This manipulation is impossible only in cases where

$$\sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2} + \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2} = 0.$$

It is clear that this case takes place if and only if $v_k^{i_1} = v_k^{i_2} = w_k$, for any k in $1 \le k \le N_0$. This means that in this case the signs of differences $\hat{I}_{15}^{i_1} - \hat{I}_{15}^{i_2}$ and $\tilde{I}_{15}^{i_1} - \tilde{I}_{15}^{i_2}$ coincide. So we can assume that

$$\sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2} + \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2} > 0.$$

Then,

$$\hat{I}_{15}^{i_1} - \hat{I}_{15}^{i_2} =$$

$$\frac{\sum_{k=1}^{p} (w_k - v_k^{i_1})^2 + \sum_{k=p+1}^{N_0} (w_k - v_k^{i_1})^2 - \sum_{k=1}^{p} (w_k - v_k^{i_2})^2 - \sum_{k=p+1}^{N_0} (w_k - v_k^{i_2})^2}{\sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2} + \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2}}.$$

Similarly to the case of the first pair of differences, the following relationships can be obtained:

$$\sum_{k=p+1}^{N_0} (w_k - v_k^{i_1})^2 = \sum_{k=p+1}^{N_0} (w_k)^2$$

and

$$\sum_{k=p+1}^{N_0} (w_k - v_k^{i_2})^2 = \sum_{k=p+1}^{N_0} (w_k)^2.$$

Taking them into account, we will have

$$\hat{I}_{15}^{i_1} - \hat{I}_{15}^{i_2} = \frac{\sum_{k=1}^{p} (w_k - v_k^{i_1})^2 + \sum_{k=p+1}^{N_0} (w_k)^2 - \sum_{k=1}^{p} (w_k - v_k^{i_2})^2 - \sum_{k=p+1}^{N_0} (w_k)^2}{\sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2} + \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2}}$$

$$= \frac{\sum_{k=1}^{p} (w_k - v_k^{i_1})^2 - \sum_{k=1}^{p} (w_k - v_k^{i_2})^2}{\sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2} + \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2}}$$

$$= \left(\sqrt{\sum_{k=1}^{p} (w_k - v_k^{i_1})^2} - \sqrt{\sum_{k=1}^{p} (w_k - v_k^{i_2})^2}\right)$$

$$\times \frac{\left(\sqrt{\sum_{k=1}^{p} (w_k - v_k^{i_1})^2} + \sqrt{\sum_{k=1}^{p} (w_k - v_k^{i_2})^2}\right)}{\sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2} + \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2}}.$$

Now let us look at the second difference:

$$\tilde{I}_{15}^{i_1} - \tilde{I}_{15}^{i_2} = \sqrt{\sum_{k=1}^{p} (w_k - v_k^{i_1})^2} - \sqrt{\sum_{k=1}^{p} (w_k - v_k^{i_2})^2}.$$

From the previous relationships, it follows that the signs of differences $\hat{I}_{15}^{i_1} - \hat{I}_{15}^{i_2}$ and $\tilde{I}_{15}^{i_1} - \tilde{I}_{15}^{i_2}$ coincide. Therefore, complex search characteristic I_{15} also has the order preservation property. Thus, we demonstrated that characteristics I_{14} and I_{15} do indeed have the order preservation property as formulated in this section.

In conclusion, we will give an example that could be used in the future to illustrate a difference in the understanding of the functional efficiency of a document search implied in this section from the understanding of functional efficiency implied earlier. We say "could be used in the future" meaning that to make a decision regarding the correctness of using such examples for the purpose described will only be possible when we learn to distinguish in which cases complex search characteristics of type I_{14} allow for a pragmatically justified evaluation of the functional effectiveness of a search. Thus, let us assume that two different search methods were used for a search based on the same query in a search collection containing N_0 documents and these searches resulted in the following vectors:

1. $V^1 = (1; 1; 1; 0; 0; 0; 0; \ldots; 0)$
2. $V^2 = (1; 1; 1; 1; 1; 1; 0; \ldots; 0)$

Let us assume also that the analysis of the search collection documents by the user resulted in the following vector:

$$W = (1; 1; 1; 1; 0.3; 0.3; 0; \ldots; 0).$$

Next, let us determine values of complex search characteristic I_{14} based on the results of the described search:

$$I_{14}^1 = \sum_{i=1}^{N_0} |w_i - v_i^1| = 1.6;$$

$$I_{14}^2 = 1.4.$$