SLR pomocí inverzní matice

Pomocí inverzní matice řešte následující systémy lineárních rovnic:

(b)

$$\begin{array}{rcrcrcr} x & + & y & + & 2z & = & -1 \\ x & - & 2y & + & z & = & -5 \\ 3x & + & y & + & z & = & 3 \end{array}$$

A^{-1}:

$$\begin{array}{r} \\ -r_1 \\ -3r_1 \end{array}\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \sim \begin{array}{r} \\ \\ -2r_2+3r_3 \end{array}\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -3 & -1 & -1 & 1 & 0 \\ 0 & -2 & -5 & -3 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -3 & -1 & -1 & 1 & 0 \\ 0 & 0 & -13 & -7 & -2 & 3 \end{array}\right)\begin{array}{l} +13r_1+2r_3 \\ -13r_2+r_3 \\ \\ \end{array} \sim$$

$$\left(\begin{array}{ccc|ccc} 13 & 13 & 0 & -1 & -4 & 6 \\ 0 & 39 & 0 & 6 & -15 & 3 \\ 0 & 0 & -13 & -7 & -2 & 3 \end{array}\right)\begin{array}{l} -r_2+3r_1 \\ \\ \\ \end{array} \sim \left(\begin{array}{ccc|ccc} 39 & 0 & 0 & -9 & 3 & 15 \\ 0 & 39 & 0 & 6 & -15 & 3 \\ 0 & 0 & -13 & -7 & -2 & 3 \end{array}\right)\begin{array}{l} :39 \\ :39 \\ :(-13) \end{array}$$

$$A^{-1} = \frac{1}{13} \cdot \begin{pmatrix} -3 & 1 & 5 \\ 2 & -5 & 1 \\ 7 & 2 & -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \cdot \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{13} \cdot (3-5+15) \\ \frac{1}{13} \cdot (-2+25+3) \\ \frac{1}{13} \cdot (-7-10-9) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

$$K = \{(1, 2, -2)\}$$

Příklad 3.4.B17: Ve vektorovém prostoru V jsou zadány podprostory W_1, W_2. Určete dimenzi a bázi podprostorů $W_1 + W_2$, $W_1 \cap W_2$, je-li:

(a) $V = \mathbb{R}^3$, $W_1 = L(\vec{u_1}, \vec{u_2})$, $W_2 = L(\vec{v_1}, \vec{v_2}, \vec{v_3})$,
$\vec{u_1} = (1; 1; -3)$, $\vec{u_2} = (1; 2; 2)$,
$\vec{v_1} = (1; 1; -1)$, $\vec{v_2} = (1; 2; 1)$, $\vec{v_3} = (1; 3; 3)$;

$\dim W_1 = 2$

$$\begin{array}{r} \\ -r_1 \\ -r_1 \end{array}\begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 3 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ \cancel{0} & \cancel{2} & \cancel{4} \end{pmatrix} \to \dim W_2 = 2$$

$W_1 + W_2$

$$\begin{array}{r} \\ -r_1 \\ -r_1 \\ -r_1 \end{array}\begin{pmatrix} 1 & 1 & -3 \\ 1 & 2 & 2 \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{pmatrix}\begin{array}{l} \vec{u_1} \\ \vec{u_2} \\ \vec{v_1} \\ \vec{v_2} \end{array} \sim \begin{array}{r} \\ \\ \\ -r_2 \end{array}\begin{pmatrix} 1 & 1 & -3 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \\ 0 & 1 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -3 \\ 0 & 1 & 5 \\ \cancel{0} & \cancel{0} & \cancel{2} \\ 0 & 0 & -1 \end{pmatrix}$$

$\dim(W_1 + W_2) = 3$

$W_1 + W_2 = (\vec{u_1}, \vec{u_2}, \vec{v_2})$

$W_1 \cap W_2$: $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$

$3 = 2 + 2 - \dim(W_1 \cap W_2)$

$1 = \dim(W_1 \cap W_2)$

Hledáme $\vec{w}$, $\vec{w} \in W_1 \wedge \vec{w} \in W_2$

$$\boxed{\vec{w} = \alpha_1 \cdot \vec{u_1} + \alpha_2 \cdot \vec{u_2} = \beta_1 \cdot \vec{v_1} + \beta_2 \cdot \vec{v_2}}$$

$$\alpha_1 \cdot \vec{u_1} + \alpha_2 \cdot \vec{u_2} - \beta_1 \cdot \vec{v_1} - \beta_2 \cdot \vec{v_2} = \vec{o}$$

$$\begin{array}{r} \\ \\ -r_1 \\ +3r_1 \end{array}\begin{array}{c} \begin{array}{cccc} \alpha_1 & \alpha_2 & \beta_1 & \beta_2 \end{array} \\ \left(\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0 \\ 1 & 2 & -1 & -2 & 0 \\ -3 & 2 & 1 & -1 & 0 \end{array}\right) \end{array} \sim \begin{array}{r} \\ \\ -5r_2 \end{array}\left(\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 5 & -2 & -4 & 0 \end{array}\right) \sim$$

$$\sim \begin{array}{c} \begin{array}{cccc} \alpha_1 & \alpha_2 & \beta_1 & \beta_2 \end{array} \\ \left(\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -2 & 1 & 0 \end{array}\right) \end{array} \begin{array}{l} \alpha_1 + 2t - t - 2t = 0 \Rightarrow \alpha_1 = t \\ \alpha_2 - 2t = 0 \Rightarrow \alpha_2 = 2t \\ \beta_1 = t \Rightarrow -2t + \beta_2 = 0 \\ \qquad\qquad\qquad \beta_2 = 2t \end{array}$$

$$\vec{w} = t \cdot (1, 1, -3)^T + 2t \cdot (1, 2, 2)^T$$
$$= t \cdot (3, 5, 1)^T$$

