

379
N81d
NO. 730
V. 1

DIALOGO DELLA MUSICA ANTICA ET DELLA MODERNA

OF VINCENZO GALILEI:

TRANSLATION AND COMMENTARY

DISSERTATION

Presented to the Graduate Council of the
North Texas State University in Partial
Fulfillment of the Requirements

For the Degree of

DOCTOR OF PHILOSOPHY

By

Robert H. Herman, M. M.

Denton, Texas

August, 1973

Herman, Robert H., Dialogo della musica antica et della moderna of Vincenzo Galilei: Translation and Commentary. Doctor of Philosophy (Music Theory), August, 1973, 929 pp., 43 tables, bibliography, 165 titles, index.

The purpose of this study is to provide a practical English translation of Vincenzo Galilei's significant treatise on ancient and modern music (1581). In spite of the important place this work holds in the history of music, it has never before been made available in its entirety in any language other than the original Italian.

The Dialogo della musica antica et della moderna was, in effect, an expression of monodic tendencies in Italian music of the sixteenth century. While the prevailing vocal compositional style was contrapuntal, Galilei advocated the development of homophonic pieces, which could produce in their hearers whatever affections or emotions the composer desired to convey. The emphasis was on simplicity of melody, with only a few chords for accompaniment. This style, with its sparseness of means, was influenced by the music of the ancient Greeks, whose philosophies and art forms were in credit at the time, having been revived by such Renaissance humanists as Lorenzo di Medici, Marsilio Ficino, and Angelo Poliziano. Galilei, influenced by the erudite Girolamo Mei, who was

a classical philologist, drew away from his former mentor, Gioseffo Zarlino, a firm believer in traditional counterpoint. He included in his Dialogo many attacks upon Zarlino's teachings, engendering a long and bitter controversy which extended until Galilei's death in 1591.

The ideas promulgated by Galilei became the subjects of discussion by members of the Florentine Camerata of Count Giovanni Bardi. Experiments revealed the dramatic possibilities of the new monodies, which in turn led to the establishment of a new art form, the opera.

Besides the 1581 edition, the Dialogo has been reproduced in two twentieth century facsimile editions (Rome, Reale accademia d'Italia, 1934, and New York, Broude Bros., 1967). These versions, along with a number of microfilm copies, were used in preparing this dissertation. All appeared to be from the same impression.

The original Dialogo is not divided into chapters. Division headings have been supplied in the translation according to the logical separation of Galilei's material. The first section of the translation concerns itself with acoustical speculations, while the second section is a critical discussion of ancient and modern musical art. The third section is a critical discussion of polyphony and the fourth section is a history of musical instruments

and notation. The fifth and final section concerns itself with a critical discussion of the instrumental music of Galilei's time. These five headings were considered by Galilei to be the ones most pertinent to a modern treatise on music.

In the present translation every effort has been made to provide a lucid rendition of Galilei's work. The long, tedious sentences which appear in the original version have been divided, thus removing the same obstacle to English readers as would be presented to modern Italian readers of the 1581 edition. It has also been necessary, on occasion, to alter the grammatical structure of particularly idiomatic passages as well as to remove a number of confusing particles and conjunctions. In no instance, however, has the original meaning been distorted.

Because of increased demands upon the time of musical scholars, there is an even greater case for translated versions of significant treatises. It would, moreover, permit the free flow of important musical ideas, unhampered by the barrier of a foreign language, thus helping to preserve for an increasing number of readers the extensive heritage of our musical past.

© 1973

ROBERT HENRY HERMAN

ALL RIGHTS RESERVED

PREFACE

In the course of the ages, many works on music have appeared, some of which have flourished briefly before passing into obscurity. Others, possessing more enduring qualities, have been granted a more permanent place in the panorama of history. There are very few treatises, standing at the foundation of an era, which form the cultural concepts that cleave to the core of musical society. Vincenzo Galilei's Dialogo della musica antica et moderna was such a work, for at its inception it fomented a fierce controversy between the proponents of ancient and modern music. Before the strife faded, its tremors were felt in many lands by academicians of diverse disciplines.

The Dialogo is significant for several reasons. It is, initially, a valuable study, both of the Renaissance concept of ancient Greek music and the music of the Renaissance. Galilei's work is rich in source materials, for he cites nearly all of the ancient authorities on music, including many incidental references to that subject in the classical authors. The Dialogo also contains

a wealth of information on many facets of musical art including tuning theories, comparisons of Greek music with that of Galilei's time, sixteenth-century polyphony, the history of musical instruments and notation, and Renaissance instrumental music.

There are, in addition, authentic examples of ancient Greek music, the Hymns to the Muse, to the Sun, and to Nemesis, which appear in print in the Dialogo for the first time, as well as portions of the tables of Alypius, which contain the early Greek notation symbols.

Despite the importance of this treatise, no full English translation of it has been made until now. There are excerpts in collections by Oliver Strunk¹ and Bruno Nettl,² but neither of these is sufficient to realize the entire potential of the Dialogo as a valuable reference work. The purpose of this dissertation, therefore, has been to translate the Dialogo into English, transcribe its musical examples into modern notation, and provide a commentary upon its salient features.

¹Oliver Strunk, Source Readings in Music History (New York, 1950), pp. 302-322.

²Bruno Nettl, The Book of Musical Documents (New York, 1948), pp. 36-37; 48-49.

The Dialogo takes the form of a long rambling discourse, 149 pages in all. It is equipped with marginal glosses known as "postilli" which are set off from the main body of the text. Their function is to pinpoint the location of a name or subject in the text, to clarify or amplify a statement, or to serve as a repository for bibliographical information. Since there are no chapter headings or divisions in the Dialogo, the "postilli" more or less act in their place. To eliminate this inconvenience, chapter divisions and headings have been supplied in the English translation. Numbers and captions have also been added by the translator to all musical examples and diagrams to facilitate reference to them. In order to expedite comparisons with the 1581 edition of the Dialogo, the original page numbers are given in brackets in the margin of the translation. The traditional catchword, like the musical "custos", stands guard at the foot of each column except where the text artistically dwindles down to a point, heralding the approach of a large illustration on the succeeding page.

Except for the use of indirect discourse, occasional archaic or interchangeable spellings ("i" for "y", "h" either retained or dropped, and so forth), and a few obsolete verb forms, the Italian of the

Dialogo is essentially the same as that of today, and, as such, poses no monumental problems to the translator. Galilei generally manages to say what must be said, but his style is rather verbose, full of redundancies and tautologies. Only occasionally does it border upon the elegant, being more often commonplace and sometimes decidedly awkward. He has a pronounced tendency for complexity of sentence structure and almost a penchant for parenthetical expressions. Sir John Hawkins describes his writing as "clear and nervous but negligent," adding that "nice judges say that it is in some instances ungrammatical."³ In order to provide a more lucid English translation, many of the lengthy sentences have been divided into shorter ones, and many of the unwieldy particles and conjunctions have been eliminated. The references to "questa" and "quella" (this and that) and "cosa" (thing), proper enough in Italian, but vague and confusing in English, have been replaced, where unclear, by the words they originally modified. While it has been necessary to deviate in some instances from the formal grammatical structure of the Dialogo, in no case have Galilei's statements been materially altered.

³Sir John Hawkins, A General History of the Science and Practice of Music, 2 vols., facsimile ed. (New York, 1965), I, 404.

There are, in addition, frequent technical errors or inconsistencies. The marginal glosses were particularly prone to printing problems. In several instances, one digit of a page or chapter number is missing. Similarly, it is not always clear who is referred to by the citation in a marginal gloss due to the omission of his name. If there are several successive glosses, it is also difficult to determine the object of each one. There is an instance (Dialogo, p. 124) in which the final letters of several lines were cut off in printing. The same error was apparent in comparing the 1934 and 1966 facsimile editions. A number of Galilei's musical examples came without sharps and, in one case, even without notes (Dialogo, pp. 37-38).

Galilei's recalcitrant Venetian printers are probably to blame for some of the errors in the Dialogo. They held his manuscript, ostensibly at Zarlino's order, forcing him to resort to a Florentine firm where a mere two-thirds of the original was published under very rushed conditions. It is not known what was done with the other third; presumably Galilei incorporated it later in another work.

The Dialogo contains many comparisons of Greek and Renaissance music, which necessitates the maintenance of several divergent nomenclatures. For reasons of clarity

and in order to maintain them in the proper historical perspective, a number of technical terms from both the Greek and Renaissance eras have been retained in lieu of replacement with modern equivalents (e.g. proslambanomenos, diesis X , f fa ut, major hexachord, etc.).

The commentary to this translation takes the form of footnotes, which serve not only as a source of bibliographical information, but also as an amplification and explanation of the text. The authors mentioned by Galilei have been cited whenever possible, in both their original form and in modern translation. The translations of quoted materials, unless otherwise indicated, are by this author. These often take the form of paraphrases, rather than literal renditions, in order to avoid unnecessary repetition of material. In quoting the translations of other authors, their spellings and punctuations have been preserved.

TABLE OF CONTENTS

	Page
PREFACE	iv
LIST OF DIAGRAMS.	xi
LIST OF TABLES.xiii
LIST OF MUSICAL EXAMPLES.	xiv
Chapter	
I. INTRODUCTION.	1
Vincenzo Galilei's Life and Works	
The Ancient-Modern Controversy	
Background for the <u>Dialogo</u>	
TRANSLATION AND COMMENTARY	
DEDICATION.	29
II. THE TUNING CONTROVERSY: DITONIAION VS SYNTONON	35
III. A CRITICAL DISCUSSION OF ANCIENT AND MODERN MUSICAL ART.	307
IV. A CRITICAL DISCUSSION OF POLYPHONY.	467
V. A HISTORY OF MUSICAL INSTRUMENTS AND NOTATION.	553
VI. A CRITICAL DISCUSSION OF CONTEMPORARY INSTRUMENTAL MUSIC.	849
INDEX	930
BIBLIOGRAPHY.	961

LIST OF DIAGRAMS

Diagram	Page
I. Intervals of the Diatonic Syntonon Applied to the Monochord.	55
II. Interval Ratios According to the Superparticular Genus	96
III. Example of the Epigoneion, Instrument of Forty Strings, Discovered by Epigon of Ambraciota.	259
IV. Example of the Simikion, Instrument of Thirty-five Strings.	260
V. Demonstration of the Thirteen Modes, According to the Opinion of Aristoxenus	310
VI. Demonstration of the Thirteen Modes, According to the Opinion of Aristoxenus, with Two Added in the High Part by his Followers, Which in All Make the Number of Fifteen	334
VII. Demonstration of the Eight Modes, According to the Opinion of Boethius.	336
VIII. Demonstration of the Eight Modes, According to the Opinion of Ptolemy	373
IX. A Spherical Demonstration of the Modes of Ptolemy.	384
X. A Mixed Division of the Eight Modes, According to Gaffurio	423

XI.	Demonstration of the Eight Modes, According to the Opinion of Boethius, to the Strings of Which Are Applied the Charac- ters According to the Usage of the Ancient Greeks	582
XII.	The Lyre of Mercury, Tuned According to Various Opinions	689
XIII.	Heptachord Synemmenon of Terpander.	693
XIV.	Octochord Diezeugmenon of Lycaon.	706
XV.	Enneachord of Prophastus Periotos	710
XVI.	Endecachord of Timotheus the Milesian	717
XVII.	Decachord	723
XVIII.	Decatescarachord of an Unknown Author	726
XIX.	Greater Perfect System, Disjunct.	728
XX.	A Composite View of All Strings Added to the Lyre, with Their Inventors	732
XXI.	Greater Perfect System, Conjunct.	735
XXII.	The Conjunction of Tetrachords According to Ptolemy.	740
XXIII.	The Lyre of Mercury Described by Homer.	769
XXIV.	Form of the Ancient Lyre.	789
XXV.	Form of the Ancient Lyre Described by Hyginus in Book Three of the <u>Constellations</u>	791
XXVI.	The Plectrum.	795
XXVII.	The Proportions of Various Sounding Bodies.	825
XXVIII.	The Tuning of the Irish Harp.	891

LIST OF TABLES

Table	Page
I. Excerpts from the Books of Alypius.	564
II. Description of the Hypaton Tetrachord	664

LIST OF MUSICAL EXAMPLES

Example	Page
1. Location of the Tones and Semitones in the Diatonic Syntonon of Ptolemy.	70
2. A Comparison of the Conjunct and Disjunct Tetrachords of the Diatonic Syntonon.	186
3. Divisions of the Major Thirteenth and Minor Tenth	206
4. The Minor Hexachord and Ditone of the Distribution of Dydimus	234
5. An Uncommon System of Notation.	242
6. <u>Clangat Hodie</u>	248
7. Order of the Consonances According to the Greeks.	342
8. Order of the Consonances According to the Romans.	344
9. Dissonant Intervals and Their Resolutions	395
10. Description of the Eight Ecclesiastical Modes	410
11. Description of the Twelve Modes	451
12. <u>Sit Nomen Domini</u>	584
13. Greek Hymns to the Muse, to the Sun, and to Nemesis.	589

CHAPTER I

INTRODUCTION

Vincenzo Galilei's Life and Works

The Dialogo della musica antica et moderna, whose first edition bears the date of 1581, is the largest and most important of the theoretical works of Vincenzo Galilei. Relatively little is known concerning the life of this noted theorist-musician. Fortunately, however, his output is more familiar, since many of the fruits of his genius have endured, perhaps even the greater part.

According to Fabio Fano,¹ Vincenzo Galilei was born of an impoverished noble Florentine family around 1520 at S. Maria Monte in the Arno valley near Florence. His grandfather had come there to live at the end of 1472, since he was no longer financially able to maintain a home in Florence. Vincenzo's parents were Michelangelo Galilei and Maddalena di Carlo di Bergo. He had an older sister named Lucrezia.

¹Fabio Fano in the preface to the facsimile edition of Vincenzo Galilei, Dialogo della musica antica et moderna (Rome, 1934).

Galilei went to Florence, probably around 1540, in order to study music.² His lute playing brought him to the attention of patrons of music like Bernardetto di Medici and Giovanni Bardi di Conti di Vernio. Having been encouraged by Bardi and the members of his Camerata, who supported him with his lute playing, he undertook the study of music theory. Aided by Bardi,³ he went to Venice in order to study with Zarlino. The precise time of these studies is not accurately known, but we know from Zarlino's own statements⁴ that he became the successor to Cypriano de Rore, choirmaster of San Marco in Venice, shortly before 1565, having transferred to that city in 1541 to study music with Adrian Willaert. According to Fano,⁵ Galilei's father died between 1540 and 1542, and his mother also after 1544, leaving him to fend for himself. After 1550, he moved to Pisa, where he

²Claude V. Palisca, "Galilei," Die Musik in Geschichte und Gegenwart, Vol. IV (Kassel and Basel, 1955), p. 1265.

³The dedication to the Dialogo contains Galilei's acknowledgement of Bardi's support in those studies to which he devoted himself since his youth. See p. 30 below.

⁴Palisca, MGG, IV, 1266; XIV, 1017-1022.

⁵Fabio Fano, Vincenzo Galilei, La Camerata Fiorentina, espressione de nuove idealità musicali, Vol. IV of Istituzioni e monumenti dell' arte musicale italiana (Milan, 1939), p. xxiv.

taught lute to distinguished merchants and foreign students at the university.⁶ In 1562, he married Giulia del fu Cosimo degli Ammannati, who bore him six, perhaps seven, children. The first-born was the great Galileo, born in 1564.⁷

Very little is known of Galilei's creative and speculative musical efforts prior to 1574.⁸ Of his works composed during those years, only the following are extant:

(1) the first book of the Intavolature di Liuto (1563)

(2) the dialogue, Fronimo (1568), a work on the art of intabulation for the lute⁹

(3) the Primo libro di Madrigali a 4 e a 5 voci (1574).¹⁰

In 1568, Galilei went to Venice, on account of the

⁶Palisca, MGG, IV, 1266.

⁷See p. 1, n. 1.

⁸Fano, op. cit.

⁹The second edition of this Fronimo is probably what Galilei means by his "other work" which, in his dedication, he promises will appear in addition to a Latin translation of the Dialogo. See p. 33 below.

¹⁰Only the tenor part remains. Fano, op. cit., p. xxv.

printing of the first edition of Fronimo.¹¹ In 1572, he moved to Florence, leaving his family under the protection of Muzio Tedaldi, the husband of his sister-in-law, until they rejoined him in 1574. Beginning in 1572, he began his correspondence with Girolamo Mei,¹² a philologist interested in Greek music. "It was Mei who revealed to him the importance of the principle of monody."¹³ Galilei made two journeys to Rome to see Mei, probably between 1572 and 1577. During this time, he was working on a didactic musical treatise, the Compendio nella theorica della musica, which was probably intended as a handbook for his pupils and the members of the Camerata.¹⁴ It consisted, essentially, of a shortened adaptation of Zarlino's Le Istitutioni harmoniche. Galilei discarded this work, however, when he determined that Zarlino's teachings were not in conformity with Mei's views on Greek music. Mei had formulated his opinions on that subject on the basis of ancient manuscripts he had studied in the

¹¹Palisca, MGG, IV, 1266.

¹²See Claude V. Palisca, Girolamo Mei: Letters On Ancient And Modern Music To Vincenzo Galilei And Giovanni Bardi (Rome, 1960), for reprints of parts of this correspondence and an exhaustive treatment of Mei's influence on Galilei.

¹³Ibid., p. 1.

¹⁴Palisca, MGG, IV, 1266.

Vatican library. The Dialogo della musica antica et moderna (1581) was the result of Galilei's discussions with Mei. Prior to the publication of the Dialogo, forces not in favor of Galilei's ideas were at work behind the scenes. Henriette Martin¹⁵ relates, according to Arteaga,¹⁶ that a veritable cabal was conducted against the author, which attempted to suppress the edition, having begun by taking away the original manuscript. Zarlino, in Arteaga's opinion, was the instrument of these intrigues. The victim himself alluded to this situation in the dedication of his Dialogo,¹⁷ complaining of the Venetian printers who held his manuscript to please some envious person. Galilei took his revenge in 1583 in a new version of Fronimo,¹⁸ where he engaged in a violent polemic with Zarlino on the subject of the Dimostrazioni harmoniche.¹⁹

¹⁵Henriette Martin, "La 'Camerata' du Comte Bardi et la musique florentine du XVI^e siècle," Revue de musicologie, XIII (1932), p. 65.

¹⁶Stefano Arteaga, Rivoluzioni del Teatro musicale italiano (Bologna, 1793).

¹⁷See p. 33 below.

¹⁸Vincenzo Galilei, Fronimo, dialogo sopra l'arte del bene intavolare et rettamente suonare la musica (Venice, 1583).

¹⁹Gioseffo Zarlino, Dimostrazioni harmoniche (Venice, 1571).

Zarlino later replied to Galilei's affronts with the Sopplimenti musicali.²⁰

In 1582, Galilei employed the principles he had postulated in the Dialogo in some original compositions, the first examples in the new monodic style.²¹ They were entitled as follows:

(1) The Lamentations of Jeremiah

(2) The Responses for Holy Week

(3) and the Lament of Ugolino from Dante's Inferno.

Galilei himself sang the Lament of Ugolino, accompanied by viols, at the home of Bardi. None of the pieces, unfortunately, are extant. It is, however, possible to gain certain insights into their probable structure from other "pseudo-monodies" arranged by the same composer. Palisca²² relates that Galilei left manuscript arrangements of favorite madrigals, villanelle, and similar pieces for solo voice and lute in a copy of the Fronimo printed in 1568. These dated from about 1570, the time of Galilei's first interest in monody.

²⁰Gioseffo Zarlino, Sopplimenti musicali (Venice, 1588).

²¹Palisca, MGG, IV, 1267.

²²Claude V. Palisca, "Vincenzo Galilei and Some Links between Pseudo-monody and Monody," Musical Quarterly, XLVI (1960), p. 365.

There are thirteen melodies for bass or baritone--this was apparently Galilei's own voice range--twelve of which are provided with lute reductions of the four and five-voiced pieces from which they were taken. Galilei evidently was accustomed to entertaining the Camerata and other noble groups by singing the bass part of well-known madrigals and part-songs while accompanying himself with an intabulation of the original composition. This would seem to establish the fact that Galilei was already attuned to the principle of monody, only awaiting the catalyst which later appeared in the form of Mei's vision of reviving ancient Greek music.

In 1584, Galilei published a contrapuntal duo,²³ a copy of which was unearthed in a private Florentine library by Alfred Einstein.²⁴ "Galilei here wavers between melodic expressiveness and scholarly treatment once more Here again we have that 'contradictoriness' in Galilei's historical position."²⁵

²³Vincenzo Galilei, Canto de contrappunti a due voci (Florence, 1584).

²⁴See Alfred Einstein, "Vincenzo Galilei and the Instructive Duo," Music and Letters, XVIII (1937), 360-368, for a discussion of this brief, but significant work.

²⁵Ibid., p. 367.

It should not be forgotten that this enigmatic musician was not only a promoter of expressive music and a pioneer of monody, but also was a master of sixteenth-century practice in the area of madrigals and of lute music.²⁶

In 1587, Galilei produced his second book of madrigals. It is his only collection in which all the part-books are preserved intact. These madrigals, although conventional in character, made the point that the medium of expression was itself worn out and trite, for example, the melodic and harmonic chromaticism, the cross-relations, and the dissonant suspension chains. Some of these madrigals, which were more inclined to homophony and declamation, came closer to fulfilling the ideal which Galilei approved as a theorist.²⁷

Gioseffo Zarlino brought out his principal refutation of Galilei's earlier attacks in 1588. The Sopplimenti musicali²⁸ consisted mainly of a systematic series of quotations from the Dialogo or other writings

²⁶Palisca, MGG, IV, 1268.

²⁷Ibid.

²⁸See p. 6, n. 20.

of Galilei, followed immediately by a none-too-subtle set of reproofs. Galilei responded in 1589 with his Discorso intorno all' opere di Gioseffo Zarlino, in which he refuted Zarlino's belief that musical law must be based on "natural" mathematical principles. He wished to end the dogmatism and pedantry of music theory and to replace the strict rules with simple procedures by which physical perception and practical musical experience are taught.²⁹

During the last year of his life (1591), Galilei completed a treatise on counterpoint which has been termed his most significant achievement.³⁰ In its original form the work was in two parts: Il primo libro della prattica del contrapunto intorno all' uso delle consonanze ("The First Book of the Practice of Counterpoint with Regard to the Use of Consonances") and Discorso intorno all' uso delle dissonanze ("Essay Concerning the Use of Dissonances"). He later decided to append a third essay to his work, entitled Discorso intorno all' uso dell' enharmonio, et di chi fusse

²⁹Palisca, MGG, IV, 1268.

³⁰See Claude V. Palisca, "Vincenzo Galilei's Counterpoint Treatise: A Code for the 'Seconda Pratica'," Journal of the American Musicological Society, IX (1965), p. 81.

autore del cromatico ("Essay Concerning the Use of the Enharmonic and Concerning the Author of the Chromatic"), and ultimately, he added a short supplement to this last essay, called Dubbi intorno a quanto io ho detto dell' enharmonio con la solutione di essi ("Doubts Concerning What I Have Said about the Use of the Enharmonic, with Their Resolution"). The work, as a whole, is not titled, but the first words of the initial essay reveal the main subject to be "L'arte et la pratica del moderno contrapunto" ("The art and the practice of modern counterpoint").³¹

All of the parts of the counterpoint treatise may be found in the Biblioteca Nazionale of Florence.³² Galilei set aside all the rules except the most important ones. He both advocated and practiced the free employment of dissonances and chromatic interweavings, which revealed new possibilities for expression and anticipated the essence of the "seconda pratica" of the seventeenth-century. "The examples in his treatise on counterpoint contain harmonic progressions whose boldness was unequaled in the music of his time."³³

³¹ Ibid., p. 83.

³² MSS Galileiani, Anteriori a Galileo, Vols. I-III.

³³ Palisca, MGG, IV, 1268.

Before Galilei died, he conducted a number of acoustical experiments. Rather than accept the numerical dogmas of the Pythagoreans and Platonists, Galilei experimented with strings of various materials, with tubes, and other sounding bodies, to ascertain how universal the sacred numbers were.³⁴ His interest in such experiments is evident from the discussion of Bardi and Strozzi on the subject of organ pipes in the Dialogo.³⁵ Up to the very last, Galilei remained a strong supporter of the theories he had proposed in the Dialogo, seeking always to strengthen them. He was forever mindful of current trends in music, unlike Zarlino and other more reactionary musicians who preferred to remain a generation behind, contenting themselves with the dissemination of the traditions of their learned predecessors.

The Ancient-Modern Controversy

With the coming of the Renaissance, mankind was at last released from the spiritual subjugation of the medieval era. The spirit of humanism, a belief in the intrinsic worth of the individual, became prevalent

³⁴See Palisca, "Vincenzo Galilei's Counterpoint Treatise," p. 84.

³⁵See pp. 821-829 below.

throughout western civilization as the Roman church became more tolerant of secularisms. For two centuries prior to the time of Galilei, the Italian humanists had been reviving the glories of antiquity.³⁶ The art of painting had been restored by artists like Giotto and Cimabue, Dante Alighieri had written his Divina Commedia, and Petrarch had penned his sonnets. Everywhere, artists and musicians pursued their callings in relative freedom, each one supported by a wealthy noble patron like Lorenzo di Medici, or perhaps even the Pope himself. The old Greek and Latin classics were avidly read and discussed by learned gentlemen who habitually assembled for that purpose in private homes. One such group in Florence was Count Bardi's renowned Camerata,³⁷ another the Alterati.³⁸ Long before Galilei, musicians had also been probing the ancient ruins for new outlooks. Most of the possibilities had proved futile. The return to

³⁶Warren Dwight Allen, Philosophies of Music History (New York, 1962), p. 39.

³⁷See Manfred F. Bukofzer, Music in the Baroque Era (New York, 1947), pp. 5-9, with regard to the origin and aims of the Camerata.

³⁸See Claude V. Palisca, "The Alterati of Florence, Pioneers in the Theory of Dramatic Music" in William W. Austin, ed., New Looks at Italian Opera (New York, 1968), pp. 9-38, for an account of this lesser-known but significant group.

chromaticism and enharmonicism advocated by Plutarch³⁹ had been explored in vain by Don Nicola Vicentino,⁴⁰ working on the false premise that a monophonic method could be applied to a polyphonic composition. The Dodecachordon of Glarean was no better, for the mild sixteenth-century church modes could not be used to recreate the legendary moral effects of the Greek "tonoi", although humanists believed this possible. Another attempt, the "vers mesurés à l'antique", emulated only one external characteristic of ancient music, which was also insufficient to realize its effects. While a few works of intrinsic beauty were produced, the modern musicians had made little progress in recapturing the lost music of the ancient Greeks.⁴¹

All these mistakes stemmed from misunderstanding, poor method and inadequate knowledge. Musicians went to the shell, rather than the core of Greek art.⁴²

³⁹See Plutarch, On Music, translated by Benedict Einarson and Phillip H. DeLacy (Cambridge, Massachusetts and London, 1967), pp. 441-445.

⁴⁰See Nicola Vicentino, L'antica musica ridotta alla moderna prattica (Rome, 1555). Vicentino constructed an instrument known as the archicembalo which was designed to play in the chromatic and enharmonic Greek scales as well as the diatonic. His experiments were not successful and did not gain acceptance, because it proved impractical to employ the chromatic and enharmonic in part-music.

⁴¹See Palisca, Girolamo Mei, pp. 1-2.

⁴²Ibid.

The viewpoint of the "moderns" differed greatly from that of the "ancients". The contrapuntal style taught by Zarlino was founded on the teachings of his own master, Adrian Willaert, who, in turn, came from a long line of polyphonic composers such as Josquin and Ockeghem. As such, Zarlino was a defender of the "prima prattica",⁴³ which incorporated a rich tradition of discourse inherited from the Middle Ages, both speculative and practical, with a sixteenth-century accumulation of practical theory occasioned by particular problems arising from contemporary musical questions and usages. Both of these factors combined with the growing Renaissance interest in the musical speculation and practices of Antiquity to produce a tradition of encyclopedic music learning. Gioseffo Zarlino's Le Istitutioni harmoniche, published in 1558, met all the qualifications for such a comprehensive endeavor. It gave as much attention to such details as classical legends of the efficacy of music, general speculations concerning the nature of mathematical proportion, similarities between tonal patterns,

⁴³See Bukofzer, Baroque Music, pp. 1-4, concerning the classifications of music according to styles or practices. Ibid., p. 16, contains an apt table showing the difference between Renaissance and Baroque styles.

the elements and the humors, etc., as it did to the considerations of polyphonic composition.⁴⁴ Zarlino profited by the humanist revival of Greek texts on music, but he was no idolator of antiquity, and what he sifted from the mass available to him is remarkably pertinent to his age.⁴⁵ He exemplified the mid-sixteenth-century attitude toward Greek music. Because the conventions of "musica theorica" demanded an explanation of the Greek musical systems and a discussion of what they implied for modern music, comparisons are found throughout the Istitutioni between the Greek and modern theories. These are not so much encouragement to musicians to emulate the Greek models as they are compromises with the current humanist atmosphere.⁴⁶ Zarlino discussed briefly the nature of the Greek "harmonies" and scales.⁴⁷ He observed that there were many conflicting opinions about them, but decided that these "harmonies" were of little use in the modern practice, and therefore speaking any more about them would be futile. He

⁴⁴John Hollander, The Untuning of the Sky (New York, 1970), p. 20-21.

⁴⁵See Gioseffo Zarlino, The Art of Counterpoint, translated by Guy A. Marco and Claude V. Palisca (New Haven and London, 1968), p. xiv.

⁴⁶Palisca, Girolamo Mei, p. 42.

⁴⁷Zarlino, Istitutioni, pp. 293-300.

considered it a vanity to try to follow the ways of the ancients, as certain restorers of the chromatic and enharmonic--he was alluding to Vicentino--demonstrated clearly by their failures. It was apparent to Zarlino that those ancient systems could not be applied to part-music, and since he knew of no examples of Greek music, he was certain that the ancient practice had disappeared. Further, Zarlino was not disturbed that modern music did not arouse its hearers in the manner attributed to ancient music. He thought it unnecessary to move anyone to tears and laughter, but favored music which would instill men with virtue. Zarlino would in no case have been willing to abandon polyphony. To him, the object of music was to convey honest delight while passing one's leisure time. Because he was so imbued with these characteristics, as well as being antagonistic to the ideals promulgated by Galilei, Zarlino was entirely unwilling to alter any of his conceptions of music. Galilei, on the contrary, wished to emulate the musical art of the Greeks since he felt it was superior to the contemporary practice of counterpoint, which tended to obscure the words and prevent, with its unwarranted mixture of modes, the communication of any desired affection or emotion to the listener. The question uppermost in his mind was this: "Did the

Greeks have elaborate, many-voiced harmony, or were they content with a single line of melody?"⁴⁸

In 1581, having formulated his theories with the aid of Girolamo Mei, Galilei's reply to Zarlino was voiced in the Dialogo della musica antica et moderna. With this assumption of the defense of the ancients, the strife with Zarlino had begun, a controversy which would continue for centuries under different guises, extending into many different localities. It was a quarrel which would cause the name of Vincenzo Galilei to be remembered well.

Background for the Dialogo

No study of Vincenzo Galilei would be complete without some insight into his rather enigmatic, equivocal personality. It has been said that he stood half in the sixteenth and half in the seventeenth century with regard to his musical perception.⁴⁹ As a madrigalist and lutenist, he was a veritable master of sixteenth-century practice. It was, moreover, his desire to benefit the younger generation of composers through his rich experience. On the contrary, he promoted expressive music, and as a pioneer of monody, he envisioned the ideals of the seventeenth century, the era of accompanied solo music. One

⁴⁸Allen, op. cit., p. 39.

⁴⁹Palisca, MGG, IV, 1267-1268.

finds in Galilei's Dialogo a manifestation of these monodic tendencies, at least in theory, if not in actual practice. Although this noble Florentine did not show how to implement his neo-Hellenic ideas, he did state clearly two rudimentary conceptions of the so-called "seconda prattica":⁵⁰

(1) A melody's emotional quality is dependent upon the range of the voice it uses--high, medium, or low--and upon the tempo and meter.

(2) Other independent parts should not be allowed to interfere with voices, which express the text.

These principles constituted a clarion call to combat against the very musical thinking which had itself nurtured Galilei since his early years. The personification of such thought was the Reverend M. Gioseffo Zarlino, with whom his relationship was destined to deteriorate drastically with each passing year. Zarlino, born in 1717, was not much older than his "loving disciple", as he aptly termed his rebellious pupil, Vincenzo. The Sopplimenti musicali⁵¹

⁵⁰See Palisca, "Links Between 'Pseudo-monody' And Monody," p. 346.

⁵¹Zarlino, Sopplimenti musicali, p. 85.

(1588), his reply to the polite attacks of the Dialogo, was full of such sarcasms. If the irascible choirmaster of St. Mark's in Venice was less than cordial to his one-time scholar, Galilei's replies in the Discorso (1589) compensated with equal aplomb. It was evident that with the publication of the Dialogo, Galilei was opening up a new horizon of musical endeavor. One may well question the "display of rather pedantic erudition and the somewhat petulant arguing over tiny points of detail," as Pirotta⁵² puts it, but through such sciolisms "we can see that the dominant feature of the book is the unrelenting criticism of the musical language of the time--a language based on counterpoint."

Galilei's abstruse style is that of a philologist, and rightly so, for his association with Girolamo Mei, a man of that profession, is well-known through a long correspondence between them. Only Mei's letters are extant, having been preserved in the Vatican (Vatican MS Regina Lat. 2021), but a comparison of these letters with Galilei's Dialogo easily manifests an unmistakable similarity. Clearly, Mei had provided Galilei with the bulk of material for his work, but the only acknowledgment of indebtedness to him in the Dialogo takes the

⁵²Nino Pirotta, "Temperaments and Tendencies in the Florentine Camerata," Musical Quarterly, XL (April, 1954), 172.

form of a general tribute, naming Mei as a "worthy man to whom all musicians and learned men should render thanks and honor."⁵³ He is mentioned there once again, merely as "one of our Florentine gentlemen".⁵⁴ Notwithstanding, Galilei's dependence upon Mei was generally known. Artusi, an exponent of Zarlino, attacked the Dialogo in a pamphlet,⁵⁵ referring to it as "vostro non vostro Dialogo della musica antica et moderna", and Zarlino also claimed that it was not his work at all, but was written by several gentlemen, friends of Galilei.⁵⁶ Rising angrily to his own defense, Galilei claimed in his Discorso (1589) that the work was all his, saying: "he who believes differently tells an untruth; and he who says differently tells a lie."⁵⁷ It is certain that, despite the invaluable aid of Girolamo Mei, Galilei's work nevertheless

⁵³See p. 38 below.

⁵⁴See p. 588 below.

⁵⁵Giovanni Maria Artusi, Trattato apologetico in difesa dell' opere del Rdo Zarlino. This treatise is not extant.

⁵⁶Zarlino, Sopplimenti musicali, p. 131.

⁵⁷Palisca, Girolamo Mei: Letters on Ancient and Modern Music, p. 77.

possesses a high degree of originality. Perhaps Mei was also content in knowing that his ideas had found fertile ground.

In conceiving the Dialogo, Galilei was concerned, not only with the development of monody, but with the re-grouping of contemporary music into five branches, which were

- (1) the problem of tuning,
- (2) the theory of the modes,
- (3) the rules of counterpoint,
- (4) the history of music,
- (5) and instrumental performance practice.⁵⁸

Galilei elected to use the venerable dialogue form, which had first been employed by Plato and the ancient Greeks, since it suited his didactic purposes better than discourse. He disavowed the acceptance of ideas on sheer authority, preferring the empirical approach of Aristotle⁵⁹ by means of which he might introduce

⁵⁸See Palisca, MGG, IV, 1268.

⁵⁹Ingemar Düring, "Greek Music, Its Fundamental Features and its Significance," Journal of World History, III (1956), 318. "Aristotle was an absolute innovator with his historico-critical method which enabled him to fit his results into a wider perspective. In dealing with a problem he regularly begins with those of his predecessors, with special emphasis on their solutions and the difficulties they present. His own solution of a problem is often a compromise resting on a common sense appraisal of different possibilities."

opposing views and pertinent facts for subsequent evaluation and logical conclusion. His chosen interlocutors, Count Bardi and Piero (or Pietro) Strozzi, pursued their protracted polemic, presumably in a plush parlor of Bardi's dwelling, where the Camerata habitually "passed their days in commendable discussions with well-bred repose."⁶⁰ It would have been somewhat incongruous, certainly, for the noble Bardi actually to have uttered the vast profusion of acoustical theories for which Galilei made him responsible in the Dialogo, for Bardi himself would have been more at home with the adages of Aristotle or the proverbs of Plato, which did, however, appear frequently enough in that work. The "lofty principles" which Galilei gleefully and glibly expounded through the mouth of Bardi often set Strozzi in search of his "safe port" before foundering upon the treacherous reefs of his constant confusion. Galilei, obviously enough, enjoyed the role of guide through the maze of clever, complex "entanglements" which, through Bardi, he promised, at the beginning of the Dialogo,⁶¹ to set up for

⁶⁰See p. 39 below.

⁶¹See p. 45 below.

Strozzi's eventual edification and enlightenment. As the dialogue unfolds, Strozzi seems one time to comprehend, another to be overwhelmed with perplexity, but he is invariably prepared with another doubt (*dubbio*) which launches his "faithful pilot" Bardi onto yet another series of sesquipedalian terminology. Underlying the entire Dialogo is an unmistakable aura of aristocratic smugness, based on the disdain of common people as vulgar and ignorant incompetents. This attitude seems even more appropriate for Galilei than for Bardi, since the former, though a noble, through the decline of his family fortune, stood uncomfortably close to the economic apanage of the proletariat. Bardi, however, being affluent, could afford at least a studied tolerance.

Galilei possessed, first of all, a love for the popular musical idiom of his day and a desire to be of service to the practical musician through his great wealth of experience as a lute player and composer. Galilei was constantly warning the "prattico" to cease his malpractice and turn into better paths. Secondly, he was an erudite scholar, and wished to bring about a reform of serious vocal music. He therefore refuted Zarlino, once his mentor, for advocating polyphony, which Galilei felt was ruinous to compositions for

voices. The Dialogo, in which his primary aesthetic values and musical ideals were embodied, was the means by which these goals could be achieved.

Galilei devotes the first section of his Dialogo to tuning theories. He refutes Zarlino's hypothesis that the tuning used at that time in singing is the diatonic syntonon of Ptolemy, or "just" intonation. He advances the opinion that the vocal tuning must be, in practice, a compromise between the Pythagorean tuning with its pure fifth and the Ptolemaic diatonic syntonon. The consonant third is to be included. He evolves a practical system of equal temperament for the lute, using semitones in 18:17 proportion.

In the second section, Galilei examines the distinction between the musical art of the Greeks and that of his own time. He criticizes the systems of Glarean and Zarlino, considering the church modes to be ineffectual and inappropriate as representatives of the ancient Greek tradition.

The third section is a critical discussion of polyphony in which Galilei mocks the naive imitations of the meaning of individual words by the contrapuntists and their fastidious attention to the declamation of the text. He deplores the confusion which originates in

church music through the independence of voices.

In the next section, Galilei gives a history of instruments and notation, publishing for the first time the tables of Alypius, which contain the ancient Greek notation symbols and some authentic Greek songs.

The final section of the Dialogo is a critical treatment of the instrumental music of Galilei's time.



