 0.3\ \text{grams/m}^3 \times 1.4 \times 10^{10}\ \text{m}^3 = 4.2\times10^9$ grams. But 1 kg = 1000 grams, so **M = 4,200,000 kg**. This also equals **4200 metric tons**!

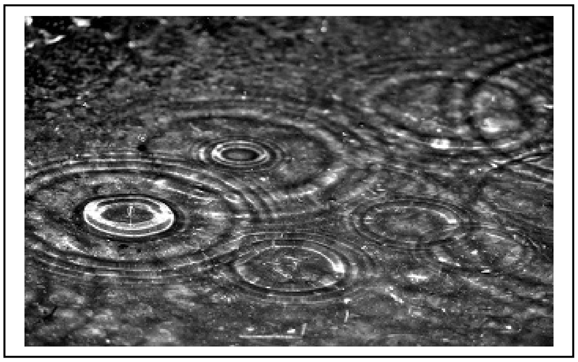
Problem 4 – You spot two clouds in the sky. The cumulus cloud is 1/5 the diameter of the cumulonimbus cloud, and the cumulonimbus cloud has 8 times the density of the cumulus cloud. What is the ratio of the mass of the cumulus cloud to the cumulonimbus cloud if both clouds are spherical in shape?

Answer: Mass = Density x Volume.

V(Cumulus)/V(CN) = $(1/5)^3$ and D(Cumulus) = 1/8D(CN) so

Mass(Cumulus) = $(1/5)^3$ x (1/8) x Mass(CN) = 1/1000 Mass(CN) and so

Mass(Cumulus)/Mass(CN) = **1/1000**



After a local rain storm, your news station might announce that 0.5 inches of rain fell during the morning hours before Noon.

Have you ever wondered just how much water fell out of the sky to cause so much trouble to people trying to get to work or stay dry outside?

Meteorologists classify rain rates for different levels of activity as you can see in the table below:

Type of Storm	Rate
Light Rain	2 - 4 mm/hr
Moderate	5 - 9 mm/hr
Heavy	10 - 40 mm/hr
Violent	more than 50 mm/hr

Problem 1 – Suppose the local news said that 1.6 inches of rain fell between 8:00 am and 1:00 pm. What type of storm was this? (1 inch = 25 mm)

Problem 2 – If 1 mm of rainfall equals 1 liter of water over an area of one square meter, how many liters of water will fall over a town that has an area of 100 km^2 during a light rain shower that lasted 3 hours at a rate of 2 mm/hr?

Problem 3 – About 500,000 cubic kilometers of rain falls on the surface of Earth every year. What is the average rate in mm/hr if the surface area of Earth is 500 million km^2?

Common Core Math Standards:

Grade 6 – *CCSS.Math.Content.6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Grade 7 – *CCSS.Math.Content.7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*

Optional: Grade 8 - *CCSS.Math.Content.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology*

Problem 1 – Suppose the local news said that 1.6 inches of rain fell between 8:00 am and 1:00 pm. What type of storm was this? (1 inch = 25 mm)

Answer: The depth of the rain was 1.6 inches x 25 mm/ 1 inch = 40 mm. This fell in the time between 8:00 am and 1:00 pm which is 5 hours, so the rate was 40 mm/5 hours = 8 mm/hr. This type of storm would be considered a **moderate storm**.

Problem 2 – If 1 mm of rainfall equals 1 liter of water over an area of one square meter, how many liters of water will fall over a town that has an area of 100 km^2 during a light rain shower that lasted 3 hours at a rate of 2 mm/hr?

Answer: First we have to calculate the total number of millimeters that fell, which is 2 mm/hr x 3 hours or 6 millimeters.

Then we calculate the rate in terms of liters/meter^2 which will be 6 mm x (1 Liter/meter^2) = 6 Liters/meter^2.

Next we convert the area of the town into square meters, which is 100 km^2 x (1000 m/1km)x (1000 m/1 km) = 10^8 meters^2.

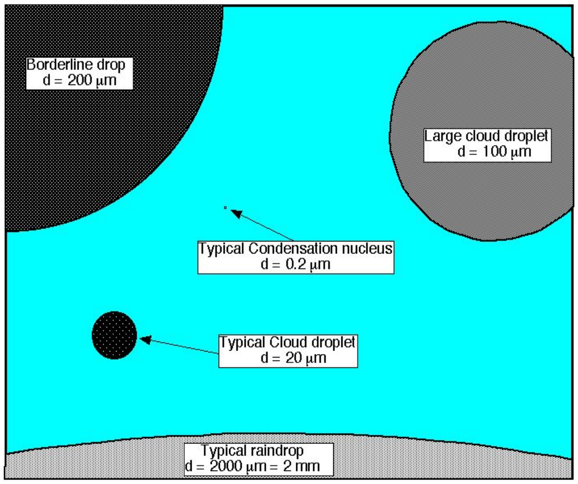
Finally we multiply the rate by the area to get 6 liters/meter^2 x 10^8 meters^2 = **6.0x10^8 Liters**.

Problem 3 – About 500,000 cubic kilometers of rain falls on the surface of Earth every year. What is the average rate in mm/hr if the surface area of Earth is 500 million km^2?

Answer: Volume = Height x Area so
500,000 km^3 = height x 500 million km^2 and so
height = 500,000/500,000,000 = 1/1000 km or 1 meter.

This falls in one year. 1 year = 365 days x 24h/1day = 8760 hours so the rate is

R = 1 meter/8760 hours
= 1000 mm/8750 hrs
= **0.11 mm/hr**.



Without droplets of water, most clouds would be transparent! If you were to look inside a cloud you would see droplets of water of many different sizes because droplets constantly grow in size once they are formed.

The figure to the left shows some typical kinds of water droplets you might find in a cloud along with their diameters in micrometers (microns). Recall that one micrometer=1/1000000 or 10^{-6} meters.

The following exercises let you explore some of the properties of water droplets. In all cases, assume that the droplet is a perfect sphere!

Problem 1 – Water droplets are made out of water (of course!) and water has a density of 1000 kg/m^3. What is the mass, in grams, of each of the five types of droplets described in the figure?

Problem 2 – To the nearest 1000, about how many typical cloud droplets have to be combined to form one large cloud droplet?

Problem 3 – To the nearest 1000, about how many large cloud droplets have to combine to form one typical raindrop?

Problem 4 – Suppose that it takes about 2 minutes for a large cloud droplet to double in mass. How long does it take a large cloud droplet to grow into a raindrop and leave the cloud?

Common Core Math Standards:
CCSS.Math.Content 6.RP.3.d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
CCSS.Math.Content.8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
CCSS.Math.Content.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology

Problem 1 – Water droplets are made out of water (of course!) and water has a density of 1000 kg/m^3. What is the mass, in grams, of each of the five types of droplets described in the figure?
Answer: Recall Volume = $4/3 \pi R^3$ and Mass = Density x Volume.

Raindrop: r=2 mm = 0.002 meters
$V = 4/3 \pi (0.002)^3 = 3.3 \times 10^{-8} m^3$
Mass = 1000 kg/m^3 x $3.3 \times 10^{-8} m^3$ x (1000gm/1kg) = **0.033 grams**

Borderline Drop: r = 200 microns. = 0.0002 meters
$V = 4/3 \pi (0.0002)^3 = 3.3 \times 10^{-11} m^3$
Mass = 1000 x 3.3×10^{-11} x (1000gm/1kg) = **3.3×10^{-5} grams** (= 33 micrograms)

Large Cloud Droplet: r = 100 microns = 0.0001 meters
$V = 4/3 \pi (0.0001)^3 = 4.2 \times 10^{-12} m^3$
Mass = 1000 x 4.2×10^{-12} x (1000gm/1kg) = **4.2×10^{-6} grams** (4.2 micrograms)

Typical Cloud Droplet: r = 20 microns = 0.00002 meters
$V = 4/3 \pi (0.00002)^3 = 3.3 \times 10^{-14} m^3$
Mass = 1000 x 3.3×10^{-14} x (1000gm/1kg) = **3.3×10^{-9} grams** (3.3 nanograms)

Typical Condensation Nucleus: r = 0.2 microns = 0.0000002 meters
$V = 4/3 \pi (0.0000002)^3 = 3.3 \times 10^{-20} m^3$
Mass = 1000 x 3.3×10^{-20} x (1000gm/1kg) = **3.3×10^{-14}** grams

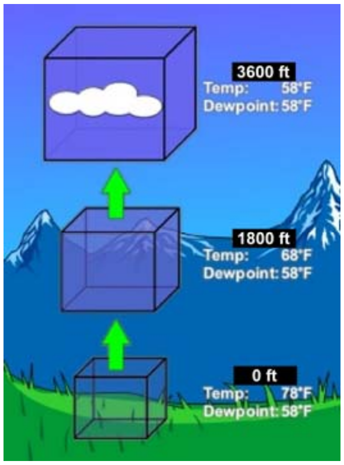
Problem 2 – To the nearest 1000, about how many typical cloud droplets have to be combined to form one large cloud droplet? Answer: N = Mass of large droplet / mass of typical cloud droplet = 4.2×10^{-6} gm/3.3×10^{-9} gm = 1272 typical droplets or **about 1000**.

