of differences $\hat{F}^{i_1} - \hat{F}^{i_2}$ and $\tilde{F}^{i_1} -_s \tilde{F}^{i_2}$ coincide, then complex search characteristic F has the order preservation property.

We will explain why this formulation of the order preservation property and the one used earlier are essentially similar. Let us assume that the search methods, mentioned in the new formulation of the order preservation property, are ordered, for instance, by the decreasing order of obtained values of $\tilde{F}$. If the complex search characteristic F has the stated property, then the order of the preceding search methods will be the same also in the case where these methods will be ordered in decreasing order of the obtained values of $\hat{F}$, which is clear from the discussed property and the similar reasoning in Section 10.8, "Order Preservation Property." In other words, if it is necessary to order applied search methods using the results of search and analysis in collection D in the sense of a certain CSC having the stated property, we can do this in the following way: we can order these methods using the results of search and analysis in subcollection h and use the obtained order as a desired result. On the other hand, a similar ordering of the same search methods, but in the sense of the CSC possessing the order preservation property according to the former formulation, can also be done using the results of search and analysis in subcollection h. This is what we meant when we stated that the latter and the former formulations of the property are essentially similar.

Now, we will show that complex search characteristics I_{14} and I_{15} have the property formulated in this section. For this purpose, assuming that the condition of the property is satisfied for each of the above CSCs, let us consider following pairs of differences for any two search methods (i_1-th and i_2-th), specified in the property condition, with the following pairs of differences:

1. $\hat{I}_{14}^{i_1} - \hat{I}_{14}^{i_2}$ and $\tilde{I}_{14}^{i_1} - \tilde{I}_{14}^{i_2}$;
2. $\hat{I}_{15}^{i_1} - \hat{I}_{15}^{i_2}$ and $\tilde{I}_{15}^{i_1} - \tilde{I}_{15}^{i_2}$.

Let us assume, for the purposes of convenience, that subcollection h is the first p documents in collection D (keeping in mind that the ordering of the collection does not affect the proof). Thus, the first difference of the first pair is

$$\begin{aligned}\hat{I}_{14}^{i_1} - \hat{I}_{14}^{i_2} &= \sum_{k=1}^{N_0} |w_k - v_k^{i_1}| - \sum_{k=1}^{N_0} |w_k - v_k^{i_2}| \\ &= \sum_{k=1}^{p} |w_k - v_k^{i_1}| + \sum_{k=p+1}^{N_0} |w_k - v_k^{i_1}| \\ &\qquad - \sum_{k=1}^{p} |w_k - v_k^{i_2}| - \sum_{k=p+1}^{N_0} |w_k - v_k^{i_2}|.\end{aligned}$$

Note that all $v_k^{i_1}$ and $v_k^{i_2}$, when $p + 1 \leq k \leq N_0$, are equal to zero, because they correspond to the documents not included in subcollection h. (Any document, corresponding to a nonzero value of $v_k^{i_1}$ or $v_k^{i_2}$ is included in subcollection h.) Therefore,

$$\sum_{k=p+1}^{N_0} |w_k - v_k^{i_1}| = \sum_{k=p+1}^{N_0} |w_k|$$

and

$$\sum_{k=p+1}^{N_0} |w_k - v_k^{i_2}| = \sum_{k=p+1}^{N_0} |w_k|.$$

Taking this into account, we have

$$\begin{aligned}\hat{I}_{14}^{i_1} - \hat{I}_{14}^{i_2} &= \sum_{k=1}^{p} |w_k - v_k^{i_1}| + \sum_{k=p+1}^{N_0} |w_k| - \sum_{k=1}^{p} |w_k - v_k^{i_2}| - \sum_{k=p+1}^{N_0} |w_k| \\ &= \sum_{k=1}^{p} |w_k - v_k^{i_1}| - \sum_{k=1}^{p} |w_k - v_k^{i_2}|.\end{aligned}$$

Now let us look at the second difference:

$$\tilde{I}_{14}^{i_1} - \tilde{I}_{14}^{i_2} = \sum_{k=1}^{p} |w_k - v_k^{i_1}| - \sum_{k=1}^{p} |w_k - v_k^{i_2}|.$$

It follows that signs of differences $\hat{I}_{14}^{i_1} - \hat{I}_{14}^{i_2}$ and $\tilde{I}_{14}^{i_1} - \tilde{I}_{14}^{i_2}$ coincide. Hence, complex search characteristic I_{14} has the order preservation property. Now let us consider the first difference of the second pair:

$$\begin{aligned}\hat{I}_{15}^{i_1} - \hat{I}_{15}^{i_2} &= \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2} - \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2} \\ &= \left(\sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2} - \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2}\right) \\ &\qquad \times \frac{\sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2} + \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2}}{\sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2} + \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2}} \\ &= \frac{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2 - \sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2}{\sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_1})^2} + \sqrt{\sum_{k=1}^{N_0} (w_k - v_k^{i_2})^2}}.\end{aligned}$$