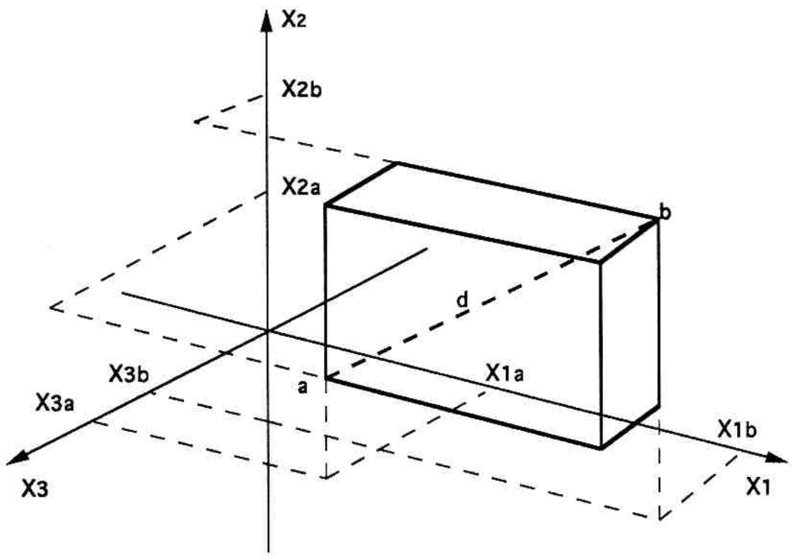

Figure 1.2
Distance between points in space.

state of the system. Nevertheless, very important inferences about properties of systems can be made from arguments using the representation of the state of a system in the form of points in a corresponding space even in cases when $n > 3$.

Although the concept of an n-dimensional space is abstract, its properties in many respects are derived from properties familiar to us from one-, two-, and three-dimensional spaces. In particular, one of the basic geometrical concepts—the distance between two points—can be introduced in four-dimensional space in the same way it is done in three-dimensional space. The distance d between points a and b in three-dimensional space is just the length of a diagonal of a parallelepiped (see Figure 1.2) whose vertices include a and b and whose edges are parallel to the coordinate axes.

It is known that the length d of the diagonal of a right parallelepiped with edges

$$X_1 = X_{1b} - X_{1a}, \qquad X_2 = X_{2b} - X_{2a}, \qquad X_3 = X_{3b} - X_{3a}$$

is found by the expression

$$d = \sqrt{X_1^2 + X_2^2 + X_3^2}\,. \tag{1}$$

Analogously, for an n-dimensional space, the distance between point a with coordinates $(X_{1a}, X_{2a}, \ldots, X_{na})$ and point b with coordinates $(X_{1b}, X_{2b}, \ldots, X_{nb})$ can be defined as the quantity

$$d = \sqrt{X_1^2 + X_2^2 + \ldots X_n^2}\,, \tag{2}$$

where $X_1 = X_{1b} - X_{1a}$, $X_2 = X_{2b} - X_{2a}$, . . . , $X_n = X_{nb} - X_{na}$. The quantities $X_1, \ldots X_n$ are represented by the edges of an n-dimensional right parallelepiped analogous to the three-dimensional parallelepiped illustrated in Figure 1.2.

The space in which each state of the system is represented by a specific point is called the *state space of the system.* The number of dimensions of the state space equals the number of independent properties (which often are called "parameters") that determine the state of the system. Each state of the system is characterized by a set of specific values of variables $X_1, X_2, \ldots, X_n$, which identify a point in the state space. This point is called the representative point (it "represents" the given state of the system), and variables $X_1, X_2, \ldots, X_n$ are called coordinates of the system.

In real systems, not all of the coordinates can be changed in unbounded limits (i.e., $-\infty < X < \infty$). Most coordinates can take only values lying in a bounded interval, that is, those satisfying the condition

$$a_i \le X_i \le b_i,$$

where a_i and b_i are boundaries of the interval of possible values of coordinate X_i. The region of the state space in which a representative point can be found is called the *region of admissible states.* In the following, when speaking about a state space, we will have in mind only its admissible region.

However, even within the limits of the region of admissible states, it is not always true that any point represents a possible state of the system. Only a *continuous state space* has this property, corresponding to a system whose coordinates can take any values (within the admissible limits). Some systems, however, are *discrete,* in which coordinates can take only a finite number of fixed values. The state space of such systems is discrete as well. In this case, a representative point can occupy only a finite number S of positions

$$S = s_1 \cdot s_2 \cdot \ldots \cdot s_n,$$

where s_i is the number of discrete states of the ith coordinate.

1.5 Input and Output Quantities

The motion of a system—the change of its state—can take place under the influence of external actions or as a result of processes occurring within the system itself.

An infinite set of various external actions, strictly speaking, exerts influence on each system, but not all of these actions are essential. Thus, it is obvious that the force of attraction of the moon does not exert an essential influence on the motion of an automobile with respect to the earth, although in principle such an influence does occur. From the set of all external actions, we choose