Problem 3 – To the nearest 1000, about how many large cloud droplets have to combine to form one typical raindrop? Answer: N = Mass of raindrop/mass of large droplet = 0.033 grams/4.2×10^{-6} grams = 7857 or about **8000 large droplets**.

Problem 4 – Suppose that it takes about 2 minutes for a large cloud droplet to double in mass. How long does it take a large cloud droplet to grow into a raindrop and leave the cloud?

Answer: In Problem 3 we saw that about 8000 large cloud droplets equals a raindrop. Since 8000 is about 2^{13}, we need 13 doubling times to grow this large, which takes 13 x 2 minutes = **26 minutes**.

Note: Students may want to make a scaled model of droplet sizes using styrofoam balls or other round objects.



Courtesy Katie R. Roussy (2006)
University of Illinois at Urbana

Air can carry water vapor, and warm air can carry a lot more water vapor than cold air. That's why your skin feels wet and clammy in the summer, and you often have problems with dry skin during the winter.

When the temperature of air reaches a critical temperature called the **dewpoint**, water vapor begins to condense as droplets. For large masses of air, hundreds of droplets can form in every cubic centimeter and you see a cloud begin to appear.

The diagram to the left shows what happens to an 'air mass' with a dewpoint temperature of 58^{o}F as it rises to cooler altitudes. When the local air temperature equals the dew point, the cloud appears.

Sometimes, the local temperature near the ground can be slightly above the dew point. When this happens, the air remains clear, but droplets of water can form on windows or on cars. When the local ground temperature is below the dew point, droplets will condense in the air and you get ground fog!

Problem 1 – It's a warm sunny day and the air is rather humid with a dewpoint of 75^{o}F .The ground temperature is 85^{o}F. If the air temperature decreases at a rate of 3.5^{o}F/1000 feet (called the **lapse rate**), at what altitude will a cloud begin to appear?

The dewpoint temperature is very complicated to calculate exactly because it depends on the local atmospheric pressure and temperature, and the amount of water vapor in the air. There are some ways to estimate dewpoint temperature that give a rough idea of what to expect. The following formula is one of these methods:

$$T_{dewpoint} = T_{air} - \frac{100 - P}{5}$$

If the humidity of the air is 60% you will feel uncomfortable (P = 60) and if the outside temperature is 90^{o}F (Tair=90^{o}F) then the dewpoint temperature is 82^{o}F.

Problem 2 – On a summer day, the humidity is 50% and the outside temperature is 80^{o}F. At what altitude might clouds start to form overhead if the air temperature is decreasing at a lapse rate of 2.0^{o}F/1000 feet?

Common Core Math Standards:

Grade 6 – *CCSS.Math.Content.6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Grade 7 – *CCSS.Math.Content.7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*

Problem 1 – It's a warm sunny day and the air is rather humid with a dewpoint of 75^{o}F .The ground temperature is 85^{o}F. If the air temperature decreases at a rate of 2.5^{o}F/1000 feet, at what altitude will a cloud begin to appear?

Answer: The rising air near the ground has to drop in temperature by 85^{o}F – 75^{o}F = 10^{o}F. It is decreasing by 2.5^{o}F every 1000 feet, so the dewpoint temperature of 75^{o}F will be reached at an elevation of 10^{o}F/2.5^{o}F = **4000 feet**.

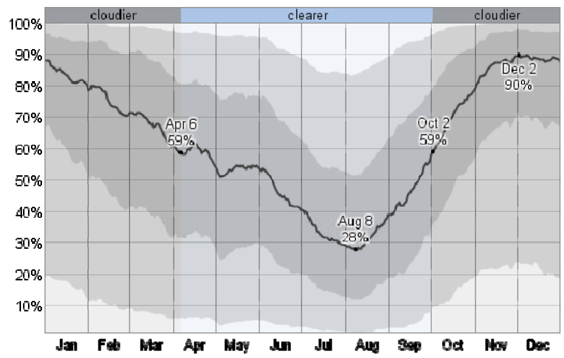
Problem 2 – On a summer day, the humidity is 50% and the outside temperature is 80^{o}F. At what altitude might clouds start to form overhead if the air temperature is decreasing at a lapse rate of 2.0^{o}F/1000 feet?

Answer: First use the dewpoint formula to calculate Tdewpoint.
For P=50 and Tair=80^{o}F we get Tdewpoint=80 – (100-50)/5 so Tdewpoint=70^{o}F.

Next, calculate the altitude from the lapse rate.

The difference between the ground temperature and the dewpoint temperature is 80^{o}F – 70^{o}F = 10^{o}F.

The altitude will be A = 10^{o}F/(2.0^{o}F/1000feet) = **5000 feet**.



This plot describes the cloud cover percentage over Two Harbors, Minnesota over the course of an average year. This region has a humid continental climate with warm summers and no dry season. It is covered by forests (48%), water (38%), croplands (9%), and grasslands (4%).

Many homeowners now use solar panels to collect sunlight and convert it into electricity on their rooftops. This is a good idea when there are no clouds in the sky, but what happens on a cloudy day?

There is a simple formula to predict how much sunlight reaches the ground for different amounts of cloud cover:

$$P = 990\ (1-0.75F^3) \quad \text{watts/m}^2$$

where F is the fraction of sky cloud cover on a scale from 0.0 (no clouds) to 1.0 (100% complete coverage).

Problem 1 – For what percentage of the year are conditions considered cloudy in Two Harbors?

Problem 2 – Based upon the trend in the black line on the graph, what is the average cloud cover during the year?

Problem 3 – From the formula for solar power, for what percentage of sky cover will the homeowner get more than 50% of the maximum solar power from their electric system?

Problem 4 – On the cloud cover graph, shade in the region that represents the condition that the homeowner will get more than 50% of the available electrical power.

Problem 5 – About what percentage of the year will the homeowner be able to generate more than 50% of the available solar power?

Common Core Math Standards:
CCSS.Math.Content.6.RP.A.3c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
CCSS.Math.Content.8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

More about the cloud cover formula can be found at:
http://www.shodor.org/os411/courses/_master/tools/calculators/solarrad/

The data from Two Harbors is from
http://weatherspark.com/averages/31818/Two-Harbors-Minnesota-United-States

Problem 1 – For what percentage of the year are conditions considered cloudy in Two Harbors? Answer: January, February, March and October, November and December so P = 100% x (6/12) = **50%**.

Problem 2 – Based upon the trend in the black line on the graph, what is the average cloud cover during the year? Answer: The highest is 90% and the lowest is 28% so the average of these two is **59%**. Students may also calculate the average monthly coverage (January=85%, February=75%, March=67%, April=60%, May=55% June=45% July=32% August=32% September=45% October=70% November=87% December=87%) and get **62%**.

Problem 3 – From the formula for solar power, for what percentage of sky cover will the homeowner only get 50% of the maximum solar power from their electric system? Answer: The maximum solar power occurs for F=0 and equals 990 watts/m^2. Half of this is 495 watts/m2 so we want $495 = 990(1-0.75F^3)$. This means that $0.50 = 0.75F^3$ and so $F^3 = 0.67$ and so solving for F we get $F = (0.67)^{1/3} = 0.87$. **So 87% cloud cover produces a reduction of 50% in electrical power.**

Problem 4 – On the cloud cover graph, shade in the region that represents the condition that the homeowner will get more than 50% of the available electrical power. Answer: **Draw a horizontal line across the graph at '87%'. And shade in all the area below this line to indicate 'more than 50% of available power'.**

Problem 5 – About what percentage of the year will the homeowner be able to generate more than 50% of the available solar power? Answer: Only three months have more than 87% cloud cover: January, November and December, so there are 9 months producing more than 50% : P = 100% (9/12) = **75%**.