TRANSLATION AND COMMENTARY



DIALOGUE OF VINCENZO GALILEI
NOBLE FLORENTINE
CONCERNING ANCIENT
AND MODERN
MUSIC



To the Most Illustrious
Lord, and my Patron
The Most Worthy
Lord Giovanni Bardi,
Count of Vernio



It is truly very difficult to repay those from whom we have received many extraordinary benefits, for it is necessary that the one to whom we are in debt comes to value our work. We agree that, given the inspiration, we can fulfill these obligations. It happens that such insights now occur less often, and therefore, many are accused of being ungrateful. This is not the case, however, because the giving and receiving of benefits should not take precedence in such a noble endeavor. For if that were so, carrying it out properly might not be in our power--a most improper thing to think, let alone to say. If, moreover, that is true--and it really is, Signor Giovanni--I could still with clear conscience sustain the debt which I owe to Your Most Illustrious

Lordship, who as a connoisseur of good literature, and versed in the arts which are proper to a true Gentleman, has never asked of me anything which exceeds my powers, since they are weak. I have never been able to engage in the service of Your Most Illustrious Lordship in a way which leaves me any chance at all to avoid the name of ingrate, which I just now said pertains to one who lacks inspiration and ability. And yet, if something done by me were of value, I would still accomplish nothing in comparison with the many exceptional benefits received continually from the kind hand of Your Illustrious Lordship. How could I even in the least repay the convenience that you have given me of being able with calm mind to attend to those studies to which I devoted myself since my youth, which, without your aid, would never have reached their present conclusion. To that is added your thoughtfulness in having procured at my request from the most remote parts of Europe, various books and instruments, without which it would have been impossible to obtain that information about music which has been obtained through their agency. In order that this science could be shown by me to the world much more clearly than it has been shown, perhaps since the loss of music, has it not seemed important to

you to give me the convenience of travel provisions and to bestow on me your favor in every other matter necessary for searching many places and afterwards deriving the most exhaustive and reliable information possible from the customs of the inhabitants, from ancient records, and from men intelligent in musical science? Although some others may perform such acts of generosity, these suppliers of riches and protections, either through natural instinct or desire of glory, attach no importance to similar expenses, and they are quite often abundantly and willingly accustomed to making them. However, they are so much more regarded for them to the extent that, accompanied by equal humanity, they exceed the common custom of such things. What greater sign of courteous and benevolent spirit could you give me, having left many times your more serious and important negotiations, than to tell me enthusiastically the obscure sentiments of the ancient and serious writers? Although few have understood these concepts, you stated them so accurately that one could well believe that you were in those happy centuries in which fullest knowledge of the art of music had been accomplished. Music, afterwards, in the course of many ages, has fallen into such neglect

[ii]

that no one has been found up to today who has given knowledge of it conforming to the truth and to that which these ancient men of yours have left written. This situation has moved me to try--with that talent with which nature has endowed me and with the labors of many years spent in that pursuit, aided also by the favor and liberality of Your Most Illustrious Lordship--to return this art if not into its original state, at least into greater prominence. Up to now, this has not been done, to my knowledge, by the moderns that have written of it. I have judged that it is the duty of a sincere man who is not at all envious of the well-being of others, to give public account of this art, since he can bring both benefit and delight to those who will follow him, in order that those who will read about it can ascertain if anything there is good, and exert themselves to provide where he has lacked completeness. I do not wish, however, to believe that I have so dispelled the darkness of the ancient writers, that no one else can, having been solicited and aided by my efforts, add to them any greater clarity. Having written it according to my abilities and being obliged to publish it for the reasons just now given, this labor of mine in honor of Your Most Illustrious Lordship has just

been completed. I owe you this work since I have received from you very great conveniences, which were sought in order to complete a similar work which it was, without doubt, my obligation to send to you. And it has been requested that the uplifted and benefitted one renders to the benefactor the necessary reward. If it does not come to you with that corrected speech which is proper, and if it does not come to you translated into Latin--as it will soon come, together with my other work--blame the little faith of some Venetian printers. These men have, contrary to every obligation, detained me many months in order to please someone who forbade this labor of mine to be published, either impelled by envy or to honor himself by my long waiting. They have at last little less than denied me the manuscripts, consigning them at the end of last October in order that they would have to publish it before I could copy it for you by hand. I have, for the alleged cause, printed here two-thirds of it, extracted from a sketch left to me. In conclusion, I kiss your hand with every reverence, asking you to accept with pleasure this work of mine, having no

regard for myself but only for your justness and your most singular humanity. In Florence the first day of June, 1581.

Your Most Illustrious Lordship's

Most affectionate and obligated servant,

Vincenzo Galilei

[CHAPTER II]

[THE TUNING CONTROVERSY:

DIATONIAION VS

SYNTONON]

Music was numbered by the ancients among the arts that are called liberal, that is, worthy of a free man; and deservedly among the Greeks, its masters and inventors--like almost all the other sciences--it was always in much esteem. And it was ordered by better legislators that it must be taught, not only as delightful to life, but also as useful to virtue to those who were born to achieve the perfection and the human bliss, that is the purpose of the state. As time went on, the Greeks lost music and the other doctrines once more, together with the Empire. The Romans possessed knowledge of music, deriving it from the Greeks; but they employed mainly that

[1]

part suitable to theaters, where tragedy and comedy were presented, without much valuing that which is concerned with speculation; and continually occupying themselves with wars, did not devote much attention to that again, and thus they easily forgot it. Since Italy for a long period of time had suffered great inundations of barbarians, every light of science was spent; and as if all men had been overtaken by a grave lethargy of ignorance, they lived without any desire to learn. As for music, they had the same knowledge of that as of the West Indies; and they persisted in such blindness until first Gaffurio,¹ then Glarean,²

¹Franchino Gaffurio was born at Lodi, Italy in 1451 and died in 1522. His writings exercised a powerful influence on the musical studies of his day, and most of his successors quote his opinions as authoritative. His main theoretical works are the Theorica musicae (1480) and the Practica musicae (1496). Galilei upholds many of his ideas in the Dialogo, although Gaffurio had been previously attacked by Spaturo and Aaron, on account of some of his proportions of intervals which fomented a long bitterly contested strife in which many joined. For an English translation, see Franchinus Gaffurius, Practica musicae, translated by Irwin Young (Madison, Milwaukee and London, 1969).

and after him Zarlino³--truly the princes in this modern practice--began to investigate what music might really be, and to seek to extricate it from the darkness where it had been entombed. That part which they understood and valued, they brought little by little to the end result in which it is found; but it does not seem to any men of intelligence that

²Heinrich Glarean (1488-1563), actually Loris (Glarus was the region in Switzerland where he was born), wrote the important, controversial theoretical work known as the Dodecachordon. The aim of this treatise was to propose that there were twelve, not eight Greek modes which corresponded with the twelve church modes. Galilei, in the Dialogo, takes great exception to his statements, defending Gaffurio against Glarean's criticisms. For an English translation, see Glarean, Dodecachordon, translated by Clement A. Miller (Rome, 1965).

³Gioseffo Zarlino, born in 1517 at Chioggia, died in 1599. One of the most famous theorists of his day, he held the post of organist at St. Mark's in Venice. Sansovini, one of his contemporaries, calls him "unequaled in theory and composition", and another, Foscarini, terms him the most famous restorer of music in all Italy. Zarlino's most famous theoretical works are the Istitutioni harmoniche (1588), Dimostrationsi harmoniche (1571), and Sopplimenti musicali (1588).

they have restored it to its ancient state. From what can be understood from the endless passages of the ancient histories, from the poets and the philosophers, they had neither attained true nor perfect knowledge of it. Perhaps this was caused by the coarseness of the times, the difficulty of the subject and the scarcity of good interpreters. Nevertheless, these writers deserve the highest praise, and the world owes them a perpetual obligation, if for nothing else, at least for having given many the occasion of developing it with greater dedication, in order to see it brought to its perfection. It appears that this--only as much as pertains to theory--has been attained in our times by Girolamo Mei,⁴ a worthy man, to whom all musicians

⁴Girolamo Mei (1519-1594), a native of Florence, became recognized as early as 1540 as one of the most promising young literary men of Florence. He was by training what today would be termed a classical philologist. In 1561, his attention was drawn to a book of ancient authors on music, reawakening a former desire to investigate the nature of Greek music. Galilei's reading in 1572 of his writings on that subject engendered a

and all scholarly men should render thanks and honor, and after that in our city, the Most Illustrious Lord, Giovanni Bardi, Count of Vernio. Bardi, who has long made a study of music and being much delighted with it, as with all the other sciences, has greatly ennobled it and made it praiseworthy; having by his example aroused the nobles to the very same study. Many of them are accustomed to going to his house and there passing the time in delightful songs and in commendable discussions with well-bred repose. Since I am much obligated to the courtesy of this most cultured gentleman, and yet desirous of showing him by some outward sign the inward wish that I have of serving him, I have

lengthy correspondence with Mei, who then lived in Rome. The valuable ideas gleaned from his letters were incorporated in Galilei's Dialogo della musica antica et moderna (1581). For a complete study on Mei and his influence, see Claude V. Palisca, Girolamo Mei: Letters On Ancient And Modern Music To Vincenzo Galilei And Giovanni Bardi (Rome, 1960).

judged myself not to be able to spend my time more profitably than to dedicate my labors to such a subject; because in so doing I had the hope of being able to give him some sign of gratitude, and to extend to the world no little aid in leading it out of the darkness, in which, since the aforesaid loss, it has, up to now, been enveloped. I say this, however, without arrogance, and with every respect for those who from the time of Guido d'Arezzo⁵ until now have written about such matters. However, if I were to give myself a great deal of glory for this deed,

⁵Guido d'Arezzo, or Aretino, was an eleventh-century Benedictine monk, who made important contributions to the development of musical theory in the Middle Ages. His pedagogical skill is evident in his use of innovative devices such as solmization, the Guidonian hand, and a multi-colored musical staff. The Latin texts of his works, the Prologus in Antiphonarium, Micrologus, and Epistola de ignoto cantu, may be found in Martin Gerbert, Scriptores ecclesiastici de musica (1784), 3 vols., facsimile ed. (Milano, 1931), II. Portions of his works are translated into English in Oliver Strunk, Source Readings in Music History (New York, 1950), pp. 117-125.

perhaps I would not deserve rebuke. Because of the inclination given me by nature for these liberal studies and the continual diligence I have exercised for the space of many years in preparing it, it would justify my discussing them with good cause; but the judgement of this must be reserved purely to intelligent men.

[2]

For this reason, other than what I said a moment ago, and in order that I might not defraud the world of any usefulness it might draw from my labors, I was pleased to publish some of my ideas about ancient music and about that of our times, which until this day have been--in my opinion--little understood by whoever has dealt with them. This matter, without my further testimony, is clear evidence of the difficulty of the subject. Therefore I desire of the reader that he prepare himself to give judgement or make a comparison of my writings with those of the other moderns, with the very greatest attention and with mind liberated from

every human emotion. It is clear that whoever has a mind not entirely purged of every passion cannot give perfect judgement of anything that exists. Every suggestion that may be made to me by an intelligent man and a lover of the truth I will receive with pleasure, and will remain obliged to him without ever being ashamed of learning from one who may understand better than I. Now, since long and continuous speaking, while running like the gush of a torrent, does not seem to have that power and vigor in concluding sentences and arguments that dialogue has, I have considered it greatly relevant to treat my present discourses in such a way. This, I believe, was one of the potent causes that prompted Plato to treat the matters of divine philosophy in such a way. Consequently, I have chosen the Most Illustrious Lord Giovanni Bardi--mentioned a little while ago--and with him, Signor

Piero Strozzi,⁶ to discuss this thoroughly, since these men are most studious of the true music, great lovers of such speculations as these, and suitable also to sustain this and greater weight. On this occasion, moreover, Signor Piero Strozzi wishes to see which of the diatonic species is the authentic one in which the modern contrapuntists compose and the singers of their songs sing. Therefore, he speaks to Signor Giovanni in this manner:

Strozzi: It seems an important thing to me, Signor Giovanni, that of so

⁶Piero Strozzi, the scion of a very old, established Florentine noble family, was undoubtedly an apt choice for the speaking partner of Bardi in the Dialogo. There is a Strozzi chapel in the Santa Maria Novella today, noted for its art treasures, which attests to the affluence and prestige of Piero's people (see p. below). Little is known of his life, however, but Jacopo Peri mentions him as a learned gentleman who had heard Peri's new style of singing with interest and pleasure. See Strunk, Source Readings, p. 375. Federico Ghisi, Alle fonti della monodia (Milan, 1940), p. 60, contains a "pseudo-monody" by Piero Strozzi, the madrigal Fuor dall'humido nido.

many excellent men, who have from Guido d'Arezzo to the present written of the art of music, not by chance, but as professors of it, there has not been anyone, as far as I know, who has explained the diatonic species, in which one composes and sings today, so that a thousand difficulties and contradictions are not engendered. Nonetheless, among the principal matters, I rank this the most principal, the most important, and the most necessary to be known. Nor can I succeed in being unashamed, when I realize how little knowledge is universally found among the materials available to our practicing contemporaries. They make a profession of knowing and understanding about the power and nature of so little, being content to be thus esteemed by the unskilled masses. Finding myself so encumbered, I desire greatly, with your aid, to free myself from such a defect.⁷

⁷Literally, "finding myself in such pitch, I, soiled, desire to purge myself of such a defect."

Bardi: You continually place under consideration the most discerning and uncommon questions, which give everyone who hears them an indication of your fine talent. In order to clarify the problem given me, I will need to develop many intricate techniques. This, however, since it pleases you, will not be a burden to me.

Strozzi: If it will not be wearisome to you to reveal them, it will give me the greatest satisfaction to understand them. Whenever you please, I am ready to listen with the greatest attentiveness you could possibly desire.

Bardi: It is necessary, on account of this noble purpose, to examine diligently every interval of the diatonic species about which such a controversy results, to see which of our diatonic species conforms with those that are

What species is the one sung today according to Zarlino⁸ in chapter 16 of the 2nd book of the Istitutioni harmoniche and in the 5th discussion at the 3rd definition in the Dimostrationsi.

⁸See Zarlino, Le Istitutioni harmoniche (Venice, 1558), pp. 83; Dimostrationsi harmoniche (Venice, 1571), p. 268.

composed and sung today. The knowledge of this, without doubt, will lead us into a safe port. Since all of the practitioners of our time, under the influence of Reverend M. Gioseffo Zarlino, universally concur that it is important, we will examine before every other species that which is called the intense syntonic of Ptolemy.⁹ After he examines this--if it occurs to him--with equal care, he will agree with what all the other moderns like Guido d'Arezzo, Gaffurio, Glarean, LeFèvre,¹⁰ Valgulio,¹¹ and other serious

⁹Claudius Ptolemy (2nd century A.D.), the celebrated Greek astronomer, the events of whose life are virtually unknown, was the author of a three-part treatise on music. This work gives a clear exposition of the transposition of the Greek modes in addition to other important information with regard to ancient music. The followers of Ptolemy were very much concerned with reconciling Pythagorean musical theory, which relied upon numbers, with that of the Aristoxenians, which depended upon the judgement of the ear. There is a German translation of Ptolemy's Harmonics in Ingemar Düring, Ptolemaios und Porphyrias über die Musik (Göteborg, 1934).

¹⁰Jacques LeFèvre d'Étaples (Jacobus Faber Stapulensis), the author of Musica libris quatuor demonstrata (Paris, 1551),

writers, have openly admitted. All of these men of common opinion affirm that what is sung today is the most ancient diatonic ditoniaion, the proportions of which were investigated with shrewd deliberation by the strict Pythagoras of Samos¹² in the sixtieth olympiad.

The Chronicles¹³
of Eusebius.

was a sixteenth-century Pythagorean. He made no real innovations, but his treatise was a standard work, much in demand long after his death, as a clear exposition of the ancient musical number theories.

¹¹Carlo Valgolio Bresciano is known for his translation and commentary of the pseudo-Plutarchian treatise On Music. A copy of this work, Plutarchi Dialogum de musica ad Titum Pyrrhinum (Brescia, 1507), may be found in the British Museum. There is an Italian translation of both the text of Plutarch and the commentary in one of Vincenzo Galilei's manuscripts (Florence, B.N.C., MSS anteriori a Galileo, Vol. VII).

¹²Pythagoras of Samos (fl. 528 B.C.), the noted Greek philosopher, left no writings of his own. His teachings, disseminated by his many followers, undoubtedly suffered numerous modifications. His strict number theories, which limited the musical consonances to the octave, fourth, and fifth, necessitated a dissonant major third, and precluded the possibility of modulation. Pythagoras taught that numbers were the soul of the universe. He related the numerical proportions to the heavenly bodies, thus

Strozzi: Before your Lordship begins to expose the heart of the proposed problem, I desire that we always exclude in those matters attainable by the intellect-- as Aristotle¹⁴ says in the eighth book of the Physics--not only the authority, but also the prejudiced argument which would be contrary to our desired appearance of

Aristotle in the 8th [book] of the Physics at heading 22.

promulgating the familiar doctrine "music of the spheres". The authority of Pythagoras was invoked for hundreds of years until equal temperament became the standard in the eighteenth century.

¹³See Eusebius of Pamphilus, Chronici canones, ed. by John Knight Fotheringham (London, 1923), pp. 184-185. Eusebius shows that Pythagoras flourished in the sixty-second, not the sixtieth olympiad. The olympiads were the four year intervals between the Greek olympic games. The sixty-second olympiad in our system would be 528 B.C. (Christ was born in the 194th olympiad).

¹⁴See Aristotle, Physics, in Basic Works of Aristotle, ed. by Richard McKeon (New York, 1941), pp. 354-394. Galilei is evidently referring to Aristotle's empirical approach in examining the arguments of others and logically eliminating those which are not pertinent in order to reach a logical conclusion.

truth. For it seems to me that those who, in order to prove some conclusion of theirs, wish it to be believed exclusively on simple authority without presenting any other valid arguments, are doing a ridiculous thing, in order not to say with the Philosopher, "like fools". One finds that this privilege has not been granted to others except to the all-wise Pythagoras--just now mentioned by you--by his followers.

Furthermore, I want you to concede that it is lawful for me to question you freely and answer you without any kind of flattery, since this is truly suitable among those who seek the truth of things.

Privilege of Pythagoras. Boethius, in the 1st book On Music, chapter 33.¹⁵

Cicero¹⁶ Concerning the Nature of the Gods.

¹⁵Calvin M. Bower, "Boethius' The Principles of Music, An Introduction, Translation, And Commentary," unpublished doctoral dissertation, School of Music, George Peabody College for Teachers, Nashville, Tennessee, 1967, p. 100. "For when the master Pythagoras said something, no one thereafter dared challenge his reasoning. The authority of the teacher was reason enough for them."

¹⁶See Marcus Tullius Cicero, On the Nature of the Gods, trans. by Hubert M. Poteat (Chicago, n.d.), p. 198.

Bardi: All is conceded. First of all, it is necessary to memorize--still according to the syntonon of Ptolemy, however, as I have said--the numbers in lowest terms, between which are separately contained each one of the intervals which are encompassed by the queen of the consonances. These, according to the opinion of those who write about such things, do not exceed the number of fifteen. Beginning with the smallest one, I say that the comma is contained in its root terms by the proportion called sesquioctogesima, between these numbers.....81:80
the minor semitone, between.....25:24
the major semitone, between.....16:15
the minor tone, between.....10:9
the major tone, between.....9:8
the minor third, between.....6:5
the major third, between.....5:4
the fourth, between.....4:3
the tritone, between.....45:32
the semidiapente, between.....64:45

[3]

The octave is called queen of the consonances.

These numbers contain the musical intervals of the syntonon.

the diapente, between.....3:2

the minor sixth, between.....8:5

the major sixth, between.....5:3

the minor seventh, between.....9:5

the major seventh, between.....15:8

and the queen of the consonances called

today octave, between 2:1. Perhaps the

Greeks gave it the name of diapason

since it contains in itself, as its name

implies, each of the intervals named, and

because it is valid only through each of

these other intervals. I warn you that

these intervals of the fourth, fifth,

octave, and the sesquioctava (or tone)

were not known by those names to any of

the ancient or modern musicians, except,

however, by the originators [of such

terms] and their followers. The cor-

ruption of these, because of the dispar-

ity which they have with the intervals

already understood and known by such

appellations, generates no little con-

fusion and disturbance in the minds of

those who read their writings. They

Why the octave
is called dia-
pason.

Names of the
intervals
corrupted.

have taken these names quickly from the diatonic ditoniaion and used them, in order to substantiate some of their schemes, to disguise the syntonon of Ptolemy. You will understand this better in due course. However seldom this practice is repeated, and to whatever extent we disagree about it, we will relegate it sensibly to its place.

Strozzi: Why did Pythagoras construct the fifth rather than the octave between 3:2 thereby assigning these numbers as the limits of the diapente, or else between 4:3 in which he constructed the diatessaron?

Bardi: This was not a human work or invention, but rather one of nature. It is quite true that Pythagoras--as I have said--was the first to consider it.

Strozzi: What induced him to do that, please?

Bardi: Listen! Stretch two strings of the same length, size and quality over a plane surface, and tune them in unison.

Method of hearing any interval in its true proportion.

Then divide one of them with the compass into enough identical parts to equal the larger number of the interval you wish to hear. Next divide one of them by means of an immobile bridge, in such a way that the distance of the greater number exceeds the smaller. Then, when the two strings are plucked simultaneously, you will hear from the whole--which is represented by the larger number--and from the parts--which are represented by the smaller number--the dissonance or consonance which corresponds to the proportion which has thus been applied to these strings. Anyone who wishes to hear some interval on a single string can accomplish it in another manner. First, divide the string into enough identical parts to equal the sum of the lowest terms of the desired proportion. Next, place the bridge as a divider between one term and the other. When two parts of the whole string which has been divided by the bridge are plucked separately or at the same time,

Another manner.

the expected intervals will be heard. For better understanding, here is a clear example of the second manner. Suppose that we want to hear, in this second manner, the diatessaron, which is contained in its true form by numbers in sesquitertia proportion. Added together, its lowest terms--which are 4:3, as I have shown you--make seven. Next, divide the proposed string into that many equal parts, and place the bridge over the point which separates the four parts from the three. When these parts are plucked together, or one after the other, the consonance diatessaron will be heard. In order to show you in this situation some natural effects of the proportions of numbers, I wish to put them, as in singing, in their lowest terms, in order to hear in both ways each of the consonant intervals which the diapason contains. From these simple consonances spring the compounds. Considering them carefully will provide light and knowledge of many things

Example.

There are seven species of diapason.

[DIAGRAM I]
 [INTERVALS OF THE DIATONIC SYNTONON APPLIED TO THE MONOCHORD]

Octave	Unison	Fifth	Twelfth	Fourth	Fifteenth
1	1	2	3	4	4
1	1	2	1	3	1
			Octave	Twelfth	Twelfth

Major third	Fifteenth	Seventeenth	Minor third	Major seventeenth	Nineteenth
4	5	1	5	6	6
4	1	1	5	1	1

Major sixth	Fifth	Major tenth	Minor sixth	Major sixth	Eleventh
3	5	2	5	8	8
3	2	2	5	3	3

pertinent to our discussions. You will find, in addition, that those within the diapason are as often consonant as dissonant--suitable, however, for singing--and do not exceed, rationally considered, the number of their species.

Strozzi: I have understood very well the cause of everything. Therefore, you may proceed whenever you wish.

[4]

Bardi: Those who are seeking to persuade us that the diatonic in which they compose and sing today is the syntonon of Ptolemy have shown, moreover, that within the aforesaid limits is found each singable interval. I want us to prove to these men, with the very same principles, that this species, so contrived, is not in any way that which they say it is. Instead, it consists of a greater number and variety of intervals than the ones proposed. They have not wanted to mention these greater numbers, not because they were somewhat inaccurate, but because they were not exactly purposeful, and hindered their schemes. First,

Other intervals, in addition to the first ones, are contained in the syntonon.

Paradoxes.

let us observe that in the syntonon of Ptolemy, the note d la sol re is sharper through h durum than it is through b molle [Figure 1.]. From F fa ut and C sol fa ut naturals to the same notes altered by this sign [i.e. X], and also from b fa to h mi [Figure 2.] is not the same interval as it is from E la mi to its b molle [i.e. E flat] and from G sol re ut to its diesis [Figure 3.]. In addition, I say that one does not find the same distance between D sol re and E la mi as there is between a la mi re and h mi [Compare Figures 4 and 5.]. Therefore, there is not the same interval between C sol fa ut and D sol re as one finds between G sol re ut and a la mi re. When ascending, there is not the same interval between D sol re and F fa ut [Figure 6.] as there is between E la mi and G sol re ut [Figure 7.]. Such an interval does not even exist between F fa ut and C sol fa ut [Figure 8.], when they have been altered by the sign X, to a la mi re and e la mi [respectively].

From a la mi re, to c sol fa ut [Figure 9.], when it has been altered by the same sign X, is not the same distance as there is from c sol fa ut to e la mi [Figure 10.]. From a la mi re to d la sol re [Figure 11.] there is not the same interval as there is from G sol re ut to C sol fa ut [Figure 12.]. The interval between F fa ut and h mi, when considered as a fourth and a minor semitone, is not a tritone [Figure 14.]. The dissonance which occurs between h mi and f fa ut [Figures 15 and 16.], when considered as two minor thirds, is not a semidiapente. Between D sol re and a la mi re there is not a fifth [Figure 13.] It is not even possible to have a similar interval by adding together the semidiapente and the minor semitone of the minor tone, which is the one generally recognized up to the present day. There is not the same interval from G sol re ut, altered by the sign X, and e la mi [Figure 17.] as there is between E la mi and c sol fa ut [Figure 18.]. Between F fa ut and d la sol re, and C sol fa ut and a la mi re [Figure 19.] there is not

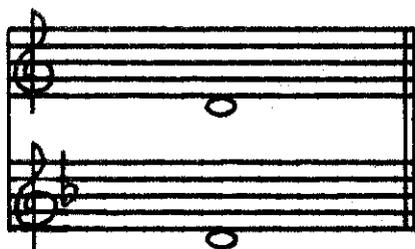
a major sixth. Between D sol re and c sol fa ut [Figure 20.] there is not a minor seventh. Also, that interval which is found between E la mi and D la sol re, considered, however, in two sesquitercias, cannot be a similar interval [i.e. minor seventh]. Such an interval does not occur, even if it is considered as a fifth in the lower portion and a minor third in the higher, between G sol re ut and f fa ut. Between D sol re and d la sol re, and between C sol fa ut and c sol fa ut [Figure 21.] there is not an octave. There is the same distance between b fa and h mi through b molle as there is between b fa and h mi through h durum [Figure 22.]. It is the same distance from h mi to c sol fa ut [Figure 23.] as it is from c sol fa ut to b fa altered by either of these signs, h or X. However, this h is the proper sign. Finally, I say that each tone does not have semitones of the same proportion and distance, but different kinds.

Strozzi: Your lofty principle represents to me, instead of a secure cove, that part of the earth under the south pole which is called unknown, because the things which you just now mentioned are so new to my ears that if I did not have so faithful a pilot as my guide I would abandon hope of the port.

Bardi: Be of good courage and do not be frightened of these few reefs which we will pass most securely with our little craft. In order to reassure you and ease your way, observe in practice the example of all that has been said. It will easily clarify everything for you, proceeding step by step. First, permit me to explain that in this second, carefully constructed enumeration concerning the disparity which the first intervals have with the second ones, I did not mean to say that such intervals are not found among the notes of the syntonon, but rather that all are not their regular members, not even the first

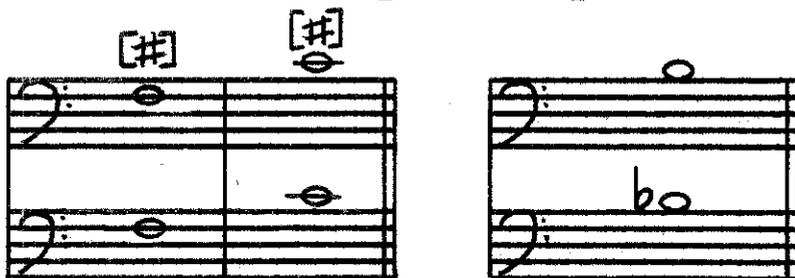
ones, and none of them are under consideration by our practicing contemporaries. I would also like to tell you that those of which they possess knowledge are sung only in their demonstrated proportions, as you will soon see. I will prove to you, therefore, according to what I have proposed, that the two notes placed below are not in unison.¹⁷

[5]



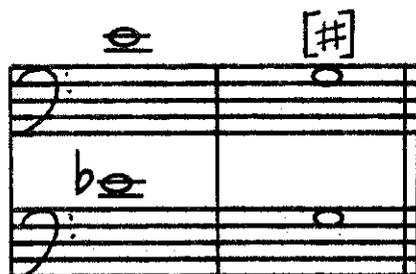
[Figure 1.]

I will prove to you also that the following [Figure 2.] are not the same distance apart as these are [Figure 3.].



[Figure 2.]

¹⁷See pp. 185-186 below for Galilei's explanation of this enigma.



[Figure 3.]

These [Figure 4.] are not distant one from the other by the very same interval as these [Figure 5.].

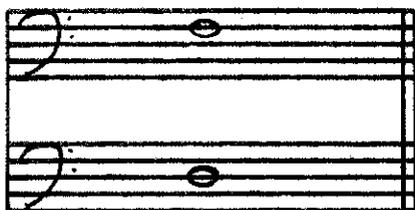


[Figure 4.]



[Figure 5.]

I say that these [Figure 6.] are less distant, but not these [Figure 7.].



[Figure 6.]



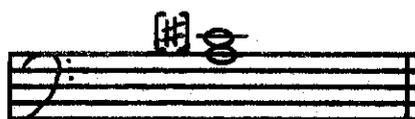
[Figure 7.]

I say that these two intervals are similar to that which is found between D sol re and F fa ut; each of these is the same as the ancient semiditone, and necessarily dissonant.



[Figure 8.]

I say, in addition, that this [Figure 9.] is a major interval, but this [Figure 10.] is not.



[Figure 9.]

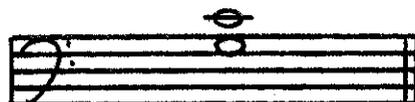


[Figure 10.]

I say that the present [Figure 11.] is dissonant and greater than this [Figure 12.].

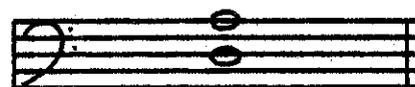


[Figure 11.]



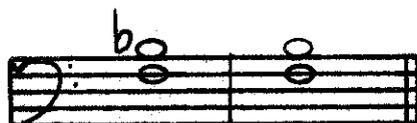
[Figure 12.]

I say that this [Figure 13.] is not distant by a diapente.



[Figure 13.]

The two second notes in the following example [Figure 14.] are not distant by a tritone, considered, however, in the manner previously described, that is in a fourth in the low part and in a minor semitone in the high part.



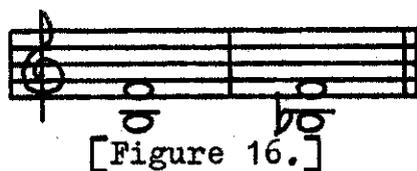
[Figure 14.]

These [Figure 15.] are not distant by a semidiapente, considered in two minor thirds.



[Figure 15.]

I say that the two second notes are not distant one from the other by a diapente when, however, they are considered in a semidiapente in the high part and in a minor semitone in the low, according to what the example below shows.



[Figure 16.]

I say that there is not the same distance between these [Figure 17.] notes as between the present ones [Figure 18.].

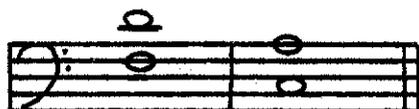


[Figure 17.]



[Figure 18.]

I say that the two intervals of the example which follows are not major sixths.

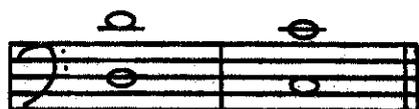


[Figure 19.]

It is also not true that the present notes [Figure 20.] are distant from each other by a minor seventh, nor is anyone of these [Figure 21.] distant by an octave.

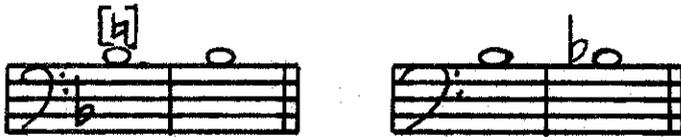


[Figure 20.]



[Figure 21.]

These [Figure 22.] are equally distant.



[Figure 22.]

These [Figure 23.] are the same distance apart.¹⁸



[Figure 23.]

Since you now possess a detailed understanding of these numbers and know why each of the first sixteen intervals demonstrated is itself contained among them, we may now discuss the names of the proportions which contain them, how these proportions are constructed, what parts of them are more or less remote or near, and to what degree one proportion exceeds the other. By means of these principles I will come to answer the questions just now posed, and by following

[6]

¹⁸See pp. 185-186 below.

the proposed order, we will see all these particulars in the minor and major semitone, more than in any other interval. However, since I desire that you understand me with the greatest facility I can possibly engender, it is first necessary to know--according to the common opinion of practical musicians and theorists of our times who believe that what is sung today in the diatonic genus is the syntonon of Ptolemy--their proper position and thus the position of major and minor tones, and by what order they proceed through each octave, going as much through h durum as through b molle. The example which follows shortly [Example 1.] will give you this information. In this example annotations will be made between the pitches according to the opinion of the same Ptolemy. Nor can they be arranged in any other order, because--as it appears in the distribution of Didymus¹⁹--they would be

more clearly displayed in convenient signs. Moreover, the nature of the diatonic syntonon of Ptolemy is to proceed from low to high in each of its tetrachords, by sesquiquindecima, sesquioctava (or tone), and sesquiquinta. In order to be better understood by our practicing contemporaries, we will say that each tetrachord proceeds from low to high by major semitone, by major tone, and by minor tone, and on the contrary, from high to low by minor tone, by major tone, and by major semitone. And such, according to what pleases Reverend M. Gioseffo Zarlino, is the genus in which one composes, sings, and plays today. To the reproof of his opinion, we will dedicate the beginning

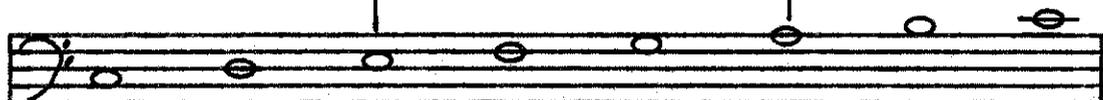
Opinion of Zarlino re-proved by the author in the Institutioni harmoniche, book 2, chapter 16.²⁰

¹⁹J. Murray Barbour, Tuning and Temperament (East Lansing, 1951), p. 21. "Dydimus' diatonic contains the same intervals as Ptolemy's syntonic ditoniaion, but with the minor tone (10:9) below the major tone (9:8) instead of the reverse."

²⁰See Zarlino, Istitutioni, p. 83.

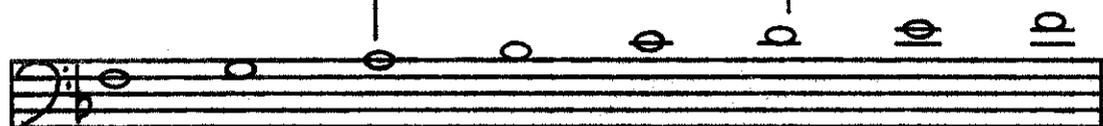
Zarlino,²¹
Institutioni
harmoniche,
 book 2,
 chapter 39.

Tetrachord Meson of
 the Diatonic Syntonon
 of Ptolemy



Major tone Minor tone Major semi-tone Major tone Minor tone Major tone Major semi-tone

Tetrachord Synemmenon
 of the Diatonic
 Syntonon of Ptolemy



Major tone Minor tone Major semi-tone Major tone Minor tone Major tone Major semi-tone

[Example 1--Location of Tones and Semitones in the Diatonic Syntonon of Ptolemy.]

²¹See Zarlino, Istitutioni, pp. 120-122.

[Minor Semitones of the Syntonon]²²

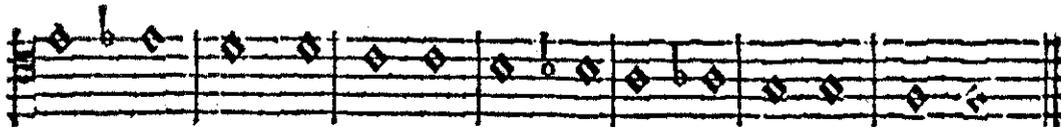
Minor Minor Minor Minor Minor Minor Minor
 semi- semi- semi- semi- semi- semi- semi-
 tone tone tone tone tone tone tone



Minor Minor Minor Minor Minor Minor Minor
 semi- semi- semi- semi- semi- semi- semi-
 tone tone tone tone tone tone tone

[Example 1--Continued]

²²In the original example of minor semitones, the sharps were omitted, probably through a printing error. Compare Zarlino, *Sopplimenti musicale*, p. 137, where the sharps are present in a quotation of the identical example from Galilei, *Dialogo*, p. 6.



Semit. min. Semit. min. Semit. min. Semit. min. Semit. min. Semit. min. Semit. min.



Semit. min. Semit. min. Semit. min. Semit. min. Semit. min. Semit. min. Semit. min.

of our discourse, as I have promised, in order to satisfy our first demand. Here, I warn you that I do not understand that there are intervals in the syntonic other than the pure diatonic ones and the others called chromatic by modern contrapuntists. One sees, according to examples of the modern practice, that the major semitone is indicated [by a sign] on various pitches and the minor semitone remains on the same pitch with some sign adjoined [i.e. by key signature]. From this it necessarily follows that the *b molle* never occurs on a minor semitone in the low register, just as the diesis \times does not ever occur in a minor semitone in the high register. The reverse happens to the major semitone, which happens to be exactly the opposite of that which occurs in the ancient diatonic ditoniaion. Coming at last to discuss the quality and size of the intervals, I say that among the different ones which the musicians

B molle never occurs in a low minor semitone nor the diesis \times in a high minor semitone.

and theorists commonly keep in a safe ratio, two of them are dissonant, having been called semitones by those theorists. In order to distinguish one from the other, I accompany them, as you know, with one of these words: major or minor. They say that the minor semitone is that which is contained between the numbers 25:24. This interval has, according to its authors, a proportion of sesquivicesimusquartas. Similarly, from the proportion sesquiquindecima [16:15] they construct the major semitone. On the contrary, in the diatonic, the minor semitone, which is also called limma, is contained between the limits of 256:243; half of this they then call the diaschisma. The major semitone, also called apotome, is found between these other limits, that is, 2187:2048.²³

[7]

The significance of limma.

The meaning of of diaschisma.

²³See Willi Apel, ed., Harvard Dictionary of Music (Cambridge, Massachusetts, 1969), p. 710, for an explanation and an illustration of the limma, the apotome, the schisma, the diaschisma, and the Pythagorean and Ptolemaic commas. These terms are also explained in Barbour, Tuning and Temperament, pp. ix-xii.

Strozzi: For what reason did those first speculative musicians construct the major and minor semitones between such numbers in the diatonic scale?

Bardi: Because of this, the ancient musicians called minor semitone that portion of the diatessaron left over after two tones had been subtracted from it, and because when two sesquioctaves had been subtracted from the sesquitertia, there remained the proportion of 256:243. In this way they constructed such a semitone and called it minor because two of them added together did not fill up the void of the tone. Therefore, they subtracted the minor semitone from the tone and called the remainder a major semitone; two of these exceed the tone.

Why the major and minor semitones are within those numbers.

Strozzi: What is the meaning of that term, limma?

Bardi: Limma means the same as remainder. This was very appropriate since the limma was nothing else, as I have said, but that remainder of the diatessaron after two tones had been

subtracted. The Greeks also called limma that part of a thing which taken twice does not fill up the whole thing. They also called the major semitone apotome, which means in that language, "detached". For example, when an apotome has been taken from the ditone, there remains the semiditone. There have been others who have understood as a major semitone the superquintaspartiente 76, which is the form of the second interval of each tetrachord of the ancient chromatic. This, in lowest terms, comes within these numbers, 81:76. Having this knowledge, we can understand, when we subtract the sesquivicesimusquartas [25:24] from the sesquiquindecima [16:15]--these are the forms of the syntonic semitones--how much the major semitone exceeds the minor. According to Boethius,²⁴ the manner of subtracting one musical interval from

The meaning of apotome.

Method of subtracting one interval from another.

²⁴See Bower, Boethius, pp. 395-453, with regard to Boethius' musical mathematics and Pythagorean number theories.

another is this. First, one arranges the least terms of their proportions placing those numbers which contain the minor interval below, and those which contain the major interval above.

16	15	Form of the major semitone
	X	
25	24	Form of the minor semitone

[Figure 24.]

When these numbers have been placed in such a fashion, they represent, at first glance, a way contrary to that used by the arithmetician in subtracting one number from another. Nevertheless, the effect is the same. The music theorist, however, does not consider the mere value of numbers like the arithmetician, but the quantity of sound which is included between them. Because, most of the time, the smaller numbers contain the larger intervals, it happens that the opposite of what the intellect understands appears to the senses.

Strozzi: Is it not true, therefore, that the smaller terms always contain the larger intervals?