$W_1 \cap W_2 = ((3, 5, 1))$

(b) $V = \mathbb{R}^4$, $W_1 = \langle\{\vec{u_1}, \vec{u_2}, \vec{u_3}\}\rangle$, $W_2 = \langle\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}\rangle$,
$\vec{u_1} = (1;2;0;2)$, $\vec{u_2} = (1;2;1;2)$, $\vec{u_3} = (3;1;3;1)$,
$\vec{v_1} = (1;1;1;1)$, $\vec{v_2} = (1;-1;1;-1)$, $\vec{v_3} = (1;3;1;3)$;

$$\dim W_1: \begin{array}{r} \\ -r_1 \\ -3r_1 \end{array}\begin{pmatrix} 1 & 2 & 0 & 2 \\ 1 & 2 & 1 & 2 \\ 3 & 1 & 3 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & -5 & 3 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & -5 & 3 & -5 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\dim W_1 = 3$$

$$\dim W_2: \begin{array}{r} \\ -r_1 \\ -r_1 \end{array}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 3 & 1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 \\ \cancel{0} & \cancel{2} & \cancel{0} & \cancel{2} \end{pmatrix} \quad \dim W_2 = 2$$

$$W_1 + W_2: \begin{array}{r} \\ -r_1 \\ -3r_1 \\ -r_1 \\ -r_1 \end{array}\begin{pmatrix} 1 & 2 & 0 & 2 \\ 1 & 2 & 1 & 2 \\ 3 & 1 & 3 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & -5 & 3 & -5 \\ 0 & -1 & 1 & -1 \\ 0 & -3 & 1 & -3 \end{pmatrix} \sim \begin{array}{r} \\ \\ -5r_2 \\ \\ -3r_2 \end{array}\begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & -5 & 3 & -5 \\ 0 & 0 & 1 & 0 \\ 0 & -3 & 1 & -3 \end{pmatrix}$$

$$\sim \begin{array}{r} \vec{u}_1 \\ \vec{v}_1 \\ \\ \vec{u}_2 \\ \\ \end{array}\begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & -1 & 1 & -1 \\ \cancel{0} & \cancel{0} & \cancel{2} & \cancel{0} \\ 0 & 0 & 1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{-2} & \cancel{0} \end{pmatrix} \rightarrow \dim(W_1 + W_2) = 3$$

$$W_1 + W_2 = (\vec{u}_1, \vec{u}_2, \vec{v}_1)$$

$$W_1 \cap W_2: \quad \dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$$

$$3 = 3 + 2 - \dim(W_1 \cap W_2)$$

$$2 = \dim(W_1 \cap W_2)$$

Opět: $\vec{w} \in W_1 \cap W_2 \Leftrightarrow \vec{w} \in W_1 \wedge \vec{w} \in W_2$

$$\vec{w} = \alpha_1 \cdot \vec{u}_1 + \alpha_2 \cdot \vec{u}_2 + \alpha_3 \cdot \vec{u}_3 = \beta_1 \cdot \vec{v}_1 + \beta_2 \cdot \vec{v}_2$$

$$\begin{array}{r} \\ -2r_1 \\ \\ -2r_1 \end{array}\left(\begin{array}{ccccc|c} \alpha_1 & \alpha_2 & \alpha_3 & \beta_1 & \beta_2 & \\ 1 & 1 & 3 & -1 & -1 & 0 \\ 2 & 2 & 1 & -1 & 1 & 0 \\ 0 & 1 & 3 & -1 & -1 & 0 \\ 2 & 2 & 1 & -1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 1 & 3 & -1 & -1 & 0 \\ 0 & 0 & -5 & 1 & 3 & 0 \\ 0 & 1 & 3 & -1 & -1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{-5} & \cancel{1} & \cancel{3} & \cancel{0} \end{array}\right) \sim \left(\begin{array}{ccccc|c} \alpha_1 & \alpha_2 & \alpha_3 & \beta_1 & \beta_2 & \\ 1 & 1 & 3 & -1 & -1 & 0 \\ 0 & 1 & 3 & -1 & -1 & 0 \\ 0 & 0 & -5 & 1 & 3 & 0 \end{array}\right)$$

$$\alpha_3 = r, \ \beta_2 = s$$

$$\alpha_2 + 3r - (5r - 3s) - s = 0 \qquad \beta_1 = 5r - 3s$$

$$\underline{\alpha_2 = 2r - 2s}$$

1. řádek:

$$\alpha_1 + (2r - 2s) + 3r - (5r - 3s) - s = 0$$

$$\alpha_1 = 0$$

$$\vec{w} = (2r - 2s) \cdot (1,2,1,2)^T + r \cdot (3,1,3,1)$$

$$= r \cdot (5,5,5,5) + s \cdot (-2,-4,-2,-4)$$

$$= 5r \cdot (1,1,1,1) - 2s\,(1,+2,1,+2)$$

$$W_1 \cap W_2 = ((1,1,1,1),(1,+2,1,+2))$$

$$= (\vec{v}_1, \vec{u}_2)$$