Courtesy Aaron McNeeley
mmcneely@nd.edu

When a cloud is dense enough with water droplets that it appears fleecy white, it is also dense enough that it can cause a shadow.

The amount of light a cloud reflects is called its **albedo**. The amount of light that passes through the cloud is called its **transmission**.

These two properties of a cloud can be measured in terms of percentage.

Problem 1 – Suppose that a cloud has an albedo of 100%. How much light is transmitted through the cloud to the surface of Earth?

Problem 2 – Albedo and transmission are linearly related to each other. Write a formula that relates albedo, A, and transmission, T to each other.

Instead of transmission, scientists prefer to use the term opacity, x, because it can be more easily calculated from the actual properties of the cloud. For example, x = kL, where L is the thickness of the cloud and k is a constant that describes the density of droplets in the cloud and droplet sizes. Transmission, T, and opacity are related by the formula:

$$T = 100\% \ 10^{-0.69x}$$

Problem 3 - Graph the function T(x) for opacities from 0.0 to 5.0. To the nearest percentage, what is the range of cloud transmission and albedo for opacities covered by your graph?

Problem 4 – A cumulus cloud is 2.5 kilometers thick and its opacity constant, k = 0.5, what is the albedo of this cloud, and how much light is transmitted through the cloud to the ground?

Common Core Math Standards:
CCSS.Math.Content.HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

CCSS.Math.Content.HSF-LE.A.4 For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

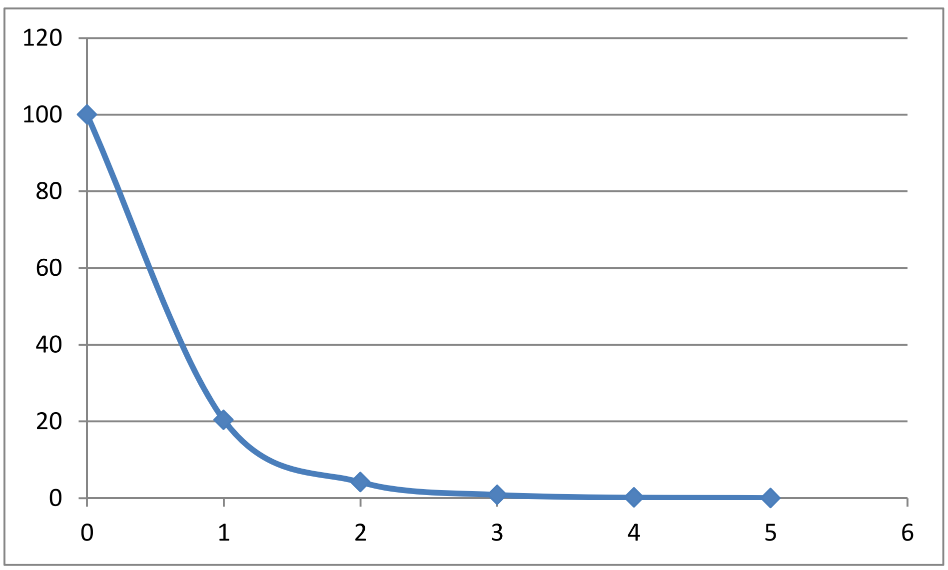
Problem 1 – Suppose that a cloud has an albedo of 100%. How much light is transmitted through the cloud to the surface of Earth?

Answer: If all the light is reflected, then no light passes through the cloud so its transmission is 0%

Problem 2 – Albedo and transmission are linearly related to each other. Write a formula that relates albedo, A, and transmission, T to each other.

Answer: A = 100% - T

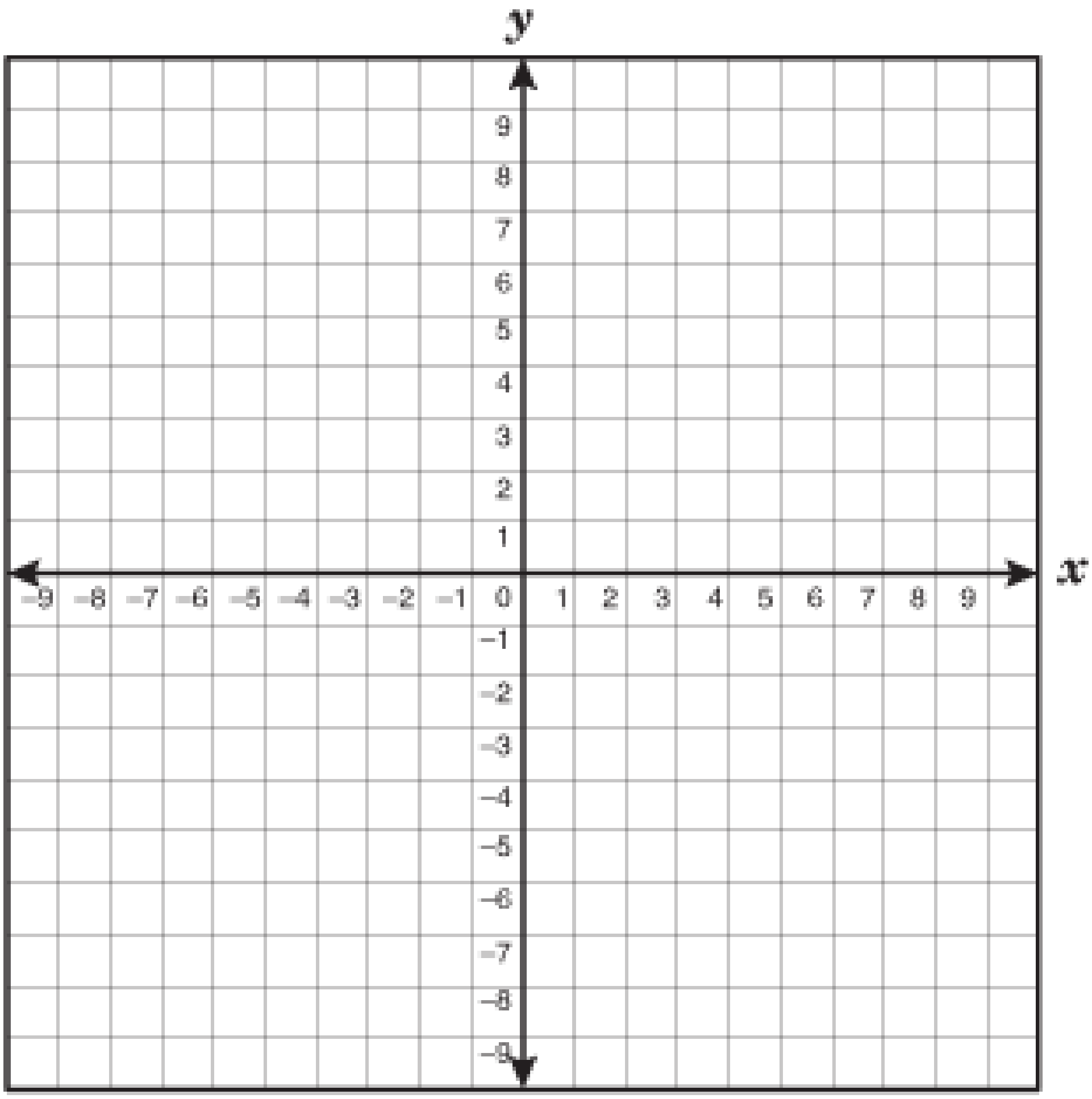
Problem 3 - Graph the function T(x) for opacities from 0.0 to 5.0. To the nearest percentage, what is the range of cloud transmission and albedo for opacities covered by your graph?



Opacity 1 to 5
Transmission: 20% to 0%
Albedo: 80% to 100%

Problem 4 – A cumulus cloud is 2.5 kilometers thick and its opacity constant, k = 0.5, what is the albedo of this cloud, and how much light is transmitted through the cloud to the ground?

