

“optimal” does not mean “good”; it just means that other possible choices are even worse.

It was noted earlier that the problem of any control is to actively exert influence on the object with the purpose of improving its behavior. But in order to compare various forms of behavior in controlled systems, and to identify the best of them, it is necessary to propose some standard appropriate for this purpose, a quantity characterizing the effectiveness of the control—an *effectiveness criterion J*.

Depending on the purpose of the system and its operating conditions, various quantities can serve as effectiveness criteria. Thus, for a system designed to control the motion of a train, the effectiveness criterion can be the time  $T$  of its motion from its departure station to its destination station; for an irrigation control system, it might be the increase  $\rho$  from realization of the yield taken from irrigated lands. Each control alternative has a corresponding specific value of effectiveness criterion  $J$ , and the problem of *optimal control* is to find and realize the control alternative for which the corresponding criterion takes the most favorable value. In the illustrated examples, the problem is finding a program for changing the traction of the locomotive for which the duration of the trip is minimal or finding an irrigation program for which maximal yield would be obtained from the realization of harvest. In this connection, it is necessary to take into account that controlling actions can change only within specific bounded limits; they must not go outside the limits of the region of possible (admissible) actions. In addition, other limitations can be placed on a system, for example, a limitation on the complexity of the control algorithm.

Thus, by *optimal control we mean a collection of controlling actions that provides the most favorable value of the effectiveness criterion (criteria)*.

In many systems, in particular in information system, problems associated with the optimization of processes often arise, such as the problem of making a system progress from some initial state  $X_0$  to an assigned state  $X_o$  with the help of a controlling action  $Y$ . In a state space, points  $a_i$  and  $a_0$  correspond to states  $X_i$  and  $X_0$ , and transition of the system from  $a_i$  to  $a_0$  is a trajectory connecting these points. Using controlling actions  $Y$ , it is possible to select many alternatives of control that will satisfy the requirement of a transition of the system from state  $a_i$  to  $a_0$ . Each of these control alternatives has a corresponding trajectory connecting  $a_i$  with  $a_0$ . However, in relation to the effectiveness criterion  $J$ , these trajectories are nonequivalent, for each of them corresponds to a specific value of  $J$  equal to  $J_1, J_2, \dots$ . The problem of finding an optimal control may in this case be treated as the problem of choosing from a set of possible trajectories connecting  $a_i$  with  $a_o$  that for which the effectiveness criterion would have the most favorable value.

Any control is carried out on the basis of information. However, properties of control systems essentially depend on the kind of sources of information that are used in the control structure for the formation of control signals. Sys-

tems exist in which the information  $Z$  obtained by the control structure does not include data on the state  $X$  of the controlled object. Then a program of a sequence of changes of control actions  $Y_0(t)$  or data on perturbing actions  $M(t)$  can enter into  $Z$ . In the latter case, to obtain a control signal  $S$  the controlling structure must contain data on what the value  $Y$  must be for each value  $M$ , which must be known in order to achieve the goal of control.

Much more common are systems in which information on values of controlling quantities is used for the formation of controlling actions. It is these systems that are the basic objects of consideration in cybernetics. In fact, these systems are called systems with feedback.

The originality and importance of the cybernetic approach to control are seen in the application of a general principle of control—the principle of feedback. *Feedback in a more general form is the mechanism of calculating the difference between the goal of a system's operation and its result. Wiener characterized feedback as “the property allowing [one] to regulate future behavior by the previous fulfillment of directives”* (Wiener, 1961). A generalized diagram of a system with feedback is depicted in Figure 1.8.

Feedback usually consists of a feedback line (FL) as well as a feedback mechanism, which is realized in the control structure. As seen in Figure 1.8, the input-control action  $Y$  on the object of control (OC), because of feedback through the control structure (CS), depends on its output quantity  $X$ . In other words, there is a flow of information from output to input.

Feedback is one of the most important concepts of the systems approach. It helps to explain many phenomena taking place in controlled systems of various natures. Feedback can be detected by investigating the processes taking place in living organisms, economic structures, and technical systems, to name a few. Feedback that increases the effect of input actions on an output quantity is called *positive*, and feedback decreasing this effect is called *negative*. Negative feedback enables equilibrium to be restored to a system when it has been broken down by an external action, and positive feedback causes a still greater deviation

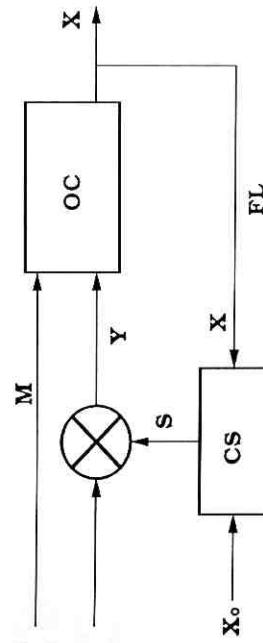


Figure 1.8  
A control system with feedback.