Bardi: No, Signor. In fact, the larger intervals are contained by superparticular ratios, except the dupla and tripla, which are multiples. These smaller intervals, besides some others which appear as superpartient ratios, are the limma and the apotome. When the numbers have been set out in the demonstrated manner, we will multiply the 16 (larger term of the sesquiquindecima) by the 24 (smaller term of the sesquivicesimusquartas) and the 25 (larger term of the sesquivicesimusquartas) by the 15 (smaller term of the sesquiquindecima). From those multiplications we will have these products, 384:375. By finding their common divisor we will come to reduce them into their lowest terms because one can more easily comprehend the quantity of the interval and sound when it is contained in its lowest terms. To do that, I first measure the 384 term,

Manner of finding the divisor of proportions.

which is the larger number, by the smaller, which was 375. The remainder is 9. Since this is not of common measure with them, it cannot, consequently, be the desired divisor. Therefore, I measure again the smaller term, which was 375, by the 9. The remainder is 6. Having considered this, I find that not even this is common measure with each term, but only with the greater, which was 384, into which it goes 64 times. However, I measure once more the first and greater remainder, which was 9, by the lesser, which was 6, and 3 is left. This is necessarily the desired divisor. Because of this, when the two larger numbers which were first considered are divided, the result is 128:125. Since these numbers can in no way be reduced into smaller numbers, they come to be in lowest, or root terms. The interval which is contained by these is something more than a comma and a half. Thus, the minor semitone is exceeded by the major by such a quantity. We can easily verify this by

adding this quantity with the proportion and numbers which contain the minor semitone, because the product they will give will have the same form as that which contains the major. In order to do this, the same order is maintained, first arranging the numbers of their forms in this manner.

Method of adding proportions together.

128	125	Form of the supertripartiente 125
25	24	Form of the minor semitone [sesquivicesimusquartas]

[8]

[Figure 25.]

When one then multiplies the 128 (larger term of the supertripartiente 125) by the 25 (larger number of the sesquivicesimusquartas) and the 125 (smaller term of the supertripartiente 125) by the 24 (smaller term of the sesquivicesimusquartas), the following products will be obtained, 3200:3000, which are related to the proportion sesquiquindecima [16:15]. Within these numbers is included the major semitone, not in its root terms. If one

wants to reduce them, the aforementioned rule is to be observed.

Strozzi: I hope it will not be any trouble for you to answer me in this case.

Bardi: I subtract from the larger term, which is 3200, the lesser term, which is 3000; the remainder is 200. This excess, since it is of common measure with each term, is also their divisor. Therefore it is not necessary to seek any further. In that way, when the 3200 has been divided by 200, the results are 16 and 15. When the 3000 is divided by the same 200, these smallest terms (which are 16:15 as I have said), contain virtually the same major semitone, but between prime, not between composite and communicant numbers, like those larger ones. Regarding the reduction of multiple and superparticular intervals into lowest terms, I could also produce the multiples through dividing them by the smaller term and the superparticulars by dividing them

Prime numbers are those which are not numbered by other than one. The composites can be measured by other numbers.

by the difference, according to the rule. When ascending by step toward the high register (according to the promised order) the proportion sesquinona follows the sesquiquindecima. What is contained by the sesquinona is called today the minor tone by our practicing contemporary musicians and theorists. After the sesquinona comes the sesquioctava which is called major tone to differentiate it from the minor tone. I say that the minor tone is contained in its true form by the sesquinona proportion between the numbers 10:9. The major tone is contained by the sesquioctava proportion between different numbers, that is 9:8. We can easily see, with only this small amount of information, which of these consists precisely of the major and minor semitone without either a deficiency or a surplus in any of its parts. It will be one whose form is entirely similar to the root terms of the product which results from adding them and then

Which tone consists of the major and minor semitones.

reducing them into lowest terms. Because I have already spoken sufficiently about the manner of adding, subtracting, and finding the divisor of the proportions, it will suffice for the future--when these needs occur--that I form the example of what I must demonstrate so that you will understand everything easily. I now want to demonstrate--in the manner just now prescribed--which one of the two tones is not suited, due to its extremes, to contain an interval larger than the product obtained by adding together the terms and forms of both semitones. We will see this in the example below.

	25	24	Form of the minor semitone
	16	15	Form of the major semitone
	<hr/>		
	400	360	Form of the minor tone, not in lowest terms
40			
	10	9	In lowest terms

[Figure 26.]

We have derived the minor tone from two semitones added together. This agrees with what Zarlino proves in his Dimostrationsi.

Strozzi: This fact does not exactly agree with the opinion of the pure practical musician, nor is it at all surprising that the major tone is expected from these [numbers]. However, please proceed.

Bardi: Now when the minor tone has been subtracted from the major, we will be able to see clearly how much larger the major is than the minor. This we will understand from the next example.

9	8	Form of the major tone
	X	
10	9	Form of the minor tone
<hr/>		
81	80	Form of the sesquioctagesima, called comma today

[Figure 27.]

Zarlino,²⁵ at proposition 19 of the 1st discussion and at proposition 33 of the 3rd discussion [in the Dimostrationsi armoniche].

How much the major tone exceeds the minor.

²⁵Gioseffo Zarlino, Dimostrationsi armoniche (Venice, 1571), p. 173.

The sesquioctagesima results from subtracting the minor tone from the major. The resultant quantity is called comma by our practicing contemporaries.

Strozzi: Is this the same as the ancient comma?

Bardi: No, Signor.

Strozzi: What is the difference?

Bardi: This! The ancient musicians understood the comma to be the remainder of the apotome, after the limma had been subtracted, or, we should say, the remainder of the tone, when two minor semitones have been subtracted. Today the comma is understood--as you have heard--to be that excess by which the tone exceeds the sesquitone. The moderns did not wish to derive it from the difference of the semitones like the ancients for the reason which will be given shortly afterward. The ancient comma, however, is contained within these numbers, 531441:524288. The half of this they then called schisma.

The difference between the ancient comma and that of today.

The meaning of schisma.

Strozzi: Is our comma greater or is that of the ancients greater?

Bardi: The modern one exceeds the ancient one by the following interval, 32805:32768.

Strozzi: How many of our commas do the major and minor tone and the major and minor semitone contain?

Bardi: I will tell you this briefly for now. The sesquivicesimusquartas consists of three commas plus more than a fourth but less than a half of another comma. The sesquiquindecima consists of five commas, plus a little over an eighth of another comma. The sesquinona exceeds eight commas by a little less than half a comma, and the sesquioctave exceeds nine commas by the same amount the minor [tone] exceeds eight commas. Boethius tells us very well how many ancient commas were contained in the ancient major and minor semitones. You can also

[9]

How many commas comprise the major tone, the minor tone, the major semitone, and the minor semitone.

Boethius,²⁶
book 3, chapter
14 and 15 of
De Musica.

²⁶See Bower, Boethius, pp. 202-205; 205-207.

derive true knowledge of how much the tone exceeds the major and minor semitones separately from the example given above [Figure 27.], which proves that the minor tone contains no more than the major and minor semitones. From having seen it, it is understood that the comma added to the minor tone produces the major [tone]. Thus we know how much greater the major tone is than the minor tone. Therefore, concerning the intervals we have discussed up to the present, it only remains to inform you how much the major tone exceeds the minor and the major semitones. The two examples which follow [Figures 28 and 29.] will give you this information.

9 8 Form of the major tone

X

25 24 Form of the minor semitone

216 200 Form of the superbipartiente
 25, not in lowest terms

8

27 25 In lowest terms

[Figure 28.]

The minor semitone of the major tone is exceeded by the superbipartiente 25, which consists of a major semitone plus a comma. This is so very clear that other proof is not necessary. However, we will come with the example which follows to subtract the major semitone from the major tone in order to see what the remainder is.

9	8	Form of the major tone
	X	
16	15	Form of the major semitone
135	128	Form of the super 7 partiente 128

[Figure 29.]

The major semitone of the major tone is exceeded by the super [7] partiente 128. This interval--as one can comprehend from what we said earlier--contains in itself a minor semitone plus a comma. I want--since I am proving these truths to you in many ways--to convince you of the true opinion that one should have of the intervals concerning their value, their

contents, and how much one exceeds the other. We can see very well--from what has been explained up to the present--and comprehend the distance from b molle, when it has been placed on a la mi re or on e la mi, to the diesis X when it has been placed on G sol re ut or on d la sol re, according to what is seen notated in this example.

The diesis X, when placed on G sol re ut and d la sol re is this much lower than b molle when it is placed on a la mi re and e la mi.



[Figure 30.]

In order that honest and upright judgment be made, we will consider--besides that which has been said previously--the quality of the interval in which such accidental signs occur. In addition, we will consider the power which these signs have to exercise in such a place. Therefore, I say first that both intervals where these effects are

employed are in essence minor tones, each one of which is diminished a minor semitone in either extreme by both the diesis X or the b molle. Now let us see what remains of a minor tone after two minor semitones have been extracted. Because part of this operation happened previously when we were examining the quality of semitones, this single admonition will suffice. When taking away two minor semitones from a minor tone, there will remain to that same interval of a minor semitone what is left of the major semitone when the minor semitone has been subtracted from it. The b molle of a la mi re and of e la mi is sharper than the diesis X of G sol re ut and that of D la sol re by the same amount.

Strozzi: This has indeed been a new and gratifying speculation which I have many times desired to understand, not so much for the sake of counterpoint as for that of keyboard instruments.

Bardi: I want to advise you, in addition, that when such a case arises-- although it seldom happens--in major tones, the interval which remains will be a comma less than a major tone, provided, however, that its minor semitones are considered in the form and measure which we will say, at the proper place, are suitable. The minor third, or sesquitone, which has also been called by moderns semiditone, I said to be that interval which is contained in its lowest terms by the proportion sesquiquinta, between these numbers, 6:5. This semiditone, in the diatonic syntonon of Ptolemy--where for the present we will mainly consider all the intervals that it will be necessary for us to measure in order to culminate as much as we have taken into consideration--contains in itself a semitone and a tone, both of them major.

Admonition

The meaning of sesquitone, LeFèvre²⁷ book 3, chapter 24 of the Elements of Harmony.

²⁷See LeFèvre, Musica, p. 26.

Thus, when the numbers which include the intervals just mentioned are added together, the true form of the semiditone will be obtained from their product, as appears through the example which follows.



[10]

	9	8	Form of the major tone
	16	15	Form of the major semitone
	<hr/>		
	144	120	Form of the minor third not in lowest terms
24	6	5	In lowest terms

[Figure 31.]

Strozzi: Is our semiditone the same as that of the ancients?

Bardi: It is in no way the same, because ours is consonant, as you know, and is produced in the superparticular genus by the sesquiquinta proportion. The ancient semiditone, as all musicians affirm, is dissonant since it is contained in the

The meaning of semiditone.

superpartient genus between these numbers, 32:27.

Strozzi: How have so many different things been understood and distinctly known by the same name?

Bardi: They are known with the greatest difficulty and confusion, certainly, for the one who deals with them and for the one who listens. However, I will proceed with the greatest distinction and facility of which I am capable, in order to be more clearly understood by you, although my voice is rough in nature. Since it is true, moreover, that the minor third consists of a major tone and a major semitone and, consequently, is contained by the sesquiquinta--as Zarlino²⁸ particularly affirms in proposition twenty-six of the second discussion of his Dimostrationsi--it necessarily follows, contrary to the opinion of the practical musician, that the examples below [Figure 32.] are not actually minor thirds of the same proportion and measure as the first two shown.

The minor third is not found between D and F fa ut.



[Figure 32.]

That only happens because each of these contains in itself no more than a minor tone and a major semitone. These intervals together are not suited to produce a minor third of the measure and proportion of those first intervals, but only a dissonant semitone of the ancient diatonic ditoniaion. Although the senses, according to what has been said, accept it without repugnance, we will nevertheless prove it to the intellect in this manner. Add together the numbers which contain both intervals, according to the example which follows [Figure 33.], and then notice what product results. When the two aforementioned intervals have been added together, the result is

²⁸ See Zarlino, Dimostrazioni, p. 130.

the superquintapartiente 27, which is the true form of the semiditone of the diatonic ditoniaion.

	10	9
	16	15
	160	135
5		
	32	27

[Figure 33.]

This semiditone is dissonant only because-- as has been said on a previous occasion-- it is a comma less than the minor third of the syntonon of Ptolemy. In addition, its form is found in the superpartient genus, which is unsuitable, according to Pythagoras, for a consonant interval, and [is found] outside of the senary number.²⁹

²⁹The senary number, or six, held a mystic significance for Renaissance musicians. They considered six to be the first perfect number, that is, the sum of all the numbers of which it is a multiple (1+2+3=1x2x3=6). Its musical significance was the fact that all the primary consonances could be expressed as superparticular ratios, employing only numbers from one to six. For a fuller explanation of the ramifications of the senary number, see Zarlino, Istitutioni, pp. 23-28.

Strozzi: Where do you suppose that the musicians of today have obtained such a fine consideration that between the parts of the senary number is contained each simple musical interval and part of the consonant compound intervals?

Bardi: In considering the order in which the proportions are placed in the second genus of greater inequality called superparticular, I firmly believe that this occasion was brought about by having joined the ten prime intervals together two by two in natural order, afterwards reducing them into their lowest terms.

Strozzi: How, may I ask?

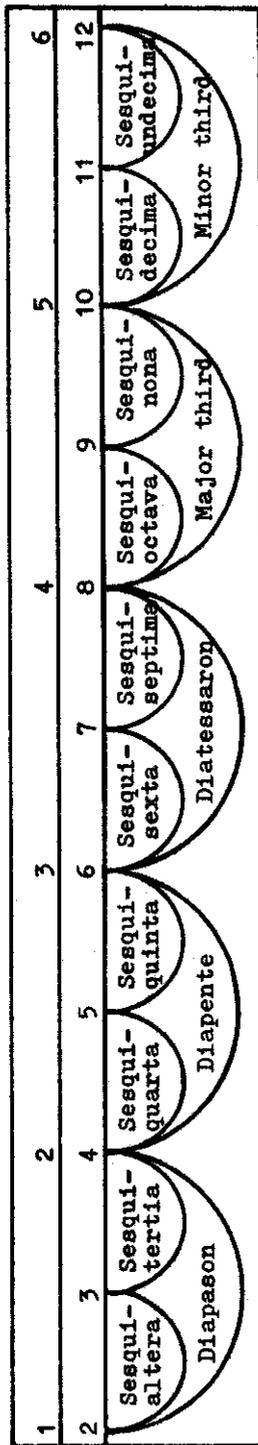
Bardi: Here is a convenient example which, with nothing more, will inform you of everything I feel about it [Diagram II]. That consideration also may have been derived from the eighth chapter of the third book of the Harmonics of Ptolemy or from the fourteenth chapter of the first book of his Quadripartite [or Tetrabiblos]

The origin of the power of the senary number.

[11]

[DIAGRAM II]
 [INTERVAL RATIOS ACCORDING TO THE
 SUPERPARTICULAR GENUS]

Numbers arranged according to the nature of the superparticular genus, between which one finds in use not only the form of any desired simple musical interval but in power each one of the mixed and compound intervals. He who proceeds further will also discover those which contain the major and minor semitone. When their numbers are considered otherwise, one will obtain the form of any other desirable interval.



where he ingeniously proceeds to compare the aspects of the planets to the forms of the musical intervals of his times, speaking thus:

Ptolemy³⁰
compares the
aspects of
the planets
to the musical
intervals.

The tetragon or square, compared to the trine, makes sesquitertia. Compared to the hexagon (or sextile, as we should say), it makes sesquialtera. Compared to the opposite side, it makes dupla, and with the entire circle of the Zodiac, it makes diapason plus diapente. All of this, again compared to the square, makes disdiapason. Finally, three squares compared to two trines have the same relation as nine has to eight.

Strozzi: Are the forms of the imperfect consonances not found also among these aspects?

Bardi: No, Signor, because Heaven does not permit or tolerate imperfection. Whoever wishes to investigate more carefully may also derive from each of the two places cited the forms of all the other musical intervals of our times. However, enough has been said.

Strozzi: I believed that this power of the senary number was entirely a new

³⁰ Compare Ptolemy, Tetrabiblos, translated by F. E. Robbins (Cambridge, Massachusetts and London, 1940), p. 73-75.

discovery, and I see that I was probably not correct. This causes me to doubt that any of the other things (concerning inventions) which are very old are represented as new [things] by one [writer] or another.

Bardi: Do not doubt it at all, because the simple-minded many times, in reading some book about any subject, believe, through little experience, that those things are found in no other book but that one. In most cases, the same things have been written in many books a good many years before.

Strozzi: There is a division of the square into three equal parallelograms, the middle one of which is separated into two equal parts, then bisected by a line which runs from one of the angles of the square and is placed over the opposite side so that it is divided into two equal parts. These different portions of the line, compared with one another, have power to give the forms of a greater number of musical intervals

than the senario and its parts. Is this parallelogram an ancient or, rather, a modern invention?

Bardi: The parallelogram is question is taken from the bulk of the second chapter and book of the Harmonics of the same Ptolemy.³¹ Concerning this, he jokingly tells as much as his purpose requires in order to denote the musical intervals of those times. From what I will now tell you, you will easily be able--when you have an interval outside of its true or lowest terms--to perceive if it is enlarged or reduced from its normal condition. I warn you, however, that if its smaller term is diminished from that which is appropriate to its natural state, it is a clear indication of being enlarged. If the smaller term

³¹Strozzi and Bardi are speaking of Ptolemy's helicon, a device for illustrating the consonances by means of intersecting straight lines and for showing the relative string lengths for each pitch. See Düring, Ptolemaios, pp. 60-63. There is an explanation of the helicon in Cecil Adkins, "The Theory and Practice of the Monochord," unpublished doctoral dissertation, Department of Music, State University of Iowa, Iowa City, Iowa, 1963, pp. 53-56.

is larger than usual, such an interval is in reduced form. How much it is enlarged or reduced can be ascertained by subtracting from it or adding to it whatever amount you consider to be more appropriate. The opposite effect would occur in its larger term.

Strozzi: I have not completely understood this last conception of yours.

Bardi: Now I shall make myself clearer. We have instead of the sesquiquinta--since it has been assigned by contemporary teachers of counterpoint as the form of the minor third of the syntonon of Ptolemy--the superquintapartiente 27. Since we now wish to see if this proportion is larger or smaller than the sesquiquinta, the excess with which the 32, its larger term surpasses the lesser term which is 27, is able to explain it sufficiently. This excess is 5. Now consider how many times this 5 will go

Warning.

into the smaller term of this proportion; it goes five times with a remainder of 2. In the sesquiquinta, however, the excess by which the larger number exceeds the smaller, which is 1, goes in five times exactly. It clearly appears that what I have said is true, that is, since the smaller term of the super 5 partiente 27 is larger, when compared, however, to that of the sesquiquinta, it is consequently smaller by as much, then, as has been said previously.

Strozzi: I have understood everything very well, but tell me another detail. Since the semiditone is not the same thing as the minor third, with which of these does the trihemitone agree? I have found that this same trihemitone has been given three different names by various authorities.

Bardi: The trihemitone, as you know, is the highest interval of each tetrachord of the ancient chromatic

The meaning of trihemitone.

and does not agree with either the semiditone or the minor third of the syntonon because it falls under the proportion supertriptiente 16, between these numbers, 9:16, when in lowest terms. Thus it happens to be less than the minor third and greater than the semiditone. Zarlino³² agrees with this opinion in chapter thirty-four of the second part of his Institutioni harmoniche. It remains for us to see in this case how much the minor third exceeds the major and minor tone. This will be explained in the two examples which follow [Figures 34 and 35].

6	5	Form of the minor third
	X	
9	8	Form of the major tone
48	45	Form of the major semitone not in lowest terms
3		
16	15	

[Figure 34.]

³²Zarlino, Institutioni, p. 113-114.

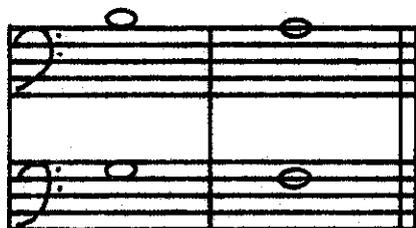
The minor third is larger than the major tone by a major semitone. No further proof of this is necessary, since it has just now been seen in combining both of the semitones. However, let us see how much the minor tone is exceeded by the minor third. [12]

	6	5	Form of the minor third
		X	
	10	9	Form of the minor tone
	<hr/>		
2	54	50	Form of the superbipartiente 25 not in lowest terms
	27	25	In lowest terms

[Figure 35.]

The minor tone is exceeded by the minor third by the superbipartiente 25, which consists of a major semitone, and a comma, according to what was proved above. They say that the major third is that imperfect consonance which arises from the proportion sesquiquarta and which is contained in lowest terms by the numbers 5:4. In addition, it consists of the major and minor

tone, which we will see, according to this [next] example, by adding them together and noting the result.



9	8	Form of the major tone
10	9	Form of the minor tone
<hr/>		
90	72	Form of the major third not in lowest terms

18

5	4	In lowest terms
---	---	-----------------

[Figure 36.]

The major third results from adding together both tones. From this knowledge, together with subtracting the minor third from the major third, we will be able to learn how much one is exceeded by the other [Figure 37.].

5	4	Form of the major third
	X	
6	5	Form of the minor third
<hr/>		
25	24	Form of the minor semitone

[Figure 37.]

The minor third is exceeded by the major third by a minor semitone. Further proof of this would be adding to it the minor semitone in question. The simple practical musician possesses other species of major thirds through the use of these signs, X and b, as accidentals of different pitches. They are considered by him to be of the same importance as the first demonstrated. This, in my opinion, is one of the greatest abuses he perpetrates--among many many others, besides the ones I have already mentioned--which I have selected to demonstrate for you.

Strozzi: I am greatly surprised that among so many excellent men who have written about the art of music so cleverly and wisely, there still have

not been any, as far as I know, who have, for mutual benefit, revealed and discovered such obvious errors as these.

Bardi: That is not exactly surprising. It happens that such a consideration could never occur to those who believe in singing the diatonic ditoniaion, because the cause of that [situation] was not found there in any way, but only the effect. Regarding those, then, who have said that the species which is sung today is the syntonon of Ptolemy, I can easily believe, because of my many encounters with them, that they were very well aware of the problem. Because revealing it did not precisely suit their purpose, however, as I said on another occasion, they have passed over it in silence since, impelled by a vain, ambitious thought, they valued more highly the impertinent novelty of their whims than the convenience and usefulness which they would have been

able to give to the public by explaining the truth. In this way, they would not have corrupted and spoiled the names of the most important things pertaining to the art of music. Moreover, the thirds which I have mentioned are contained between the notes of the present example.



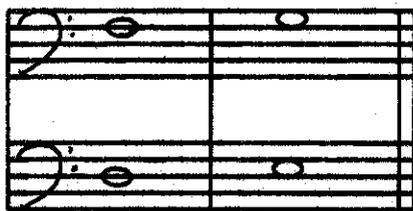
[Figure 38.]

The major third is not found between these pitches.

Since each one of these [intervals] is made up of two major tones, as a result they all happen to fall under the proportion 81:64. When it has been well considered, one will find in this 81:64 proportion each one of those particular accidentals which are found in the superquintaspertiente 27. This cannot in any way be the natural major third of the diatonic syntonon, but only the dissonant ditone of the diatonic.

ditoniaion--as Zarlino³³ affirms in the fourth discussion of his Dimostrationsi at the third definition. This proceeds from high to low in each of its tetrachords by tone, tone, and limma. It is quite true that the examples which follow are contained by their proportion of sesquiquarta. Consequently, they are consonances, because the interval which has first place in the low register is a minor tone. That which is found in the second [place] between F fa ut and G sol re ut is also a minor tone. In addition, I wish to warn you that you should consider the excess by which the larger term of the proportion which contains the ancient ditone, which is 17, exceeds the smaller term.

[13]



[Figure 39.]

³³Zarlino, Dimostrationsi, p. 268.

You will find--according to the rule given you a little earlier--that because it surpasses its smaller term, which was 64, close to four times, there is clear indication that the aforesaid proportion remains the true state of the sesquiquinta.

I say that the diatessaron is that middle interval which springs from the sesquitertia proportion, which is contained in lowest terms by these numbers, 4:3.

It is not inappropriate that I mentioned the fourth to you with the name of middle interval, for it happens that among five consonant intervals, the forms of which are found between one and six, and the other intervening numbers which are placed between them in natural order, it comes third. Two of the five consonant intervals, the octave and the fifth, are in the low register. Two others, the major and minor third are in the high register. In addition, the sound which issues from the extremes of the fourth is, in its simplicity of nature, not at

Diatessaron is an interval midway between consonance and dissonance.

all offensive to the ear as the dissonances are, nor does it have the quality to delight it as the other consonances do. Therefore, deservedly, it can be stated to be midway between dissonance and consonance; no one should wonder about that [fact] because Nature is not accustomed in her undertakings to pass from one extreme to the other without touching the middle. As the smallest of the perfect intervals, it is less consonant than the other, being farther from perfection. In the same way, the smaller imperfect consonances are more harmonious than the larger ones, being farther from imperfection. Exactly the opposite happens to dissonances when sevenths are applied to perfect consonances and seconds are applied to imperfect consonances, as we will clearly demonstrate in the appropriate place. I now return to the fourth, saying that principally it contains in itself a major semitone, a major tone, and a minor tone. It may be considered as composed of the minor third

and the minor tone, which we will see, according to the example, from the product which results from adding together the lowest terms of both of these intervals.



6	5	Form of the minor third
10	9	Form of the minor tone
<hr/>		
60	45	Form of the fourth, not in lowest terms
5	4	3
		In lowest terms

[Figure 40.]

The fourth may also be obtained in the same proportion and measure from the product of the major third and the major semitone. We will see this in the following example [Figure 41.] where their forms are added together.



5	4	Form of the major third
16	15	Form of the major semitone
<hr/>		
80	60	Form of the fourth, not in lowest terms
20	4	3 In lowest terms

[Figure 41.]

Since the fourth--because it is consonant and in its true proportion--contains the amount I have said and proved, it necessarily follows--contrary to the opinion of the simple practical musician--that the examples below are neither consonant fourths, nor in the same proportions as the first ones. This agrees with what Zarlino³⁴ says in proposition twenty-eight of the second discussion of his Dimostrationsi. This happens because each one of the fourths contains two major

In proposition 28 of the 2nd discussion.

³⁴Zarlino, Dimostrationsi, p. 131.

tones, and a major semitone which separates one tone from the other. When the proportions which contain these intervals have been added together, we will have from their product a proportion which is a comma larger than the sesquitertia.



[Figure 42.]

The extremes of this will necessarily be dissonant. Even the simple practical musician can ascertain through the example given that the tones are what I have said they are. However, I will prove it to you as a theorist, in the example which follows, by means of arithmetic.

The diatessaron is not found between these pitches.

6	5	Form of the minor third	[14]
9	8	Form of the major tone	
<hr/>			
54	40	Form of the super 7 partiente 20, not in lowest terms	
2			
27	20		

[Figure 43.]

From this calculation we have obtained the super 7 partiente 20 which, as I told you, consists of a fourth plus a comma. This can be seen easily by subtracting the comma from the fourth. Besides, if this were not so, it would follow that in descending into the low register with a tone of the quality which is contained between the notes G sol re ut and a la mi re, the extremes would not sound as a whole fifth. This is not exactly true, as will be seen at the appropriate place. Now, I want us to see if the examples below [Figures 44 and 45.], when considered with a third uppermost and a tone below, are whole sesquitercias or not. I also want to know how the theorist considered it and why. I have proposed this question because I desire to appease each difficulty and doubt which could possibly arise concerning this important topic of our discussion. A little earlier, I proved to you in regard to the minor

third that when ascending between D sol re and F fa ut, such an imperfect consonance was not actually contained in the sesquiquinta, but only the ancient semiditone.



[Figure 44.]³⁵

At present, when we consider the fourth of this example--as much through b molle as through h durum--and each one of these [fourths] has been composed of the interval which I have mentioned [i.e. semiditone] and the major tone, we will find

³⁵The pitch "a" in the original example [Dialogo, p. 14] has been emended to an "f" by an unknown hand.



that when the numbers which contain them have been added together, they will give a whole sesquitertia, as you will see from the following example.

	32	27	Form of the semiditone
	9	8	Form of the major tone
	<hr/>		
	288	216	Form of the fourth not in lowest terms
72	4	3	In lowest terms

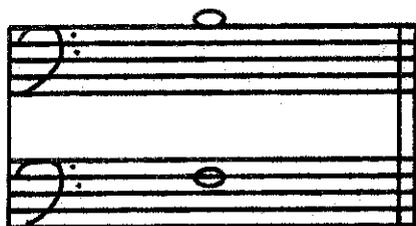
[Figure 45.]

We have obtained the fourth in its true form by adding together two intervals. This did not occur previously, because the third of this next example [Figure 46.], in comparison with the first one shown, has been diminished by the same amount the tone was augmented in the earlier example [Figure 44.]. The fourth in the ancient diatonic also may be considered as composed of such parts. This concludes our consideration of the fourth, since I am informing you how much it exceeds the major and minor thirds. The two examples [Figures 44 and 45.] given you when we

considered the fourth as composed of the intervals just mentioned, and of the major semitone and the minor tone, will show this fact. Now, I come to tell you some particulars about the tritone, which our practicing contemporaries say is contained by the proportion super 13 partiente 7 between these numbers, 45:32.

It contains in itself three tones and took its name, perhaps, from those contents. However, this only refers to the diatonic ditoniaion where these tones are equal. One can consider this dissonant interval to be composed of a major third and a major tone in the low register as well as in the high. We can ascertain that the aforementioned proportion is the true form of the syntonic tritone in many ways. Among them, the one which is the plainest and easiest, I will now show you. Add together the numbers which contain the intervals mentioned above, its propinquous parts, which will give the same numbers which I said contained it.

The origin
of the name
"tritone".



5 4 Form of the major third

9 8 Form of the major tone

45 32 Form of the tritone

[Figure 46.]

The tritone in its true form and in lowest terms is obtained from the product of the major third and the major tone. This interval, together with its derivative, since their forms and extreme sounds are very disproportionate, so to speak, and consequently are understood with difficulty by the intellect, they cannot be considered nor are they found naturally in the octave in any more than two pitches. This is not at all surprising since it happens that nature seldom produces either disproportionate or monstrous things. We can immediately

Nature seldom produces either perfect things or monstrous things.

see how much the tritone exceeds the fourth by subtracting one from the other.

45	32	Form of the tritone	[15]
	X		
4	3	Form of the fourth	
135	128	Form of the super 7 partiente 128	

[Figure 47.]

The tritone exceeds the fourth by the super 7 partiente 128. This proportion consists of a minor semitone and-- contrary to the opinion of the practical musician--a comma besides, as has been demonstrated previously in another place. It seems impossible to the practical musician that when the fourth has been subtracted from the tritone more than an ordinary minor semitone must remain. All the difficulty that he has in understanding what is to him a novelty results from not having the knowledge which he should have of the intervals he has continually contrived. This

knowledge calms those who have it perfectly. That does not escape his reproach, since it is impossible for those who do not understand the propriety and virtue of the matter, whatever it may be, to exercise it well. And this is said by many--even all the wise men and scholars--to be one of the principal causes, among many others, that the musical practice of our times does not have that power to operate in the minds of the hearers any of those marvellous and virtuous effects which the ancient practice had. Returning to the tritone, however, I say that it cannot be obtained by adding together the form of the fourth and that of the minor semitone, since a major tone must be used as the divisor in order to accomplish that. The major tone is capable of containing a comma in addition to the major and minor semitone, according to what has been proved another time, and as one can prove again by adding them together. On some occasion a proportion could occur to you between large numbers

with so little difference between them that you would not know--in a manner of speaking--what the difference was between one interval and another, although you very much wished to see this difference. It is also not impossible in this manner of manipulating the numbers, as in some other cases, to subtract a very large number from a very small one. When you have been warned, however, by what I am about to tell you and show you with an example, you will ascertain which of these contains the greater [quantity] and which the smaller. Therefore I say, first, that a large interval can be subtracted from a small one. For example, a major tone can be subtracted from the comma, although the comma is much smaller than the major tone. And in order that that may be done, we will prove it to the senses with the present example [Figure 48.].

Warning.

81 80 Form of the comma

X

9 8 Form of the major tone

648 720 Form of the subsesqui-
nona not in lowest
terms

72

9 10 In lowest terms

[Figure 48.]

It seems, at first glance, from having subtracted a minor tone from the comma, that, consequently, the comma exceeds the tone by a certain quantity. This offends each one who knows about it, in regard to the intellect and the sense. This is to warn you, however, that the smaller number of that interval which we have sought to subtract from the comma has come to the place of the greater number, and, on the contrary, the greater to the smaller. For this reason they do not otherwise come to have the form of the sesquinona, since they are contained, as I told you another time between 10:9 and not, on the contrary, between 9:10.

Warning.

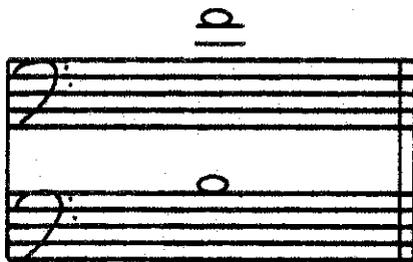
Strozzi: What interval, therefore, will be that which is contained by such a proportion?

Bardi: That is a subsesquinona, which manifests, in such a place, how much the interval from which it was derived is exceeded by the sesquioctave and not how much the comma exceeded it. Therefore, such proportions justly come to be called privatives and rationals, and those others just mentioned, positives and reals.³⁶ This will suffice for all other such cases which may occur to you. I come now to discuss with you the semidiapente, which--according to the opinion of our practicing contemporaries--is that interval which consists of a major tone, a minor tone, and two major semitones. Perhaps it took such a name because its extremes sound a diminished diapente, having been divided, however, into four intervals by five

Why the semidiapente is so called.

³⁶For a fuller explanation of privatives, rationals, positives, and reals, see Zarlino, Istitutioni, pp. 22-23.

limits and pitches. It is contained, moreover, in its lowest terms by such numbers as these, 64:45, which we will consider as composed of a fourth and a major semitone, as much in the low portion as in the high. From these facts one can be capable of [doing] it in many ways, but it is simplest and briefest to add together the numbers which contain the two aforementioned intervals and see if their product will give the form which we have said.



[16]

4	3	Form of the fourth
16	15	Form of the major semitone
<hr/>		
64	45	Form of the semidiapente

[Figure 49.]

One may not consider or compose this interval, as was true of the previous interval, between natural diatonic tones in any octave from any of its parts except the two mentioned. If the practical musician told me he could compose this from two minor thirds of the same form and of equal proportion, I would respond and prove that he is wrong in this, as in many other things, since his goal is only to appease, not the intellect or the sense of hearing, but frequently that of the sight. This is completely, or more than the others, deceived with ease, and the hearing has the same part, or only a little greater one, in distinguishing sounds than the ear has in discerning the differences of colors. The senses are particularly deceived with regard to the smallest differences of ordinary things and usual objects. These, the rational intellect understands with great difficulty. The fact

The goal of
the practical
musician.

that the semidiapente in its demonstrated form cannot be composed of two minor thirds of the same proportion is first manifested to us by the number of its larger term. Although it is squared, this larger term needs to be capable of [containing] the geometric mean.³⁷ The other necessary condition is not found in the larger term because it must also be capable of producing the larger term of one of those intervals into which one will want it to divide equally. This does not happen with the larger number and term of the semidiapente, because it is not related to the six which is the largest term capable of containing the minor third. However, we will put all this into actual practice with a clear example, which will consist of adding together the terms of such intervals. The form of the semidiapente will in no way be derived from their products, but only that of the super 11 partiente 25.

Warning.

³⁷ For an explanation of the geometric, arithmetic, and harmonic means, see Bower, pp. 128-130.

6	5	Form of the minor third
6	5	Form of the minor third
<hr/>		
36	25	Form of the super 11 partiente 25

[Figure 50.]

The interval which we have obtained by adding together two minor thirds consists of a semidiapente plus a comma. If this is true, one may see it by subtracting the super 19 partiente 45 (form of the semidiapente) which will leave a comma. This is the reason. When the theorist considers that the semidiapente is composed of two minor thirds, he is unable to divide any interval of the first three simple genera of proportions rationally into equal parts and consequently not the quadrupla and its compounds. Since he understands everything in terms of arithmetic, he perceives, necessarily, that one is larger and different from the other. This fact I have already shown you clearly with regard to the

minor third. It only remains to see, when such diverse minor thirds are added together, if their product will give the true form of the semidiapente. From this example [Figure 51.] it will be evident that the ancient Ptolemaic musicians had such species under consideration also, because, if they had considered them to be in the same measure, they would not have assigned to the semidiapente--which was capable of containing both--the demonstrated proportion. Instead, they would have assigned the proportion which was derived previously from the product of the two minor thirds, each one of which had been contained by the sesquiquinta. The example which follows will make it clear that the thirds are, therefore, different in form.

32	37	Form of the semiditone
<u>6</u>	<u>5</u>	<u>Form of the minor third</u>
192	135	Form of the semidiapente not in lowest terms
3 64	45	In lowest terms

[Figure 51.]

Here is the semidiapente in its true form composed of the two aforementioned intervals. It is not necessary to see any more of this, unless it is how much it exceeds the tritone. The example which follows will inform us of this, in which the tritone will be subtracted from the semidiapente.

64	45	Form of the semidiapente
	X	
<u>45</u>	<u>32</u>	<u>Form of the tritone</u>
2048	2025	Form of the super 23 partiente 2025

[Figure 52.]

The tritone is exceeded by the semidiapente [17] by the super 23 partiente 2025, which consists of approximately half a comma.

Strozzi: Are the semidiapente and tritone of the ancients of the same measure and proportion as ours?

Bardi: No, Signor, because the interval which occupied the space of the tritone, according to the ancients, fell under these numbers, 729:512, which exceeds ours by a comma. Their semidiapente fell under these other numbers, 1024:729, which, on the other hand, happens to be less than the super 19 partiente 45, called semidiapente by the moderns, by such an interval as one can clearly see by subtracting one from the other.

Strozzi: So that the greater part of names of the musical intervals of today, if not all, are corrupt and spoiled?

Current names
of the intervals.

Bardi: There is no doubt of it at all.

Strozzi: Would it not be wise to provide them before the thing ages and becomes more confused in the minds of men?

Bardi: Yes, indeed. But this is the province of a man of authority and of great worth. However, we will proceed in regard to the names of the intervals, following, although it is poor in quality, the common usage, if for no other reason than to be understood. However, let us proceed at last to the examination of the diapente which they say is contained in its true proportion by the sesquialtera, called also sesquipla, between these numbers in its lowest terms, 3:2. I say that it contains two major tones, a minor tone and a major semitone, which may be considered in many ways. The most usual and important way, however, is being composed of the major and minor third as the product of the numbers which contain it will demonstrate; this will be seen in the example which follows [Figure 53].

Glarean,³⁸ in chapter 12 of book 3, calls the sesquialtera sesquipla.

³⁸See Glarean, Dodecachordon, translated by Clement A. Miller (Rome, 1965), p. 62.



	5	4	Form of the major third
	6	5	Form of the minor third
	30	20	Form of the fifth not in lowest terms
10	3	2	In lowest terms

[Figure 53.]

It will be in its true harmonic form every time the major [third] is in low position and the minor [third] is uppermost. I have said that the fifth is composed of such intervals before each of the others because those are its nearest parts. So that you may also understand the cause of this well, I say that in the opinion of theorists, the nearer parts of intervals are the ones from whose product, when the numbers which contain them have been added together, can be derived not only the same proportion of the whole, but a

The meaning of near and far parts of the intervals.

proportion which is quite close to its same lowest terms, and sometimes--as you have seen with the tritone and the semidiapente--actually in them. On the contrary, the farthest parts [of intervals] are the ones which in their least terms all give the remotest product. One has been derived from their most imperfect and unequal parts and the other from their most perfect and equal parts. Since we are about to consider the other parts of the fifth and to construct it from them, I say that whatever consists of a fourth and of a major tone will be a fifth, because, when the numbers of their proportions have been added together, its demonstrated form will be produced by the product, although it will be a little more remote from its smaller terms [Figure 54.].

4	3	Form of the fourth	
9	8	Form of the major tone	
<hr/>			
36	24	Form of the fifth not in lowest terms	
12	3	2	In lowest terms

[Figure 54.]

Since the fifth originates from those two intervals demonstrated, as has been said, one may argue with Zarlino³⁹ that those of the following example may not result from anything except from different proportions and genera, which is contrary to the opinion of the practical musician no matter how much it is proved to them that they are dissonant.

Zarlino in the 30th proposition of the 2nd discussion of his Dimostrationsi.



[Figure 55.]

They are of different proportions than those first ones because they contain a

³⁹See Zarlino, Dimostrationsi, pp. 132-133.

fourth and a minor tone. These two intervals, when added together, cannot produce a fifth of the same proportion as the two first demonstrated, but another which will be a comma less than these, and therefore dissonant. In order to see how much less it is, here is an example which will verify it.

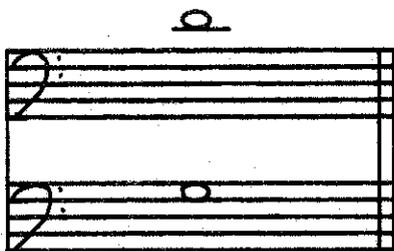
The diapente is not found between these pitches.