Answer: x= kL so x= (0.5)(2.5) = 1.25 then the transmission $T = 100\%\ 10^{-0.69(1.25)}$
Then T = 100%(0.137)
And so T = **13.7%** and the albedo = 100% - 13.7% = **86.3%**



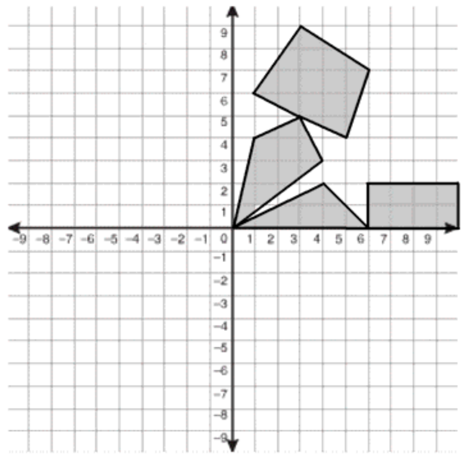
Snowflakes have a symmetrical shape that often follows a simple pattern that is replicated to form the full shape that you see.

Problem 1 - Graph the following points to make a design in the First Quadrant:

(10,0), (10,2), (6,2), (6,0), (4,2), (0,0), (4,3), (3,5), (5,4), (6,7), (3,9), (1,6), (3,5), (1,4), (0,0)

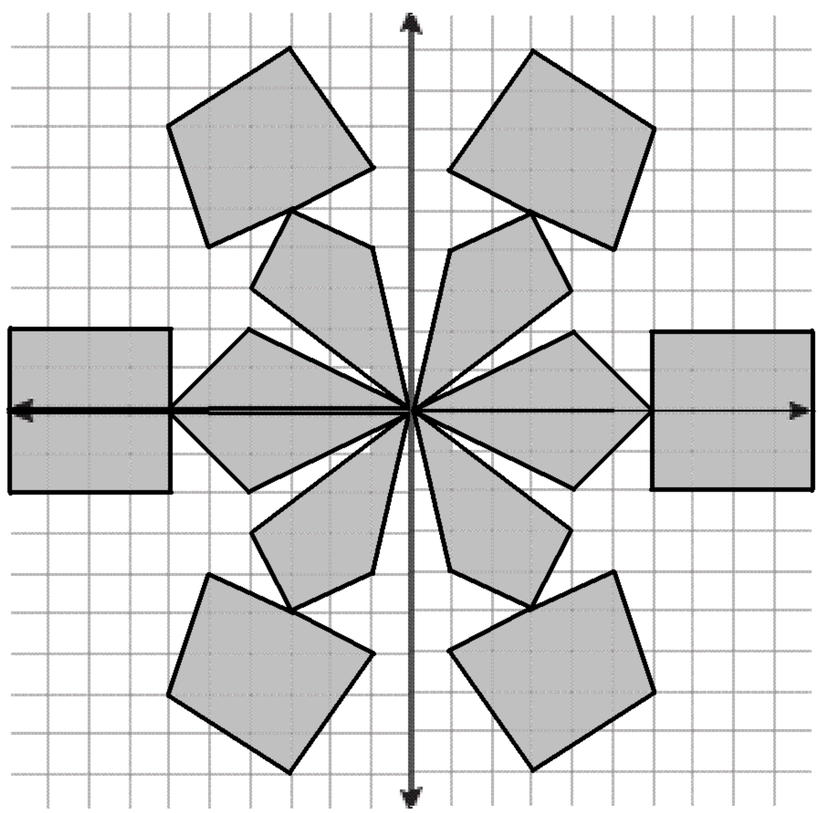
Problem 2 - Connect the points with line segments in the order given.

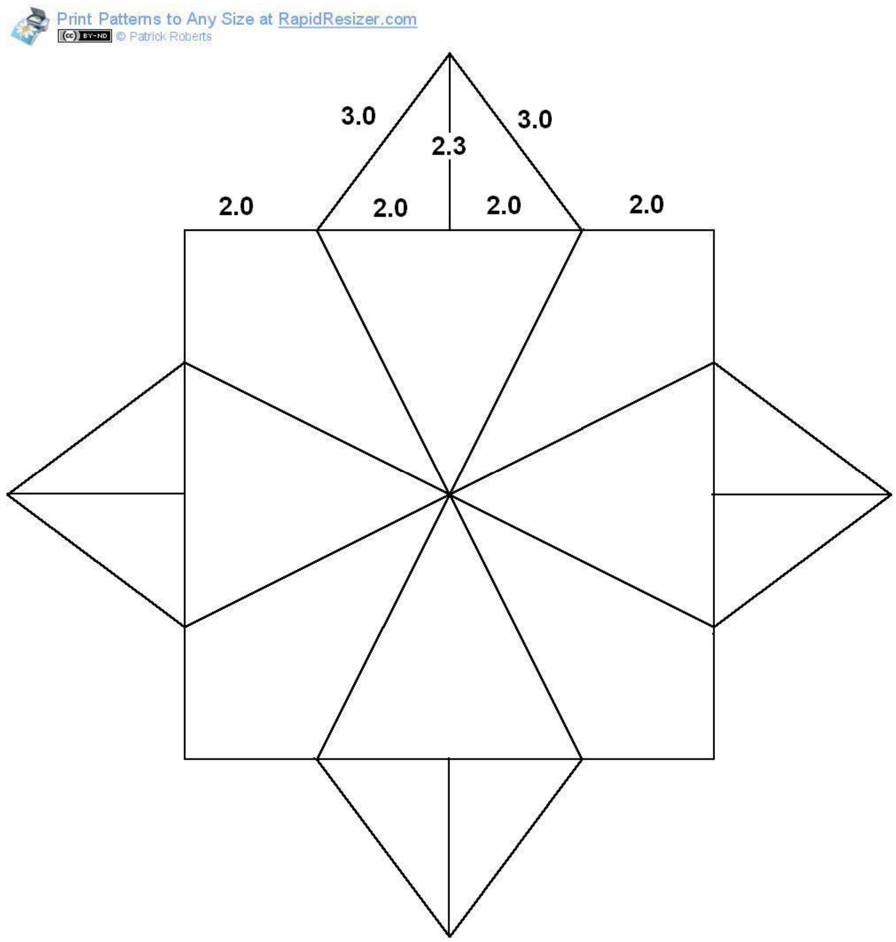
Problem 3 - Reflect the pattern that you drew into the Second Quadrant, then complete the pattern in Quadrants Three and Four to form the full snowflake shape!



Problem 1 and 2 -

Problem 3 - Students may either place 'mirrors along the X and Y axis and redraw the shape in the First Quadrant, or use the following symmetry idea: To reflect the figure into Quadrant Two, plot the points in Quadrant One with the sign of the x coordinates replaced by their negative : (x,y) becomes (-x, y). For Quadrant Three use (x,y) becomes (-x,-y) and for Quadrant Four (x,y) becomes (x,-y). The full figure is shown below:





The diagram above shows the basic plan for one common type of snowflake. The detailed pattern within each polygonal area has been removed to show the regular areas. The numbers at the top are the measured line segments in millimeters.

Problem 1 - Using the geometric clues in the diagram, what is the total area of this pattern in square millimeters, rounded to the nearest integer?

Problem 2 - If all measurements were doubled in length, what would be the total area of the pattern to the nearest integer in square-millimeters?

Problem 1 - Using the geometric clues in the diagram, what is the total area of this pattern in square millimeters, rounded to the nearest integer?

Answer:

The pattern consists of a main square with a side length of 2.0mm+2.0mm+2.0mm+2.0mm = 8.0 mm and an area of $(8.0\text{mm})^2$ = 64 mm^2.

