

SLR pomocí inverzní matice

Pomocí inverzní matice řešte následující systémy lineárních rovnic:

(b)

$$\begin{cases} x + y + 2z = -1 \\ x - 2y + z = -5 \\ 3x + y + z = 3 \end{cases}$$

A^{-1} :

$$\begin{aligned} & \begin{pmatrix} 1 & 1 & 2 & | & 1 & 0 & 0 \\ 1 & -2 & 1 & | & 0 & 1 & 0 \\ 3 & 1 & 1 & | & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & | & 1 & 0 & 0 \\ 0 & -3 & -1 & | & -1 & 1 & 0 \\ 0 & -2 & -5 & | & -3 & 0 & 1 \end{pmatrix} \xrightarrow{+13r_1+2r_3} \begin{pmatrix} 1 & 1 & 2 & | & 1 & 0 & 0 \\ 0 & -3 & -1 & | & -1 & 1 & 0 \\ 0 & 0 & -13 & | & -7 & -2 & 3 \end{pmatrix} \xrightarrow{-13r_2+r_3} \\ & \begin{pmatrix} 13 & 13 & 0 & | & -1 & -4 & 6 \\ 0 & 39 & 0 & | & 6 & -15 & 3 \\ 0 & 0 & -13 & | & -7 & -2 & 3 \end{pmatrix} \xrightarrow{-r_2+3r_1} \begin{pmatrix} 39 & 0 & 0 & | & -9 & 3 & 15 \\ 0 & 39 & 0 & | & 6 & -15 & 3 \\ 0 & 0 & -13 & | & -7 & -2 & 3 \end{pmatrix} \begin{matrix} :39 \\ :39 \\ :(-13) \end{matrix} \end{aligned}$$

$$A^{-1} = \frac{1}{13} \cdot \begin{pmatrix} -3 & 1 & 5 \\ 2 & -5 & 1 \\ 7 & 2 & -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \cdot \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{13} \cdot (3 - 5 + 15) \\ \frac{1}{13} \cdot (-2 + 25 + 3) \\ \frac{1}{13} \cdot (-7 - 10 - 9) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

$$K = \{(1, 2, -2)\}$$

Příklad 3.4.B17: Ve vektorovém prostoru V jsou dány podprostory

W_1, W_2 . Určete dimenzi a bázi podprostorů $W_1 + W_2, W_1 \cap W_2$, je-li:

(a) $V = \mathbb{R}^3, W_1 = L(\vec{u}_1, \vec{u}_2), W_2 = L(\vec{v}_1, \vec{v}_2, \vec{v}_3)$.

$\vec{u}_1 = (1; 1; -3), \vec{u}_2 = (1; 2; 2)$.

$\vec{v}_1 = (1; 1; -1), \vec{v}_2 = (1; 2; 1), \vec{v}_3 = (1; 3; 3)$.

$\dim W_1 = 2$

$$\dim W_2: \begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 3 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{pmatrix} \rightarrow \dim W_2 = 2$$

$$W_1 + W_2: \begin{pmatrix} 1 & 1 & -3 \\ 1 & 2 & 2 \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{pmatrix} \xrightarrow{\substack{r_2-r_1 \\ r_3-r_1 \\ r_4-r_1}} \begin{pmatrix} 1 & 1 & -3 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \\ 0 & 1 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -3 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$\dim(W_1 + W_2) = 3$

$$W_1 + W_2 = (\vec{u}_1, \vec{u}_2, \vec{v}_1)$$

$$\begin{aligned} W_1 \cap W_2: \dim(W_1 + W_2) &= \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2) \\ 3 &= 2 + 2 - \dim(W_1 \cap W_2) \\ 1 &= \dim(W_1 \cap W_2) \end{aligned}$$

Hledáme vektor \vec{w} , $\vec{w} \in W_1 \wedge \vec{w} \in W_2$

$$\vec{w} = \alpha_1 \cdot \vec{u}_1 + \alpha_2 \cdot \vec{u}_2 = \beta_1 \cdot \vec{v}_1 + \beta_2 \cdot \vec{v}_2$$

$$\begin{aligned} & \alpha_1 \cdot \vec{u}_1 + \alpha_2 \cdot \vec{u}_2 - \beta_1 \cdot \vec{v}_1 - \beta_2 \cdot \vec{v}_2 = \vec{0} \\ & \begin{pmatrix} 1 & 1 & -3 & | & 0 \\ 1 & 2 & 2 & | & 0 \\ 1 & 1 & -1 & | & 0 \end{pmatrix} \xrightarrow{\substack{r_2-r_1 \\ r_3-r_1}} \begin{pmatrix} 1 & 1 & -3 & | & 0 \\ 0 & 1 & 5 & | & 0 \\ 0 & 0 & 2 & | & 0 \end{pmatrix} \xrightarrow{r_2 \leftrightarrow r_3} \begin{pmatrix} 1 & 1 & -3 & | & 0 \\ 0 & 0 & 2 & | & 0 \\ 0 & 1 & 5 & | & 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \alpha_1 + 2\alpha_2 - \beta_1 - \beta_2 &= 0 & \alpha_2 - 2\alpha_2 &= 0 & \beta_1 &= t \\ \alpha_1 + 2\alpha_2 - \beta_1 - \beta_2 &= 0 & \alpha_2 - 2t &= 0 & -2t + \beta_2 &= 0 \\ \alpha_1 &= t & \alpha_2 &= 2t & \beta_2 &= 2t \end{aligned}$$