4	3	Form of the fourth	[18]
<u>10</u>	<u>9</u>	Form of the minor tone	
40	27	Form of the super 13 partiente 27	

[Figure 56.]

The interval which results from adding together the fourth and the minor tone is the super 13 partiente 27, which consists of a fifth lessened by a comma. The man [i.e. Zarlino] could assure himself more greatly of this fact by adding it together with itself, because he would obtain from both the sesquialtera, form of the diapente. From what we have said, he might wonder

whether the following [example]--considered as a fourth plus a tone with the fourth in high position and the tone in low position--is a fifth of the first or the second species. I would respond to this saying that it belongs to the first species and not in any way to the second.



[Figure 57.]

If he then replied that the tone which is in low position is minor, when it should be major--according to the way I have demonstrated the composition of such intervals--I would tell him that this is completely true. For this reason, it happens to be consonant in that situation because the fourth, which is in high position in that [fifth], is greater by the amount which is lacking to the minor tone, which is in low position, in order to be made equal to the major [tone]. Therefore,

since it is augmented by one interval and diminished by the other because the excess of the first is equal to the deficiency of the second, the fifth can be derived from the intervals mentioned in its true form and proportion. This can be seen clearly by everyone by adding together their lowest terms. The fifth can also be obtained in its true form from the sum of the tritone and the major semitone, but certainly not from that of the semidiapente and the minor semitone, as the following example makes clear.

64	45	Form of the semidiapente
25	24	Form of the minor semitone
<hr/>		
1600	1080	Form of the super 13 partiente 27 not in lowest terms
40		
40	27	In lowest terms

[Figure 58.]

The super 13 partiente 27 is obtained from such a sum. This consists, as has been said, of a fifth diminished by a comma, so that from the semidiapente and from

the minor semitone, one does not obtain, as the practical musician believes, the same product as from the major semitone and the tritone. This does not exclude the knowledge of how much the fifth exceeds the tone and of how much it exceeds the semidiapente. The two examples which follow will give us complete knowledge of this [Figures 59 and 60.].

3	2	Form of the fifth
	X	
45	32	Form of the tritone
96	90	Form of the major semitone not in lowest terms
6	16	15 In lowest terms

[Figure 59.]

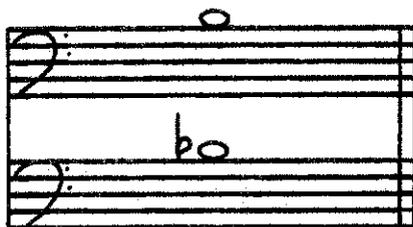
The tritone is exceeded by the fifth by a major semitone, additional proof of which would be adding together the tritone and this major semitone. By adding these one undoubtedly should obtain the fifth, but this is not the case for the matter is clear in itself.

However, we will proceed to demonstrate in the example which follows how much the semidiapente is exceeded by the fifth.

3	2	Form of the fifth
	X	
64	45	Form of the semidiapente
135	128	Form of the super 7 partiente 128

[Figure 60.]

The semidiapente is exceeded by the fifth by the super 7 partiente 128, which-- according to what I proved to you earlier in examining the major tone--consists of a minor semitone plus a comma. Therefore, the practical musician, in addition to being surprised, does not have to tell me that I intend for the same interval to contain first one form, then the other.



[Figure 61.]

The minor semitone in this example exceeds the comma by the sesquivicesimusquartas.

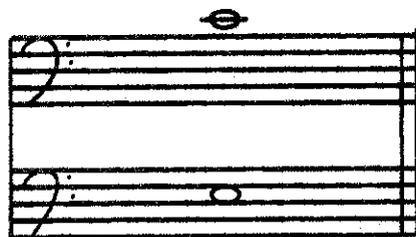
I say that I previously considered the present semitone to be contained by the sesquivicesimus quartas and finally to be contained by the super 7 partiente 128 in which position it is actually capable of as much as I said. I add to this that the interval which is enclosed within the confines of the super 7 partiente 128 is the true form of the minor semitone of the major tone, as I will prove to you a little later.

Strozzi: This will be very unusual to me as well as new and agreeable.

[19]

Bardi: In this proposition I will deal with the smallest possible case desirable regarding the form of the minor sixth, also known by modern practical musicians as the minor hexachord. They say that this interval is that imperfect consonance which is contained by the supertripartientequintas between these numbers, 8:5, in its root terms. This interval consists of two major tones, a minor tone, and two major semitones, and

one may consider that, initially, it is composed of a fourth and a minor third, as we will see from that product which results when the smallest numbers of their proportions are added together. In regard to this, there are some who reprove Franchino [Gaffurio] because he said that the minor sixth is composed of the diatessaron and the larger chromatic interval. He understood this last interval to be the trihemitone, taking it instead of the minor third or, to say it better, the semiditone. These intervals, when added together, are not able in any way to produce the minor sixth in the form in which it is contained in the example which follows shortly [Figure 62.]. They cannot even produce it according to the diatonic ditoniaion, in which Franchino considered it. This happened, perhaps, because he did not regard it as imperfect, as many others do.



4	3	Form of the fourth
6	5	Form of the minor third
<hr/>		
24	15	Form of the minor sixth not in lowest terms
3	8	5 In lowest terms

[Figure 62.]

With regard to this imperfect consonance, there is another consideration which is what the common man believes, and which should not be ignored. According to his opinion, there is found in each musical interval, a species which is not less than the number of its pitches. This is not true, because the species of the minor sixth and of the major sixth are not more than three. In addition such a rule is not verified in any intervals which for brevity's sake are not mentioned. Returning now to the consideration of the minor sixth, I say that

Warning.

it is also that quantity which consists of a semidiapente and a major tone, because when the numbers which contain them have been added together, their product will produce its true form. The example which follows immediately demonstrates this fact.



64 45 Form of the semidiapente

9 8 Form of the major tone

576 360 Form of the minor sixth
not in lowest terms

72

8 5 In lowest terms

[Figure 63.]

Since this is true, it follows--contrary to the opinion of the practical musician--that none of the present examples [Figure 64.], nor any others like them are minor sixths of the same proportions as those first demonstrated.



Minor sixths are not found between these pitches.

[Figure 64.]

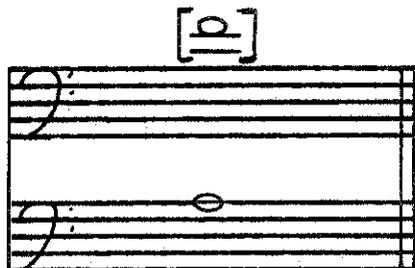
It therefore results that both the tone and the semidiapente cannot produce the minor sixth between the demonstrated pitches as between those above, since the tone in the latter situation is minor, while, on the contrary, it is major in the former. Therefore, the latter necessarily becomes less than the former by a comma. Whatever interval results from these, we will now see from the product which we will obtain by adding together the numbers of their lowest terms.

64	45	Form of the semidiapente
10	9	Form of the minor tone
<hr/>		
640	405	Form of the super 47 par-
		tiente 81 not in lowest
5		terms
128	81	In lowest terms

[Figure 65.]

The interval which has been obtained from these is the super 47 partiente 81, which contains the true minor hexachord of the diatonic ditoniaion in its lowest terms, which is a comma smaller than the super-tripartientequintas. The supertripartientequintas is called today the form of the minor sixth, although some also call it the minor hexachord. It is consonant in differentiation from that ancient one, which is dissonant, since it is, as has been said, a comma smaller. Anyone who takes the trouble to subtract the ancient hexachord from the modern one will ascertain this fact. One can also obtain this imperfect consonance from the fifth and the major semitone, as we will see from the result of adding their lowest terms together according to this example [Figure 66.] which will indicate how much the minor sixth exceeds the fifth.

[20]



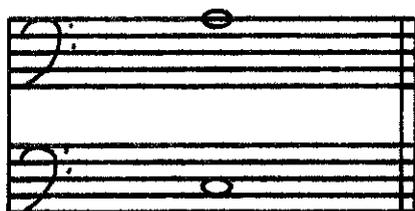
3	2	Form of the fifth
16	15	Form of the major semitone
<hr/>		
48	30	Form of the minor sixth not in lowest terms
6	8	5 In lowest terms
[Figure 66.] ⁴⁰		

Where the tritone intervenes, one may not obtain the minor sixth unless by means of more than two intervals or through the agency of anyone of these [just named]; however, I pass over this in silence. I wish for us to see now the same occurrences

⁴⁰Galilei's musical example is erroneously a fourth instead of a major sixth.



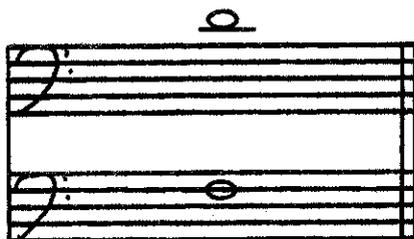
in the major sixth, whose lowest terms the practical musicians say are between these numbers, 5:3, and which they say consists of two major tones, two minor tones, and one major semitone. It can be considered, initially, to be composed of a fourth and of a major third, so that such intervals will entirely fill up the void of its terms. I ascertain this fact every time the result of adding together the numbers of their proportions is contained by the same numbers which contain the major sixth. The example which follows immediately clarifies this for us.



4	3	Form of the fourth
5	4	Form of the major third
<hr/>		
20	12	Form of the major sixth not in lowest terms
4	5	3 In lowest terms

[Figure 67.]

Thus, the major sixth in its true form has such a product, but this will not be the case, when it is considered with similarly appearing intervals between the pitches of F fa ut and d la sol re. This is because the fourth which is found between them in the upper portion does not fall under the form of the first one demonstrated, but under another form, 27:20. This, according to what was proved in another place, was greater by a comma. Whatever proportion we will obtain from such intervals, you will notice in this example where their least terms have been added together.



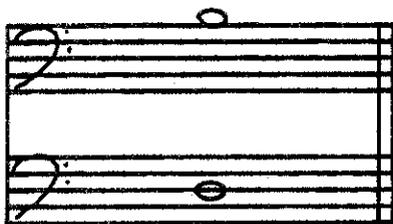
The major sixth is not found between these pitches.

27	20	Form of the super 7 partiente 20
5	4	Form of the major third
<hr/>		
135	80	Form of the super 11 partiente 16 not in lowest terms
5		
27	16	In lowest terms

[Figure 68.]

Thus, as you see, the super 11 partiente 16 has been obtained. This is the true form of the major hexachord of the diatonic ditoniaion. Since it exceeds the major sixth of the syntonic of Ptolemy by a comma, it is necessarily dissonant. For this reason and no other, Zarlino⁴¹ stated in proposition thirty-five of the second discussion of his Dimostrationsi that, when adding the major tone to the fifth, a consonance cannot be the result. One can clearly see that the major hexachord exceeds the major sixth by a like quantity, as I have said, by subtracting one from the other. The major sixth will be produced in very consonant form from the product of the fifth and the minor tone, as one can clearly see from the next example [Figure 69.] in which the pitches that contain them are added together.

⁴¹See Zarlino, Dimostrationsi, pp. 138-139.



3	2	Form of the fifth
10	9	Form of the minor tone
<hr/>		
30	18	Form of the major sixth not in lowest terms
6	5	In lowest terms

[Figure 69.]

This imperfect consonance may also be obtained in its true form from the minor sixth and from the minor semitone, with the aid, however, of the present sign, *b*. It may also be found in the following example [Figure 70.].



	8	5	Form of the minor sixth
	25	24	Form of the minor semitone
	200	124	Form of the major sixth not in lowest terms
40	5	3	In lowest terms

[Figure 70.]

While one may also make judgement, not only of how much the minor sixth is exceeded by the major, but that these [examples] are major hexachords, and not major sixths as the practical musician believes, this situation only results because the minor semitone is derived from the major tone.



[Figure 71.]

This [major tone] is necessarily a comma greater than what is obtained from the minor tone. This, as I have promised, will be seen more clearly in its place. It is not

my intention to omit this other consideration, for as the minor sixth could not occur between those pitches where the tritone intervened, one likewise may not have the major [sixth] between those where the semidiapente could fall. You, yourself, will clarify the cause of this the moment you begin to investigate.

Warning.

We come at last, however, to the examination of the minor seventh, which our practicing contemporaries say contains in itself three major tones, one minor, and two major semitones. We will consider, initially, that this interval is composed of the fifth and of the minor third, because its form is constituted in the superquartapartientequintas between these numbers, 9:5. The sesquialtera and the sesquiquinta added together give the abovementioned dissonance. Here is a clear example [Figure 72.].

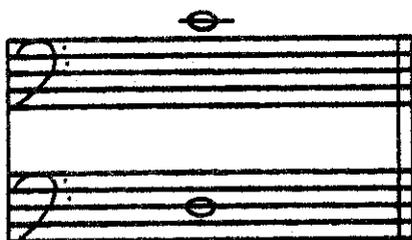


The minor seventh
is not found be-
tween these pitches.

	3	2	Form of the fifth
	6	5	Form of the minor third
	18	10	Form of the minor seventh not in lowest terms
2	9	5	In lowest terms

[Figure 72.]

The minor seventh has just been derived in its true form. This, considered between these pitches according to the nature of the fifth which is found between D sol re and a la mi re or in a major sixth in low position and a major semitone in high position, will not be of the same proportion as the first. The same occurs in the two lower examples [Figures 73 and 74.] because they have, like the other example, need of the major tone, since they need to become octaves.



[Figure 73.]



[Figure 74.]

On the contrary, a minor [tone] sufficed that [minor seventh] of the first example [Figure 72.]. These matters, since they are ignored by the practical musician, cause in him, when they are discovered, the stupor many times mentioned. You, yourself, will clarify that they are not minor sevenths by adding together the lowest terms of their designated parts. The product of these will give the interval 16:9, which includes between its extremes the super 7 partiente 9, the true form of the minor heptachord of the diatonic ditoniaion. Nevertheless, some

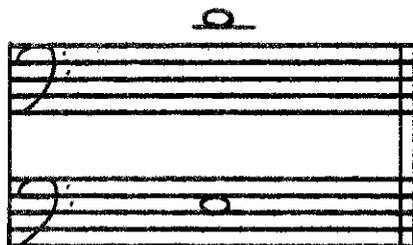
of our practicing contemporaries have called the minor seventh of the syntonon, which is contained, as you have seen, by the superquartapartientequintas under such a name, without having had regard for the difference between the two and for the confusion it was capable of generating in the minds of men.

	5	3	Form of the major sixth
	16	15	Form of the major semitone
	<hr/>		
	80	45	Form of the minor heptachord not in lowest terms
5	16	9	In lowest terms

[Figure 75.]

This heptachord is a comma less than the minor seventh, as anyone who takes the trouble to subtract one from the other will clearly see. One cannot even obtain the minor seventh from the product which will result from adding together two sesquitertias. This will seem impossible to the practical musician because he prefers to figure the case between the pitches of the first example [Figure 72.]

which was the same as the below-placed example, but divided and considered otherwise.



[22]

4 3 Form of the fourth

4 3 Form of the fourth

16 9 Form of the minor heptachord

[Figure 76.]

The minor heptachord has been obtained in lowest terms by adding together two sesquitertias. I proved earlier, in this regard, that the minor seventh was contained between the same pitches. I said, besides, that when the parts of any interval are added together after they have been divided in any manner you wish, they must necessarily, since they have been added together, render the whole interval in the same form and measure which first existed. On the contrary, the simple

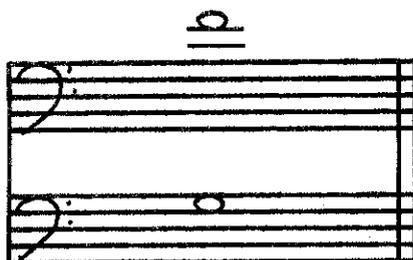
practical musician perceives in this last example [Figure 76.] the opposite effect from that which I first stated. He does not see the reason for it, because he considers that two fourths of the same proportion and measure are contained between the demonstrated pitches. They are actually different. Thus, despite his rule, he comes to err in the beginning and is surprised because the calculation does not prove favorable, as it should do upon completion. This is because the fourth which lies in high position falls under this proportion, 27:20, as has been proved earlier. When this has been added to that which has been established by the theorist, the result will be the minor seventh in the form which I said had been assigned to it in the beginning of our discussion.

Strozzi: I think I am beginning to perceive the port. Every time you make me see clearly, without any other authority, that the ditone, the semiditone, the major

hexachord and the minor hexachord are dissonant--in addition to the diapentes and the diatessarons which you have previously called dissonant--I will believe absolutely that what is sung--that is as it is customarily done today--is in no way the diatonic syntonon of Ptolemy. Although such intervals agree in the instruments and voices of the men of our times, I still do not consider, however, that they are different in effect and nature from those of the ancients.

Bardi: Let us continue our voyage now that the wind is propitious and the waves calm, because, if it pleases the Giver of all good things, I want you to see clearly in your mind the truth of each smallest detail, freed from any authoritarian influence. We should not delay any longer because we are already many miles away from port. I shall now reveal another abuse of the practical musician, who considers the fifth and the minor third of the following example [Figure 77.] to occupy the same space as

the semidiapente and the major third combined. Consequently, when the numbers which contain their forms have been added together, one set will give the same result as the other. This, according to what we will see now, is very far from the truth.



64	45	Form of the semidiapente
5	4	Form of the major third
<hr/>		
320	180	Form of the minor heptachord not in lowest terms
20		
16	9	In lowest terms

[Figure 77.]

The minor heptachord has been produced by adding together the two demonstrated intervals. Previously, the minor sixth was obtained by combining the fifth and the minor third. Therefore, it is not true that such intervals, when combined,

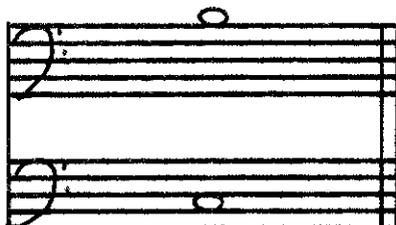
occupy the same space. The practical musician is greatly amazed by this, which happens, as I have said, only because he wishes to derive everything from one source, since he has no knowledge of the intervals. In order to satisfy him, I say that when he newly examines and understands the size of the minor third which the minor heptachord has in its upper part--as I last showed you--he will accept it. All wise men of common opinion say that nature has not made anything in vain. I want to prove to you with this proposition that the demonstrated heptachord does not lack the proper form of the minor seventh of the syntonon, which is the superquarta-partientequintas. The procedure will be this. Anyone with reasonable judgment will know from what I have proved up to now and said, that neither the major tone nor the minor [tone], and also the major semitone, have ever been altered to a form different from that which was assigned to them in the

beginning of our discussion, according to the opinion of the theorist, but only the intervals of the octave and, later, the major seventh. This also resulted because he had not examined them. These [examples], when they have been considered by us carefully, will not surpass the others in excellence in this case. We find, moreover, two different kinds of minor sixths from which, by means of two different tones, one can compose the octave in its true form. One can compose the octave, [23] initially, from the minor heptachord plus the sesquioctave (or tone). It can also be composed from the minor seventh plus the sesquinona, which will be clearly understood by adding together their lowest terms and noting the result. The many lines drawn from the center to the circumference of a circle all remain in the center of the circle. In the same way, each musical interval in the octave looks as if into a mirror like the stars also look into the sun. Thus, each of these receives its being and perfection,

according to its capacity, in the same way. Among the minor thirds, we have two--since we do not wish to investigate more deeply than in the other cases--which are contained under different proportions. The rest of these were found later in order to complete the octave. These were the two major sixths, contained under different form. Among the former, however, we enumerated the semiditone and among the latter, the major hexachord, intervals which are common to the syntonon and to the diatonic ditonaion, as many of the others. We also found the major third and the ditone, and shortly afterwards the minor sixth and the minor hexachord, since from these the octave could be composed far from any imperfect extreme. We found the diatessaron contained in different ways and afterwards the diapente was seen to be contained by eight different numbers. This was done, as we will see at the proper time, only

in order that the octave might be constructed in one single form from these combined. And all this happens because courteous Nature wishes to show, in all her operations, her immense wisdom and liberality. Nature does not tolerate a vacuum, nor does she do anything or make anything otiose or in vain. There remains to show you, in this regard, how much the minor seventh exceeds the major and minor sixth, which you can easily see by subtracting them from it. We will now hastily consider the major seventh, and then pass on in order to conclude the proposed material with that of the octave. The masters of this modern practice of counterpoint say, therefore, that the major seventh is that dissonant interval which is contained in its lowest terms by the proportion super 7 partiente 8 between the numbers 15:8, and that it consists of three major tones, two minor tones, and a major semitone. This--for the reasons many times said--we will

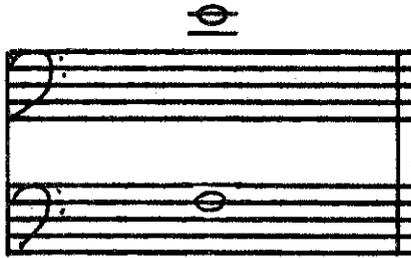
consider to be composed, initially, of a fifth and of a major third, according to what appears in the following example.



3	2	Form of the fifth
<u>5</u>	4	Form of the major third
15	8	Form of the major seventh

[Figure 78.]

One can also obtain such an interval from numbers which contain the form of the tritone when it is combined with those of the fourth, but it still will not produce the same proportion--although this is contrary to the opinion of the practical musician--as the minor tone and the major sixth when they are added together in the manner which is seen in the following example [Figure 79].



5	3	Form of the major sixth
10	9	Form of the minor tone
<hr/>		
50	27	Form of the super 23 partiente 27

[Figure 79.]

The super 23 partiente 27 has been obtained from the product of the aforesaid two intervals. It is a comma less than the major seventh, as you will clearly see by subtracting it from the major seventh. One may also obtain the major seventh from the product of the major sixth and the major tone. We will see now, by subtracting the minor seventh from the form of the major seventh, how much one is greater than the other. The minor seventh, according to this example [Figure 80.], is thus exceeded by the major [seventh] by a semitone. From this

one may comprehend that the major seventh also may be composed of the minor seventh and a minor semitone.

15	8	Form of the major seventh
	X	
9	5	Form of the minor seventh
<hr/>		
75	72	Form of the minor semitone not in lowest terms
3	25	24 In lowest terms

[Figure 80.]

Strozzi: Is not the form of the major heptachord still the super 23 partiente 27? [24]

Bardi: No, Signor, because that falls under the proportion super 15 partiente 128 within these limits, 243:128, and exceeds the super 23 partiente 27 by the quantity 6561:6400. I come, finally, to deal with the octave, which our practicing contemporary contrapuntists say is that most perfect interval which consists of three major tones, two minor tones, and two minor semitones and which is contained, besides, by the proportion

dupla between the numbers 2:1 when in lowest terms. We will consider this interval, initially, composed (as its nearest parts) of a fifth and of a fourth. We also will verify that the consonance diapason is entirely capable of containing these two intervals by adding together the numbers of their lowest terms.



3	2	Form of the fifth	
4	3	Form of the fourth	
<hr/>			
12	6	Form of the octave not in lowest terms	
6	2	1	In lowest terms

[Figure 81.]

We will also derive the octave in such a form from the product which is obtained by adding together the numbers which contain the major sixth and the minor

third, from summing the lowest terms which contain the major third and the minor sixth, and also from combining those of the minor seventh and the minor tone. It follows, necessarily, that those of the example below, when considered as a minor seventh and as a major tone of the proportion which was shown in the beginning to have been assigned by the theorist, are not consonant octaves, but only dissonances, which is contrary to the opinion of the practical musician.



[Figure 82.]

For fear that they may not be as I have said, we will experience this (according

to this example), by adding together the numbers of the proportions which contain them.

9	5	Form of the minor seventh
<u>9</u>	<u>8</u>	Form of the major tone
81	40	Form of the duplasesqui- octagesima

[Figure 83.]

The duplasesquioctagesima, which consists of an octave plus a comma, is obtained from this addition, as anyone will see clearly who takes the trouble to subtract the octave from it, or actually to place a number between these limits, which are in duple proportion with the minor, in this manner, 81:80:40. The reason that one may not obtain the octave from the major tone and the minor seventh is that the intervals which we have considered as minor sevenths in those places do not actually fall under the proportions which they were assigned in the beginning of our discussion--the one concerning the opinion of the authors that what is sung

today is the syntonon of Ptolemy--but only under the minor heptachord of the diatonic ditoniaion. This, when added together with the sesquioctave (or tone) will produce the dupla in its true form, as the following example makes clear.

	16	9	Form of the minor heptachord
	9	8	Form of the minor tone
	<hr/>		
	144	72	Form of the octave not in lowest terms
72	2	1	In lowest terms

[Figure 84.]

The octave also may be obtained in its true proportion from the result of adding together the terms of the forms which contain the major seventh and the form of the major semitone. Thus, similarly, the octave may be obtained from that of the tritone and the semidiapente. Finally, in order to know how much the octave exceeds the major and minor seventh, one will see this by subtracting one [seventh] from it and then the other.

Strozzi: I believe that your dis- [25]
course has led me by now into a safe
port so that I no longer fear the blast
of unfriendly winds. It has so removed
from my intellect the thick fog of
ignorance that, without other reasons
and examples to influence me, I am almost
certain that in the singing of today
none of the inconveniences mentioned are
present. Nevertheless, I am also sure
that when tuning the string of an instru-
ment according to the demonstrated forms,
it is necessary for each one of the
demonstrated cases to be found between
these strings. From this I argue that
the intense syntonon of Ptolemy is
probably neither sung nor played.

Bardi: You are beginning to discuss
and understand it very well, but in order
better to remove each difficulty which
may recur concerning that, I want to
clarify for you two other most important
topics, and then we shall see clearly
what genus and species is sung and played
today. Therefore, I say first, that

between the notes of the example placed below there are none distant from each other by a minor semitone of the proportion which I assigned, according to the opinion of the theorist, in the beginning of our discussion.



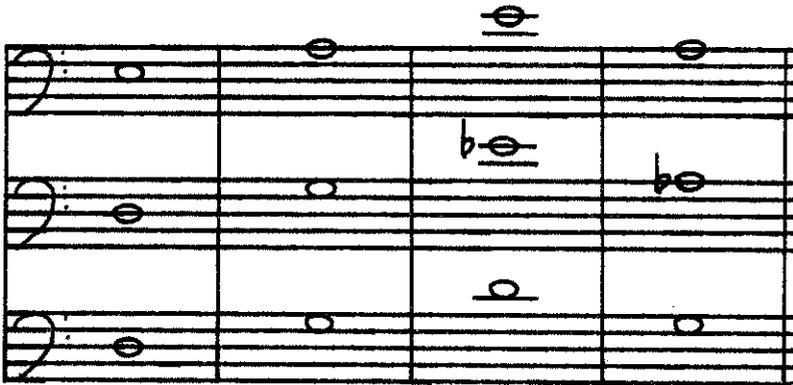
[Figure 85.]

There is also no lack of those who have vainly doubted concerning the major [semitone], but they are greatly deceived. The reason that they are not what I have said only results because both of them together cannot fill a space greater than that which contains the minor tone, as was proved in the beginning. However, both will easily be found between the pitches of this other example [Figure 86.], since the place is capable of containing

sufficient numbers to fill the spaces without anything lacking or anything being left over. Therefore, since we do not wish to proceed haphazardly and without any reason, it will be necessary to find new measure and form for both those first ones, or at least for one of them, which is not only suited to fill the interval of the major tone, but is actually that which the ancient musicians had under consideration, provided such consideration was necessary.

Since, however, there is no record or particular example of such things being done, these doubters, as many others, can only be persuaded with greatest difficulty and with powerful reasons.

One should not be dismayed by this and permit them to resort to such plausible and apparent means, even when there are no others, in order that they may learn the truth of the matter.



[Figure 86.]

I say--according to my ability, however-- that the ancient musicians did not observe any difference between the major and minor tone, unless it was the minor semitone. This minor semitone, according to what I indicated to you, is contained by the super 7 partiente 128 between these numbers in its root terms, 135:128, and this interval consists of a minor semitone plus a comma, as has been said previously. The major semitone is the same in every tone. In order that this truth also is seen clearly in the diatonic syntonon of Ptolemy between the pitch tritesynemmenon and the paramese, I confirm it to you, without running afoul of authority, in yet another manner in addition to that which I

Minor semitone of another proportion than the first one shown.



45 32 Form of the tritone

X

135 128 Form of the minor semitone
 of the major tone

5760 4320 Form of the fourth not
 in lowest terms

1441

4 3 In lowest terms

[Figure 88.]

Or else, thus, which is the same that is
seen above in proving to you how much the
tritone is greater than the fourth:

[26]

45 32 Form of the tritone

X

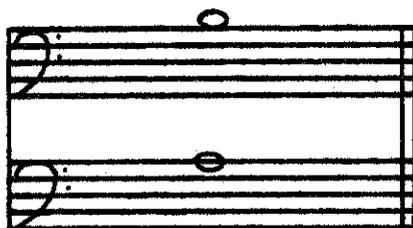
4 3 Form of the fourth

135 128 Form of the minor semitone
 of the major tone

[Figure 89.]

And [to confirm] that this minor semitone,
together with the major, has power to fill
entirely the void of the major tone, one
adds together the numbers of their lowest

terms, so that the result one will have from them will be certain. In addition, if, following this example [Figure 90.], we wish [the interval from] a la mi re to h mi to be a major tone, as it actually is, and if we also wish this other [interval] to be the same distance, it is necessary to concede a minor semitone of the demonstrated proportion to the major tone, provided that the place from which it is subtracted is, like the former, capable of containing it.



[Figure 90.]



[Figure 91.]

Let us also consider that the minor semitone which is between b fa and h mi in the usual proportion not only will not fill the void of the major tone, but that note signed as an accidental on the note h mi of the first example [Figure 90.] will be a comma higher than that of the second [Figure 91.] since h mi is higher than a la mi re by a major tone. This has been said many times. If, in that last example [Figure 91.], one wishes in departing from the pitch a la mi re to ascend two semitones to h mi, one may well consider that the distance which is found from this a la mi re to b fa is that of the major semitone. However, that space which remains between b fa and h mi is necessarily less than the proportion which contains the major tone, which is 135:128. From here, perhaps, some of our contemporary contrapuntists have had occasion to place the b quadratum [b] on f fa ut when it has had to sound a fifth with h mi, and the diesis X when it has had to form a

major third with D sol re or its replicate [an octave higher] with d la sol re. This constitutes a silent argument that the fifth would not become "just" with the diesis X as with h quadratum, but a little flat and submissive. Now, please turn your attention from those matters to only these two items which I wish to tell you in addition to many others I know but omit for brevity.



[Figure 92.]

Whether the diesis X or the h quadratum should be placed on F fa ut.



[Figure 93.]

If between these notes [Figure 92.] is found a major third, according to what we have demonstrated previously, and similarly,

a minor semitone is found between these other notes [Figure 93.], which also affirm it, it necessarily follows, because of what was proved earlier, that the extremes of the two thirds, when sung in the manner they describe and believe--and which actually is proper--sounds with a very "just" fifth, since it is contained by the sesquialtera proportion. Therefore, if the h quadratum has the power they say it has to make the note on which it is placed higher than diesis X can, the fifth, as a result, will become larger than its normal state. Similarly, the fourth above it will become smaller by the same amount the h quadratum exceeded the diesis X in power to augment the fifth. That appears, however, to be not exactly true. It is quite necessary, if one wishes the major third, its replicate, and also the fifth to be in their true forms and proportions, to have a minor semitone of the proportion

and size which I have just now described, but taken, however, from a place which is capable of containing both the major semitone and this [minor semitone]. Otherwise, any actions and labors you might undertake would be in vain, because the power to put each one of these consonances in its perfect genera consists principally in the placement rather than in the capacity of the place, and not in the difference of the numbers, as those men believe. The *h durum* was not used as a symbol unless it was introduced after the *b molle* appeared as a mode. The diesis *X* was established sometime prior to both, however, speaking always in terms of the modern practice. Together with the *b molle*, it was suitable with its [power to] increase and diminish to render the imperfect consonances and the dissonant intervals in that form [i.e. its perfect genera] and to bring them into the highest state of perfection which could be desired from their nature. At least they have been

introduced for such a reason, although badly used. If I do not deceive myself, the b molle was invented primarily as a symbol by modern practical musicians or, to say it better, put into use for no other reason than because the note F fa ut, like the others, also had a fifth in the low register and a fourth in the high and that of G sol re ut had a major sixth in the low and the minor third in the high. Also, the diesis X was introduced in both F fa uts so that the note h mi sounded with the high one as a fifth and not as a semidiapente and with the low one as a perfect fourth and not as a "hard" fourth. In addition, it made the intervals of major and minor sixth with both d and D. After the b molle had been placed in use, the h durum came into consideration as a symbol by good, modern professors of music for no other purpose than to be able, primarily, to transform one system into another, according to which pleased them and was most convenient. However, they never [27]

placed this h on any note other than its usual one. In the others, when such a need arose, [they placed] the diesis X, which finally had been perfected in its demonstrated form. One sees this very well observed in their works, and if someone answers me that it is still the custom for everyone commonly to place the b molle on the note a la mi re, and on that of the e la mi which is farther removed, I will respond that this results from the small number of characters which [we have] today in comparison with the ancients, who had for each note an appropriate sign to demonstrate whatever desirable and convenient sound they wished. But it is not, however, such a scarcity of symbols in our times--concerning demonstrating the difference of the notes--which causes any deficiency or imperfection in regard to the purpose for which the music of today is used. If that were so, some would have been designed before now which would have remedied their defect and deficiency.

But this has not occurred since those [symbols] were sufficient to give any note in the low and high registers the form of every consonant interval which man has been known to devise up to now, and particularly in the two first genera of harmony, which are the diatonic and the chromatic. In the enharmonic, then, the present sign, x, has been uselessly added--because it has not been put into practice by anyone--in order to be able to divide the major semitone into two parts, called enharmonic dieses by the ancients. The moderns, however, understand for this diesis the present symbol, X, as you know. This, in their opinion, is worth two of the former. This is not exactly true, because the ancients used to divide the limma, or minor semitone, of the diatonic into two enharmonic dieses and the moderns say to divide the sesquiquindécima, called by them major semitone. What difference there is between them has already been demonstrated.

What we have said could be restated that there is the same distance between h mi and b fa through h quadratum as there is through b rotundum. In the same manner, the identical distance is found between c sol fa ut and h mi through h quadratum as there is through b rotundum. And I say, in conclusion, that d la sol re is sharper through h quadratum than it is through b rotundum.

Strozzi: Clarify this last topic for me.

Bardi: I say that d la sol re is sharper through h durum than through b molle in comparison with this. If we move away from a la mi re, which is a fixed note and common to both systems, and we want to ascend by conjunct diatonic steps to d la sol re, we will find by means of the disjunct system--that is to say, through h durum--that there are two major tones and a major semitone. By means of the conjunct system--that is through b molle--there is a major tone, a minor tone, and a major semitone.

Therefore, using the conjunct system, d la sol re necessarily becomes a comma lower than in using the disjunct system, as the following example shows.

Major tone Major semitone Major tone

Major semitone Major tone Minor tone

[The note] d la sol re is sharper through h quadratum than through b rotundum.

[Example 2--A comparison of the conjunct and disjunct tetrachords.]

Strozzi: I have understood very well and it is actually so.

Bardi: Because there is the same distance from c sol fa ut to h mi through h durum as through b molle, it follows that the major semitone of each tone is always of the same quantity and proportion in one as in the other. For the same reason, there is the same interval from h mi to b fa through b quadratum as there is through b rotundum.

Strozzi: If it is not too much trouble, please tell me why this 16:15, more than any other proportion, is that which contains the major semitone; why this 25:24 contains the minor semitone more than another proportion; why this 65:45 more than another proportion contains the semidiapente; and finally, why this other proportion 45:32, more than another, contains the tritone? Why also, can they not be contained by other numbers, according to the syntenon of Ptolemy?

Bardi: I believed that I had satisfied you with what I had told you before, but I see that this is not so. However, I am still under obligation to you and I want you yourself to be the judge whether they can be otherwise. With this, however, you must concede to me that the true forms of the minor sixth, of the major and the minor third, of the fourth and of the sesquioctave tone are actually those which were assigned, according to the opinion of the theorist, in the beginning of our discussion. I offer, then, at the necessary time and place, to

make you clearly see and hear everything else that you desire by means of the monochord.

Strozzi: I concede you everything for now.

Bardi: First, I ask you how much greater the major third is than the minor?

Strozzi: A minor semitone.

Bardi: In that way, when subtracting the minor third from the major, those numbers which remain necessarily contain the minor semitone in its true form; is it not so?

Strozzi: It certainly is.

Bardi: Now we come once again with this example to the actual experience.

5	4	Form of the major third	[28]
	X		
6	5	Form of the minor third	
25	24	Form of the minor semitone	

[Figure 94.]

Here, as you see, since we have subtracted the sesquiquinta from the sesquiquarta, the

remainder is the sesquivicesimusquartas, assigned us as the form of the minor semitone. The verification of this fact would be adding it to the sesquiquinta and seeing if between the two of them they gave the form of the sesquiquarta. Since we saw that on another occasion, however, it would be foolish to do it [again]. Therefore, I ask you another question. How much greater is the minor sixth than the semidiapente?

Strozzi: By a sesquioctave tone.

Bardi: Therefore, in subtracting one interval from the other, must the amount which is left consequently contain the semidiapente?

Strozzi: Yes, it must.

Bardi: Therefore, here is a very clear example.

8	5	Form of the minor sixth
	X	
9	8	Form of the major tone
<hr/>		
64	56	Form of the semidiapente

The semidiapente is contained within these numbers, 64:45.

[Figure 95.]

In this, the semidiapente is in its usual lowest terms.

Strozzi: I am very much satisfied for the present. Continue, however, and tell me the rest.

Bardi: I ask you, in addition, how much greater the fifth is than the tritone?

Strozzi: A major semitone.

Bardi: So that when someone subtracts the major semitone from the fifth, will what remains necessarily be the tritone?

Strozzi: That would necessarily follow.

Bardi: We come, therefore, to the example.

3	2	Form of the fifth
	X	
16	15	Form of the major semitone
<hr/>		
45	32	Form of the tritone

Why the tritone is within these numbers, 45:32.

[Figure 96.]

Finally, I ask you how much greater the fourth is than the major third?

Strozzi: A major semitone.

Bardi: Thus, when a major third is subtracted from the fourth, those numbers which remain will actually contain the major semitone in its true form, is it not so?

Strozzi: This is true.

Bardi: We come, therefore, to demonstrate the truth of the matter with this example.

4	3	Form of the fourth
	X	
5	4	Form of the major third
16	15	Form of the major semitone

Why the major semitone is within these numbers, 16:15.

[Figure 97.]

Here is the major semitone in its lowest terms.

Strozzi: I am entirely satisfied.

Since, however, everything you have told me and proved in many ways about the forms of the intervals is true, what was it that induced Plutarch⁴² to say in his "Table

⁴²Plutarch, Table Talk, trans. by Paul A. Clement and Herbert B. Hoffleit, Vol. I of Moralia, 16 vols. (Cambridge, Massachusetts and London, 1969), pp. 266-269. "Now

Talk" that three and one are the limits of the diatessaron? This is such a simple thing and yet so far from the truth.

just as the experts in the musical theory of the lyre assert that among ratios that of 3:2 gives the concord of the fifth, 2:1 the concord of the octave, and the concord of the fourth (which is weakest) consists in the ratio 4:3; so the musicologists of Dionysus observed three concords of wine and water, fifth, third, and fourth, for in their song they say this:

Drink five or three, not four.

'Five,' indeed, is in the ratio 3:2, three parts of water being mixed with two parts of wine; 'three' is in the ratio 2:1, two parts of water being mixed with one of wine; and four,--three parts of water being poured into one of wine, this is a ratio of 4:3, a drink for some group of sensible magistrates in the prytaneion, or logicians, their brows contracted as they meditate upon syllogistic conversions, a sober and a feeble mixture. Of the two others, the mixture with ratio 2:1 brings on that disturbing and half-drunk pitch of intoxication

that plays upon
The inviolate strings of the mind,

for neither does it allow sobriety nor does it completely immerse the foolish man in strong drink. But the mixture with a ratio of 2:3 is most harmonious, a complete inducer of sleep and relaxer of care, a 'protecting and soothing governess,' in Hesiod's phrase, because it creates a profound calm and quiet among our lordly and disordered passions."