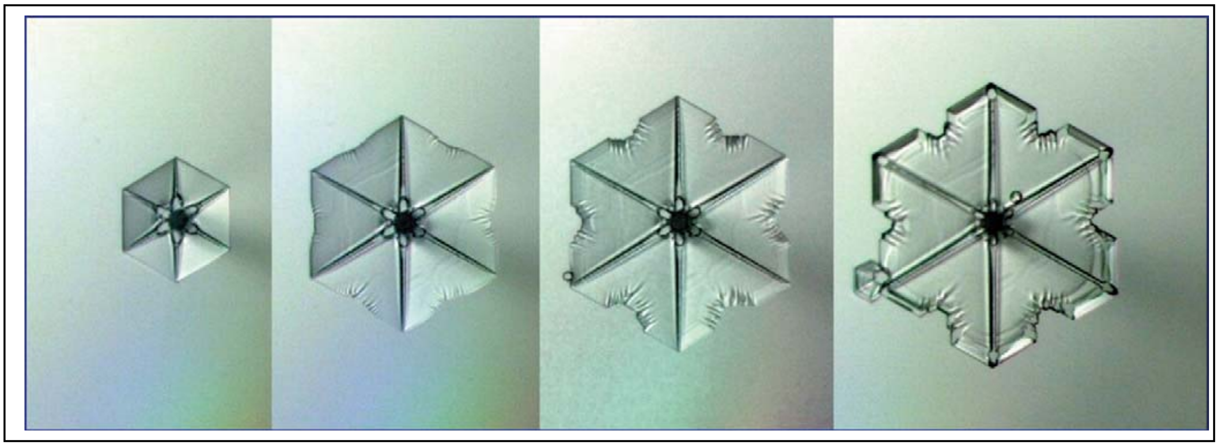
The four triangular points each have an area of 1/2(4.0mm)(2.3mm) = 4.6 mm^2, so the total area of the pattern is 64.0 mm^2+ 4(4.6mm^2) = 82.4 mm^2, which is rounded to **82 mm^2**.

Problem 2 - If all measurements were doubled in length, what would be the total area of the pattern to the nearest integer in square-millimeters?

Answer: Doubling the dimensions means that the area is increased by a factor of 2x2 = 4 so it now becomes 82.4mm^2 x 4 = 329.6 mm^2, which rounds to **330 mm^2**.

The side length of the square becomes 2 x 8.0mm = 16.0 mm and the area is then $(16\text{mm})^2$ = 256 mm^2. The four triangles each have an area of 1/2 (8.0mm) (4.6mm) = 18.4 mm^2, so the total area is 256 mm^2+ 4(18.4mm^2) = 329.6 mm^2 or rounded to **330 mm^2**.

Snowflake Growth Rates and Surface Area



A snowflake is a flat figure whose area doubles over time as liquid droplets condense on its surface. For average cloud conditions, the area doubles every 2 hours.

No matter what the shape of a polygon, the area of a polygon will increase by a fixed amount as the size of the polygon increases.

Problem 1 - Suppose the time to double its area is 2 hours. How many doublings in area will have occurred in 8 hours?

Problem 2 – If the area of the snowflake at the start of its growth is 1 square millimeter, what will its area be after 8 hours? To organize your thinking about snowflake growth, create a table for the snowflakes size and area.

Problem 3 – If the size of the snowflake was 1 millimeter at the start of growth, what will be its size at the end of a snow storm that lasted 8 hours if the area doubling time is 2 hours? To organize your thinking about snowflake growth, create a table for the snowflakes size and area.

Problem 1 - Suppose the time to double its area is 2 hours. How many doublings in area will have occurred in 8 hours?

Answer: The snowflake has been growing for 8 hours which is 8/2 = **4 doubling times.**

Problem 2 – If the area of the snowflake at the start of its growth is 1 square millimeter, what will its area be after 8 hours?

Doubling	**1**	**2**	**3**	**4**	**5**	**6**
Area	**2**	**4**	**8**	**16**	**32**	**64**
Size	**1.4**	**2**	**2.8**	**4**	**5.7**	**8**

Answer: It will have an area that is 2 x 2 x 2 x 2 = 16 times larger or **16 square millimeters.**

Problem 3 - If the size of the snowflake was 1 millimeter at the start of growth, what will be its size at the end of a snow storm that lasted 8 hours if the area doubling time is 2 hours?

Answer: 8 hours = 4 doubling times so it has increased in area by 16 times. Because area = length x length, since 16 = 4 x 4, the snowflake has increased its size by 4 times so it is now 1 mm x 4 = **4 millimeters in diameter.**

Snow to Water Ratios

The amount of snow from a storm can look impressive when it covers your house and cars, but if you melted the snow you would discover that very little water is actually involved. The 'snow to ice ratio' or Snow Ratio expresses how much volume of snow you get for a given volume of water. Typically a ratio of 10:1 (ten to one) means that every 10 inches of snowfall equals one inch of liquid water.

Problem 1 - During a winter storm called 'Snowmageddon' in 2010, the Washington DC region received about 24 inches of snow fall. If this was dry, uncompacted snow, about how many inches of rain would this equal if the Snow Ratio was 10:1 ?

Problem 2 - The Snow Ratio depends on the temperature of the air as shown in the table below:

Temp (F)	30^o	25^o	18^o	12^o	5^o	-10^o
Ratio	10:1	15:1	20:1	30:1	40:1	50:1

If 30 inches of snow fell in Calgary, Alberta at 18^oF, and 25 inches of snow fell in Denver, Colorado where the temperature was 25^o F, at which location would the most water have fallen?

Problem 1 - During a winter storm called 'Snowmageddon' in 2010, the Washington DC region received about 24 inches of snow fall. If this was dry, uncompacted snow, about how many inches of rain would this equal if the Snow Ratio was 10:1 ?

Answer: 24 inches of snow x (1 inch water/10 inches of snow) = **2.4 inches of water**.

Problem 2 - If 30 inches of snow fell in Calgary, Alberta at 18^{o}F, and 25 inches of snow fell in Denver, Colorado where the temperature was 25^{o} F, at which location would the most water have fallen?

Answer - In Alberta, the Snow Ratio for 18^{o} F is 20:1 and in Denver at 25^{o} F it is 15:1.

The amount of water that fell in Alberta is then 30 inches of snow x (1 inch water/20 inches snow) = 1.3 inches of water. In Denver it is 25 inches of snow x (1 inch water/15 inches snow) = 1.7 inches of water. **So more water fell in Denver, even though there was less snow on the ground!**

Snow Density and Volume

This scientist is collecting cylindrical snow cores to study snow density from the wall of a snow pit. This pit was carefully dug into the Taku Glacier, in the Juneau Icefield of the Tongass National Forest, Alaska.

The density of snow tells scientists a lot about the history of the snow, and whether it is safe for skiers.

Density is defined as the amount of mass that an object has compared to the volume that it takes up. On average, a cubic meter of freshly-fallen snow has an average mass of about 50 kilograms. Snow that has been compacted by its own weight at a depth of 3 meters can have 200 kilograms in the same volume.

Density is defined as mass/volume. Freshly-fallen snow has a density of 50 $kg/meter^3$ = 50 kg/m^3, while the compressed snow described above has a higher density of 200 kg/m^3. Let's explore some other examples of estimating snow density!

Problem 1 – A scientist uses a cylindrical gauge to sample the snow in a trench wall. The cylinder has a radius of 5 centimeters and a length of 60 centimeters, and it has a mass of 50 grams. After filling the cylinder with snow, the cylinder is again weighed and now has a mass of 520 grams. What is the density of the snow that was sampled?

Problem 2 – Two scientists measure the snow density from two different mountain locations using two different snow gauges: A and B. Gauge A has a radius of 6.3 cm and a height of 40 cm, while Gauge B has a radius of 8.0 cm and a height of 40 cm. To the nearest cubic centimeter, what are the volumes of the two gauges? (use π = 3.141)

Problem 3 – If 500 grams is collected by Gauge A and 804 grams is collected by Gauge B, what are the snow densities to the nearest tenth. Are the scientists sampling different kinds of snow, or similar kinds of snow at the two locations?

Problem 1 – A scientist uses a cylindrical gauge to sample the snow in a trench wall. The cylinder has a radius of 5 centimeters and a length of 60 centimeters, and it has a mass of 50 grams. After filling the cylinder with snow, the cylinder is again weighed and now has a mass of 520 grams. What is the density of the snow that was sampled?

Answer: For a cylinder, the volume is given by the formula $V = \pi r^2 h$,where r is the radius and h is the height. The snow gauge volume is then

$V = (3.141)(5cm)^2 (60 \text{ cm}) = 4{,}711 \text{ cm}^3$.

When empty, the snow gauge had a mass of 50 grams and when full of snow it had a mass of 520 grams, so the actual mass of the snow was

M = 520 gm - 50 gm = 470 grams.