$$\begin{aligned} \vec{w} &= t \cdot (1, 1, -3)^T + 2t \cdot (1, 2, 2)^T \\ &= t \cdot (3, 5, 1) \end{aligned}$$

$$W_1 \cap W_2 = L((3, 5, 1))$$

(b) $V = \mathbb{R}^4$, $W_1 = \{(\vec{u}_1, \vec{u}_2, \vec{u}_3)\}$, $W_2 = \{(\vec{v}_1, \vec{v}_2, \vec{v}_3)\}$.
 $\vec{u}_1 = (1; 2; 0; 2)$, $\vec{u}_2 = (1; 2; 1; 2)$, $\vec{u}_3 = (3; 1; 3; 1)$,
 $\vec{v}_1 = (1; 1; 1; 1)$, $\vec{v}_2 = (1; -1; 1; -1)$, $\vec{v}_3 = (1; 3; 1; 3)$

$$\dim W_1: \begin{matrix} -r_1 & \begin{pmatrix} 1 & 2 & 0 & 2 \\ 1 & 2 & 1 & 2 \\ 3 & 1 & 3 & 1 \end{pmatrix} \\ -3r_1 & \sim \begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & -5 & 3 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & -5 & 3 & -5 \\ 0 & 0 & 1 & 0 \end{pmatrix} \rightarrow \dim W_1 = 3 \end{matrix}$$

$$\dim W_2: \begin{matrix} -r_1 & \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 3 & 1 & 3 \end{pmatrix} \\ -r_1 & \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 \\ 0 & 2 & 0 & 2 \end{pmatrix} \rightarrow \dim W_2 = 2 \end{matrix}$$

$$W_1 + W_2: \begin{matrix} -r_1 & \begin{pmatrix} 1 & 2 & 0 & 2 \\ 1 & 2 & 1 & 2 \\ 3 & 1 & 3 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix} \\ -r_1 & \sim \begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & -5 & 3 & -5 \\ 0 & -1 & 1 & -1 \\ 0 & -3 & 1 & -3 \end{pmatrix} \\ -r_2 & \sim \begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \end{pmatrix} \\ -r_2 & \sim \begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \end{pmatrix} \rightarrow \dim(W_1 + W_2) = 3 \\ & W_1 + W_2 = (\vec{u}_1, \vec{u}_2, \vec{u}_3) \end{matrix}$$

$$W_1 \cap W_2: \dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$$

$$3 = 3 + 2 - \dim(W_1 \cap W_2)$$

$$2 = \dim(W_1 \cap W_2)$$

Hier ähne Vektor $\vec{w} \in W_1 \cap W_2$
 $\vec{w} = \alpha_1 \vec{u}_1 + \alpha_2 \vec{u}_2 + \alpha_3 \vec{u}_3 = \beta_1 \vec{v}_1 + \beta_2 \vec{v}_2$
 $\alpha_1 \vec{u}_1 + \alpha_2 \vec{u}_2 + \alpha_3 \vec{u}_3 - \beta_1 \vec{v}_1 - \beta_2 \vec{v}_2 = \vec{0}$

$$\begin{matrix} \alpha_1 & \alpha_2 & \alpha_3 & \beta_1 & \beta_2 \\ -2r_1 & \begin{pmatrix} 1 & 3 & -1 & -1 & 0 \\ 2 & 2 & 1 & -1 & 1 \\ 0 & 1 & 3 & -1 & -1 \\ 2 & 2 & 1 & -1 & 1 \end{pmatrix} \\ -2r_1 & \sim \begin{pmatrix} 1 & 3 & -1 & -1 & 0 \\ 0 & 0 & 5 & 1 & 3 \\ 0 & 1 & 3 & -1 & -1 \\ 0 & -5 & 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & -1 & -1 & 0 \\ 0 & 1 & 3 & -1 & -1 \\ 0 & 0 & -5 & 1 & 3 \\ 0 & 0 & -5 & 1 & 3 \end{pmatrix} \end{matrix}$$

$$\alpha_1 + (2r - 2s) - (5r - 3s) - s = 0 \quad \alpha_2 + 3r - (5r - 3s) - s = 0 \quad \begin{matrix} \alpha_3 = r \\ \beta_1 = s \end{matrix}$$

$$\alpha_1 = 0 \quad \alpha_2 = 2r - 2s \quad \beta_2 = 5r - 3s$$

$$\vec{w} = (2r - 2s) \cdot (1, 2, 1, 2)^T + r \cdot (3, 1, 3, 1)^T - 5r \cdot (1, 1, 1, 1)^T + 3s \cdot (1, -1, 1, -1)^T$$

$$= 5r \cdot (1, 1, 1, 1)^T - 2s \cdot (1, 2, 1, 2)^T = 5r \cdot \vec{v}_1 - 2s \cdot \vec{u}_2$$

$$W_1 \cap W_2 = (\vec{v}_1, \vec{u}_2)$$