Bardi: As far as I am concerned, Plutarch wishes, in that place, to be considered as a drunkard and good companion rather than as a serious mathematician. He wishes to be considered in keeping with the musical consonances, and not exactly according to the proportions of the quantity of numbers. Above all, he intends it as a pleasant topic for dinner conversation, which has the same effect but not the same result. Anyone who logically reads the whole of that question easily ascertains this through the words of the writer himself. The consonance diatessaron, therefore, as the name implies, is contained by four notes, which makes it the sesquitertia. Although it is numbered among the perfects, it is nevertheless the least perfect of all and is more distant from the nature of the unison than the sesquialtera and the diapason. It is the least certain of all to the sense, and the least

delightful, and if any small amount of it is diminished, it becomes imperfect and dissonant. This, however, is according to the usage of the ancient musicians. Thus, in watering the wine, adding three out of four parts of water is almost the least consonance and accord which is pleasing to the taste because now, as then, in diluting the wine almost to the color of water, and causing it to lose all its strength, it becomes no wine at all. This is not agreeable to people who are together for an enjoyable time at the table, but for serious men who need to remain somewhat sober while attending to some [affair] where they are the judges and the disputants, or similar persons. Therefore, the proverb of the joyful men did not mean (says the writer) that the four [parts of wine] interfered with their task in any way. That is one of the basic premises of the sesquitertia, which was the last and the most remote consonance of wine. Thus,

[29]

it was the sesquitertia and diatessaron of their drinking. But it was not so delightful to the taste, since they were seeking the good disposition which has as its goal the joy and almost the recreation of the spirits without danger of drunkenness, which, perhaps, can persist long after leaving the table from the diapason. And yet, the disputant exalted greatly the diapente, considering in it, as well as in the diapason, the terms of its forms added together and alone greater than the diatessaron. I believe that I need to interpret the place thus, since I want to follow the intention of the writer and not consider it as one compared to the three-faced sesquitertia, because this, clearly, is impossible. Therefore, the intention of Plutarch in that place is to consider only the larger terms of each proportion of the three simple, prime consonances. When one single unit representing wine has been taken away, it means that the

others which remain are the parts of water. Now, because the larger term of the sesquitertia is four, from which--as has been said--one has been subtracted as the part of wine, that which remains as the part of water is--as you know--three. And therefore, the author proceeds in regard to the sesquitertia, comparing the one to the three. It can also be said thus, according to what you have heard: three out of four parts of water. And thus we come to make mention of proper limits for this sesquitertia. This mode of comparison used in the other consonances is easily remembered. Therefore, tempering the wine according to the sesquitertia, they should consist--as has been said--of three parts of water and one of wine. This mixture makes the one who drinks it become like its nature, which is languid and mild. Next, according to the sesquialtera, [its proper limits] ought to consist of three parts of

Different ways
to temper wine
with water
according to
the usage of the
ancient musician.

water and one of wine. This, drunk often in this proportion, makes the drinker become cheerful and thus [he reflects] the nature of the diapente. Finally, according to the dupla, the proper limits are in two parts, one of water and one of wine. When this potion is drunk thus, it makes the man become like the nature of the diapason, which is most joyful. But one must consider the place, the time, the persons, and the quality of the wine that is drunk where these effects were caused, if one wishes to unite the strings of this cithara perfectly. The ancient speculators of musical proportions had other, different methods of tempering wine with water, but enough has been said about it.⁴³ Returning now to my former topic, I say that you yourself, if you have understood all I have discussed with you, may discover where the proportion of any musical interval is obtained, and not

⁴³See Zarlino, Sopplimenti musicali, pp. 106-110, where an entire chapter is devoted to the refutation of Galilei's interpretation of Plutarch's passage concerning the tempering of wine with water.

only this, but [you may also clarify for yourself] any other problem which may occur to you concerning the difference in size which exists between one interval and another in any genus and species of harmony. This is as much as is necessary for me to tell you and prove to you concerning the principal topic of the problem proposed to me, but tell me frankly how you feel about that.

Strozzi: I am more confused than ever by your explanation--now that I expect confusion less--concerning, however, the knowledge of which species is the one which they sing today. This is the main topic of the discussion we have had up to now.

Bardi: What is your new difficulty?

Strozzi: Listen. If any one of our practicing contemporary contrapuntists chances to use in any of his compositions the tone and semitone in any pitch without being, for all of that, capable of any of the demonstrated considerations, I cannot imagine how it can happen that so many discrepancies are not manifested to the Doubt.

cultivated ear. You have indeed proved to me with convincing arguments that these irregularities must be occurring frequently. And if this does not happen, a still greater inconvenience will follow. This will be that many of the things proposed to us by the theorist as a rule would fall under the two already mentioned concerning the position and difference of the location and value of the tone and the major, minor, and medial semitone. On these two topics--as I understand it, however--has been based, principally, all that which you have discussed with me up to now, unless we wish to say that the quantity of the comma, since it is so small, when subtracted from or added to any interval, does not have the power to alter the nature of its original state. I do not believe this in any way. M. Gioseffo Zarlino⁴⁴ claims, in particular, that half a comma has power, when added to or taken from any consonant interval,

In chapter 43 of the 2nd [part] of his Istitutioni armoniche.

⁴⁴See Zarlino, Istitutioni, pp. 128-131.

of making it dissonant, although he later adds, for a joke, I believe, that the consideration of the difference of major and minor tones must be omitted.⁴⁵ This consideration when taken away bears with it that of the various species of semitones. When this sole speculation is carried to the diatonic which is sung today--granting that it is the syntonon of Ptolemy--it becomes a different consideration through which what is granted is perhaps true.

Bardi: You discussed it very well and have the greatest reason to doubt, but here is the solution of the problem. If the genus in which we have considered each of its intervals with such exactness is the syntonon of Ptolemy, I say that the one in which they compose, sing, and play today is not the same at all. It

In chapter 13 of the 3rd [part] of the same Institutioni.

Solution of the problem.

⁴⁵Gioseffo Zarlino, The Art of Counterpoint, translated by Guy A. Marco and Claude V. Palisca (New Haven and London, 1968), p. 29. "We are ignoring in this [the diapente] and in the other consonances the distinction between the large and small whole tone, which, if taken into account, produce still other species."

cannot even be the ancient diatonic ditoniaion.

Strozzi: Are both sects therefore deceived--so to speak--in this topic of so much importance? Tell me this, please!

Bardi: They are fooled, certainly, but listen! In order to clarify for you the present status of the matter, I will reveal to you simultaneously, using new, authoritative examples, what species is the one which is played on keyboard instruments, which species is played on the lute, and, finally, which of these approaches nearer to perfection and why. This will neither be useless to you nor irrelevant.

[30]

Strozzi: These will be among the most valuable things that you could possibly clarify in this matter.

Bardi: I find through long observation that natural voices and instruments made by art do not actually play or sing in this modern music practice any of the nine ancient diatonic species in their simplicity. Our practicing contemporaries inadvertently use only three

Observations
of the author.

of them today, mixed together in different ways. These are the intense [diatonic] of Aristoxenus, the ancient diatonic ditoniaion, and the [diatonic] syntonon of Ptolemy.

Among stringed instruments, I consider that the viola d'arco, the lute, and the fretted lyre play the intense diatonic of Aristoxenus. Hearing and seeing in these instruments the uniformity of tones equally divided into two identical semitones impels me to believe that this is true, for the aforementioned intense [diatonic] of Aristoxenus was distributed in such a way, as you will understand in due course. However, the organ, the gravicembalo, and the modern harp--modern only in terms of the newly added strings and not concerning the instrument in its original form, which I believe is very ancient--more or less detach themselves from those [other species]. In the division of tones, for example, the tones are separated into two unequal semitones.

The wind instruments like direct flutes, transverse flutes, cornets, and other

The species of harmony which stringed instruments play.

The species which the organ and other keyboard instruments play.

The species which wind instruments play.

similar ones have power, due to the distribution of their holes and aided by the fine facility of their wise, expert players, to adjust to one [species] or another according to the need and their [player's] wishes.

This is equally true of voices, but only when they are not willing to go against their nature and adapt to those [species].

Moreover, concerning the composing and singing of today, I am persuaded, because of what I told you and am about to tell you, that the diatonic ditoniaion is mixed with the syntonon of Ptolemy. These are the causes which impel me to believe this. It is certain that if the plain syntonon is sung, then the tones and the minor semitone--in accordance with what I have proved is their nature in this species--must be unequal and different in size. Because of this inequality, the following variety of intervals would be sung, as you have seen according to the different proportions: two kinds of fifths; two kinds of fourths; three, perhaps four, kinds of minor thirds,

The species one composes and sings today.

and as many kinds of major sixths; at least two species of minor sixths; and two species of major thirds. There would be the same number of dissonances [i.e. two] and finally [that many] octaves. The netesynemmenon is sharper than the paranete diezeugmenon by an amount you know. In addition, the semitone which is found between b fa and h mi, and all those [intervals] which are derived as minor [semitones] from the major tone are in a different proportion, which is neither the sesquiquindecima [i.e. 16:15] nor the sesquivicesimusquartas [i.e. 25:24]. We have not only told of these things on another occasion, but one finds also--as I realized--that we have been warned of some of them by masters of this art. However, there is no one at all who, in singing these many airs together which were introduced a hundred and fifty years ago, has ever heard or hears now such a confused diversity of intervals, because, actually, [such intervals] never happened then, nor do they happen today. This seems to me a strong argument to persuade us that

this is true. And in order to be better acquainted with the variability of the intervals of this distribution, would you tell a few things to those who believe that [this distribution] is what is sung today? Tell them to divide, using any method they please, the major thirteenth, which is contained according to the syntonon by the numbers 10:3, into three sesquialteras, which are what they say it contains. Tell them also, according to the example which follows [Example 3.], to divide into three sesquitercias the dupla superbipartientequintas, form of the minor tenth. Then, ask them how much the second interval is exceeded by the first. It would not be very difficult to persuade us that the diatonic genus which is sung today is actually the ditoniaion--I mean the ancient diatonic--as many have believed and still believe today. In the first place, its ditone--as I have said--contains two sesquioctave tones. This interval, accompanied or

[31]

Inconvenience which would arise from singing the syntonon of Ptolemy.

$d||$
 ϕ
 Diapente Diapente Diminished diapente
 Geometric mean
 27 18 12 8
 10 Tripla sesquitertia, form of the major thirteenth 3

ϕ
 Diatessaron Augmented diatessaron Diatessaron
 Geometric mean
 64 48 36 27
 12 Duplasuperbipartientequintas, form of the minor tenth 5

[Example 3--Divisions of the major thirteenth and minor tenth]

alone, is dissonant. Its semiditone is of the same nature and so is the major and minor hexachord. In addition, the first and lowest interval of each tetrachord of the said species is a minor semitone (or limma) and between the tritesynemmenon and the paramese is the major [semitone] called apotome. On the contrary, in what is sung today, the first and lowest interval of the tetrachord is major, which is not what is found between the paramese and the tritesynemmenon. One finds also in the diatonic that the tritone exceeds the semidiapente and, on the contrary, in what is sung today the semidiapente exceeds the tritone. Now, just because none of the above-mentioned particulars are found in our diatonic--they are even contrary to these in a certain way--it does not necessarily follow that this [diatonic] is in any way that [which is sung]. The diatonic of today agrees well in some respects with the syntonon of Ptolemy, as I am about to explain. Initially, I do not believe I would err

in saying that the imperfect consonances-- omitting the consideration of dissonances for now--almost fall under the proportions of the diatonic ditoniaion, but I am still not of the opinion that they are connected with parts similar to it. For example, I believe that the major third is contained by an irrational proportion quite near the sesquiquarta, but not to the point where its sides--so to speak--are the sesqui-octave tone and the sesquinona, but only to that point where they are two equal parts of the said third. This is exactly the way the third is divided in the manner of the tetrachords of Aristoxenus, but it is not precisely the same thing. Moreover, I believe that the minor third is composed of a tone of the same proportion as those of the major and of another interval somewhat larger than the sesquiquindecima [16:15]. The other intervals are all composed in such a way and of such parts.

I maintain, therefore, that none of the other intervals except the octave is contained in its assigned proportion; it is

The octave is the only interval sung today in its true proportion.

understood that I am referring to the manner which is commonly used today in singing. We will shortly demonstrate this clearly. There is another point of agreement between them, that the tones in whose extremes our practicing contemporary contrapuntists place the diesis X and the b molle as accidentals, are minor [intervals] among the notes of the syntonon. This tends to warn us, not without reason, that if we used such signs on major tones, some of these semitones would become different in proportion, since they would be derived from the entirety of such a major tone. Our diatonic also has some conformity with the ditoniaion since each of their intervals consistently is found in the same measure and proportion, although they [all] have diverse forms. In addition, the diatessaron is found four or five times in each species of diapason, and the diapente at least three or four. I do not know if this situation can occur in another species or genus as

those of Aristoxenus, for instance, where major is plentiful, since they do not necessarily have either the tritone or the semidiapente as all the other string distributions do. Thus the perfect consonances in the mode which they are sung today happen to have greater affinity with the diatonic ditoniaion, and the imperfect consonances more closely relate to the syntonon of Ptolemy, rather to that of Dydimus--as you will learn--but always with the same proportion and equality of tones. One could add to this and say that [Dydimus] agreed with neither the syntonon nor with the diatonic, since neither the apotome nor the super 7 partiente 128 were ever found in use in the ditoniaion, nor the sesquivicesimus-quartus [25:24] in the syntonon, but only the first two as a related quantity in comparing the tritesynemmenon of the conjunct system to the paramese of the disjunct [system]. In addition, no interval of any kind except the octave

falls--as I have said--under the proportion and measure of the first or second species. This applies, however, when they are sung in the manner which is customary today.

Strozzi: Too many important things at one time, Signor Giovanni! My mind is so slow that I cannot follow your lofty conceptions with the speed and promptness with which you explain them. Therefore, you need to slacken the pace, if you wish me to keep up with you. However, I clearly know, thanks to your most subtle, ingenious, enthusiastic, true reasons that the genera in which one sings today in the diatonic species is not simply the diatoniaion or the syntonon, but a third thing mixed and composed of both. Nevertheless--because of the little experience which I have in these things--new difficulties continually occur to me (ones of no little importance) in understanding well what you so greatly comprehend. However, do not let it be a burden to you to answer me whatever is

necessary for me to ask you for greater understanding of this important affair.

Bardi: Speak freely, as always.

Strozzi: In the first place, I do not know in what manner Aristoxenus distributed the strings of his tetrachords, nor do I know what difference exists between the division of the lute and that of the keyboard instrument and consequently, which of these approaches nearer to perfection. I do not even know why the thirds and the sixths of the diatonic are dissonances or why the syntonon is by Dydimus, rather than by Ptolemy, or even less why the two sects were led to believe a thing so far from the truth. I cannot fathom how it is possible to change the diatonic singing of the ancients into our manner, which is so different with regard to the quantity and size of the intervals. Finally, [I do not understand] how, of [all] the intervals which are sung today, there are not any, from the octave on, which are contained in the proportions assigned them by the theorist.

[32]

Bardi: In order to take away all the difficulties you have mentioned, this is what we will do. Let us approach a keyboard instrument and, first of all, you must tune its fifths as excellently as you possibly can, but in the order which I tell you.

Strozzi: Will it not be easy for the ear to be deceived?

Bardi: Very easily, as much as any other sense, and more so than [the ear] which is unaccustomed to such speculations. Nevertheless, the discriminating ear of one who is well versed, accompanied also by natural judgement and by some good standard, is not so easily deceived. On the contrary, there are some [with ears] so perfect, accompanied by other circumstances, who understand each difference, however minimal, in an instant. However, let us come to the actual practice without undue verbosity.

First, tune A re and a la mi re in octaves. Tune above this the octave higher than A la mi re. Now loosen the D sol re which is below Aa la mi re, so that it sounds

Method of ascertaining what discriminating hearing is.

with it as a fifth in the most perfect way desirable. Tune in the same perfection that which is above the same a la mi re, which will be e la mi and thus, similarly, [tune] d la sol re below Aa la mi re.

Extract also in the same perfection the interval which E la mi makes above A re.

We want to see now if the sense [of hearing] has been deceived. Here is the way. Among the consonances, there are none better understood by the ear and [there is no place] where it can be deceived less than in the octave. Therefore, let us see if the notes D sol re and E la mi respond as a diapason with their higher counterparts. If they are perfectly in tune in this way, it will be evident that the ear has not been deceived at all in the temperament and perfection of the fifths.

Strozzi: They are excellently in tune.

Bardi: Proceed by tuning the others according to your experience with these [fifths].

Strozzi: It is done.

Bardi: Now play.

Strozzi: This is music which actually would make the meek angry to hear it. The manner which Timotheus⁴⁶ used to anger Alexander the Great and stir him to arms must not have been any different. Although I consider that those imperfect [intervals] which are already consonances are actually dissonances, I am unable to convince myself by this where that originates, or where the strings are stretched according to the species diatonic ditoniaion.

Effects of
the tempera-
ment of the
ditoniaion.

Bardi: Do you remember between which strings the diatonic had the major and minor semitones?

Strozzi: Yes, Signor.

Bardi: Please tell me.

⁴⁶Timotheus of Miletus, the most infamous of the ancient musicians due to his several innovations, is mentioned by numerous ancient authorities. See Bower, Boethius, pp. 36-39; Plutarch, On Music, translated by Benedict Einarson and Phillip H. DeLacy (Cambridge, Massachusetts and London, 1967), pp. 381, 419-423, 426; and p. 547 below.

Strozzi: The minor semitone in the diatonic is found--according to what you have told me--in each tetrachord between the two lowest strings, and the major semitone is found between b fa and h mi.

Bardi: Therefore, consider in such a distribution how small the one that formerly was major has become. On the contrary, consider how much larger the one that was minor [has become].

Strozzi: You are quite right. Has this happened, perhaps, because the tones have acquired that quantity which has been taken away from each octave by two major semitones?

Bardi: Actually, yes. The divine Pythagoras could hear the ditone, the semitone and both hexachords in this way and no other, since this diatonic distribution alone was in use at the time. Those who said, after him, that these intervals were dissonant were induced [to say this] for the very same reason,

Zarlino⁴⁷ in
the preface to
his Dimostrationsi.

⁴⁷Zarlino, Dimostrationsi, pp. 2-6.

although some have dreamed that [such a thing] could happen. It is because they have not heard [these intervals] in their true, legitimate places--which, according to these men, are above the octave--as if in those times they had not had knowledge of that quantity of strings which was necessary to do that, or as if within the octave, they did not make, when well placed, a pleasanter sound than they make outside the octave. It is also not exactly surprising, since Pythagoras heard those [intervals] which are contained in the syntonon and did not value them, that the musicians of those times did not use them in their songs. Pythagoras, due to the inconstancy [of things like these], refuted and banished from the world everything mixed, impure and variable, since he knew that in them caprice and rashness ruled. But like the bee, when he gathers the purest part of the celestial dew is fed and nourished by it, thus the divine philosopher

What things were valued and not valued by Pythagoras and why.

likewise tolerated among his effects the flower of the simplest and noblest things.

Strozzi: Before we dispense with our instrument, since it is already clear that in the ordinary distribution the fifths become flatter and, on the other hand, the fourths become sharper by a little more than that which would suit their proportions, I desire to know why such adjustments are necessary and if, on the contrary, the fourths could become diminished and the fifths augmented if the strings of this instrument could be suitably arranged? If this is not possible because of some inconvenience, demonstrate it and then tell me the reason. Show me, in addition, how much the fifths become diminished and the fourths become augmented and if it is possible to remedy such inconveniences and make each of the intervals--the consonant ones as well as the dissonant ones--fall under its true form and proportion.

[33]

Bardi: In order to clarify well your new doubts, it has been very much to the point to have tempered the instrument according to the diatonic. In that distribution--as you have heard--those intervals which--according to our practicing contemporaries--have the name of imperfect consonances, become dissonant not due to the perfection of the fifths, as countless dare to say, but because of the largeness of the tones and the smallness of the semitones.



[Figure 98.]

Since I now want to temper on this instrument the strings of the present diapason so that each ditone and semiditone, together with each major and minor hexachord, becomes consonant, it is necessary to reduce them to their original state. That distribution is similar to the order and proportion of the syntonon. It is neither the same--as some believe and write--nor did its authors ever think such

a thing. Instead, it was done by accident when they were seeking to tune the intervals nearer to perfection than the nature of the instrument--rather the quantity and quality of the strings--and their experience with the instrument and knowledge of those devices would allow. They always avoided the inequality of tones--as has been said before--together with each inconvenience which could possibly proceed from such inequality in this modern practice. At present, in order that we avoid such an inconvenience, we will subtract--mainly according to the method of Aristoxenus, since we cannot separate any superparticular interval into equal parts by another method--four seventh parts of a comma of our times from the interval which is between the note F fa ut and G sol re ut, thus compressing that interval by such a quantity. And because the very same distance which is found between F fa ut and G sol re ut remains between G sol re ut and a la mi re after having lowered

How much the intervals become altered in the temperament of the ordinary keyboard instruments.

the first interval by the quantity which has been mentioned, we will flatten the said a la mi re by an entire comma plus a seventh part of a comma. Afterwards, we will make the note h mi lower than its original state by a comma plus five sevenths of a comma. Thus the very same space will come to be contained between this and a la mi re as is contained between the two lower tones combined. Now, leaving between the note h mi and that of c sol fa ut the entire remainder which has been subtracted at the rate of three tones (which concurs with the composition of the tritone) is not suitable at all, because not only is the minor third which is found between a la mi re and c sol fa ut dissonant compared with the acquisition made--as you can hear--but also, when one desires to augment the limma, which originally was between h mi and c sol fa ut, as far as the limit of a sesquiquindécima--or a little more, as I have said--it is made

a half comma larger. Because, however, the fifth which is found between F fa ut and c sol fa ut does not remain diminished by such a quantity, and because the aforesaid minor third--in coming a little nearer to its least imperfect state--becomes less languid and more pleasing to the ear, we will lower c sol fa ut two seventh parts of a comma. Each fifth necessarily will become diminished by this amount. When this is done it will be necessary, if one desires the very same distance which is found between F fa ut and G sol re ut and the other diminished tones to remain between this c sol fa ut and d la sol re, to lower d sol re six seventh parts of a comma. And because e la mi does not exceed the latter, we will lower it a whole comma plus three seventh parts of a comma. This remainder we will leave entirely to the interval which remains between this e la mi and F fa ut. Thus both minor semitones [limmas] will be augmented by a comma plus three sevenths

of a comma. Although the major semitone of this new distribution exceeds the sesquiquindecima by the same amount, it is not unsuitable, since it was derived from a whole greater than the sesquinona. We will then proceed according to this order distributing all the other strings which this diapason has below, above and within its limits. From what has been said, you can now comprehend clearly that the fifth, especially, becomes lower than the usual state of its proportion and the fourth becomes sharp from that proportion. In addition, no matter how much one wishes, on the contrary, to make the diatessaron diminished and the fifth augmented, it is impossible, because the principal cause of it is inherent in the quantity of tones which these consonances contain, in the amount by which these tones are diminished, and in whether the semitones which contain such intervals are augmented. You have also been able to see how much it is, between what pitches, why, and how [it is] distributed. But I warn you that

The source of the aforesaid imperfection.

in this temperament the pitches which first contained the ditone now contain the major third and those that first contained the semiditone now contain the minor third. Between those pitches in which the major hexachord was found in the diatonic, one finds there at present the major sixth, and the minor [sixth] comes to be contained between those which enclosed the minor hexachord. One consequently finds that each tone in the demonstrated temperament becomes diminished by four seventh parts of a comma, the ditone by one whole comma plus the seventh [part] of a comma, the fifth by two seventh parts of a comma, and the major hexachord by six seventh parts of a comma. On the contrary, the minor hexachord comes to be augmented by a whole comma plus a seventh of a comma, the fourth by two seventh parts, and the semiditone by six. From this one can argue that those who say that the comma is not perceptible are greatly deceived. They have confessed, for all of this, that in keyboard instruments, the sound of which they profess to

understand thoroughly, the fifths are found diminished and the fourths augmented. This I will more readily believe they have said [34] for politeness rather than through knowing that the fact is actually so.

Strozzi: I want to take this opportunity to tell you what happened to me some time ago while arguing with one such player. This fellow believed unequivocally that sounding only those two registers distant by a fifth--which are ordinarily found in each church organ above the fifteenth--made excellent listening without having the octave and the principal down. He did not notice that the pipes were so arranged that the fifth and ninth were always together and the fourth and seventh were also paired. With all this it was not possible to convince him by means of reason, so that he almost forced me to accompany him up into an organ loft. When he had opened the two registers mentioned and had begun to play, he strove for excellence in all the ways which the fifths, the fourths, the major thirds and the minor

Dissonant registers.

thirds would unite with the fifth above or below, without having the usual foundation of the octave and the principal underneath. This defect is hidden at present from the ears of the uneducated, because of the great noise made by the simultaneous playing of so many different sounding pipes. If the authority of one of his wretched fellows, who was pumping the air for the pipes, had not interfered, he would have put me in a bad way with his importunity.

[The organ pumper] spoke to him thus:

"You have come up here, I believe, to mock me. Either play properly or I will let up on the bellows." Still another of these "sages" once tried to persuade me that the said two registers were distant from each other by a most perfect fifth but that each one separately, between its particular pipes, had flat fifths, according to the custom of the other keyboard instruments. I replied to him [and asked] if the G sol re ut of the lowest register was in unison with the c sol fa ut of the

highest register. "You know that perfectly well," he responded, whereupon I told him that between two things equal in length one could not in any way be shorter than the other and, on the contrary, if one were larger than the other, naturally they could not possibly be equal. After this, he shut up and said no more about it.

Bardi: It is indeed an important matter that most men speak so freely and willingly of those things which they understand the least. However, let us leave [such matters] aside and return to our string distribution. Since we wished to apply it to the diatonic syntonon, we will now need to subtract from each major tone four seventh parts of a comma and will have augmented the sesquinona interval--also called minor tone--by three of these. Because of this, they will come to be made equal. Each sesquiquarta (form of the major third) also becomes diminished by a seventh part of the comma and the sesquiquinta (form of the minor third)

becomes reduced by the same amount. The diapente remains reduced by two seventh parts of the said comma, so that the sesquiquindecima (called today major semitone) becomes increased by three seventh parts of the very same interval [i.e. comma]. Consequently, the sesquivicesimusquartas (called minor semitone) comes to remain in its original form, exactly the opposite of what occurs to voices. Then, if everything I have said is true, the superbipartientetertia (form of the major sixth) will have been increased by as much as the minor third has been reduced, and the supertripartientequintas (form of the minor sixth) becomes similarly increased by as much as the major third has been diminished. The fourth comes to be augmented by two seventh parts of the comma, [the same amount] which the fifth is diminished. The octave, always removed from any corrupt extreme (for the reason which will be stated below), remains with the dupla in its usual perfect form.

There have been others who, in distributing the same strings in the very same species, tuned them flatter than the previous opinion. They have wanted, instead of the two seventh parts of the comma which have been taken from the diapente and added to the diatessaron, to remove the fourth part [of a comma] from the diapente in order to bring the diapente and diatessaron to their least imperfect state--mentioned earlier--by a twenty-eighth [part] of this comma. But afterwards, the minor sixth and the major third have remained in the same proportion as they have in the syntonon, because they have taken a half comma from the major tone and have given it to the minor [tone]. Since this made them equal, I think the matter is worthy of consideration at that point, but since they are in fact the same as they were at first we will relegate it among their other impertinences.

Strozzi: How is it, Signor Giovanni, that when the strings of this instrument are distributed according to the order of

the ditoniaion, where the consonances called perfect today have been placed in their true proportions, among which none of the thirds nor the sixths agree, that in this [situation], where the diatessarons become augmented only into their true form, one does not hear between the extremes of these, either in the low register or in the high, any of those harsh effects which were heard in that [other distribution].

Bardi: Such a thing cannot occur inside or outside the extremes of the fourth, since by diminishing the tones and augmenting the minor semitones, the cause of the disagreeable sound is removed from the thirds and the sixths. These cases, and not the perfection of the fifths and fourths--as has been said--were hindering their concord. In addition, one can quite easily discover in keyboard instruments the fifths and the fourths in their true proportions without otherwise hindering the imperfect [intervals], as anyone clearly can see and hear in the temperament

we have newly discovered, which brings into practice the greatest excellence and perfection one could possibly desire.

Strozzi: I am entirely satisfied, but let us proceed to the distributions of Dydimus and Aristoxenus.

Bardi: Dydimus, a most noble Pythagorean musician, lived some years before Ptolemy and made a new string distribution in each one of the three genera of harmony. Among the others, that which he made in the diatonic proceeds in each of its tetrachords in the manner which you see in this one, which is the lowest one in the system, known as hypaton.

[35] How the syntonon was distributed by Dydimus.

E la mi	72	_____	
		Sesquioctave (or tone)	9 difference
D sol re	81	_____	
		Sesquinona	9 difference
C fa ut	90	_____	
		Sesquiquindecima	9 difference
h mi	96	_____	

[Figure 99--] Hypaton tetrachord of Dydimus.

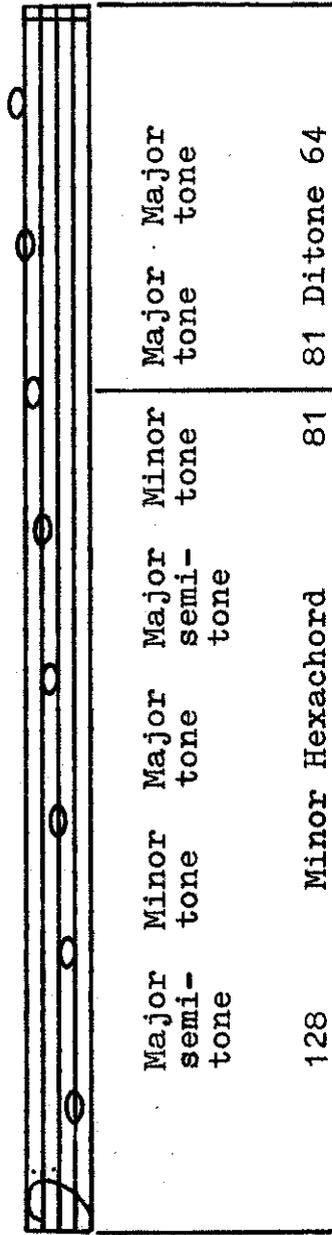
Afterwards, came Ptolemy, who changed the order of the two middle intervals of each tetrachord, placing the upper middle one in the position of the lower middle and vice versa, saying that it did not suit the larger one to be placed there, but only suited the other, which was smaller than it, but greater than the lowest. From this you can understand what Ptolemy's stand is regarding the syntonon and can know who ought to be given the honor and the palm.

Strozzi: Why, do you suppose, did those who have sought to persuade us that what is sung today is wholly syntonon--that is, diatonic syntonon--say that it belongs to Ptolemy rather than to Dydimus? According to what I have seen, one distribution does not apply any greater convenience or inconvenience to our way of composing and singing than the other.

Bardi: That which has failed to annoy you and many others has perhaps spoiled the designs of the authors of these things.

Strozzi: How [could that happen],
please?

Bardi: The interval which is found
between G sol re ut and h mi [i.e. B \sharp]
in the distribution of Dydimus is a ditone
and not a major third of the variety which
most people believe is sung today. That
which is found between h mi and this G sol
re ut is a minor hexachord and not a minor
sixth. Therefore, having first stated in
their writings that such intervals were
dissonances (as you have heard they really
are), they came too extremely and too
suddenly to their conclusions to provide
[anyone] opportunity to impede their
schemes. [To demonstrate] that these in-
tervals appeared thus, here is the first
species of the diapason, distributed ac-
cording to the intention of Dydimus
[Example 4.]. When you have examined it
carefully, you will find that what I said
is true. Although the same has occurred
in those of Ptolemy--as you have seen--
it has not been so clarified to the sense



The image shows a musical staff with a treble clef and a key signature of one flat (B-flat). The staff contains six notes: B-flat, C, D, E-flat, F, and G. Below the staff is a table with two columns of text and two columns of numbers.

Major semi- tone	Minor tone	Major tone	Major semi- tone	Minor tone	Major tone	Major tone
128	Minor Hexachord	81	Ditone	64		

[Example 4--The minor hexachord and ditone of the dis-
tribution of Dydimus.]

and judgement of the uneducated, and until now, it was possible for such a thing to defraud the general public more easily.

And this is the reason why they have said that the diatonic species which is sung today belongs more to Ptolemy than to Dydimus, unless we still wish to say--I do not believe this in any way--that they have ignored the difference which is found between these [species]. Also, as a result of this, they gave the name of comma to the excess by which the minor tone is exceeded by the major tone, and not, like the ancients, to that [excess] by which the major semitone surpassed the minor, in order to obtain more kinds of these, as you have seen.

Why the syntonon is attributed to Ptolemy.

Strozzi: What do we believe was the reason which persuaded so many of that first sect--if not in the beginning as I believe, [at least] from [the time of] Guido d'Arezzo on--that what was sung was the ancient diatonic? Among these, there have been many learned and renowned writers like LeFèvre, Gaffurio, Glarean and Valgulio,

[all of whom were] highly literate and ingenious men (and much more), for many great men like Aristoxenus, Euclid, and Vitruvius have first told them expressly that the intervals smaller than the diatessaron and those which are found between the diapente and the diapason were all dissonances. I do not know how they could say it any more clearly in their

Aristoxenus⁴⁸
in the 1st and
2nd [books] of
the Elements.

Euclid⁴⁹ in the
Introduction.

Vitruvius⁵⁰ in
the 4th chapter
of the 5th
[book].

⁴⁸Henry Macran, The Harmonics of Aristoxenus (Oxford, 1902), p. 179. "Though many smaller intervals than the Fourth occur in melody, they are without exception discords." Ibid., p. 198. "We assume then eight magnitudes of concords; the smallest, the Fourth--determined as smallest by the abstract nature of melody; for while we can produce several smaller intervals, they are all discords."

⁴⁹The Eisagoge of Cleonides was formerly attributed to Euclid. There is an English translation in Strunk, Source Readings, pp. 34-46. Cleonides calls the octave, fourth and fifth symphonic intervals because they blend together. Intervals less than the fourth are diaphonies, where the two notes refuse to combine, but grate harshly on the ear. See Ibid., p. 38.

⁵⁰Vitruvius, De architectura, translated by Morris Hicky Morgan (Cambridge, Massachusetts, 1926), p. 142. "For there can be no consonances either in the case of the notes of stringed instruments or of the singing voice, between

writings, and anyway the matter is easily manifested to the sense of hearing.

Bardi: It was difficult for Guido d'Arezzo to consider such a conception, for he was a monk and his goal was to teach his [fellow] monks to sing chants with facility. Since these chants were in unison and no harmonies of any kind were involved, he could not doubt that [conception] in any way. Therefore, the reason which led him to designate the notes in his Introduction with the same numbers⁵¹ which Plato used in his Timaeus (which had first been assigned by Pythagoras and later perpetuated by Boethius) was--as far

[36] Guido d'Arezzo did not know about the imperfect consonances.

two intervals or between three or six or seven; but, as written above, it is only the harmonies of the fourth, the fifth, and so on up to the double octave, that have boundaries naturally corresponding to those of the voice; and these concords are produced by the union of the notes."

⁵¹By numbers, Galilei apparently means the string lengths designed by Aristides Quintilianus (third or second century A.D.), a Greek writer on music, as a convenient method of expressing the Pythagorean tuning. Zarlino, Istitutioni, p. 104, cited by Galilei (see p. 250 below) incorporates these Aristidean numbers in a diagram of Guido's gamut.

as I am concerned--in order to give the thing reputation. He took not only the most ancient and famous distribution of strings that ever had been known, but that which had been given to mortals by Nature. For the same reason he did not wish to use the natural sign [i.e. h mi] on the note B fa, because neither the Greeks nor the Romans used it in their systems. His opinion was first put into use by others and afterwards was greatly approved by countless others.

Strozzi: So that in the time of Guido d'Arezzo, you believe that they sang the lowest intervals of tetrachords through a limma and not like today through a major semitone, and that they did not sing in harmony?

Bardi: I firmly believe that they did not sing in harmony because I do not find in his Introduction any mention of imperfect consonances, but I am definitely convinced that they played [in harmony]. Since the lowest interval of tetrachords

The way they sang in [Guido's] times.

was then a minor or a major semitone, I will more easily believe that the latter, rather than the former, was sung in that place. This is the reason which impels me to believe that. In the time of Guido d'Arezzo, every light of virtue-- in a manner of speaking--had gone out once again in Italy, particularly in regulated [mensural] music. It was because of this that the mode of singing which was acquired naturally at the beginning of the world had not been maintained. This is the same as that which rustic farmers use while tilling the fields and shepherds use to drive away the boredom [while journeying] through the forests and mountains behind their herds. It diverts them from continual and heavy labors. This kind of singing has always been used among men from the creation up to our times, nor will it have an end, except together with them or with the world itself, although Athenaeus⁵² says with

the testimony of Chamaeleon of Pontus that men have learned this art in seeking to imitate the song of the birds. Moreover, up to [Guido's] time or a little before, men had wept for many continuous centuries about their miseries. Afterwards, little by little, they began to apply themselves to literature, to music, and to the other fine arts. Thereupon, the above-mentioned Arezzo, the most learned man in the art of music in those times, began to reorganize the method of singing as much as the conditions of the century and his region would allow. He not only gave new names to the notes, since they had come into use a little before him, denoting them with the very same characters [as used in] the Latin alphabet in order that people could learn to use their voices comfortably with facility and distinction, but he also

From whom singing is learned.

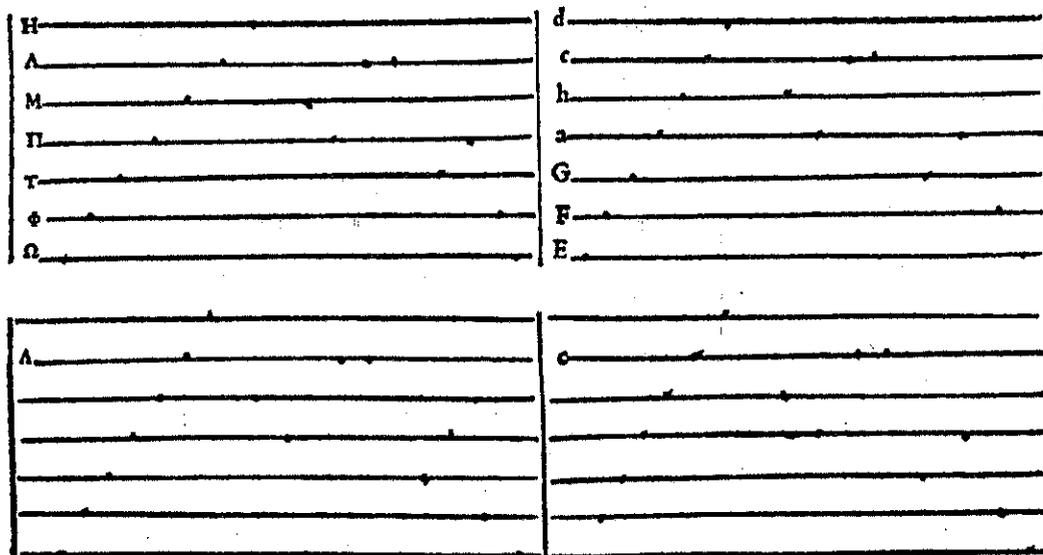
Guido d'Arezzo adds six notes to the system.

⁵² See Athenaeus, The Deipnosophists, 7 vols., translated by Charles Burton Gulick (London and New York, 1928), IV, 263.

designated, in addition to those of the Greater [Perfect] System, five [notes] in the high register and one in the low, afterwards distributing [all the notes] in seven hexachords. This was the same number as the letters he used, as it appears in his Introduction. Perhaps this was in imitation of what Demetrius of Phaleron⁵³ said with regard to the use of vowels, using the example of the Egyptian priests who pronounced the notes of their sacred songs over the sounds of these vowels, which were seven [in number]. The practical musicians who lived a little before the time of Guido d'Arezzo used, in order to signify the pitches of their compositions, the same characters which had already been employed by the ancient Greeks and also by the Romans, placing them on seven lines in this manner [See Example 5.], imitating perhaps the seven strings of the ancient cithara.

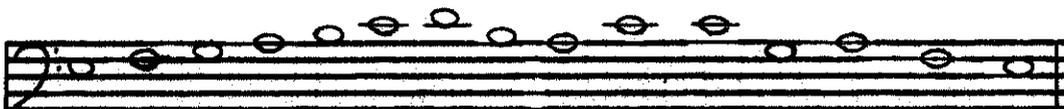
Demetrius of
Phaleron.

⁵³See W. Rhys Roberts, trans., Demetrius On Style (Cambridge, 1902), pp. 104-105.