The density of the snow is then D = M/V = 470 gms/4711 cm^3 = **0.1 gm/cm^3**

Problem 2 – Two scientists measure the snow density from two different mountain locations using two different snow gauges: A and B. Gauge A has a radius of 6.3 cm and a height of 40 cm, while Gauge B has a radius of 8.0 cm and a height of 40 cm. To the nearest cubic centimeter, what are the volumes of the two gauges? (use $\pi = 3.141$)

Answer: The volume of a cylinder is given by $V = \pi R^2 h$, so

The volume of Gauge A is V = (3.141) x $(6.3 \text{ cm})^2$ x (40cm) = 4,987 cm^3.

The volume of Gauge B is V = (3.141) x $(8.0 \text{ cm})^2$ x (40cm) = 8,041 cm^3.

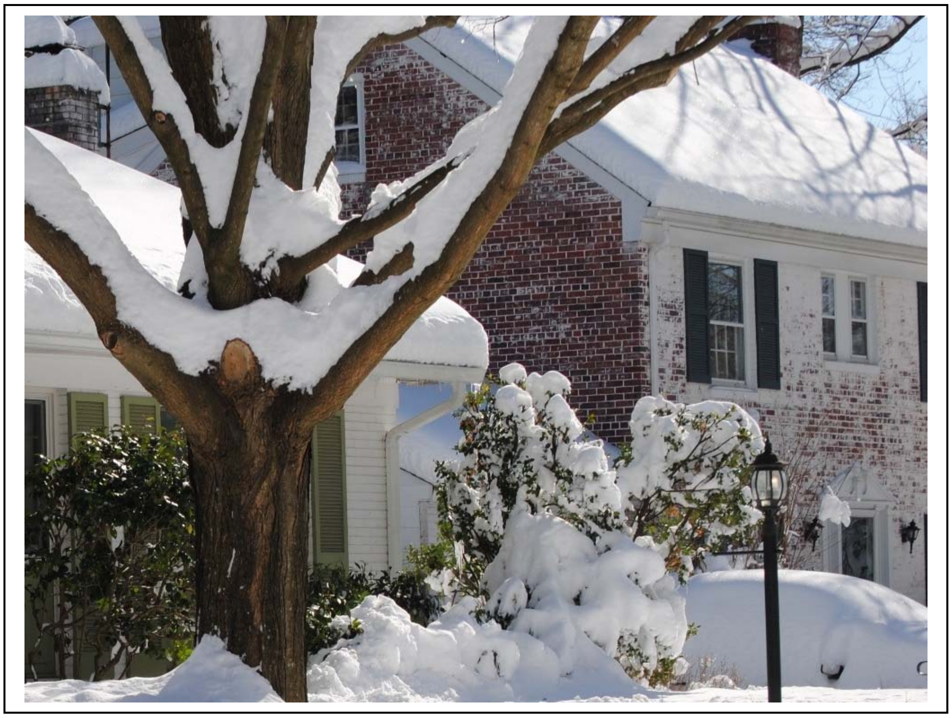
Problem 3 - If 500 grams is collected by Gauge A and 804 grams is collected by Gauge B, what are the snow densities to the nearest tenth, and are the scientists sampling different kinds of snow, or similar kinds of snow at the two locations?

Answer - The density measured by Gauge A is D = 500 gm/4987 cm^3 = **0.1 gm/cm^3**.

The density measured by Gauge B is D = 804 gm/8041 cm^3 = **0.1 gm/cm^3**

So the densities are the same and the kinds of snow are probably also the same at the two locations.

Snow Density, Mass and Roof Failure



During snowfalls, most children are excited by the accumulating snow, while many parents may worry if the weight of the snow will eventually cause their roofs to collapse. Although a small amount of snow weighs next to nothing, a few feet can weigh many pounds. How much snow is too much for the average roof on a house? Engineers estimate that 65 pounds per square foot (320 kg/m^2) is the average amount that a standard wood-framed roof can hold before it collapses. Dry snow has a density of about 50 kg/m^3 while wet snow has a density of 200 kg/m^3.

Problem 1 - Two houses are covered with a blanket of snow. House A has dry snow to a depth of 1 meter, and House B has a roof covered with wet snow to a depth of ½ meter. Which house is at greater risk of roof collapse?

Problem 2 - A snow storm of wet snow began at 6:00 am and continued steadily all day at a rate of 20 cm/hour. At what time will the snow accumulating on the roof reach the critical load for roof collapse?

Problem 1 - Two houses are covered with a blanket of snow. House A has dry snow to a depth of 1 meter, and House B has a roof covered with wet snow to a depth of ½ meter. Which house is at greater risk of roof collapse?

Answer: House A has 1 meter of dry snow covering every square meter of surface, so the mass of this snow on the roof is 50 kg/m^3 x 1 meter = 50 kg/m^2. House B has wet snow to a depth of 1/2 meter so the mass is 200 kg/m^3 x 1/2 meter = 100 kg/m^2. **House B is at greater risk even though it appears to have much less snow cover.**

Problem 2 - A snow storm of wet snow began at 6:00 am and continued steadily all day at a rate of 20 cm/hour. At what time will the snow accumulating on the roof reach the critical load for roof collapse?

Answer: The wet snow density is 200 kg/m^3. It is accumulating at a rate of 0.2 meters/hour. To reach 320 kg/m^2, which engineers say is the critical loading for roof collapse, you need to accumulate a thickness of 320/200 = 1.6 meters. At a rate of 0.2 meters/hour this will take about 1.6 meters x (1 hour/0.2 meters) = 8 hours, so by about **2:00 pm**, the roof might collapse.

In the figure to the left, the first column represents gas particles with little energy. A thermometer placed in contact with this group of particles would indicate a very low temperature. The column to the right represents particles with a high enough speed and energy to spread out inside the column. A thermometer placed in this group would show a high temperature.

When the state of matter changes its phase, the temperature and energy of matter also changes. At low temperature and energy we have a solid phase. At a medium temperature and energy we have a liquid phase, and at a high temperature and energy we have a gaseous phase.

A simple formula gives us the average speed, V, of water molecules in meters per second (m/s) for a given temperature in degrees Celsius, T:

$$V^2 = 1380(273+T)$$

Problem 1 – What is the speed of an average water molecule near A) the freezing point of water at 0° C? B) The boiling point of water at 100° C?

Problem 2 - The kinetic energy in Joules for all of the water molecules in a gallon of water, which has a mass of about M = 4.0 kilograms, and an average molecule speed of V in meters/sec, is given by the formula:

$$K.E. = \frac{1}{2}MV^2$$

To the nearest Joule, what is the kinetic energy of a 1 gallon of water at the temperatures given in Problem 1?

Problem 3 - If you heated the one gallon of water from 0^{o}C to 100^{o}C, how much 'thermal' energy would you have to add?

Problem 1 - What is the speed of an average water molecule near A) the freezing point of water at 0^o C? B) The boiling point of water at 100^o C?

Answer: From the formula:

A) $V = (1380(273+(+0))^{1/2} = (376740)^{1/2}$ = 614 meters/sec.

B) $V = (1380(273+(100))^{1/2} = (514740)^{1/2}$ = 717 meters/sec.

Problem 2 - The kinetic energy in Joules for all of the water molecules in a gallon of water, which has a mass of about M = 4 kilograms, and an average molecule speed of V in meters/sec, is given by the formula:

$$K.E. = \frac{1}{2} MV^2$$

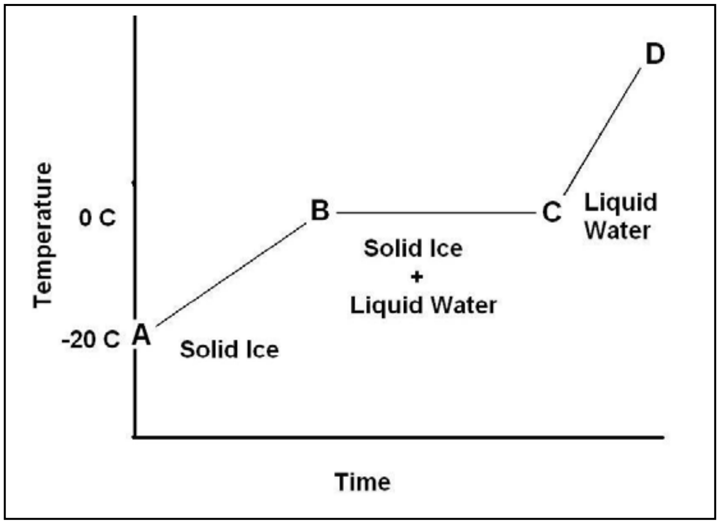
To the nearest Joule, what is the kinetic energy of a 1 gallon of water at the temperatures given in Problem 1?

