difference $I_4^{i_1} - I_4^{i_2}$ does not depend on the value of C, which means that complex search characteristic I_4 has the order preservation property.

Finally, in Case 4, which is similar to Case 2, the expression for difference $I_5^{i_1} - I_5^{i_2}$ does not contain C, as is evident from analysis of this expression. Therefore, complex search characteristic I_5, like the previously discussed CSC, has the order preservation property.

Next, let us show that the remainder of the previous complex search characteristics I_1 through I_{12}, namely, characteristics I_1, I_6, I_7, I_8, I_9, I_{10}, I_{11}, and I_{12}, do not have the order preservation property. First, let us show this for characteristics I_1, I_6, I_7, I_8, I_9, I_{10}, and I_{11}. Assume that we have two different search methods, each being used for the search on the same search request in a certain search collection. In addition, let us assume that in the case of the first search method an output is produced consisting of 20 documents, of which 8 documents are pertinent (i.e., $r^{i_1} = 8$ and $N^{i_1} = 20$), and, in the case of the second search method, the output consists of 12 documents of which 6 documents are pertinent (i.e., $r^{i_2} = 6$ and $N^{i_2} = 12$). Now, let us consider two possible situations.

1. The search collection contained 40 pertinent documents ($C = 40$).
Note that in this situation,

$$R^{i_1} = \frac{r^{i_1}}{C} = \frac{8}{40};\ P^{i_1} = \frac{r^{i_1}}{N^{i_1}} = \frac{8}{20};$$

$$R^{i_2} = \frac{r^{i_2}}{C} = \frac{6}{40};\ P^{i_2} = \frac{r^{i_2}}{N^{i_2}} = \frac{6}{12}.$$

Then,

$$I_1^{i_1} - I_1^{i_2} = R^{i_1} + P^{i_1} - R^{i_2} - P^{i_2}$$

$$= \frac{8}{40} + \frac{8}{20} - \frac{6}{40} - \frac{6}{12} = -\frac{1}{20} < 0;$$

$$I_6^{i_1} - I_6^{i_2} = \frac{r^{i_1}}{C + N^{i_1} - r^{i_1}} - \frac{r^{i_2}}{C + N^{i_2} - r^{i_2}}$$

$$= \frac{8}{40 + 20 - 8} - \frac{6}{40 + 12 - 6}$$

$$= \frac{8}{52} - \frac{6}{46} = \frac{368 - 312}{52 \cdot 46} > 0;$$

$$I_7^{i_1} - I_7^{i_2} = 2 - I_1^{i_1} - 2 + I_1^{i_2} = -(I_1^{i_1} - I_1^{i_2}) > 0;$$

$$I_8^{i_1} - I_8^{i_2} = \sqrt{(1 - R^{i_1})^2 + (1 - P^{i_1})^2} - \sqrt{(1 - R^{i_2})^2 + (1 - P^{i_2})^2}$$

$$= \sqrt{\left(1 - \frac{8}{40}\right)^2 + \left(1 - \frac{8}{20}\right)^2} - \sqrt{\left(1 - \frac{6}{40}\right)^2 + \left(1 - \frac{6}{12}\right)^2}$$

$$= \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} - \sqrt{\left(\frac{17}{20}\right)^2 + \left(\frac{10}{20}\right)^2} = 1 - \sqrt{\frac{389}{400}} > 0;$$

$$I_9^{i_1} - I_9^{i_2} = 1 - I_6^{i_1} - 1 + I_6^{i_2} = -(I_6^{i_1} - I_6^{i_2}) < 0;$$

$$I_{10}^{i_1} - I_{10}^{i_2} = 1 - \frac{1}{\frac{2}{R^{i_1}} + \frac{2}{P^{i_1}} - 3} - 1 + \frac{1}{\frac{2}{R^{i_2}} + \frac{2}{P^{i_2}} - 3}$$

$$= \frac{r^{i_2}}{2C + 2N^{i_2} - 3r^{i_2}} - \frac{r^{i_1}}{2C + 2N^{i_1} - 3r^{i_1}}$$

$$= \frac{6}{80 + 24 - 18} - \frac{8}{80 + 40 - 24} = \frac{6}{86} - \frac{8}{96}$$

$$= \frac{576 - 688}{86 \cdot 96} < 0;$$

$$I_{11}^{i_1} - I_{11}^{i_2} = 1 - \frac{2}{\frac{1}{R^{i_1}} + \frac{1}{P^{i_1}}} - 1 + \frac{2}{\frac{1}{R^{i_2}} + \frac{1}{P^{i_2}}}$$

$$= \frac{2r^{i_2}}{C + N^{i_2}} - \frac{2r^{i_1}}{C + N^{i_1}} = \frac{12}{40 + 12} - \frac{16}{40 + 20}$$

$$= \frac{12}{52} - \frac{16}{60} = \frac{720 - 832}{52 \cdot 60} < 0.$$

2. The search collection contained 10 pertinent documents ($C = 10$).
Note that in this situation,

$$R^{i_1} = \frac{r^{i_1}}{C} = \frac{8}{10};\ P^{i_1} = \frac{r^{i_1}}{N^{i_1}} = \frac{8}{20};$$

$$R^{i_2} = \frac{r^{i_2}}{C} = \frac{6}{10};\ P^{i_2} = \frac{r^{i_2}}{N^{i_2}} = \frac{6}{12}.$$

Then,

$$I_1^{i_1} - I_1^{i_2} = R^{i_1} + P^{i_1} - R^{i_2} - P^{i_2}$$

$$= \frac{8}{10} + \frac{8}{20} - \frac{6}{10} - \frac{6}{12} = \frac{1}{10} > 0;$$

$$I_6^{i_1} - I_6^{i_2} = \frac{r^{i_1}}{C + N^{i_1} - r^{i_1}} - \frac{r^{i_2}}{C + N^{i_2} - r^{i_2}}$$

$$= \frac{8}{10 + 20 - 8} - \frac{6}{10 + 12 - 6} = \frac{8}{22} - \frac{6}{16}$$

$$= \frac{128 - 132}{22 \cdot 16} < 0;$$