[Example 5--An uncommon system of notation.]⁵⁴

⁵⁴See August Ambros, Geschichte der Musik, 5 vols. (Leipzig, 1891), II, 149. The notation depicted in Example 5 corresponds to no known system. Although Galilei tries to show that it was once in general use, Ambros points out that if this were true, the case would have been more publicized. See also Athanasius Kircher, Musurgia universalis (Rome, 1650), pp. 213-214.



Afterwards, Guido d'Arezzo removed the tedium of the many lines and clefs over which, instead of the notes of our times-- which actually were discovered in Paris by the great Doctor Jean de Muris⁵⁵--he used points, placing them also within the space which is found between one line and another, as the composers do today. By using these [points], such men acquired the name of contrapuntists. This was very appropriate, through composing their songs with points, which have no other being in nature except solely in the imagination of men. They result by chance like the figures composed today by the superstitious geomants⁵⁶ regarding the judgements of

[37] Guido was the first to place notes on the spaces.

Notes were discovered by Jean de Muris.

The origin of the name contrapuntist.

⁵⁵See Nicola Vicentino, L'antica musica ridotte alla moderna prattica (Rome, 1555), p. 9. Vicentino was the first to make the extravagant claim that Jean de Muris invented musical notes. This theory was later supported by Grassineau, but refuted by Mersenne and Rousseau. See J. F. R. Stainer, "Johannes de Muris," Groves Dictionary of Music and Musicians, 5th ed., Eric Blom, ed., 9 vols. (London, 1954), V, 1008.

⁵⁶Geomants were diviners by means of dots and lines, or by clumps of dirt scattered on the ground.

problems given them. These men render their judgements of such problems according to what the aforesaid chance has caused--according to some of their few principles and terms, however--without otherwise knowing beforehand the effects which can result when one operates in one manner rather than another. With ten or more geomants, rather than only one, there will be ten different opinions about the very same problem, which they will form into ten or more figures. In the same way, entrusting ten or more contrapuntists to express the same, particular affection of mind with their music (according to the usage of this century), they would express it in ten or more different ways and in various modes. The same [thing] will happen to one alone, if he tries to express it ten or more times. [All this occurs] for no other reason than the fact that both the geomants and the composers have allowed chance to guide them in writing their

compositions, or let us say that their principles (according to what Seneca tells us) are full of wind and woven of fog and air like the dresses of lascivious Roman noblewomen of his times.

Seneca,⁵⁷
On Benefits
and also
Petronius
Arbiter.⁵⁸

Strozzi: Did the musicians prior to Guido use, for that reason, only the lines in writing their musical compositions?

Bardi: Yes, indeed. As I said before, they used different symbols to denote the clefs, and a greater number of lines, at least in the songs which were played. This was because they had no idea of the space which is found between one line and another, in order to indicate to the sense--as has

⁵⁷ Lucius Annaeus Seneca, "On Benefits," in Moral Essays, 3 vols., translated by John W. Basore (London and New York, 1928-1935), III, passim. Seneca refers continually to the lasciviousness of Roman matrons of his times.

⁵⁸ Titus Petronius, Satyricon, translated by Michael Heseltine (London and New York, 1916), p. 99. Petronius, quoting Publilius Syrus, a famous writer of farce, admonishing a cuckold: "Thy bride might as well clothe herself with a garment of wind as stand forth publicly naked under her clouds of muslin."

been said--the strings from which they were derived. They had at that time Greek characters which denoted their clefs and notes as you have been able to comprehend from those examples

[Example 5.].

Strozzi: Would it be possible to see some of those so-called early modern compositions?

Bardi: It certainly would.

Strozzi: I hope it is not a burden, but please, show me one of them.

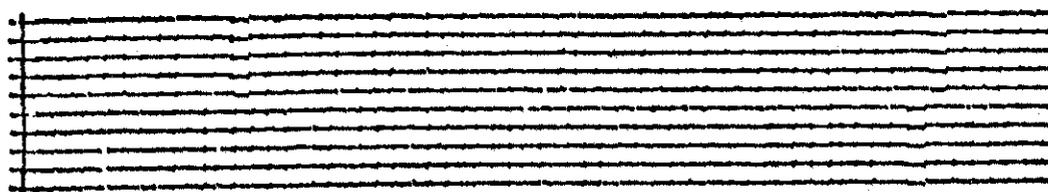
Bardi: Here is an example⁵⁹ of one [Example 6.], among others which I have secured, which was given to me by one of our Florentine gentlemen⁶⁰ who had discovered them in one of his very oldest books. It is more complete and better preserved than any I have

⁵⁹The musical example [Example 6.] is not pre-Guidonian notation, as Galilei asserts. It is actually a "Hosanna in excelsis" trope. Compare Analecta hymnica medii aevi, Vol. XLVII, ed. by Clemens Blume and Henry M. Bannister (Leipzig, 1905), p. 346. Most copies of the Dialogo have only blank staves in this example. The

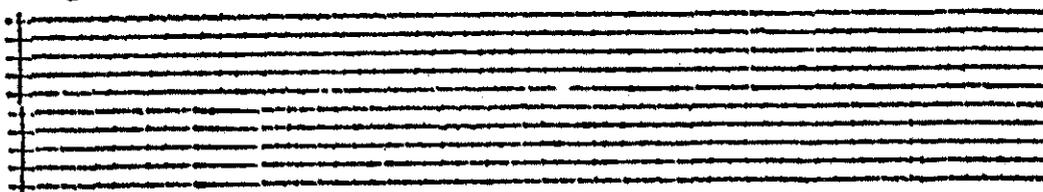
ever seen. A little later I will show you another contemporary example still earlier than this.

Strozzi: Here a long explanation will be needed since I want to know how to read it, not just sing it.

notes, apparently, were either omitted through the carelessness of the printer, or were intended to be added at a later date. See Palisca, Girolamo Mei, pp. 154-156.



clanget hodie vox nostra melodum symphonia instant annua iam quia preclara solemnia



personet nunc cinnula armonia organa musicorum choria Tonorum quem dulcia alternatim

⁶⁰Galilei is referring to Girolamo Mei, whom he mentions only once by name in the Dialogo, despite the large role Mei played in formulating that work.

Clan-gat ho - die vox no - stra me - lo - dum sym - pho - ni - a in - stant
 an - nu - a jam qui - a prae - cla - ra so - lem - ni - a Per - so - net
 nunc ti - nu - la ar - mo - ni - ae or - ga - na mu - si - co - rum
 Do - re - a To - no - rum quam dul - ci - a Al - ter - na - tim con -
 cre - pet nec - ne mo - dul - la - mina Dia - pas - on al - ti - so - na
 per vo - cum dis - cri - mi - na te - tra - cor - dis fi - gu - ra - rum
 al - ta con - ce - dens cul - mi - na su - sto - lat no - stra car - mi - na

ut cae-li fas-ti-gia im-nis an-ge-li-cis quo e-ren-da
 pa-tri me-lo-dia quo nos me-re-a-mur am-pla ca-pe-
 re per-mis-sa si-ve fru-i-tu-ri me-ra sanc-to-rum
 glo-ri-a ad quo-rum co-le-gi-a pia nos du-cunt me-ri-ta

[Example 6--Clangat hodie]

Bardi: Let it suffice you for now [to say] that it is--as I understand it-- a composition to be sung to the [sound of] the aulos rather than the lyre, and it is in the Mixolydian mode. I will interpret for you in detail each of its other particulars at the appropriate place, but let us proceed to clarify the problems you first proposed to me without placing any new ones under consideration.

Strozzi: Let it be as you please, but returning to Guido d'Arezzo, it seems to me that we owe him a great deal since he added to many fine things to the music practice, despite the roughness of the times. Before I forget it, however, I am forced, contrary to our agreement, to question you about another new problem, if you will pardon me. I do not know for what purpose Guido d'Arezzo added so many notes to the system, since they still did not sing--according to your opinion--in harmony, but only used simple

Zarlino⁶¹ in chapter 30 of the 2nd part of his Institutioni.

⁶¹Zarlino, Institutioni, pp. 103-105.

chants similar to that in the composition you showed me.

Bardi: I do not maintain that Guido d'Arezzo added any note to the system, but only that he gave each one a particular name. As for those notes which are seen in his Introduction in addition to the fifteen of the ancient Greeks, I firmly maintain that he derived them from the modes of Boethius, which are eight in number, the extremes of which have the same number of tones and [are] in the same order. Furthermore, I must say that he derived them from wind instruments like the organ, or else from those stringed instruments that were in use in those times, which were similar or perhaps even the same as the simikion and epigoneion of the Greeks. From these notes he took-- according to the diatonic species, which he understood was sung in those days-- that number which comfortably coordinated with the diversity of human voices which sang different songs with them, but [did] not [sing them] simultaneously, assigning

Origin of the notes added to the system by Guido.

eight to the low register, eight to that of the middle, and the remainder to the high, so that, in all, since the lowest of the eight in the middle was the same as the highest in the low register, and the very highest of those in the middle was the lowest in the upper register, they came to make a total of twenty-two tones and strings in three octaves, divided as I have said. Others have said that he ordered them in seven hexachords since he wished to denote to us with these numbers, which are six and seven, the excellence and perfection which is found in this distribution. It happens that the number of six is the first of the perfect [numbers] just as the second is twenty eight, which is

Pier Maria Bonini⁶²
in his [book on]
music.

Perfect numbers
[are] 6, 28, 446,
8128 and others
like them.

⁶²Pier Maria Bonini was a Florentine mathematics professor who was the author of Acutissimae observationes nobiliss. disciplinar. omnium musices (Florentiae, 1520). This treatise is apparently not extant. See Robert Eitner, Biographisch-bibliographisches Quellen-Lexicon der Musiker und Musikgelehrten der christlichen Zeitrechnung bis zur Mitte des neunzehnten Jahrhunderts, 10 vols. (Leipzig, 1899-1904), II, 112.

produced by the settenario and its parts. Others have said that they were ordered by [Guido] in major hexachords, since each one of these disposed in such a way contained some one of the three species of the diatessaron. Returning to the two stringed instruments just now named, I say that one of these had thirty-five [strings] and the other [had] forty. One can argue from the quantity of [strings] that the professional players of these [instruments] played in harmony, no longer touching them with the plectrum, which had gone into total oblivion, but with the fingers, like the harp. This species of harmony is more suited for consonances than the others, as we have already declared. This mode of composing and playing so many airs together at the same time probably had its origin from that manner of playing, as we will show at the proper time. The singers of those times produced the musical intervals according to the distribution of the strings and not otherwise and [this

Derivation of
the singing
of today.

practice] has been so conducted and used up to today, giving the name of imperfect consonances to the major and minor thirds and sixths. The surnames of these [intervals] must cause [confusion] in the mind of the general public and persuade it without thinking further to believe and say that they were the same as the ditone and semiditone and as the major and minor hexachord, according to the form in which they were contained as Pythagorean numbers in that Introduction of Guido d'Arezzo. That name of imperfect consonances was actually placed there with judgement and the greatest discretion, not only because of their indeterminate and variable nature, but because the greatest part of these--in order not to say all--appear to be, rather than actually being consonances, although they have been ascertained by our practicing contemporary contrapuntists as such, without seeking further due to necessity, having discovered that it would be a burden to them.

This opinion that they were the same as the ancient [interval] lasted in the minds of men until the coming of Reverend M. Gioseffo Zarlino, who has sought with diverse arguments to demonstrate to the sense and to the intellect that such imperfect consonances are not in any way those which are found among the notes distributed according to the diatonic ditoniaion, nor even among those same [notes] in the syntonon of Ptolemy.

Because of the novelty of this thing, he allowed himself to induce others to believe and say that the diatonic species which is played and sung today is wholly the syntonon of Ptolemy, which is not true, as you have seen. It is not valid, however, to say that the thirds and the sixths are consonances in the syntonon of Ptolemy; what we sing are the other consonances which are, moreover, the same as [those of] Ptolemy. Nevertheless, the world owes to this man, [who is so] exemplary of customs, life, and doctrine, a perpetual obligation, due to the many

[39]

This is what led Zarlino to say that the diatonic species which is sung today is the syntonon.

Praise of
Zarlino.

fine efforts which he has made, particularly in music. From these efforts one derives knowledge of [an] infinite [number of] things without which easily the greater part of mankind would be in darkness.

Strozzi: Signor Giovanni, I desire to understand how those two ancient stringed instruments, which you mentioned earlier, were made, that is, with regard to their form and the distribution of their strings.

Bardi: You are asking me about something like the platage of Archytas of which there is no record other than the name and the number of strings. Although the knowledge of these is small, I do not doubt at all that those who wish to conjecture judiciously about them will find approximately what form they had and in what proportion their strings were arranged. In order to tell you briefly what I believe about it, I consider that the material and the form consisted of a wooden frame similar to

that of the harp, within which were stretched, that number of strings which you have understood, stretched either in the same manner or in a slightly different one, and distributed among them in a certain way. I always, however, revert to the judgement of one who understands better than myself. The epigoneion was discovered by Epigon of Ambraciota, chief of a very famous sect, either a little before, a little after, or at the same time as Socrates, according to what Porphyry⁶³ says in his commentary on the music of Ptolemy. This Epigon, according to the opinion of Julius Pollux,⁶⁴ was the first who used the fingers to pluck the strings without the plectrum. This method of touching the strings, together with the number of these [strings] argue that he played harmony. This method was

The meaning of epigoneion and the name of its inventor.

Julius Pollux.

⁶³See Ingemar Düring, Porphyrios Kommentar zur Harmonielehre des Ptolemaios (Göteborg, 1932), p. 3.

⁶⁴See Eric Bethe, ed., Pollucis Onomasticon, 3 vols. (Leipzig, 1900-1937), I, 219.

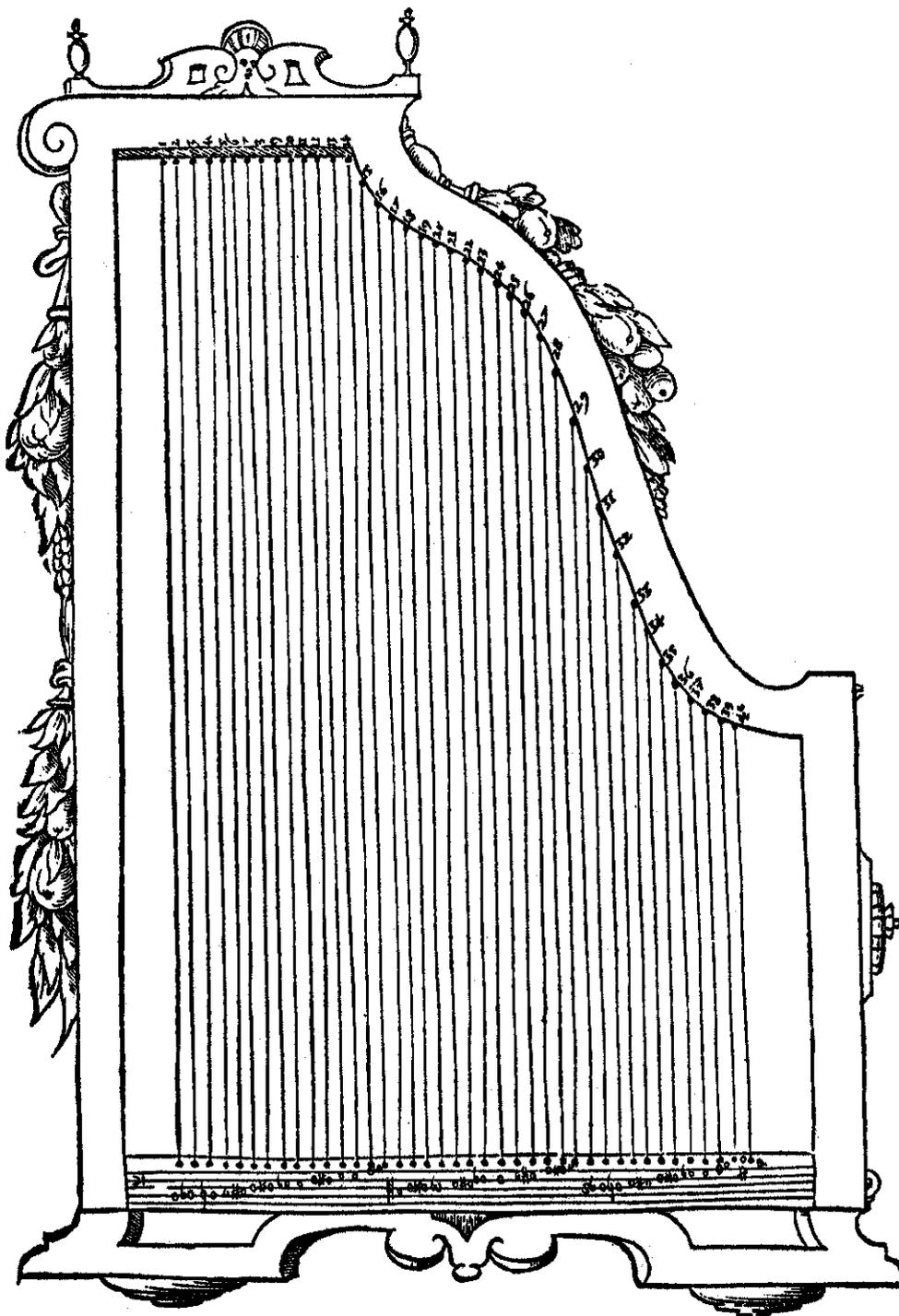
also followed later (as Suetonius Tranquillus⁶⁵ tells us) by Nero. Suetonius says in describing [Nero's] life that one time among others when he appeared publicly in the theater to sing and play on the lyre in competition with a number of professional cithara players of his times, he first made on his lyre a beautiful prelude with his fingers and afterwards began to sing. It is not known who was the inventor of the simikion, which greatly surprises me. It is probably true that, since it had thirty-five strings, it must have been discovered prior to the epigoneion which had forty [strings]. Its inventor is no less worthy of esteem by his followers than Epigon of Ambraciota.

⁶⁵See Caius Suetonius, Lives of the Twelve Caesars, translated by H. M. Bird (Argus, n.d.), p. 272. Suetonius makes no mention of plucking the strings with the fingers, but only says that Nero made the "usual prelude" before beginning to sing.

[DIAGRAM III]

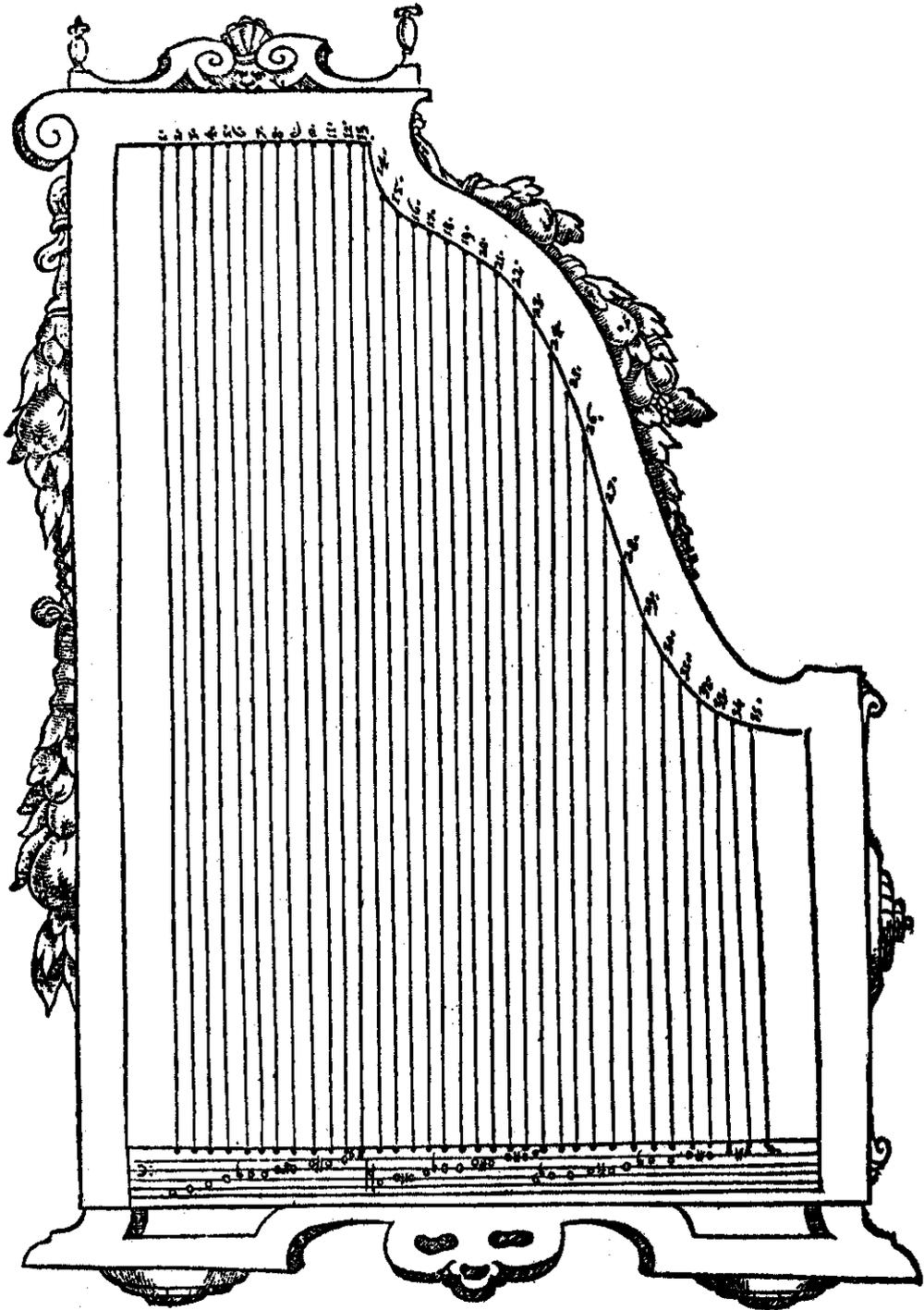
[40]

EXAMPLE OF THE EPIGONEION, INSTRUMENT
OF FORTY STRINGS, DISCOVERED
BY EPIGON OF AMBRACIOTA



[DIAGRAM IV]

EXAMPLE OF THE SIMIKION, INSTRUMENT
OF THIRTY-FIVE STRINGS



I come to the distribution of Aristoxenus and say that Aristoxenus,⁶⁶ a most noble musician and philosopher, made six different distributions of strings, that is, two diatonic, three chromatic, and one enharmonic. But we, for brevity, will deal with only those which will be needed, reserving the others for where they might be necessary. This most knowledgeable musician used to assign to each of his tetrachords that portion and quantity of sound of the diapason which agreed with [each of] their diverse genera and species. In order to do that, he first divided the diatessaron, which consisted of two tones and an entire half of [one of] these [tones], which suited his purposes, into sixty tiny equal parts. I say [that this was] with regard to the sound, not to the length of the line or string, although that quantity was also considered. [Aristoxenus] then gave twelve of these parts to the lowest interval of each of his tetrachords,

[42]

[Aristoxenus]
in the 2nd
[book] of the
Elements of
Harmony.

⁶⁶See Macran, Aristoxenus, pp. 202-204.

twenty four to that of the middle and the rest to the highest [interval]. He gave to this system, since it was so divided and ordered, the name of intense diatonic, calling the lowest interval of each of these tetrachords a semitone and calling the other two higher intervals tones. And this is the way the tones and semitones which he treated in different phases of his distributions were constructed. Of the three chromatics which he made, he named one the tonikon, which he arranged thus. He assigned to the lowest interval of each of its tetrachords twelve of the sixty tiny parts mentioned above, into which he had divided the diatessaron. He gave the same number to that of the middle and the others to the high [tetrachord].

Strozzi: Would it not have been the same to have divided the diatessaron into five equal parts rather than into sixty, and in the intense diatonic to have given one of these to the lowest interval of the

Ptolemy⁶⁷ in the 1st [book] of the Harmonics at the 12th chapter.

⁶⁷See Düring, Ptolemaios, pp. 44-46.

tetrachord, two to that of the middle, and the rest to the high? [Would it not also have been the same] in the chromatic tonikon to have given one of these to the lowest interval of the tetrachord, another to that of the middle, and the three which remained to the highest?

Bardi: It would have been [the same] in the two distributions you have mentioned, but assuming that that division of the diatessaron served each of the other four, differently distributed, he would have been unable to avoid the tedium of smaller numbers by dividing it into fewer parts. [This was] because one time it would be necessary to subtract half of the tone and another time it would be necessary to subtract a fourth of it. In the case of the semitone, it would not only be necessary to subtract a half or a fourth, but also a third. Sometimes it was necessary to augment one [interval] or the other by such an amount, in order to be able to construct conveniently any

The reason why Aristoxenus divided the diatessaron into sixty tiny parts.

one of the other intervals that agreed with his other different distributions. In order to do that he selected the number sixty as one which was capable of being divided in two, three, four, five, or six parts.

Strozzi: Could it be that the ancient musicians should have assigned tens of thousands of those [tiny parts] to the tones of the diatonic ditoniaion because the same numbers served conveniently for the chromatic and for the enharmonic?

Bardi: This was actually the case. One may consider it particularly in the ancient enharmonic, between the note tritesynemmenon, designated by the number, 4491, and the paranete of the same tetrachord, designated by another number, 4374. I tell you also, in regard to the distribution of Aristoxenus, that the intense diatonic and the chromatic tonikon--these are commonly used on the lute and the viola d'arco--more nearly

approach perfection with regard to their consonances, assuming for now that perfection is the fifth within the sesquialtera [proportion] and also the fourth in the sesquitertia. This perfection is not attained by those keyboard instruments which have been preached to us as [playing] the plain syntonon by those who have refuted the distributions of Aristoxenus without saying why. Anyone who takes the trouble to examine them by means of their particular monochords will be able to see that fact.

Strozzi: Would he not [have been] able to make me understand with ease and very great accuracy how greatly the perfect and imperfect consonant intervals of the lute and the viola become augmented or diminished from the true state in which they are contained [in] the syntonon, in order to avoid the various bad relationships and the confused and inconstant difficulties by which they are different from those of the diatonic ditoniaion?

Bardi: He would have been able [to do this] with clarity and facility, but not with greater exactness than one can show by means of the harmonic ruler.

Strozzi: Go on, please.

Bardi: Initially, the octave on the lute and on the viola (which proceed in the same manner stepwise), always being far from any imperfection, consists--as you know--of six tones, that is, twelve semitones. By conforming more than one should with the usage of our practicing contemporaries, we can also say truthfully that it consists of five tones and two semitones. We know, therefore, that each of their tones is less than the sesquioctave and greater than the sesquidona. The semitone becomes less than the sesquidecima and greater than the sesquidecimusquartas. The minor third is surpassed [in size] by the sesquiquinta; the major [third] exceeds the sesquiquarta; the diatessaron surpasses the sesquitertia; the diapente is less than the sesquialtera;

The formation of the intervals on the lute.

the minor sixth is surpassed by the super-tripartientequinta; the major [sixth] surpasses the superbipartientetertia; the tritone and the semidiapente are equal; consequently the semidiapente is smaller and the tritone greater than the ones contained in the syntonon.

Strozzi: Although I have understood the way of placing frets on the lute and viol which you have mentioned on other occasions, it seems to me, initially, that each of their tones is [a] sesquioctava, and that the minor third, in addition to many other intervals, is in the same form in which it is contained in the syntonon [43] and not exactly contrary to the form of the sesquiquinta.

Bardi: How can you maintain that such a thing can occur, since the tones are equally divided into two parts--as you have said and proved--and since the major third is consonant?

Strozzi: Listen, please! The tone, initially, falls between eighteen and sixteen since the whole is divided into

eighteen equal parts of which the tone contains two. This is the same as saying that the tone falls between nine and eight. Is all this not so?

Bardi: Proceed further, so that I fully comprehend your question.

Strozzi: Does the minor third contain three semitones which are the same value as three eighteenth parts of the whole? And do the fifteen [parts] which remain, compared to the eighteen, have the same relationship together as six has to five, which is the true form of the minor third according to the syntonon.

Bardi: Now, be advised that two eighteenth parts are only equivalent to a ninth part of the whole in this manner of measuring, because these parts are considered as a quantity of the line or string. To prove that this is true, here is the compass. By measuring with it, you yourself will find that the two first semitones on the lute do not fill up a ninth part of the length of the string, since three parts

are not a sixth part of the whole [string] but only a lesser quantity.

Strozzi: I have understood everything extremely well, but continue your original discussion.

Bardi: Because of the great agreement and similarity which the diapason, more than every other interval, has with the unison, not only can the simplest and most perfect of octaves be found, but one in which the ear can be less deceived in comprehending it. If its lowest terms are multiplied together a thousand times and then added together the upper pitch is always contained by unity. This does not happen to any of the other intervals, because they have not been taken as many times as that consonant one. For this reason, and in order to avoid both difficulty and danger of error, we will employ its aid and that of its replicates in this important undertaking. It is first necessary to know that each tone of the lute is a sixth part of the ancient comma less

than the sesquioctave. I will prove it to you in this manner. It is clear that six sesquioctave tones are greater than the diapason by one of these commas. If, moreover, six tones on the lute will fill [the diapason] entirely, without anything remaining or lacking, each one of these [tones on the lute] becomes, consequently, a sixth part of this [ancient] comma less than one of those [sesquioctave tones]. I say in addition that the sesquioctave is surpassed by each tone of the lute by three fourths of the sesquioctagesima, which is, according to our practicing contemporaries, the comma of our times because each octave is capable--as one can clearly understand from what we have mentioned above--of five sesquinonas, three commas, and two major semitones of the syntonon. These two major semitones give us a sesquinona, and approximately a comma and a half more, so that we can also consider each octave to be capable of six sesquinone tones plus four commas and a half, approximately. When these four

commas have been distributed among the aforesaid six tones, each [tone] will receive two-thirds plus a sixth part of that half [comma]. Now because two-thirds plus the sixth part of a half happens to make, when added together, three-fourths of an entire [comma], each tone of the lute necessarily comes to surpass the sesquinona by this quantity. In addition, each of the minor thirds of [the lute], compared to the sesquiquinta are diminished by three-eighths of a comma. I will prove that to you thus. The octave of the lute consists exactly of four minor thirds. Let us see now, according to the example below, in which a dupla has been subtracted from four sesquiquintas added together, what the remainder will be.

1296	625	Form of four sesquiquintas added together
	X	
2	1	Form of the dupla
1296	1250	
2	648	625 Remainder

[Figure 100.]

When a dupla has been subtracted from four sesquiquintas added together, there remains the super 23 partiente 625, which clearly demonstrates that the minor thirds of the lute, compared to the sesquiquinta, are diminished, because four of those minor thirds fill up an octave entirely. This super 23 partiente 625 consists of approximately a comma and a half. When the said interval has been distributed among four minor thirds, it will convey to each of these the quantity of three eighths of a comma. Each minor third of the lute, as I said, will be diminished by such a quantity when compared to the sesquiquinta. There is no doubt that since the octave of this instrument is in its true proportion and far from any corrupted extreme, and since each one consists of a minor third and a major sixth, each one of these major sixths will be augmented in [the octave] by the same amount as each one of those minor thirds will be diminished. Although the intellect understands very well the truth of this, I nevertheless wish to

prove it to the sense with a convenient example, which will be this. It is certain that four major sixths of the lute fill up three octaves entirely. Let us see, therefore, by subtracting three duplas from four superbipartientetertias, what will remain.

[44]

625	81	Four superbipartiente-
		tertias added together
	X	
8	1	Three duplas added
		together
625	648	Remainder

[Figure 101.]

When the octupla, form of three diapasons, is subtracted from four superbipartiente-tertias added together, the remainder is the sub 23 partiente 648. This argues that what I said concerning the augmentation of the major sixths is true. Each one of these is augmented on the lute by three eighths of a comma. In order to see now how greatly the major thirds are augmented on this [instrument], we will follow this order. It is certain that

three major thirds of the lute fill up entirely the space of an octave. Let us see now according to the example which follows what will remain to a dupla when three sesquiquartas added together have been subtracted.

$$\begin{array}{r}
 2 \quad 1 \quad \text{Dupla} \\
 \quad \quad \quad \times \\
 \hline
 125 \quad 64 \quad \text{Three sesquiquartas} \\
 \hline
 128 \quad 125 \quad \text{Remainder}
 \end{array}$$

[Figure 102.]

When three sesquiquartas have been subtracted from the dupla, the remainder is the supertriptiente 125. From this fact one may comprehend that the major thirds of the lute are actually augmented. When we examine the remainder carefully, we will see again the amount which composes the supertriptiente 125, which, as I proved to you in the beginning of our discourse, was approximately a comma and a half. This comma and a half, distributed among the said major thirds, conveys to each one a half comma. Any

major third will be augmented by half a comma and any minor sixth will be diminished [by the same amount]. This I will prove to you in this other manner. There is no one who doubts, conceding that the minor sixths on the lute consist of eight semitones [apiece], that two octaves, each containing twelve [semitones], do not amount to the same [thing] as three minor sixths. Let us see, therefore, according to the example, how much of it will remain when we subtract three supertriptientequintas from the quadrupla, form of the bisdiapason.

4	1	Quadrupla
	X	
8	5	Supertriptientequintas
20	8	First remainder
4		
5	2	
	X	
8	5	
25	16	Second remainder
	X	
8	5	
125	128	Third and final remainder

[Figure 103.]

The remainder is the subtripartiente 128, from which it clearly appears that whatever amount the major third is augmented, the minor third on the lute is found to be diminished by the same portion. If the situation were any different, it would follow that the octave, when composed of this [minor third] and that [major third], would be dissonant, but this is not so in any way. In addition, I will prove to you in this manner that the fifths of the lute are actually diminished as I have said they are. It is clear that twelve lute fifths fill the void of seven entire diapasons precisely, since four diapasons contain seven lute fifths plus a semitone. Let us see now what will remain of one interval which contains in itself neither more nor less than seven duplas, after twelve sesquialteras have been subtracted from it [Figure 104.]

128		1	Seven duplas added together
	X		
3		2	
<hr/>			
256		3	First remainder
	X		
3		2	
<hr/>			
512		9	Second remainder
	X		
3		2	
<hr/>			
1024		27	Third remainder
	X		
3		2	
<hr/>			
2048		81	Fourth remainder
	X		
3		2	
<hr/>			
4096		243	Fifth remainder
	X		
3		2	
<hr/>			
8192		729	Sixth remainder
	X		
3		2	
<hr/>			
16384		2187	Seventh remainder
	X		
3		2	
<hr/>			
32768		6561	Eighth remainder
	X		
3		2	
<hr/>			
65536		19683	Ninth remainder

[45]

65536	19638	Ninth remainder
	X	
3	2	
<hr/>		
131072	59583	Tenth remainder
	X	
3	2	
<hr/>		
262144	177147	Eleventh remainder
	X	
3	2	
<hr/>		
524288	521441	Twelfth and final remainder

[Figure 104.]

The super 2847 partiente 521441 is the remainder. It is plentiful in branches, but devoid of fruits, because it does not consist of even one entire comma as the following example [Figure 105.] will clearly demonstrate.

524288	521441
	X
81	80
<hr/>	
41943040	42236771

[Figure 105.]

The fifths on the lute therefore become flat by less than the twelfth part of a comma; the fourths necessarily become augmented by as much. I will prove that in more detail by subtracting the twelfth part of a comma from the interval which consists of five octaves, because it contains virtually twelve fourths, as the present example [Figure 106.] will show.

[46]

	32		1	
		X		
	4		3	
	96		4	
4	24		1	First remainder
		X		
	4		3	
	72		4	
4	18		1	Second remainder
		X		
	4		3	
	54		4	
2	27		2	Third remainder
		X		
	4		3	
	81		8	Fourth remainder

81		8	Fourth remainder
	X		
4		3	
<hr/>			
243		32	Fifth remainder
	X		
4		3	
<hr/>			
729		128	Sixth remainder
	X		
4		3	
<hr/>			
2187		512	Seventh remainder
	X		
4		3	
<hr/>			
6561		2048	Eighth remainder
	X		
4		3	
<hr/>			
19683		8192	Ninth remainder
	X		
4		3	
<hr/>			
59049		32768	Tenth remainder
	X		
4		3	
<hr/>			
177147		131072	Eleventh remainder
	X		
4		3	
<hr/>			
521441		524288	Twelfth and final remainder

[Figure 106.]

I proved to you above that the semidia-
pente exceeded the tritone by approxi-
mately half a comma, so that, since one
is found to be equal to the other on the
lute, the tritone on [this instrument]
is necessarily augmented by the fourth
part of a comma, and the semidiapente
[is necessarily] diminished by the
[same amount], which is half of that
remainder by which the semidiapente sur-
passes the tritone. One can comprehend
from this how discreet, diligent, and
considerate courteous Nature is in each
one of her endeavors, because those in-
tervals which more closely approach
perfection do not depart far from their
true state. This situation has not
necessarily occurred with those farther
from [perfection] and less with the
imperfect intervals and the dissonances,
because it was not so clarified to the
sense. One can also know by what we
have said clearly up to now how much
farther removed from perfection the
keyboard instrument is than the lute

[47]

Considerations
of the author.

and the viola [d'arco], and also how difficult it is (through bad relationships and false encounters) for their sounds to be united together agreeably, when these sounds continually clash in concerts where there are no others but those [keyboard instruments] which cause the disparity of semitones. This is not to omit without consideration that when the fifths acquire perfection on the lute by submitting to augmentation, the fourths necessarily come to lose part of their imperfection by being relieved of their extra size. This is the opposite of what happens to the [keyboard] instrument, as has been demonstrated. With this, I believe that I have satisfied each one of your requests.

Strozzi: Yes, that is true, I desire, however, that you next clarify another problem and afterwards show me the way I must follow when I want to construct any monochord which we have considered.

Bardi: As you see, I am ready to satisfy [your request], but tell me what doubt has newly occurred to you.

Strozzi: The doubt is this. Since the tuning of the lute is so much nearer to perfection than that of the keyboard instruments, I am very surprised that the players of these [keyboard instruments], after having learned of it, have not changed to that [new] manner [of tuning]. It happens that in addition to its excellence, [the lute] has in each fret, high and low, any desirable interval of the intense diatonic and of the chromatic tonikon of Aristoxenus, as you have told me previously. On the contrary, in the string distribution used by the keyboard instruments, which is different from the one used by the lute, many [of these intervals] are commonly lacking. Because of this the player of this [keyboard instrument], however practiced and skilled, cannot transpose a composition either into one range or another by tone or by semitone-- I say that transposition by tone is done

more often--while such a composition is very easily and usefully transposed.

Bardi: This is one of those things which I have pondered many times, and also have sought carefully [to see] if [the aforesaid tuning] could be applied to the keyboard and if it would end by being just as it is on the lute. I find, actually, that on the [keyboard] it probably does not end by being that way, but many, important defects are revealed there.

Strozzi: How can it be that the very same thing does not possess the same nature in any place and time? Why, since this particular distribution is delightful on the lute, is it unpleasant on the keyboard instrument?