Answer: A) For $+0^oC$, we calculated an average speed of 614 m/s, so the kinetic energy of the water is $KE = 1/2\ (4.0)(614)^2$ = **753,992 Joules**.
B) For 100^oC we have V = 717 m/s, so $KE = 1/2(4.0)(717)^2$ = **1,028,178 Joules.**

Problem 3 - If you heated the one gallon of water from 0^oC to 100^oC, how much 'thermal' energy would you have to add?

Answer: You have to add the difference in energy (1,028,178 – 753,992) = **274,186 Joules** to heat the gallon of water to its boiling point at 100^o C.

Note: A typical hotplate at a temperature of 400 C generates about 1000 Joules/second, so to heat the gallon of water to make it boil would take about 274186/4000 or about 4 minutes at this hotplate setting.



As energy is added to solid matter, it changes its state. The figure to the left shows what happens to water as it changes from solid ice (A to B), to a mixture of cold water and 'ice cubes' (B to C) and then finally to pure liquid water (C to D).

The energy required to change a kilogram of solid ice by one degree Celsius is called the **Specific Heat**. The energy needed to change a kilogram of solid ice at 0^oC into 100% liquid water at 0^oC is called the **Latent Heat of Fusion**.

Problem 1 - The Specific Heat of ice is 2090 Joules/kg C. How many Joules of energy do you need to raise the temperature of 1 kg of ice from -20^oC to 0^oC along the path from A to B on the graph?

Problem 2 - The Latent Heat of Fusion for water is 333 Joules/gram. How many Joules of energy do you need to melt all the ice into a pure liquid along the path from B to C on the graph?

Problem 3 - The Specific Heat of liquid water is 4180 Joules/kg C. How much energy is needed to raise the temperature of 100 grams of liquid water to $+60^oC$ for a nice warm cup of tea along the path from C to D in the graph?

Problem 1 - The Specific Heat of ice is 2090 Joules/kg C. How many Joules of energy do you need to raise the temperature of 1 kg of ice from $-20^{o}C$ to $0^{o}C$ along the path from A to B on the graph?

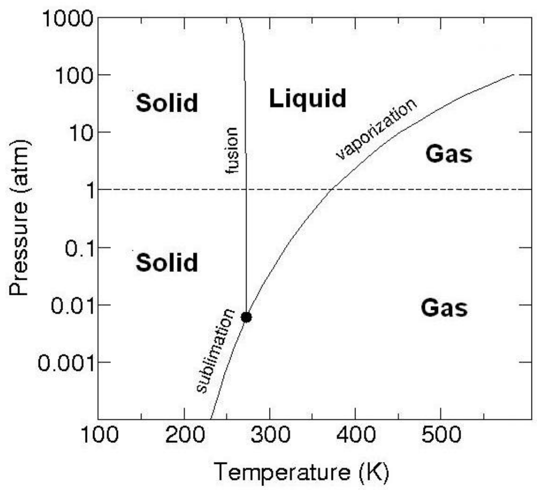
Answer: The temperature difference is $20^{o}C$, so for 1 kg of ice we need 2090 Joules/kgC x (1 kg) x ($20^{o}C$) = **41,840 Joules**.

Problem 2 - The Latent Heat of Fusion for water is 333 Joules/gram. How many Joules of energy do you need to melt all the ice into a pure liquid along the path from B to C on the graph?

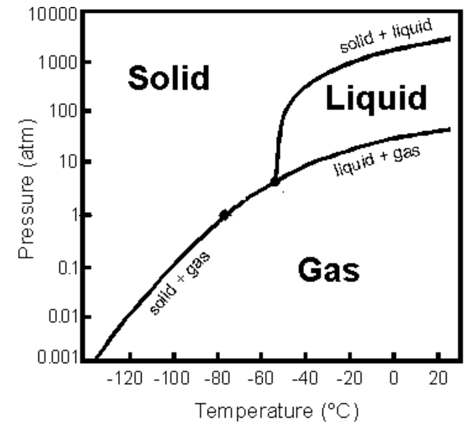
Answer: For 1 kilogram of ice ,which equals 1000 grams, we need 333 Joules/gram x 1000 grams = **333,000 Joules**.

Problem 3 - The Specific Heat of liquid water is 4180 Joules/kg C. How much energy is needed to raise the temperature of 100 grams of liquid water to +60oC for a nice warm cup of tea along the path from C to D in the graph?

Answer: 4180 Joules/kgC x 0.1 kg = **418 Joules**.



The H_2O Phase Diagram



The CO_2 Phase Diagram

These diagrams above are called **phase diagrams**. The one to the left shows all of the phases for matter for water as you change the temperature and pressure of the water in your sample. A pressure of 1.0 'atmospheres' is what we experience at sea level. This equals 14 pounds/inch^2 (or in metric units about 100 kiloPascals). As you move horizontally across the diagram towards increasing temperatures (measured in Kelvin units) at a constant pressure of 1.0 atm, the state of your water will change from solid ice, to liquid water at 273 Kelvin, to water vapor at 373 Kelvin.

Snow balls require that you create some liquid water by compressing the snow crystals so that they can glue together as the water refreezes. This will happen along the curve marked 'fusion' which is the boundary between the solid ice and liquid water phases.

The diagram to the right shows all of the phases for carbon dioxide as you change its pressure and temperature. For convenience we use the Celsius temperature scale. Note that 0^o Celsius = +273 on the Kelvin scale, and that a difference of 1^o C equals a change by 1 K on the Kelvin scale.

Problem 1 - We can make snow balls because the pressure (close to 1.0 atm) we apply with our hands at the ambient temperature (close to 273 K) is just enough to melt the ice into water and refreeze it to form a glue holding the snowflakes together. The temperature in Antarctica is typically 250 K. Can you make snowballs in Antarctica with normal hand pressure?

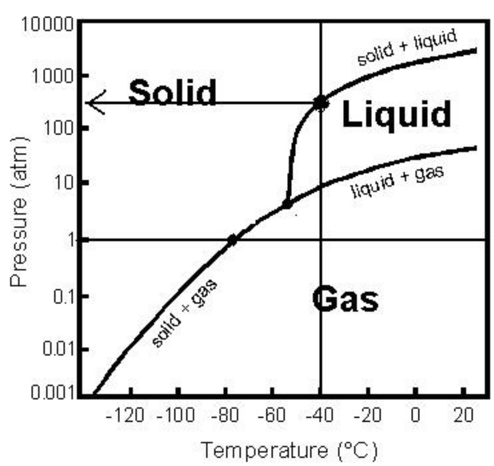
Problem 2 - On Mars, the majority of the ice is carbon dioxide ice. To make a carbon dioxide snowball, imagine applying 1 atm of hand pressure. The average temperature where the carbon dioxide snow falls is about -40^o Celsius in the daytime. Use the phase diagrams to explain why making a showball on Mars may be difficult or easy?

Problem 1 - We can make snow balls because the pressure (close to 1.0 atm) we apply at the ambient temperature (close to 273 K) is just enough to melt the ice into water and refreeze it to form a glue holding the snowflakes together. The temperature in Antarctica is typically 250 K. Can you make snowballs in Antarctica with normal hand pressure?

Answer - At normal winter temperatures near 273 K (0^oC) and 1 atm, the diagram shows that we are very, very close to the conditions needed to make solid ice turn to liquid water with a bit of extra pressure. The vertical line, which represents the ice to liquid transition crosses a pressure of 1 atm at a temperature of just +0.010 C! The diagram also shows that at 250 K (-17^oC) we are <u>far to the left</u> of the vertical line where solid ice can turn to liquid. At this temperature, we will need hand pressures higher that 1000 atm to make snowballs! So you can not make snowballs in Antarctica.

Problem 2 - On Mars, the majority of the ice is carbon dioxide ice. To make a carbon dioxide snowball, imagine applying 1 atm of hand pressure. The average temperature where the carbon dioxide snow falls is about -40^o Celsius in the daytime. Use the phase diagrams to explain why making a showball on Mars may be difficult or easy?

Answer: The main ingredient for a snowball on Mars would be carbon dioxide. The figure below shows the horizontal line representing a hand pressure of 1.0 atm and the vertical line representing the temperature of -40^o C on Mars where snowfall might occur.



The temperature is -40 C, so draw a vertical line on the CO_2 phase diagram until it intersects the solid+liquid line. This is where CO_2 can be in both the solid and liquid phases. You need the liquid CO_2 to supply the glue to hold the solid snowflakes together in the CO_2 snowball. But if you draw a horizontal line to the left, you will see that for you to get this liquid+solid phase at this temperature, you need a pressure of over 100 atmospheres. That about 1500 pounds per square inch of hand pressure, which will be impossible for you to achieve!

This diagram also shows that, on Earth, where the atmospheric pressure is 1 atmosphere, if you move from right to left along the 'pressure = 1 atm' line, a solid piece of CO_2 that you buy at the store will immediately start evaporating into the gas phase. Only if you could freeze this 'dry ice' below -80 C will it stop evaporating into a gas phase (called sublimation) and remain a stable solid.

Useful Internet Resources

Space Math @ NASA

http://spacemath.gsfc.nasa.gov

A Math Refresher

http://istp.gsfc.nasa.gov/stargaze/Smath.htm

Developers Guide to Excelets

http://academic.pgcc.edu/~ssinex/excelets/

Interactive Science Simulations

http://phet.colorado.edu/en/simulations/category/physics

My Physics Lab

http://www.myphysicslab.com/

NASA Press Releases

http://www.nasa.gov/news/index.html

NCTM - Principles and Standards for School Mathematics

http://www.nctm.org/standards/content.aspx?id=16909

Practical Uses of Math and Science (PUMAS)

http://pumas.gsfc.nasa.gov

Teach Space Science

http://www.teachspacescience.org

A note from the Author:

June, 2013

Hi again!

Here is another collection of 'fun' problems based on NASA space missions across the solar system and the universe! The 106 problems in this collection of SpaceMath@NASA problems from 2012-2013 span the gamut from fractions and percentages, to the challenges of algebraic manipulation.

This has certainly been an exciting year. We opened the year with the exciting landing of the Curiosity Rover on Mars and the steady stream of surface imagery from this car-sized super rover. A powerful gamma-ray burst lit up the sky and was captured by Fermi; the Planck mission resolved the cosmic background radiation to milky way-sized blobs of matter existing 700,000 years after the Big Bang. We also discovered a long-awaited planet orbiting the nearby star Alpha Centauri. And of course, as we entered sunspot maximum conditions in 2013, the sun popped off plenty of solar storms to keep us busy. And of course, then there was the spectacular 'Russian Meteor' of February that was captured on hundreds of dashboard cameras in Russia. Sadly over 1000 people were injured by flying glass as the shock wave shattered thousands of windows in the town of Chelyabinsk. This meteor was a near-twin to the famous 1909 Tunguska Meteor, and one shudders to think what would have happened had it entered over New York City an hour or two earlier!

This year also marked the beginning of a disturbing discussion about the future of STEM education funding across the federal government. Several years ago, a presidential Committee on STEM education (coSTEM) was set up to see what could be done to tighten-up all of the hundreds of STEM programs funded outside the Department of Education and the National Science Foundation. In April, over 2 months before the panel submitted its final recommendations, the Office of Management and Budget (OMB) moved ahead with its own ideas for consolidating STEM education in the Presidential FY14 budget. This budget ELIMINATED all of NASA's science mission education programs and gave this money to DoED and NSF. Gone would be all the classroom and teacher professional training support by NASA's science missions (Hubble, Curiosity, etc). There has now been a major protest filed by dozens of educational institutions, museums, and professional societies (NSTA, NCTM, AAAS, AAS etc) to stop this implementation. At the present time, we do not know what the future of NASA's science education programs will be, but many of its STEM teachers and scientists are making plans to look for other jobs, or retire early, as missions move ahead with losing their education funding for FY14 and beyond. As for SpaceMath@NASA, we are supported through education grants at NASA, and from individual missions that have stepped forward to support math enrichment. By Spring, 2014 as for the many NASA missions, we may well no longer exist to provide new resources to teachers and students. This will indeed be a very tense summer for all NASA STEM professionals as the FY14 budget moves through Congress and we learn more about our fates during the 2013-2014 school year and beyond.

Sincerely,
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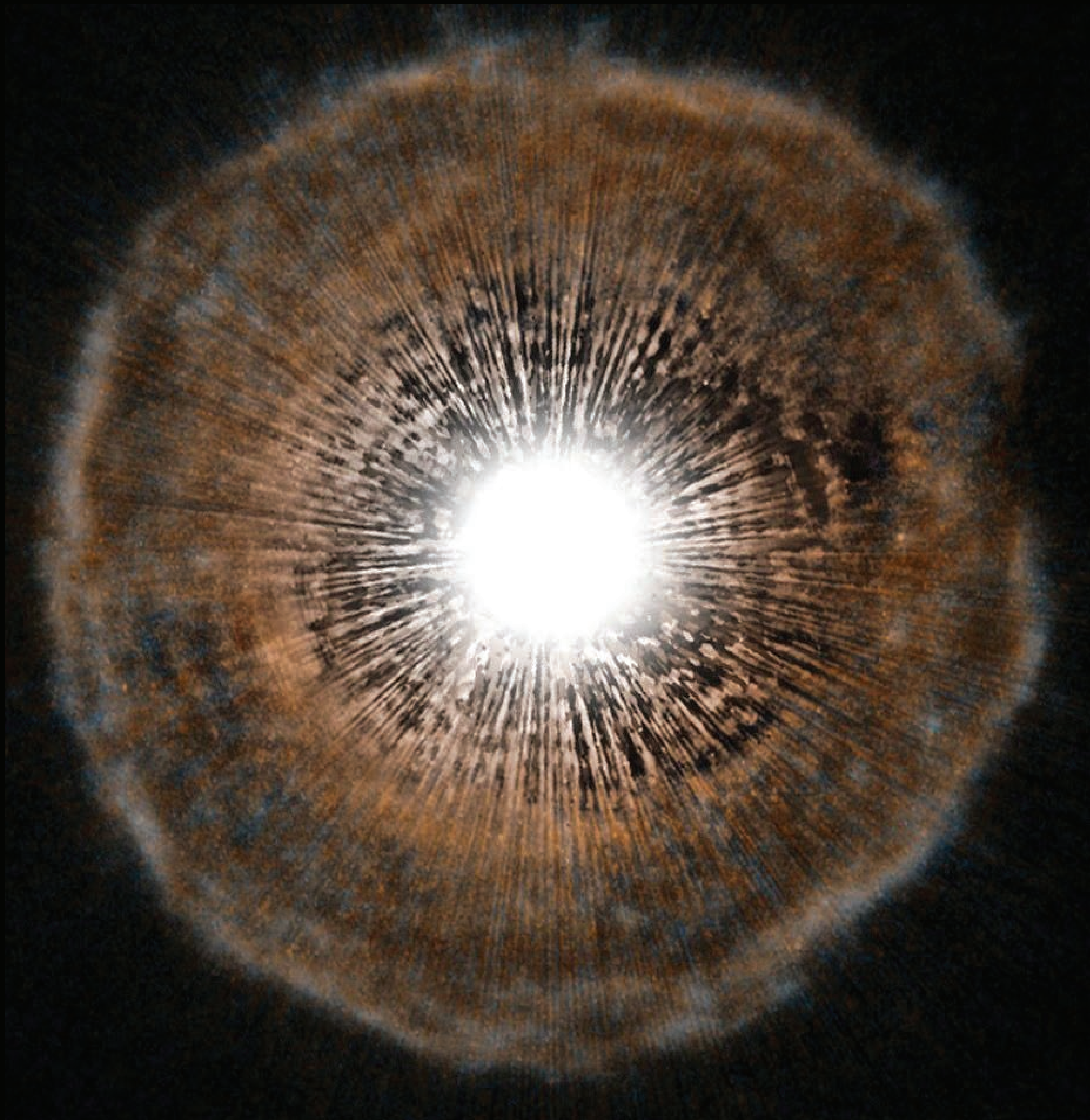

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