Bardi: It can happen very easily, as practical experience demonstrates to us. Although the cause of it is difficult to understand well, I do not wish for you to be dissatisfied because I have failed to tell you a few details which are pertinent in this case. It is possible

Why the distribution of the lute does not have a good effect on the keyboard instrument.

that this results from having accustomed the sense to hearing the intervals always in the manner which I have said they are contained. And thus, likewise, we are used to having them in another different way from the keyboard instrument. If we now wish this [keyboard instrument] to be tempered according to the usage of [the lute], it is impossible to accomplish this without offending the sense in some particular places where the discrepancy between [the intervals] is very apparent. This is because we are already inured by time to such a temperament, and the particular places are those where it is necessary to hear the difference between the major and minor semitones. It happens--as you know--that the lute has the tone divided into equal parts and the keyboard instrument has them separated into unequal parts. The most obvious places are between the diesis X of G sol re ut and the b molle of a la mi re. It [occurs] between the diesis X of d la sol re and the b molle of e la mi, because in

the keyboard instrument, when we have tuned the diesis X of d la sol re by a major tenth with h mi, and when we intend, as in the lute, for the same diesis X to perform the very same function which the b molle of e la mi performs in being used also to [form] a minor tenth with C sol fa ut, it becomes so languid and flat that it is intolerable. If, on the contrary, in sharpening it as much as suits it we tune it well in accord with C sol fa ut as a minor tenth when it is needed, then, to serve as a major [tenth] with h mi, it is so greatly stretched and tightened that it cannot be tolerated. The same happens in the other places where such a difference exists between them. Such intervals as these also can more greatly reveal their quality and nature in the keyboard instrument than they do in the lute and in the viola [d'arco], due to the conformity which the practice of singing has with its temperament, in addition to the quality and quantity of the sound compared with the diverse nature

of the situations which concur with its production. This is because the sound of the strings of the keyboard instrument assails the hearing with very great violence, but those of the lute do not. One sees clearly, therefore, that the sound of the strings of the [keyboard] instrument, tempered according to the disposition of the intervals which appear between those of the lute (including the thirds, the tenths, and also the major sixths), greatly offend the ear by their great sharpness. One also sees that the reason why the keyboard intervals manifest their quality there more than the lute-intervals is only that the material of these strings and the agent which plucks them possess more force and efficacy, because of their activity, to wound the ear with greater vehemence. This does not happen to those of the lute because of the different quality of one subject and the other, and the same difference and variety can be considered

in iron and wood when they have been inflamed, because iron heats with more force and quickness than wood and introduces its new form into that object which is fit to receive it. As a result, the harp is tempered according to the manner of the lute, where the strings and the agent which plucks them are the same. In the harp, the sounds will undoubtedly become no less endurable than in the same lute. They would be unbearable, however, in the harp every time that the agent and the material of the strings were changed, as we have experienced many times. The same also can be considered when the harp is struck by a ball of ivory or by some strong, heavy, smooth piece of wood after it has been beaten over a hard, polished stone, and when it is struck by another [object] made of cork which has been struck with the same force over a rough plank of willow. Let us also consider the comparison between a blow passing through an acute angle and one passing through an obtuse angle. Another

example of a keyboard instrument which the Elector August, Duke of Saxony once presented to the happy memory of Albert the Great of Bavaria comes to my mind in this case, which is most effective of all. This instrument⁶⁸ has strings according to the usage of those of the lute. They are bisected like those of the viola [d'arco] by an arranged skein cleverly made from the very same coarse hairs as the ones from which the strings of viol bows are made. This skein is very easily set into gyration with a foot by the same person who plays it, and it continuously cuts, by means of a wheel over which it passes, that quantity [of string] which is directed by [the player's] fingers. I tuned this instrument two years ago when I was at the court, according to the usage of the lute, and afterwards it sounded very well, not unlike a set of dulcet-toned viols. I

A beautiful, cleverly constructed keyboard instrument.

⁶⁸See F. E. Kirby, A Short History of Keyboard Music (New York, 1966), p. 26, with regard to this type of keyboard instrument.

come to the construction of monochords and say that in having distributed the strings of that [instrument], when it occurred to me, according to the diatonic ditoniaion, among the other possible ways, I adhered to this order. I first of all divided with the compass the whole line, length, or string (as we ought to call it) of the plank or ruler over which I wished to adapt it into four equal parts. I called the extreme low point A, denoting with it the pitch A re, and called the extreme high point B, without any other designation. In the point which divided the total length into two parts (intending them always to be equal), I then marked a la mi re, in that which separated A re from A la mi re, I marked D sol re, which happened to contain the fourth part of

Way of constructing the diatonic monochord.⁶⁹

Zarlino⁷⁰ teaches another way in [book 1], chapter 40 of his Istitutioni.

⁶⁹The basic string distribution discussed here is diagrammed and explained in Cecil Adkins, "The Theory And Practice Of The Monochord," unpublished doctoral dissertation, Department of Music, State University of Iowa, Iowa City, Iowa, 1963, pp. 232-233.

⁷⁰See Zarlino, Istitutioni, pp. 51-54.

order to provide this D sol re with its higher octave, I opened the compass and made two parts of the interval cut in half which was contained between this D sol re and the extreme high point B. At that point where one separated from the other, I marked d la sol re. I again divided the whole into three parts and distinguished them with four points as their limits. In the second of these (like Boethius, always calling what is in the extreme low part first), I marked E la mi and in the third [point] I marked e la mi. Placing one pole of the compass on this point, I proceeded to place the other over the line toward the high part, and in that point where it ended, I marked h mi. Taking one [compass pole] away from that [point] and holding the other firm, I moved it toward the low part and where it rested I marked h mi. When this was done, I divided into two parts the whole line which extended between d la sol re and the extreme high point, and then placing one of the poles of the compass

on this d la sol re and coming with the other toward the low part, I signed G sol re ut at that point where it ended. Wishing to find the higher octave of G sol re ut, I opened the compass again and divided the interval which was between this G sol re ut and point B into two parts in the center of which I marked g sol re ut. Without moving the space which at that moment was included by the two poles of the compass, I put one of these [poles] on G sol re ut and came toward the low part with the other. Where it rested I there marked C fa ut, and found its higher octave by dividing into two parts the distance which is between this C fa ut and the extreme high point B, in the middle of which I placed C sol fa ut. When I again divided into two parts the whole interval which remained above it toward that [high point B] and placed one of the poles of the compass on it, coming with the other [pole] toward the low part, I found the position of F fa ut

at that point where it ended. Its higher octave was acquired by dividing into two parts the space which extended from it to point B. The higher octave was in the center of that [space]. Only the note B fa was lacking to the diatonic monochord, which I found in this way. I divided the line which extends between B and f fa ut into two portions. Then I placed one of the poles of the compass on this F fa ut and came with the other toward the low part. Where it ended, there I marked b fa. I found the lower octave of that [note] by opening the compass so that the extremes of its poles contained the entire empty space which was found between this b fa and the extreme high point. Taking the pole of the compass away from this and holding the other firm, I moved it toward the low part. Where it rested, I there marked B fa, and with this I had completed the monochord of the diatonic ditoniaion. The ancient Greek musicians equipped this with a single string for no other reason than to learn and understand

Why the ancients equipped the monochord with a single string.

more exactly the quality and quantity of the intervals and to avoid at the same time the deception which could result between two or a larger number, because of their inconstancy with regard to the various situations which continually threatened them. Since we now wish to tune the said monochord according to the ancient chromatic, it will alone suffice, because the difference between the diatonic and the chromatic consists of nothing else, to arrange the third note of each tetrachord which contains the trihemitone in the upper part. The proportion of this [trihemitone] falls, as you have learned, between these numbers, 19:16. In order to do this, the same order is preserved. Divide the line which is found between B and E la mi, extreme high pitch of the hypaton tetrachord, into sixteen equal parts. When this [quantity] has been increased by three similar parts toward the low register, we will mark, at that extreme point where they end, the chromatic

[49]

Way of composing
the ancient chromatic
monochord.

pitch D sol re, which with E la mi will contain the trihemitone. The same order will be preserved in order to mark the other chromatic pitches in the other tetrachords. There are some who reprove Franchino in regard to this distribution for having varied from the [way of the] ancients the intervals of the middle and the high of each tetrachord, constituting [the middle interval] within these limits 18:17, which is the smaller part of the tone which should be the larger [one] and the high interval with the name of trihemitone between these numbers, 153:128, which is augmented from its true state. Since we also wish to temper the same diatonic monochord according to the usage of the ancient enharmonic, it will suffice merely to divide into two equal parts with the compass the distance of those two pitches which contain among them the minor semitone (or limma). At that point where they divide equally, the second enharmonic pitch will occur in each

Method of constructing the ancient enharmonic monochord.

tetrachord, and the third [pitch] will be that which was second in the diatonic and in the chromatic. Thus, we will have the smaller enharmonic diesis in the lowest interval, the larger one in that of the middle, and the ditone in the highest [interval], according to the way their authors created them. I warn you also that in the conjunct monochord of the chromatic genus, the three highest pitches of the tetrachord synemmenon become different from the disjunct, and thus, likewise in the enharmonic, although the extremes are always the same in any genera of harmony. If we want, then, to tune the monochord of the very oldest diatonic according to the diatonic syntonon of Ptolemy, it is necessary--in a way of speaking--to change from their places the third, fifth, sixth, and eighth fret. That is, they need to be sharpened. From this effect perhaps, the name of intense [syntonic] is acquired, according to Dydimus its author. In each octave, moreover, of the disjunct

Way of constructing the syntonic monochord.

The origin of the term syntonon.

system, it is necessary, when we wish to accomplish what was mentioned, to sharpen C fa ut, D sol re, F fa ut, and G sol re ut and in each of those of the conjunct [it is necessary to sharpen] b fa, D sol re, F fa ut, and G sol re ut. In order to do this, I maintain this order. I divide the whole line into six parts, and moving toward the high part where the first [part] from A re terminates, I mark C fa ut. I find the octave of this [contained] in half the space which exists between this C fa ut and the extreme high poing B, and I mark it thus, C sol re ut. In the middle of these two pitches then, I locate F fa ut and I find its higher octave at the point where the interval which is found between this F fa ut and [point] B divide into two parts. After doing this, I divide into nine parts the line which is between C sol fa ut, and I sign D sol re toward that part where the first one [of the nine] ends. I find its higher octave by making two parts out of the space which lies between this

D sol re and [point] B. In the middle of this, I mark d la sol re. I then find G sol re ut by dividing the entire line which is contained between C fa ut and [point] B into three equal parts, and in the extreme of the first one toward the high part, I mark G sol re ut. The [higher] octave of this I find at that point where the space which is contained between G sol re ut and [point] B divides into two parts. Therefore, I sign it g sol re ut. I adhere to the same order in finding b fa and its replicate as in the diatonic, and I mark them in the same manner. Thus I hastily leave the method of composing the syntonic monochord of Ptolemy. I come now to show you the way which Aristoxenus calls intense diatonic. After that [I will come to] his chromatic tonikon with which the distribution of frets upon the lute closely corresponds. This lute and also the viola d'arco, both modern instruments, have unexpectedly

Method of constructing the intense [diatonic] monochord of Aristoxenus and the chromatic tonikon.

been distributed in imitation of the chromatic tonikon. On both of these instruments the tone has been divided into two equal parts, as has been stated above. In this construction the square of the second number is greatly essential, or the double of that, which is eighteen. We will employ this, however, to avail ourselves of its effect with greater clarity and facility in the distribution we are seeking. I therefore divide the whole line A B into eighteen parts, and toward the high part (moving away from the low part) where that first part ends, I place the first fret.⁷¹ I divide once more the whole remainder of the same number of parts [i.e. eighteen] and from the very same band, I place the first fret under the second. I now proceed to

The second number squared is 9, and its duple [is] 18.

⁷¹Galilei's approximation for equal temperament, described here, is based on the principle of similar proportion, in which a correct ratio is chosen for the semitone and then applied twelve times to a string. Galilei's method was adopted and emulated by many makers of instruments and was widely discussed by writers for two hundred fifty years. See Barbour, Tuning and Temperament, pp. 57-64.

distribute the space which remains beneath the frets, always in that very same order, up to [the number of] twelve, which brings me exactly to [the point] where half of the entire string terminates. The first and lowest octave of this, I find, has been divided into twelve equal semitones and six tones, as Aristoxenus instructed. In order to do that, no other number except eighteen is suitable, which is obvious from this [fact]. It does not agree in any way with the first seventeen, because that would give us a smaller number of frets than we needed, and we would have even fewer from sixteen and from fifteen. It also does not agree with nineteen, because, on the contrary, we would have a larger quantity [of frets], and a great deal more from twenty and twenty one. Therefore, eighteen is the proper divisor, rather than a larger or smaller number, although the same thing happens to this

Zarlino⁷² contends in chapter 14 of the 1st part of his Institutioni that the aperture of this [compass] is exactly the 6th part of the circle, which is false. Warning.

⁷²See Zarlino, Istitutioni, p. 24, concerning the "sesto", or compass.

[eighteen] which also happens to the compass in wanting to measure the circumference of the circle six times exactly with its aperture, which extends--as you know--from the center to its circumference. For this reason it has become known as the "sesto". I therefore warn the industrious agent to remove, with his discretion and diligent attempts, that small inconvenience which exists between the measurers and the measured.

Why the compass, a geometric instrument, is called "sesto".

Strozzi: I have understood the whole thing very well, but please clarify another related problem.

Bardi: Tell me!

[50]

Strozzi: What order did the ancient musicians follow, with regard to the numbers, in distributing the strings of that first species which were peculiar to the chromatic and enharmonic of the Greater [Perfect] System? What induced them to leave between the first and second note of each tetrachord of the enharmonic such a space as 512:499, and [to leave] between the second and third [note] of the chromatic

this [space of] 256:243 rather than another [space]? Tell me also if these two species of harmony have any other relationship and agreement with the diatonic than the superficial appearances of numbers?

Bardi: You will recall that I showed you with numbers that which I just now have demonstrated with a line. I warn you, however, that by examining carefully the example of the hyperbolaion tetrachord of the ancient diatonic ditoniaion, every one of the difficulties mentioned will be removed. This tetrachord has been proposed by me because it has smaller numbers than the others of its species. The differences of its numbers are more easily understood by the intellect, as you see here [Figure 107.].

Hyperbolaion Tetrachord of the Ancient Diatonic

Aa	Netehyperbolaion	2304	Difference 288, its half 144; serves for the chromatic string.
g	Paranete hyperbolaion	2592	
f	Tritehyperbolaion	2916	Difference 156, its half 78; serves for the enharmonic string.
e	Netediezeugmenon	3072	

[Figure 107.]

In the first place, the extreme notes of any tetrachord in each genera and species of harmony are always the very same, as you know. They are therefore called stable, while those which have the power to change the diatonic system into the chromatic are the thirds of each tetrachord. The occasion, moreover, which induced the ancient musicians to permit that particular interval between the trite and the paranete rather than some other was taken from here. They added to the number which denotes the paranete of the diatonic, half the difference which is found between it and the nete of the same tetrachord, which is 144 since the whole (as you have been able to see in it) is 288. This half, when added together, according to what I have said, with 2592, makes 2736, and with this number they designated in that tetrachord the chromatic paranete. They left to the other notes the same numbers and positions as in the diatonic, as

What strings are stable, which are movable and which are not entirely stable or entirely movable.

appears in the example, and did likewise in the other tetrachords.

Aa	Netehyperbolaion	2304
g	Paranetehyperbolaion	2736
f	Tritehyperbolaion	2916
e	Netehyperbolaion	3072

[Figure 108.]

Now because the principal difference which exists between the diatonic and chromatic distributions, and the enharmonic consists of the position of the second note of each tetrachord, they therefore followed this procedure in order to denote it. They added to the number with which they marked the trite of the diatonic [genera], which is the same as the chromatic [one], half of the difference which is found between this diatonic trite and the netediezeugmenon, which is 78, since the whole is 156, as you have learned. This trite in the enharmonic, then, was designated with this number, 2994, and the others [were designated] with

the same [numbers], according to what is seen described here [Figure 109.].

Hyperbolaion Tetrachord
of the Ancient Enharmonic

Aa	Netehyperbolaion	2304
g	Paranetehyperbolaion	2916
f	Tritehyperbolaion	2994
e	Netediezeugmenon	3072

[Figure 109.]

And [the ancients] observed the same in all the other tetrachords in order to convert them from diatonic into chromatic or enharmonic, and the very same difference which is found between line and line of the three first monochords is found between number and number of these last ones.

[CHAPTER III]
[A CRITICAL DISCUSSION OF
ANCIENT AND MODERN
MUSICAL ART]

[Bardi:] I now come to deal with [51]
tones and the modes of the ancient musi-
cians, and omitting, in order not to be
long and tedious, the opinions of all the
other writers, which are many and diverse
with regard to their number, name, and
place, we will discuss only the three most
famous, the first of which will concern
the Aristoxenians, the second the Ptole-
maics, and for the third we will consider
that of Boethius. It was, moreover, the
opinion of Aristoxenus (according, how-
ever, to what Aristides Quintilianus,¹
Bryennius,² and Euclid³ in the Introduc-
tion which he makes of music--given, how-
ever, that this [Introduction] is his--

¹Aristides Quintilianus' work may be found in a German translation by Rudolf Schäfke. The original Greek text appears in Marcus Meibom, Antiquae musicae auctores septem, 2 vols. (Amstelodami, 1652).

tell us) that the modes must be thirteen [in number] and not seven, or eight, or another smaller number which was recognized before him. [This was] because, in considering those of which many honorable writers had made mention in his times and earlier, and which were later mentioned by Ptolemy and Boethius in addition to many others, and finding between the Lydian and the Mixolydian and [between] the Hypolydian and the Dorian the distance of a minor semitone (or limma), he proceeded--in my opinion--to ponder, thinking to himself as follows:

Since from sharpening and flattening the system by a minor semitone there results between these modes a clear and apparent difference of affection, or at least this is operated more in one than the other according to the degree of its effectiveness, how much more

There were 13 modes according to Aristoxenus, Aristides Quintilianus, Bryennius in the 1st of his 3 books. (Manuel Bryennius in the 1st [book] at the 7th chapter), Euclid in the Introduction.

Consideration of the author.

²See Manuel Bryennius, Harmonica, in John Wallis, Opera mathematica, 3 vols. (Oxford, 1693-1699), III, 417-422.

²As Galilei suggests, the Introduction is not by Euclid; the actual author is Cleonides. See Oliver Strunk, Source Readings in Music History (New York, 1950), pp. 34-46.

so would an entire half a tone make it? And since such a variety of harmony and affection is found between the aforesaid modes, for what reason will no other pitches be distant from each other by such an interval?

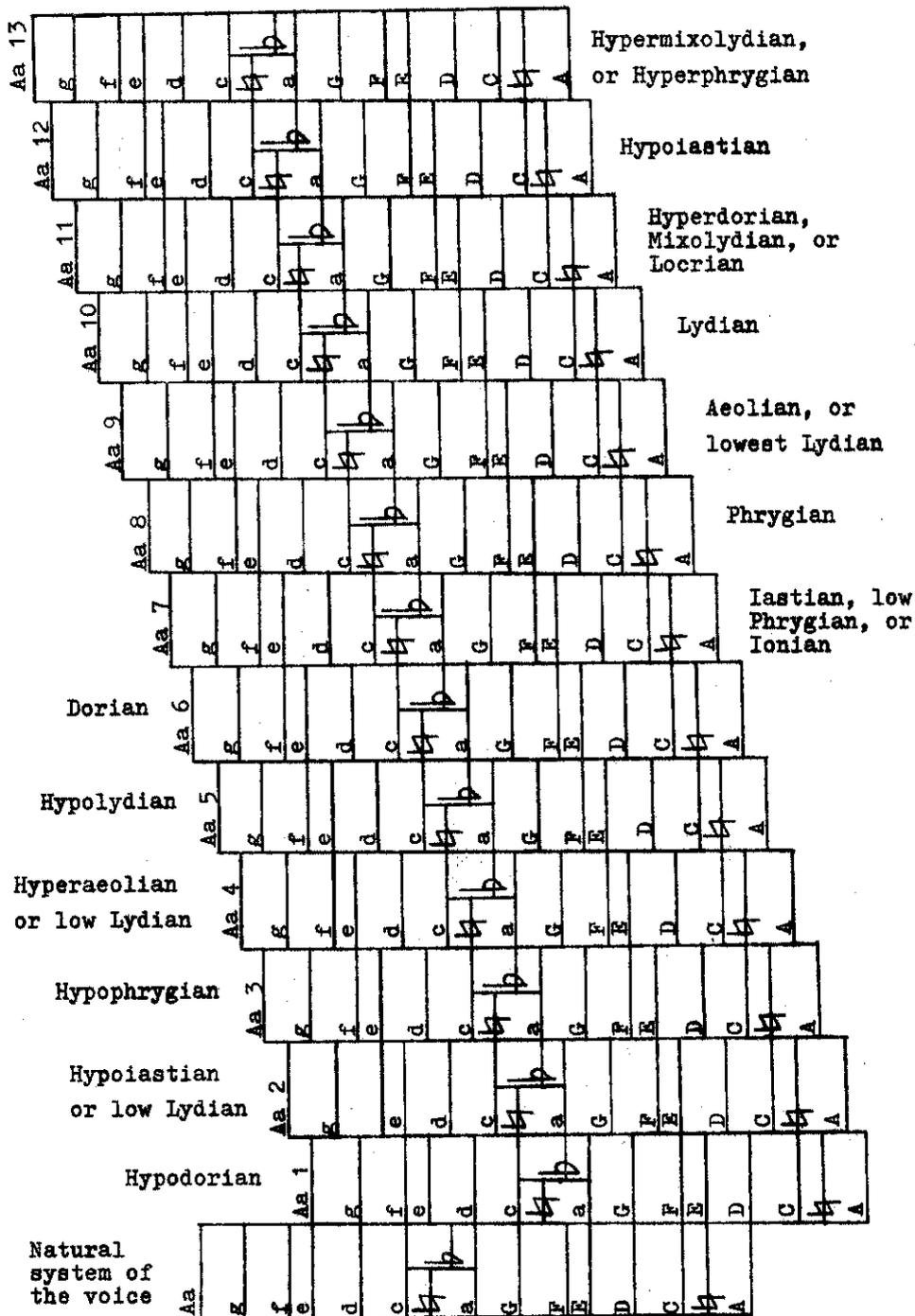
And while he was pondering these reasons, he divided the five tones and the two minor semitones which contain in themselves the species of the diapason which served the Dorian mode into twelve equal parts, and to each term of these parts, which came to be thirteen since the intervals which constituted the mean of one of these tones were twelve [in number], naming them and arranging them in the manner which is seen in the notated example [Diagram V.] which follows. From the words [of Aristoxenus], this most noble musician and philosopher, Ptolemy⁴ took occasion to reprove him for many things, among which there are three of some consideration. Therefore, I want to relate them to you and afterwards prove that such libel was unjustified.

Ptolemy reproves Aristoxenus for many things in the 1st [book] at the 9th and 12th chapters, and in the 2nd [book] at the 9th and 11th chapters.

⁴See Ingemar Düring, Ptolemaios und Porphyrios über die Musik (Göteborg, 1934), pp. 37-39; 44-46; 60-63; 79-81.

[DIAGRAM V]

DEMONSTRATION OF THE THIRTEEN MODES
ACCORDING TO THE OPINION
OF ARISTOXENUS



The first [consideration I mentioned] concerns the distribution of strings, the second concerns the division of the tone into equal parts, and the third and final [consideration] concerns the number of modes. Ptolemy, therefore, said this:

[53]

If half of a string, for example, which stretches over a plane surface is divided into twelve equal parts with the compass, it will be clear that [when one considers] the quantity of sound which the whole contains with the half, which comes to be a diapason, the eighth and ninth spaces will contain the greater part of it, rather than the first and second spaces.

With this method of measuring, one who pursued the situation very thoroughly would find that one of the last parts would contain four, five, and more times as much as the first and second. For the same reason, the frets on the neck of the lute and the viola d'arco are seen to move progressively closer and closer together, in proportion to which the sound of the same string, as it is shortened, becomes progressively higher

in pitch. Consider a string stretched over a plane surface which is, for example, an arm's length. The octave above this string is obtained from its half (which is half an arm's length) everytime [this string] is struck together with its whole or a similar string. And if [this string] were two arm lengths, the diapason above it would count one, and the diapason below it would count four. By means of this demonstration which Ptolemy makes, it appears that "he has" (as the proverb goes) "reasons for selling". But the fact is not so. It happens that Aristoxenus never understood or said that frets would have to be distributed as, for example, has been said with regard to the neck of the lute, after having first divided half the total length of the string or line into twelve equal parts. For Aristoxenus knew very well--as I have demonstrated--that he had to distribute the quantity of sound into equal parts, not the

Aristoxenus
[is] defended
by the author.

quantity of the line, string, or space, since he was acting at that time as a musician concerned with the body of sound, and not as a simple mathematician concerned with the continuous quantity [of sound].⁵ He felt, consequently, that the first fret alone would occupy the ninth part of [the string's] entire half, that is to say the eighteenth part of the whole, and the second [would occupy] the ninth part of that which remained to the same first half after the first semitone had been extracted. The third fret, then, would likewise occupy the ninth part of that space which remained of half the string after the first and second semitones had been removed, or to state it yet another way, the eighteenth [part] of the whole. Thus, the

Method of placing "just" frets on the lute and the viola [d'arco].

⁵Dividing a string into equal parts does not insure that the same quantity of sound will be contained in each of these parts, for the size of the interval sounding increases as the amount of string used decreases. This phenomenon becomes apparent immediately when one fingers an ascending scale on a stringed instrument.

others [proceed] in order, always keeping in mind the condition mentioned above, which is that the half of the string must be taken, not at that point where it terminated the first time that it was divided into two equal parts, but where it terminated after that number of semitones which was necessary had been removed. And this, as I said another time, is the rule to be observed when one wishes to distribute the frets "justly" on those instruments which are [being] investigated, like the lute, the viola d'arco, and others. Ptolemy, in addition, reproves Aristoxenus that the tone cannot be divided into two equal parts, and he wishes to prove it to him demonstrably, speaking in the following manner:

The tone is firmly contained between 18 and 16, so that no other number except 17 will enter in between [them]. This 17 when considered as the divisor of the sesquioctava, comes to divide it into unequal parts because that which is contained by the sesquidecimasexta [i.e. 17:16] is a

greater part than that which is contained by the sesquidecimaseptima [i.e. 18:17] by the following interval, 289:288. Whereupon, it follows necessarily that the tone cannot be divided into two equal parts.

There is no man, however slow-witted, who doubts this, with regard, however, to the science of arithmetic. But Aristoxenus neither said such a thing, nor understood it, except in the manner which I demonstrated to you, particularly in placing frets of the lute, in which one can actually divide each musical interval into any desired number of equal parts, [in a manner] no different from what [is done] by means of the monochord. The sound, in this case, is considered by the musician as qualitative, not quantitative, although Daniel Barbaro⁶ [in his commentary] on Vitruvius understands it to be the contrary. The demonstration of Ptolemy is the very same, for he says that it is not possible by means of numbers to accommodate

[Daniel Barbaro]
in chapter 4 of
the 5th [book].

⁶See Vitruvius Pollio, I dieci libri d'architettura, translated by Daniel Barbaro (Venice, 1629), pp. 227-243.

any other interval between smaller terms of the diapason. Nevertheless, between the hypate and the nete one finds, in addition to the other strings, the lichanos and the paramese which produce the third and the fifth in the grave register and the fourth and the sixth in the acute. Similarly, between F fa ut and f fa ut of the lute one also finds (besides the tone divided into two equal parts) the h mi, which separates it into two identical portions, as has been said. In addition, [one finds in considering] two strings, each two arm lengths in span, which are tuned in unison, that every-time, for example, that the distance of of an arm length and a half is taken from one, and three-quarters of an arm length [is taken] from the other, and those portions are sounded together, the result will undoubtedly be the octave. Concerning, then, the number of modes, [I say that] what we have discussed above in the person of Aristoxenus is so very clear in itself that there is not any contradiction.

Comparisons.

Strozzi: All is well at this point, but who is the one who persuades us to believe that at the time of Aristoxenus there were modes in use [which were] distant from each other by a minor semitone, as you have said?

Bardi: Aristotle, in consideration of Philoxenus, who--as you will soon learn-- was the inventor of the Hypodorian mode (the last to be discovered) and Ptolemy,⁷ in the tenth chapter of the second book [of his Harmonics], show the intervals which were used in them according to the design of their inventors.

Strozzi: I am surprised, therefore, that Ptolemy wrote so boldly against Aristoxenus, knowing how many volumes he had written on the science of arithmetic and [knowing] how worthy he was, since, in addition, he had not only been a pupil of Aristotle and a contemporary of Theophrastus,⁸ but had [also] acquired all

⁷See Düring, Ptolemaios, pp. 76-79.

⁸Theophrastus (?-287 B.C.), a Greek philosopher and natural scientist, belonged to the Aristotelian school.

the doctrine of Pythagoras from Xenophanes,⁹ a noble Pythagorean. Moreover, as you have demonstrated, [Ptolemy] was greatly at fault.

Bardi: This is not exactly surprising, since the same thing has happened between many others. What more can be said than that which Aristotle first wrote concerning matters of philosophy against his divine preceptor, Plato, even while he was still alive. It is true that this [man], as a friend of truth, wanted (as was suitable to a philosopher) to have more respect for it than for his master, and for Socrates as well. Ptolemy also wrote against the opinion which the Pythagoreans had of the diapason plus diatessaron; his opinion has been vigorously refuted by

Plato reproved
by Aristotle.

⁹Xenophanes is credited by many ancient writers as the teacher of Aristoxenus. He is not to be confused with the sixth century B.C. Eleatic philosopher of the same name.

Jacques LeFèvre d'Étaples.¹⁰

Strozzi: It is actually so, but may we please examine another matter concerning the distribution of modes of Aristoxenus, which causes me to wonder greatly.

Bardi: Tell me what it is.

Strozzi: You said before that Aristoxenus divided the tetrachord into sixty tiny, equal parts, twelve of which, in his intense diatonic, he gave to the first and lowest interval, twenty-four to that of the middle, and the remainder to the highest, so that the semitone did not come to occupy an entire half of the sesquioctave (or tone), but a lesser distance.

Bardi: That is true.

Strozzi: Now, for example, since in the species of diapason which is found between E la mi and e la mi, two tetrachords were separated by the tone of the disjunction, each one of these came to contain

[Jacques LeFèvre d'Étaples] in chapter 24 of the 3rd [book] of the Elements of Music.

¹⁰See Jacques LeFèvre d'Étaples, Musica libris quatuor demonstrata (Paris, 1551), p. 26.

six of these modes, which made the number of twelve. That [mode] which then divided the said tone of the disjunction into two equal parts, came to be the Ionian. This was the reason that each of its adjoining extremes was found farther removed from it than any others [which were] connected together.

Bardi: I am pleased with this particular doubt of yours, since it indicates that you are coming closer to understanding [the situation], but I must warn you that Aristoxenus did not divide one member and then the other in such a way, but, as I have said, [he divided] the entire body of the diapason. For this reason (if you recall the specific words which I used previously with regard to his distributions), I said that they, that is the diatessarons, were always arranged in conformity with his designs.

Strozzi: Does one find, moreover, in his systems that the fifths are diminished and the fourths augmented as on the lute? Similarly, does one find these intervals compared to one another?

Fifths and fourths [are] altered in the distributions of Aristoxenus.

Bardi: Yes, Signor. They are compared, however, with those [intervals] that are not contained by the sesquialtera and by the sesquitertia, which were assigned by Pythagoras in their true form.

Strozzi: This seems to me [to be] quite an inconvenience [which is] not exactly worthy of Aristoxenus that he distributed the strings of his systems so that the fourths and the fifths continually are obtained outside of their true proportions.

Bardi: Do not be so easily provoked to reprove Aristoxenus, because I am here needing to defend him from all those [men] who attempted to speak against him, since he was truly one of the most judicious and learned musicians that the world has ever known. But listen, please! How can you believe that the consonant intervals are sung today, [considering that they require] more excellent singers with discriminating hearing than can be found?

Strozzi: I believe that they are sung within their true proportions, also that some of the artificial instruments-- as you have clearly made me see--play nearer to these [proportions] and others farther away from them.

Bardi: Which [instrument] is the one which has the more perfect consonant intervals, the lute or the keyboard instrument, given that they are perfect when they are in their above-mentioned forms.

Strozzi: The lute.

Bardi: What would you say if I made you realize that the singing of today-- with regard to the perfection of the intervals--is no less imperfect than the playing?

Strozzi: To me, this would be one of the most novel things that I could ever imagine, and I could not believe it in any way.

The singing of today is little different from the distributions of Aristoxenus.

Zarlino¹¹ in chapter 45 of the 2nd part of the Istitutioni.

¹¹See Gioseffo Zarlino, Istitutioni harmoniche (Venice, 1558), pp. 135-137.

Bardi: Now, take note! Will you concede to me that, in the diatonic genera which is in use today, every musical interval is always sung in the same proportion? [Will you also concede], for the above-mentioned reasons, that the major third is consonant and the ditone is dissonant and that the space and contents of the tone are approximately one half of our comma less than the sesquioctave?

Strozzi: The reasons which you conveyed to me earlier in this matter [now] convince me to believe and say yes.

Bardi: Since this is true, therefore, it is necessary that every fourth always becomes augmented when it is sung according to the usage of the modern practice, and similarly, every fifth becomes diminished. I will prove that to you in this way. Let us ascend, for example, singing from C fa ut to F fa ut. I say that this interval is presently an augmented fourth, because when ascending, then, from there to G sol re ut with a space smaller than the sesquioctave by the amount previously

mentioned, it should follow that a fifth is found so diminished that it is dissonant, in order that the sesquitertia may exceed the sesquialtera by a sesqui-octave (or tone), as you know.

Strozzi: I do not deny that the sesquialtera does not exceed the sesquitertia by a sesquioctava, but only that C fa ut is found in such a way when singing away from G sol re ut by an imperfect fifth, because however greatly the fourth which was sung--as you have said--from C fa ut to F fa ut was augmented, the tone which is between F fa ut and G sol re ut may well be diminished, as you just now said in considering the fifth in its true form [to be] composed of an augmented fourth and a diminished tone, and so it is possible for a perfect fifth to be contained in sesquialtera proportion between this C fa ut and G sol re ut.

[55]

Bardi: It is true that I said what you say [I did], but in that place, the augmentation of the fourth was equal

to the diminution of the tone, which does not happen here. Besides, if what you say is true, which it is not in any way, it follows that when ascending from G sol re ut to c sol fa ut with a fourth of the same proportion as the first, a perfect, consonant diapason will not be found between [c sol fa ut] and C fa ut, which is impossible in the manner which is sung today. Or if this is actually the case, it will be necessary to concede that the diatessaron which is sung from G sol re ut to c sol fa ut is a different proportion and measure from that which was originally between C fa ut and F fa ut, which is not a suitable thing to say. It therefore follows necessarily--contrary to the common opinion--that the fifths are sung today diminished from their true states, and the fourths [are sung] augmented. Because of this, (according to what I said) one comes, from the octave onward, to sing any other interval outside of its true proportion, and consequently [in a manner which is]

dissimilar to those which are contained by the senario and by the syntonon, although the general public approves them as perfects and is entirely satisfied (because of not having heard the true ones) and, since every hope of being able to improve them has been taken away, one could also say that the intervals which are sung today are, due to the inequality of semitones, more conformed and similar to those which are found in the temperament of the keyboard instrument than among those of the lute. I say to you in addition that the fifth is understood with greater discrimination by the general public according to the proportion which Aristoxenus gave it than within the sesquialtera, its original form. This, I firmly believe was the result of bad usage, which corrupted the sense, because the fifth, within the sesquialtera, not only seems to possess, but actually possesses a small degree of hardness due to the extreme amount it

can be augmented, which avoids my saying (together with others of delicate hearing) that it is harsh. In the manner of Aristoxenus, however, it seems that the small degree of diminution gives it grace and causes it to become more in keeping with the taste of today, that is, soft and languid, and I do not believe that that happens for any other reason except being accustomed to hearing them continually under that form, or a similar one. From this is derived an important and effective argument to persuade [us of] that which you will soon learn at the appropriate place, which is that this mode of singing was learned from stringed instruments, and particularly from those which had no keys, like the lute and the viola d'arco.

Strozzi: This is such a valuable, new thing that nothing like it has ever been heard. Do go on!

Bardi: Now consider the imperfection Warning.
[which is caused] by this sole abuse in the music of our times and how much the general public is deceived. [Consider

also] how difficult it is [for the general public] to know the truth of things and how little knowledge it has of the true music, not even having known up to today the size, the quality, or the nature of the singable, audible intervals which are the simple elements and principles of music. These things, together with others pertaining to the profession of music (which are many) were understood by Aristoxenus and most of the ancient musicians with supreme excellence. In addition, the perfection and imperfection of the intervals did not mean a thing in the world to the way of singing of those times, since they did not employ them (as you will learn) in the manner which we use. The quality of these intervals can also be considered--as I said--in relation to highness and lowness between one and another constitution of thirteen modes and the other which he made--and not fifteen, as others have said--by comparing one pitch to another. For example, [let us consider the] proslambanomenos of the [Greater Perfect] System in the

Zarlino¹² in chapter 3 of the 4th part of his Institutioni says that Aristoxenus made 15 modes, and in chapter 16 of the 2nd [book] he disparages his distributions without any reason.

Phrygian mode. I say that [this Phrygian] proslambanomenos is higher than that of the Hypodorian by a flat fifth, that is to say, the mese of the Hypolydian is sharper by an augmented diatessaron than that of the Phrygian.

Strozzi: I have understood very well and I am now satisfied about everything, but [first] tell me, why are the flat fifth and the diminished [sic.] fourth tolerated and, on the contrary, why are the sharp fifth and the flat fourth not permitted? Why, then, does the octave not bear alteration in one part nor in the other?

Bardi: I have already told you that the fifth, with regard to bad usage, when it is in its true form represents to the ear a slight state of sharpness--in order not to say tedium, as others do--rather than anything else. Now think how much more [tedious] it would become by being stretched more greatly than the [state in] which it is contained by the sesquialtera,

¹²See Zarlino, Istitutioni, pp. 299-300; 82-86.

its proportion. In addition, in the playing and singing of today, it is not possible, for the reason which I have demonstrated to you, to proceed in a manner other than the one which you have seen previously. It happens, then, that the octave is not allowed to be diminished or augmented. This takes place not only through its perfection, but through being composed of it, since a consonance of the same quality is taken many times, and if two of these were altered, either up or down, and afterwards were added together, the extremes would be intolerable. This thing does not happen to the other consonances since only dissonant intervals are composed of these, and being so constructed, the ear does not detect that small difference which is found among them so easily.

Strozzi: You have satisfied [my
curiosity] to a great extent. Continue,

[56]

however, the discussion of modes which you have begun, according to the opinion of the Aristoxenians.

Bardi: The followers of the said Aristoxenus added two additional modes to his [original] thirteen, and placed these new modes toward the top. The extreme notes of their diapasons came toward that part [which was] outside the ordinary system. For this reason they were later refuted by many because they considered that the human voice was distinguished in three parts, that is, low, high, and middle--Martianus Capella¹³ makes particular mention of this--and that the number of thirteen could not easily be distributed in three equal portions. Therefore, they extended it

[Martianus Capella] in the 9th and final book where he deals with music.

¹³William Harris Stahl, Martianus Capella and the Seven Liberal Arts (New York and London, 1971), p. 211. "Voice production is divided into three classes: continuous, discrete, and intermediate. Continuous pitch variation is used in flowing conversation; discrete pitch variation in music; the intermediate movement of the voice is used in the recitation of poetry." Compare Adolf Dick, Martianus Capella (Leipzig, 1925), p. 500.

up to the number of fifteen, giving the name of principals to five of the middle, which are the warlike Dorian, the changeable Iastian, the religious Phrygian, the plain Aeolian, and the querulous Lydian, and [giving the names] of plagals to the five modes below them, denoting them with these titles: Hypodorian, Hypoastian, Hypophrygian, Hypoaeolian, and Hypolydian. They called the five modes [which were] above [the principals] authentics, distinguishing them as Hyperaeolian, Hyperastian, Hyperphrygian, Hyperaeolian, and Hyperlydian. Through this order, the principals became higher than the plagals by a diatessaron, and the authentics were found lower by a similar interval, so that

In book 2, chapter 10 of Glarean¹⁴ with the testimony of Apuleius in the 1st [book] of the Floridium.

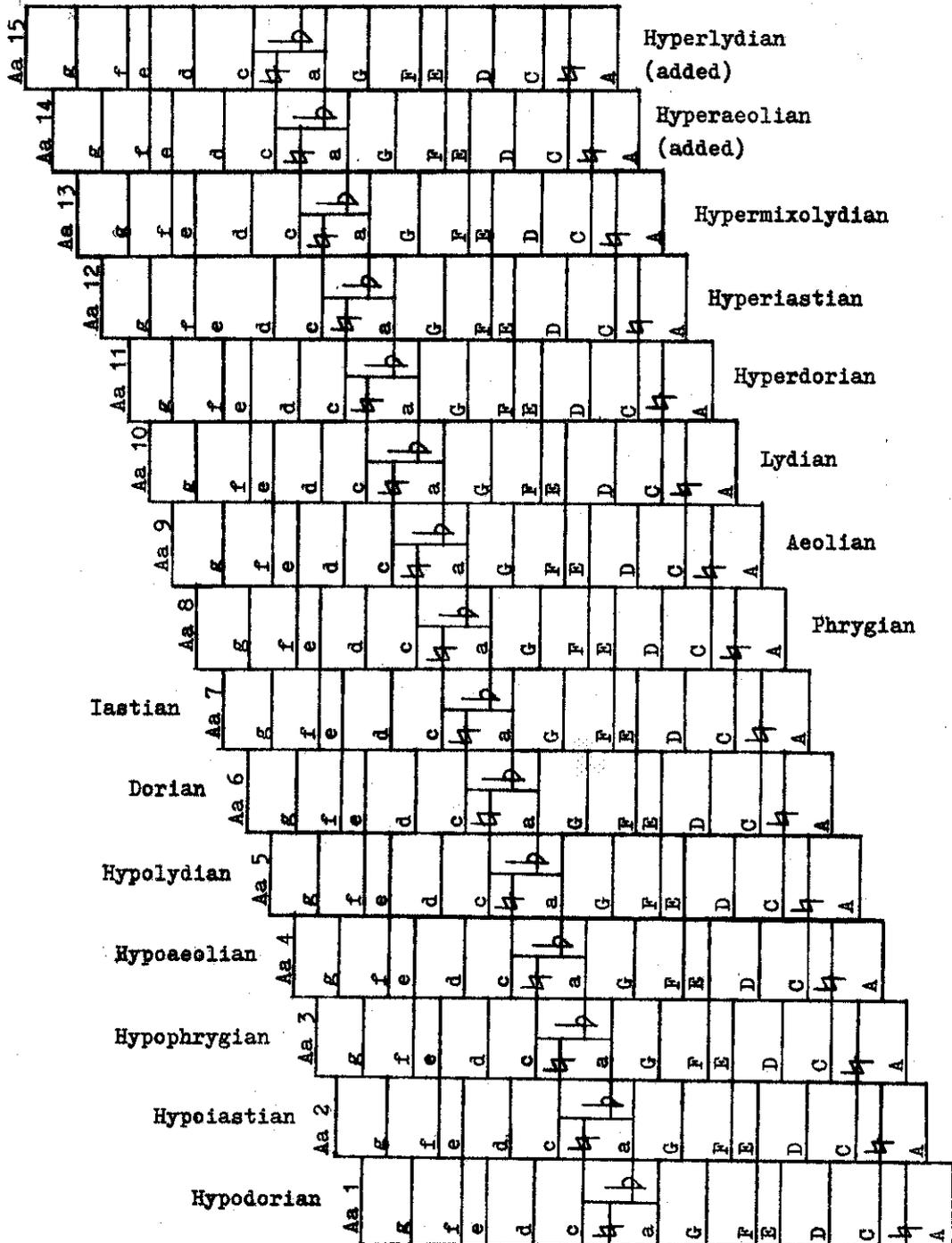
¹⁴ See Glarean, Dodecachordon, 2 vols., trans. by Clement A. Miller (Rome, 1965), I, 128. Compare Apuleius, The Works of Apuleius (London, 1853), p. 376. "Antigenidas was a certain flute-player, a honeyed modulator of every tone, and a skilled performer in every mode, whether it were the simple Aeolian, the varied Asian, the plaintive Lydian, the religious Phrygian, or the warlike Dorian."

from the Hypodorian to the Hyperdorian was the distance of a minor seventh, that is to say, five tones or ten semitones. This was very important (according to the usage of Aristoxenus, however) for it greatly favored their intention, since they had found that the extremes of the thirteen which he made responded by octave, and not by seventh, with those of Ptolemy. The highest of these, [which was] called Hypermixolydian, was none other than the replicate of the Hypodorian. They did the same [thing], moreover, to the Hypoiastian and to the Hypophrygian by raising them an octave, forming with these the Hyperaeolian and the very highest Hyperlydian. These became ordered and disposed in the manner which is seen in the demonstration below [Diagram VI.], marked clearly with their names. After these [Aristoxenians] came Ptolemy, whose order and opinion, being very clever and difficult to understand well, we will reserve in order to demonstrate it last. We will first state,

[DIAGRAM VI]

[57]

DEMONSTRATION OF THE THIRTEEN MODES, ACCORDING TO THE
 OPINION OF ARISTOXENUS, WITH TWO ADDED IN THE
 HIGH PART BY HIS FOLLOWERS, WHICH IN
 ALL MAKE THE NUMBER OF FIFTEEN

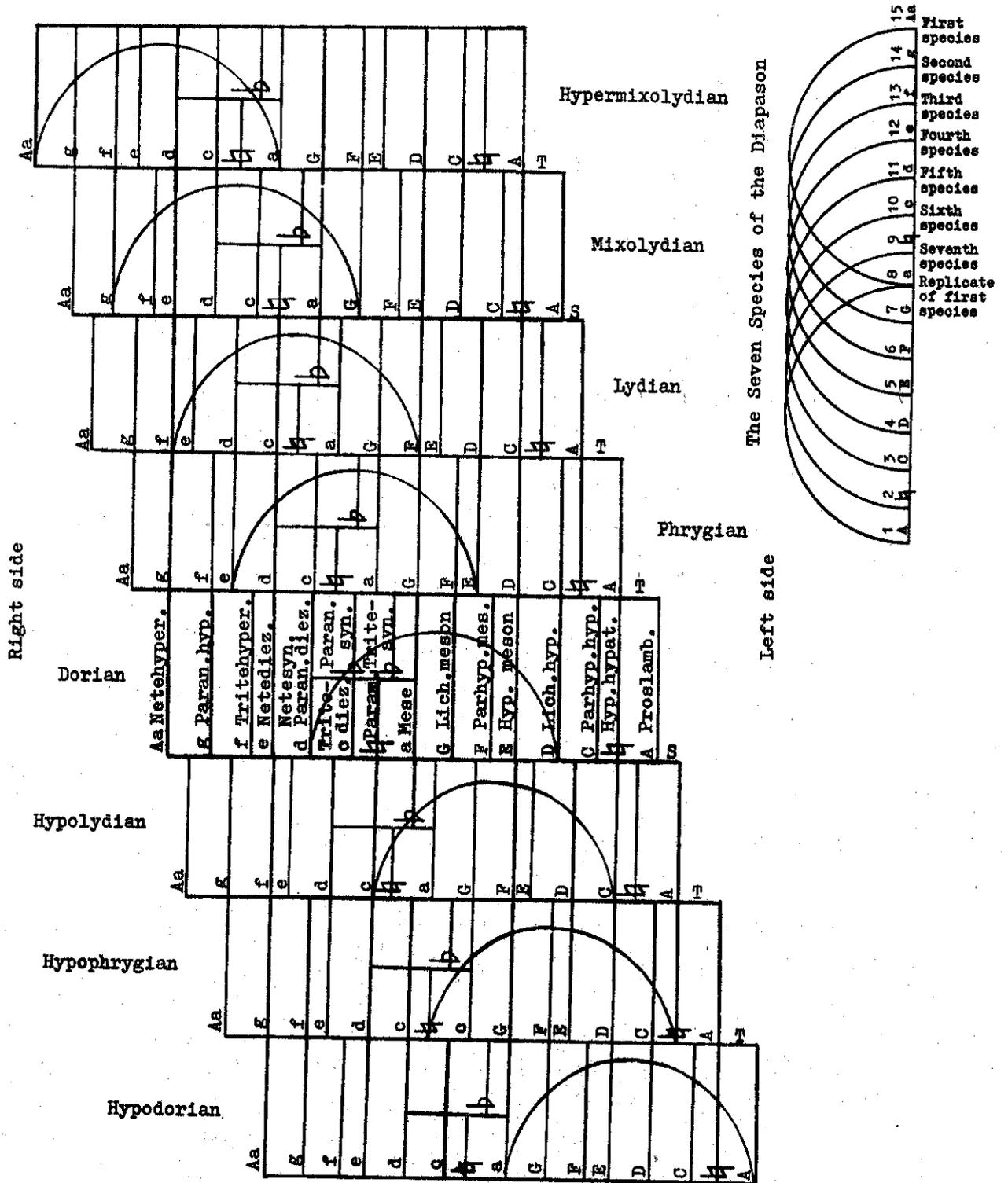


however (since it is least difficult), that which Boethius felt about it. He was of the opinion, although such a view had first been refuted by Ptolemy, that there were eight modes. He maintained, in addition, that each one's particular system proceeded from low to high in the same order, with the same steps, and with the same pitch names which were used in the natural, common [system], which is that which serves the Dorian mode. He made the Phrygian higher than the Dorian by a tone, and the Lydian higher than the Phrygian by a similar interval. He also insisted that their plagals corresponded to the diatessaron below, that the Mixolydian was a semitone higher than the Lydian, and that the Hypermixolydian corresponded by octave with the Hypodorian (in imitation of those of Aristoxenus), ordering them and displacing them, then, in the manner in which they are seen below in the demonstration [Diagram VII.] under their individual names.

[DIAGRAM VII]

[58]

DEMONSTRATION OF THE EIGHT MODES, ACCORDING TO THE OPINION OF BOETHIUS



In this demonstration, one sees clearly each smallest case which is considered by Boethius¹⁵ with regard to these modes. For example, it shows clearly what the particular species of the diapason is in each mode, where it is located in its particular system, and between which pitches [it is found]. One sees also which ones are higher than others by a tone, called "page" by him in that place, or by a semitone, which he there calls "verse". One likewise sees which of these modes is the right side and which is the left. For example, the proslambanomenos signed in the Hypermixolydian with an α is the mese of the Hypodorian signed with the same character, and the netehyperbolaion of the Hypodorian, designated with a Γ [sic.], is, on the

[59]

[Boethius] in chapter 14 of the 4th [book].

¹⁵See Calvin M. Bower, "Boethius' The Principles of Music, An Introduction, Translation, And Commentary," unpublished doctoral dissertation, School of Music, George Peabody College for Teachers, Nashville, Tennessee, 1967, p. 267-275, for a complete exposition of the modes.

contrary, the mese of the Hypermixolydian. One sees, in addition, that the system of the Dorian mode--according to what [Boethius] says--is distant from that of the Mixolydian by a diatessaron toward the low register and distant by a diapente from the Hypermixolydian, in addition to [seeing] many other, important particulars which for brevity are omitted. With all this, there is no lack of doubts, particularly concerning the characters with which the ancient Greeks used to designate the pitches of their songs. These doubts will be removed a little later. Besides, the demonstration which is found in the text of Boethius does not agree in any [of its] parts, with regard to the highness and lowness of the modes, with the words he [uses to] describe them. I doubt seriously that this was one of the potent causes which some careless, in order not to say unwise, men have

boldly stated, perhaps for their [own] convenience, that the text in that place is corrupt; this is false! The demonstration, however, is quite incorrect and battered, thanks to the little accuracy (in order not to say the little intelligence, which is more appropriate) of those who have taken it upon themselves to print it in Venice in the year 1491. Boethius easily attained that number of modes through the advice of Alypius,¹⁷ although he does not mention this, finding in this [work] (as we will show in the proper place) the characters for signing the pitches of each one of these eight modes individually. We shall see, in addition, since there are only seven [pitches], that all fifteen pitches of the system were not occupied, [just] as he said. This will suffice for the present concerning the understanding of modes according to the

Zarlino¹⁶ in chapter 8 of the 4th [book] of the Istitutioni.

¹⁶See Zarlino, Istitutioni, pp. 307-308.

¹⁷Alypius' exposition of the ancient Greek musical notation may be found in Carolus Janus [Karl Von Jan], Musici scriptores graeci (Leipzig, 1895), pp. 367-406.

opinion of Boethius. Now, in order to understand well the order and the number of those [modes] according to the opinion of Ptolemy,¹⁸ I must deviate a little from the present consideration. Therefore, I tell you this. The ancient Greek musicians, according to the authority of the same Ptolemy, had later accepted for the first of their seven species of diapason that which is contained by the pitches h mi and h mi. For the first of the four species of diapente which they possessed, they accepted that which is found between the pitch of E la mi and that of h mi. They took, for the first of the three species of diatessaron which existed, that which is located between h mi and E la mi. For the second species of the diapason, they accepted the interval which contains the pitches of C fa ut and c sol fa ut. They

[Ptolemy] in the 2nd [book] of the Harmonics at the 3rd [chapter].

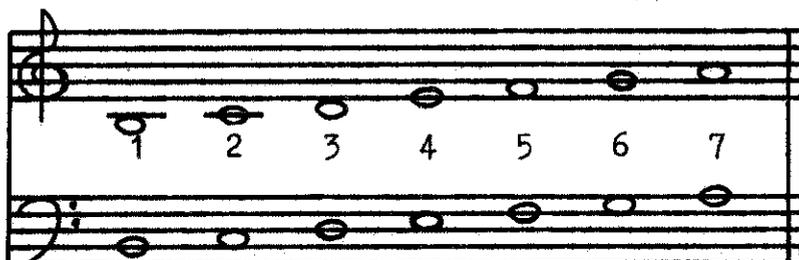
Aristoxenus¹⁹ at the end of the 3rd [book] of the Elements.

¹⁸See Düring, Ptolemaios, pp. 63-64.

¹⁹See Henry Macran, The Harmonics of Aristoxenus (Oxford, 1902), pp. 220-221.

said that the third was between D sol re and d la sol re; the fourth was between E la mi and e la mi; the fifth was between F fa ut and f fa ut; the sixth was between G sol re ut and g sol re ut. The seventh and last necessarily came to be contained between the pitch a la mi re and that of Aa la mi re. The other three species of the diapente and the two [other species] of the diatessaron were those which, ascending by conjunct steps toward the high register, proceeded following the order of the first [ones] proposed, according to what is seen notated in this example [Example 7.]. After these, the Romans accepted as the first species of the diapason that [interval] which is found between Aa la mi re and a la mi re; for the first of the diapente, [they accepted] that which is [found] between h mi and E la mi; and for the first of the diatessaron, they accepted that [interval] which is [found] between h mi and E la mi; the other species followed [them], descending by conjunct steps. This was the order of their

Species of the diapason

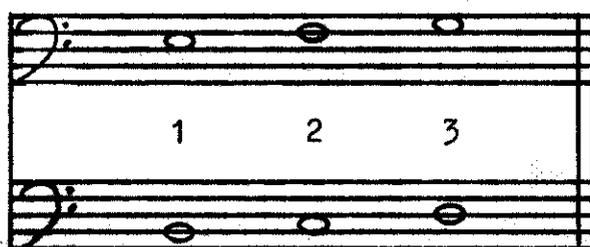


Order of the
consonances
according to
the Greeks.
Ptolemy²⁰ in
chapter 5 of
the 2nd [book].

[Species] of the diapente



[Species] of the diatessaron



[Example 7--Order of the consonances
according to the Greeks.]

²⁰See Düring, Ptolemaios, pp. 66-68.

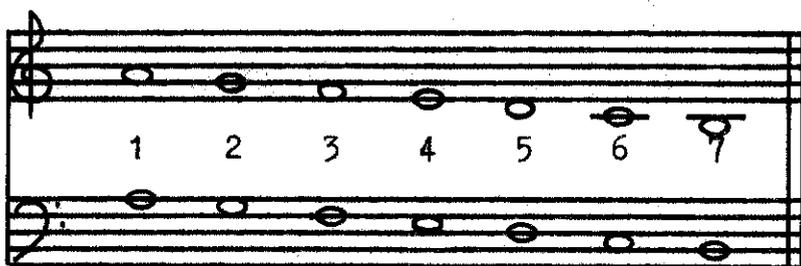
intervals, if we mean those which Boethius has written. This is the example [Example 8.]. And although some have believed and said that [these species] proceed contrary to those of the Greeks, perhaps because they feared that Boethius had not understood in what manner the lowest diapason could serve the highest tone and the lowest [tone] could serve the highest [diapason]. We will show them that this was not so. It is quite true that Boethius, in the order of numbering the consonances, seemed to some people not to have taken care of naming the semidiapente which is found between F fa ut and h mi as the first species of the diapente. In this case, it is credible and reasonable that Boethius understood that the hypatehypaton ought to form an octave with the tritesynemmenon, because each one

[60]

Zarlino²¹ is of contrary opinion in the same place.

²¹See Gioseffo Zarlino, The Art of Counterpoint, translated by Guy A. Marco and Claude V. Palisca (New Haven and London, 1968), pp. 27-29.

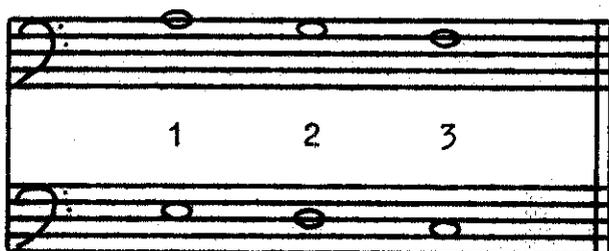
Species of the diapason



[Species] of the diapente



[Species] of the diatessaron



[Example 8.--Order of the consonances according to the Romans.]

Order of the consonances according to the Romans. Boethius²² in chapter 13 [sic.] of the 4th [book]. Zarlino²³ spoke to the contrary in chapter 13 of the 3rd [book] of his Institutioni.

²²Galilei's reference to Boethius IV, 13, is erroneous. It should read Boethius IV, 14. See Bower, Boethius, pp. 267-275.

²³Zarlino, Ibid.

of his modes, as you have seen in the demonstration, includes both systems. In addition, he named the said species (which we have applied to the modern practice by proceeding from high to low) in this order, that is: mi, mi; la, re; sol, ut; fa, mi; and fa, fa. He undoubtedly understood [the entire situation] because of his superior intellect. There have been others of the moderns who have given this particular species of the diapente the name of second, although Boethius called it fourth, and others have called it other names, according to their various designs, saying that the first species of diapasen was between A re and a la mi re; that the first [species] of the diapente was between D sol re and a la mi re; and that the first species of the diatessaron was between this D sol re and G sol re ut. The other species followed by conjunct steps toward the high register. Others have felt otherwise, according to what has turned out well for them, without

submitting as a testimony of the truth anything except their opinions. And all say that they have imitated Boethius, whose text--it must be that [same] edition which used to be procured with great difficulty from both Basel and from Paris and which was mentioned above--has not come to our notice, other than having seen it in manuscript in different, famous libraries.

Strozzi: For what reason did the ancient Greeks not want to accept as the first species of their diapason, nor of their diapente, nor of their diatessaron, that which commences with the lowest proslambanomenos, since they were (according to some opinions) accepted by the Romans, as you have said?

Bardi: I have never said that the Romans accepted for the first species of any of their consonances that which begins with the proslambanomenos. There are also others [who share] this opinion, but I do not know where it [all] began. I state emphatically, however, that if you will observe the order of the species of

consonances according to the opinion of the Greeks, you will find that they agree with those of the Romans.

Strozzi: I do not know how this could possibly be, since it happens that you said that the first species of the diapason of the Romans is that which is found between Aa la mi re and a la mi re and that of the Greeks between h mi and h mi. Besides, those of the Greeks, which begin in the low register, go by conjunct steps when proceeding toward the high, and those of the Romans, on the contrary, when moving away from the high register, proceed toward the low register.

Bardi: All is well. Now, please take warning. In the next demonstration, you will see, with regard to modes, according to the opinion of Ptolemy, that the species of diapason which is found between h mi and h mi, which the Greeks had adopted for their first [species] and which serves the Mixolydian mode, is applied to that pitch which, in the Dorian

The order of consonances according to the Greeks and the Romans is the same.

mode, was likewise known to the Romans by such a name. The same is true of the others in order, so that they agree very well together. That little difference which exists between them, with regard to the size of the interval, only arises because of the diversity of their purposes in applying either species of the diapason to either system. One could also consider that the two extreme pitches of this particular diapason serve the Hypodorian mode; however, they serve it between those [same pitches] in the Dorian system. The ancient Greek musicians were accustomed to begin to number the pitches of systems from the top, coming [then] toward the bottom. Consequently, they said that the first species of the diapason was the one which served the Mixolydian mode, and the others [went] in descending order. Their other species of consonances were treated by them in the same manner. The reason, then, that they did not accept the proslambanomenos as the first species of the diapason and [accept as] the first species of

the other consonances those which began with the very lowest pitch was because it did not intervene in any genera of harmony nor [in any] different distribution of species, nor between the stable pitches, nor among the movable [pitches] of their tetrachords, besides being last to be numbered. Although it did not intervene--as has been said--between the pitches which enclose and which are enclosed in tetrachords, it is not because the proslambanomenos is not stable in each Greater Perfect System, for the reason which will be said below. And to tell you more, not only was this pitch [i.e. proslambanomenos] the last string [to be] added to the cithara, but [it was added] after [the other pitches] had been ordered and distributed in tetrachords, and the reasons for its addition numbered two or three of not much importance. The first of these [reasons] evolved only because the mese came in the middle of this system, as its name implies.

Without the aid of the proslambanomenos in that place, this would have been impossible. The second reason was because the extreme high pitch sounded as a disdiapason (or quint decima) with the extreme low pitch. None of the intervals reputed by [the Greeks] as consonances were lacking to that [disdiapason], and then the mese would sound as a diapason (or octave) with the extremes.

I said [these were] causes of not much importance because, without [the proslambanomenos], each species of the diapason appeared in the system in the same order and without any impediment. Consequently each mode (or tone), if not perfectly enacted with regard to its entire constitution, at least appeared to be in effect concerning the species of the diapason. This aspect must be the principal cause that, while the lyre and cithara (which were the same thing to them according to some higher authorities) had been received in those early times by Mercury (its inventor) with only four strings, it had

been augmented in process of time up to the number of fifteen.

Strozzi: You are therefore of the opinion that the lyre and the cithara were the same instrument according to the ancient Greek and Roman musicians?

[61]

Bardi: I have very little doubt about it, because of the many verifications of authorities, in addition to which Pausanias²⁴ says that the lyre was discovered by Mercury, and the cithara by Apollo.

[Pausanias] in the 4th book.

Strozzi: I judge that your opinion of this is highly contrary to the common one.

Zarlino²⁵ in the Institutioni at chapter 3 (and in chapter 4 of the 2nd book) and in the 1st chapter of the 4th [book].

Bardi: I firmly believe that you and anyone else who has patience to hear me will concede [to my opinion].

²⁴J. G. Frazer, Pausania's Description of Greece, 3 vols. (London and New York, 1898), I, 258. "There is an altar of Apollo and Hermes [Mercury] in common, because there is a Greek tale about them that Hermes was the inventor of the lyre and Apollo of the [cithara]."

²⁵Zarlino, Istitutioni, pp. 61-64.

Strozzi: It will be most gratifying to understand such a thing.

Bardi: I will state briefly to you the most famous authorities which I have gathered in favor of both sides [of the question], leaving, then, the judgement of what is more pertinent to you and to everyone else of rational intellect. But listen! One finds, according to the Greeks, five names of musical instruments (among the many others for which they have different meanings) which, from what is gathered from their writings seem to mean the very same [thing]. These are lyre, chelyn, cithara, cethra, and phorminx. Hardly anyone doubts that lyre and chelyn are not the same instrument, and that either one of these, similarly, is not the very same as that which was then called testudo by the Romans. They also believe, in the same way, that cithara and cethra are the same thing, the poet calling it one name most of the time and the orator the other. It is clear that the phorminx is the same instrument as the cithara. It

How many different names the Greek lyre has been given.

happens that Suidas²⁶ confirms this, writing about it thus: "Phorminx, that is, cithara." Next, Homer²⁷ recalls in the Odyssey that when a servant at the banquet had given the cithara to Phemius he added these words, "Surely this one 'phormison'," while Dydimus, the interpreter says, "'phormison', that is, to play on the cithara." Because of this, if we now demonstrate that the lyre is the same thing as the phorminx, no one will doubt that the lyre is the same as the cithara. This is what Horace²⁸ is able to teach us sufficiently, because having translated almost word for word

Suidas.

Homer.

Horace.

²⁶Immanuel Bekker, ed., Suidae Lexicon (Berlin, 1854), p. 1103.

²⁷See Homer, Odyssey, translated by S. H. Butcher and others (New York, 1963), p. 208.

²⁸James Gow, ed., Q. Horati Flacci Carmina (Cambridge, 1896), p. 12. "Quem virum aut heroa lyra vel acri tibia sumis celebrare Clio, quem deum?"

from Pindar²⁹ that song of his which begins, "O Clio, what man or what hero will begin to celebrate you with your lyre or with the shrill tibia?", while Pindar in place of lyre says this, "anaxiphorminges". From this it is plainly known that Horace calls lyre what Pindar first called phorminx. From the words of the same Pindar, however, this very same [fact] can easily be ascertained, because a little later he writes this: "We will take the lyre." In that place Pindar calls lyre that same instrument which he earlier called phorminx.

Pindar

Strozzi: I concede to you that the phorminx was the very same thing as the lyre, but it will not follow because of this that the lyre was the same thing as the cithara, since it could easily be an ambiguous name.

²⁹Sir John Sandys, trans., The Odes of Pindar (London and New York, 1915), p. 18-19. The Greek word "anaxiphorminges" is rendered as "lyre".

Bardi: Tralascio, the interpreter of Pindar, indiscriminately uses lyre for cithara in many places. I am able, however, to prove this same [thing] to you more clearly with the authority of more serious writers. First [I shall quote] the authority of Xenophon³⁰ who, in that book he wrote [called] Concerning the Care of a Family, said this: "Those who first learn to play cithara spoil the lyre." In addition, Socrates, according to Plato,³¹ said that Alcibiades learned to play the cithara and shortly afterwards, that which he earlier had called cithara, he called lyre. Now let me gather arguments in favor of this side, since I am able with a single testimony worthy of faith (and this is Suidas)³²

[Plato] in
Alcibiades I.

³⁰See Xenophon, The Oeconomicus in J. S. Watson, Xenophon's Minor Works (London, 1905), p. 80.

³¹W. R. Lamb, Plato (London and New York, 1927), p. 108-109.

³²Bekker, Suidae Lexicon, p. 599.

to bring you into close proximity with the truth. He uses these words. "The cithara is a musical instrument which otherwise is called lyre." Could he say anything plainer and more open than this? Now hear, oppositely, what I have gathered in favor of the other side, which seems in a certain way to persuade us [to] the contrary. Plato³³ in the third [book] of the Republic says: "The lyre, moreover, remains, and the cithara is useful in the city. The syrinx aids in country pastorales." Athenaeus³⁴ in the fourth book toward the end [says]: "The magadis is an instrument like the cithara, the lyre, and the barbiton." In the same place, he alludes to a certain Anaxilas, having taken two quotes, in addition to the others, from a work entitled The Builder of the Lyre, in which one reads in the same order works

Lyre and cithara are the same thing.

³³ Compare the Plato excerpt in Oliver Strunk, Source Readings in Music History (New York, 1950), p. 6.

³⁴ Athenaeus, Deipnosophistae, 7 vols., translated by Charles Burton Gulick (London and New York, 1928), II, 309.

which correspond to these. "But I have strung barbitons, trichordons, pektides, citharas, lyres, and scindaposes."

Pollux³⁵ in the fourth book [says] that "among those, moreover, which are played are the lyre, the cithara, and the barbiton." Plato³⁶ in another place, which troubles us more, speaks always of the cithara nobly, and, on the contrary, he deals with the lyre as an instrument for fun and games. Therefore, it seems impossible to many, since the cithara--as has been shown above--is the same thing as the lyre, that it has been discussed by so many in the manner which you have learned. Nevertheless, with the sole consideration of the diversity of names with which our own keyboard instrument is known [today], every scruple is easily taken away which the words of these famous, serious writers could possibly bring us. This, as everyone knows, is

[Plato] in the Dialogue of Laches.

³⁵Eric Bethe, ed., Pollucis Onomasticon, 3 vols. (Leipzig, 1900-1937), I, 218.

³⁶B. Jowett, trans., The Dialogues of Plato, 4 vols. (New York, 1872), I, 80-81.

called by us with the names of clavichord, harpsichord, clavicembalo, spinet, buonacordo, archicembalo, and others, only because of the different quantity and quality of strings and registers and [because] of the size and form of the instrument. Yet, in essence, one instrument is the same thing as another, and whoever plays this one, plays that one likewise. There is no one, also, of rational intellect who, when comparing a soft gravicembalo to a strident spinet, does not judge the latter, in comparison with the former, to be a thing for fun and games. So, also, when comparing a sweet, sonorous, deep organ to a shrill, raucous, harsh regal or to the noisy sordunes, the result will be the same. One sees, in addition, that the viola da gamba, the harp, and many wind instruments which we use every day in serious musicals and important concerts are also used for amusement at balls and dances. But among the artifices which are employed in both the [serious] and [light] ways [of playing],

one can consider that there is the same difference which Plato supposes between the dignity of the cithara and the sport of the lyre. It may very well be also, that when Xenophon said that those who learn to play the cithara spoil the lyre, he wanted to infer that the young girls of his times, in wanting to learn to play the cithara, practiced first on the lyre, since its shape was more convenient and also its sound was more suitable, through its high pitch or [some] other [attribute], for that tender age. Thus, through their inexperience, they came in the beginning to spoil the strings, both with the plectrum and with nails. It is no different from what happens to our own [people] every day when in wanting to learn to play the organ (that is to say the harpsichord), they practice most of the time on a spinet or on a small clavichord.

Pausanias' opinion concurred also with this sentiment without exactly misrepresenting it when he said that Apollo invented the cithara, and Mercury invented

the lyre. It happens that Mercury discovered the lyre in his youth, as an instrument suitable to that age, and Apollo discovered the cithara after he had attained manhood. The point of this is to say that the invention of the cithara was attributed to Apollo, and that of the lyre to Mercury in order to distinguish the qualities of the [two] persons. Aristotle³⁷ also agrees with this opinion in dealing with the difference of the harmonies--as you will learn later, and which I will tell you at the appropriate place--what difference there actually was (if there was any) between the cithara and the lyre.

Strozzi: You have greatly satisfied [my curiosity]. If it pleases you, however, you could return to your original topic and continue the discussion already begun about tones (or modes) of the ancient Greek musicians according to the opinion of Ptolemy. Also, please pardon me [for the inconvenience].

[Aristotle] in
the 8th [book]
of the Politics.

³⁷ See p. 363 below.

Bardi: I told you above that the ancient musicians gradually proceeded to augment the strings of their cithara and lyre up to the number of fifteen. They distributed these strings in the manner which one sees ordinarily in the Greater Perfect Disjunct System of the Dorian mode. In the process of many, many years, knowing that this number of strings was comfortable suited to express any human emotion effectively, they were satisfied. I told you, in addition, that the meaning of the name of the lowest string, the proslambanomenos, since it was last to be added to the [Greater] Perfect System and last also to be numbered according to the usage of the Greeks (although first according to the Romans), was the same in that language as it is in ours, that is, "added", "taken in addition", or "surplus". This meaning--as I have said--seems to warn us that that was not done through any necessity but as a proper choice. The reason, then,

The significance of proslambanomenos.

that it is called stable in any Greater Perfect System results from the same relationship which it continually has in every genera and species of harmony with the netehyperbolaion. Whereupon, since the nete is stable, then such is the nature of all the extreme strings of any tetrachord. The proslambanomenos also is necessarily stable, because it is continually found [to be] distant from the nete and from the other stable [strings] by as much as has been said. In addition, in the Greater Perfect Disjunct System, even without the example of infinite writers, the strings are stretched--as I said before--according to the Dorian tone (or mode). This harmony was reputed, approved, and valued more than any other by every ancient musician, poet, and philosopher. This--if I am not deceived--resulted mainly because the strings in its system were stretched according to the tone (or mode) in which one normally discourses without effort. The human voice cannot sing all the

Why the Dorian harmony was more reputed than the others.

pitches of an entire system more comfortably than those which are tuned according to this Dorian mode, the excellence of which has continually caused it to be adopted as an example by every famous man who has written of the art of music. This situation has not been true of the others. We should not wonder about that, for it happens that the very high modes and those which were too low were mainly refuted by the Platonists³⁸ in their well-ordered Republic, since those [high modes] were woeful and these low ones were lugubrious. Only those middle modes were accepted by these [Platonists] since numbers and rhythms were also necessary according to them. Their opinion was later refuted by Aristotle,³⁹ who said that the mild harmonies are not to be disdained with regard to old men who, because of their years, are not able to sing the extended harmonies. The high modes then, like the Lydian, he

The Platonists refuted the harmonies [which were] too high and too low.

Rhythm means the dancing and movement of the body. [Aristotle] at the end of the 8th [book] of the Politics.

³⁸See the excerpt from Plato's Republic in Strunk, Source Readings, pp. 4-6.

³⁹See Aristotle, Politics (1342b20-43) Richard McKeon, ed., Basic Works of Aristotle (New York, 1941), p. 1316.

conceded to young girls in order (as he said) to make them both ornamental and disciplined at the same time. One can argue, in the absence of another authority, that the compositions of the ancients were actually sung according to the sound of pitches in which they were found written by their composers. The contrary happens today with our [compositions]. There is a particular example of the system of the Dorian tone [or mode], and not of the others, in Ptolemy, in Boethius, in the Introduction⁴⁰ of Guido d'Arezzo, in the Theorica and Practica of Franchino [Gaffurio], in the music of Glarean, in the Institutioni harmoniche of Zarlino, and, finally, in the Timaeus of Plato, there has been mentioned the larger and better part of the species diatonic ditoniaion, as all the other [modes] are called, in addition to the many different places [where they are so-called by] other

Ptolemy, Boethius,
Guido d'Arezzo,
Franchinus [Gaffu-
rio], Glarean,
Zarlino, Plato.

[63]

⁴⁰By Introduction of Guido, Galilei alludes to Guido's Prologus in antiphonarium. See Strunk, Source Readings, pp. 117-125.

authorities. The species of diapason of this mode is the fourth [species] which is located, according to what I said, between the pitches of E la mi and e la mi where Boethius placed the fourth species lower than E la mi and e la mi by a sesquioctave interval for no other reason than to provide room for the Hypermixolydian mode, which he could not do unless he assigned to the Dorian mode that which Ptolemy first assigned to the Phrygian. These two modes [i.e. Dorian and Phrygian] were continually used in speech by the characters of comedy and tragedy. That [procedure] was likewise followed in satire since [these characters] used it on the stage of the theater, accompanied by the sound of the tibia [aulos], in reciting their poems. Above this mode [Dorian], there were three higher modes and there were also three modes below it. The three higher ones were the Phrygian and Lydian, and the highest of all was the Mixolydian. These

Why Boethius varied the species of the diapason from the [usage of] the Greeks.

The characters in comedy and tragedy recited their poems to the sound of the aulos.

[additional modes] were then distributed with these conditions. By raising the ordinary disjunct system by a sesquioctave interval, in which system--as I have said--they sang the Dorian mode which had been discovered by the great Thamyris of Thrace,⁴¹ one obtained the Phrygian [mode], whose inventor was Marsyas, or Masses,⁴² son of Hyagnis the Phrygian and, according to the opinion of Aristotle, the teacher of the great Olympus. This Marsyas was the first who played the pipe with holes, since it had not been known before him. He also played two of them with one breath, and he was likewise the one who first combined high and low sounds. Elegies with

Thamyris of Thrace invented the Dorian harmony. Marsyas invented the Phrygian [mode].

⁴¹See Plinius, The Natural History of Pliny, 6 vols. (London, 1855), II, 231. Thamyris was also reputed to be a former lover of Hyakinthos by Apollodorus.

⁴²Plutarch, On Music, translated by Benedict Einarson and Phillip H. DeLacy (Cambridge, Massachusetts and London, 1967), p. 368-369, gives the alternate name of "Masses" for Marsyas. Marsyas is credited by Plinius with the invention of the Phrygian mode and of the double-pipe.

fixed form were sung to the sound of that instrument. The chorus in tragedies used the Phrygian harmony a great deal, as well as the Lydian, since the latter was appropriate for the angry and for the cries of the doleful, and the former was suitable for one who was rejoicing with alacrity. These [modes] were repudiated by Socrates because they were apt to introduce affections in man which were inappropriate. For this reason he repudiated also the Lydian and the deep Iastian as weak and proper for drunkards, not when they were infuriated but when they were languid. The Dorian and the Phrygian he conceded as useful to war. I say that the species of the diapason in the Phrygian mode is found between D sol re and d la sol re, and that its mese is necessarily G sol re ut, while that of the Dorian is a la mi re. Once more sharpening the system by a tone

These are the harmonies which were used by the chorus in tragedies.

[Plato] in the 8th [book] of the Politics⁴³ [sic.].

⁴³Galilei's reference to the Politics is erroneous. See Plato, Republic, in Strunk, Source Readings, p. 5-6.

more than that which serves the Phrygian mode, one will obtain that of the Lydian mode, of which Amphion⁴⁴ was the inventor, although others say Menalippides,⁴⁵ and others, Corebos.⁴⁶ Pindar claims that Anthippus at the wedding of Niobe was the first who taught the Lydian harmony.⁴⁷ The Epithalmians were especially accustomed to singing the [Lydian mode] and this is what Plato had called the intense Lydian in differentiation from its weak

Amphion [was the] inventor of the Lydian harmony. [Pindar] in the Paeans.

⁴⁴Plinius, op. cit., p. 20. "Amphion was the inventor of music." Ibid., p. 231. "Amphion invented the Lydian measures in music."

⁴⁵Menalippides was a fifth century B.C. dithyrambic poet from Melos. Plutarch, On Music, p. 385. Ibid., p. 421, alludes to a new complexity in Greek music which superseded the more conservative schools.

⁴⁶Ibid. Corebos, or Torebus, or Torrhebos, was the son of Atys, king of Lydia. It is he who, according to ancient sources, invented the Lydian mode.

⁴⁷Ibid., p. 385. "Pindar says in his Paeans [Frag. 75, ed. Turyn] that the Lydian mode was first presented at the wedding of Niobe."

plagal. The Lydian species of diapason is contained between C fa ut and c sol fa ut; its mese is f fa ut. Plutarch⁴⁸ relates, following Aristoxenus, that the first use of this particular harmony resulted from a certain mournful event, and that, first of all, Olympus, on the death of Python, played upon the aulos funeral verses according to the Lydian usage. Plato⁴⁹ spoke thus in describing its nature, cursing it. "The Lydian harmony is high, raging, and strident, and is altogether suited for laments." Now this particular system, when transposed a minor semitone (or limma) higher, produces the strings strung according to the Mixolydian mode, which is full of

Olympus, on the death of Python, played the Lydian mode on the aulos.

⁴⁸Ibid. "For Aristoxenus in his first book On Music says that Olympus was the first to perform on the auloi a lament for Python in the Lydian mode." The cited work of Aristoxenus exists only in fragmentary form (Frag. 80, ed. by Wehrli, Testim. 105, ed. Da Rios).

⁴⁹Ibid. "Hence Plato in the third book of the Republic shows distaste for such music; thus he rejects the Lydian mode, since it is high pitched and appropriate to lamentation."

many affections of harmony and is very much suited to tragedy; its mese is e la mi, and the species of its diapason is that which is found between h mi and h mi. The names of the three modes which are below the Dorian are the Hypolydian, the Hypophrygian, and the Hypodorian. These are called plagals, since they are appropriate for those who are afraid and who pray entreatingly. They are created and formed in this manner. By lowering the system of the Lydian mode by a diatessaron, or to a semitone from that of the Dorian, one will obtain the Hypolydian mode, which was discovered, together with the Orthian nome (from which the trochaeus, giver of the sign, was derived), by Polymnestus of Colophon.⁵⁰ This [Polymnestus] used more strings than

Why plagals modes are so called.

Polymnestus, inventor of the Hypolydian harmony.

⁵⁰Plutarch, *On Music*, p. 365. "There was also a poet Polymnestus, son of Melos of Colophon, who composed Polymnestian nomes." *Ibid.*, p. 373. "Polymnestus too composed nomes sung to the auloi, but whether he employed the Orthios nome in his music as the writers on harmonics assert, we are unable to say definitely, as on this point the ancients are silent." *Ibid.*, p. 417. "To Polymnestus is ascribed the mode now called Hypolydian."

any other musician of his times. The species of the Hypolydian diapason is between F fa ut and f fa ut; its mese is h mi. If the system of the Phrygian mode is once again lowered by a diatessaron, that is to say to a tone below that of the Hypolydian (which is very important with regard to the effect), one will obtain the Hypophrygian mode which Plato called mild Iastian. I have not yet been able--although I have searched diligently--to find out the name of its inventor. The [Hypophrygian] diapason is contained between the pitches of G sol re ut and g sol re ut; its mese is c sol fa ut. Finally, if we transpose the system of the Dorian mode down a diatessaron, that is, to a tone below the Hypophrygian, we will obtain the lowest [mode] of all, the Hypodorian. The invention of this is attributed to Philoxenus.⁵¹ It was the last mode to come into use. Its diapason is contained

The inventor of the Hypophrygian harmony [is] unknown to the author.

Philoxenus [was the] inventor of the Hypodorian harmony [which was] last to be discovered.

⁵¹Philoxenus of Cythera (435-379 B.C.), a dithyrambic poet, is credited with the invention of the Hypodorian

between a la mi re and Aa la mi re; its mese, consequently, is d la sol re. And this was the actual number and measure of the tones (or modes) of the ancient Greek musicians, according to the opinion of Ptolemy. These musicians were searching among all seven modes--there were only seven because there were no new species of the diapason to occupy--for twenty-one pitches and their higher and lower replicates; for example that of the Hypermixolydian, which was refuted by Ptolemy, came outside of the natural, common system. In addition, in the demonstration of Boethius,⁵² the same occurred also to the Mixolydian mode, since it was different from Ptolemy in its consideration of the pitches of the diapason of these modes. This matter is worthy of no little consideration.

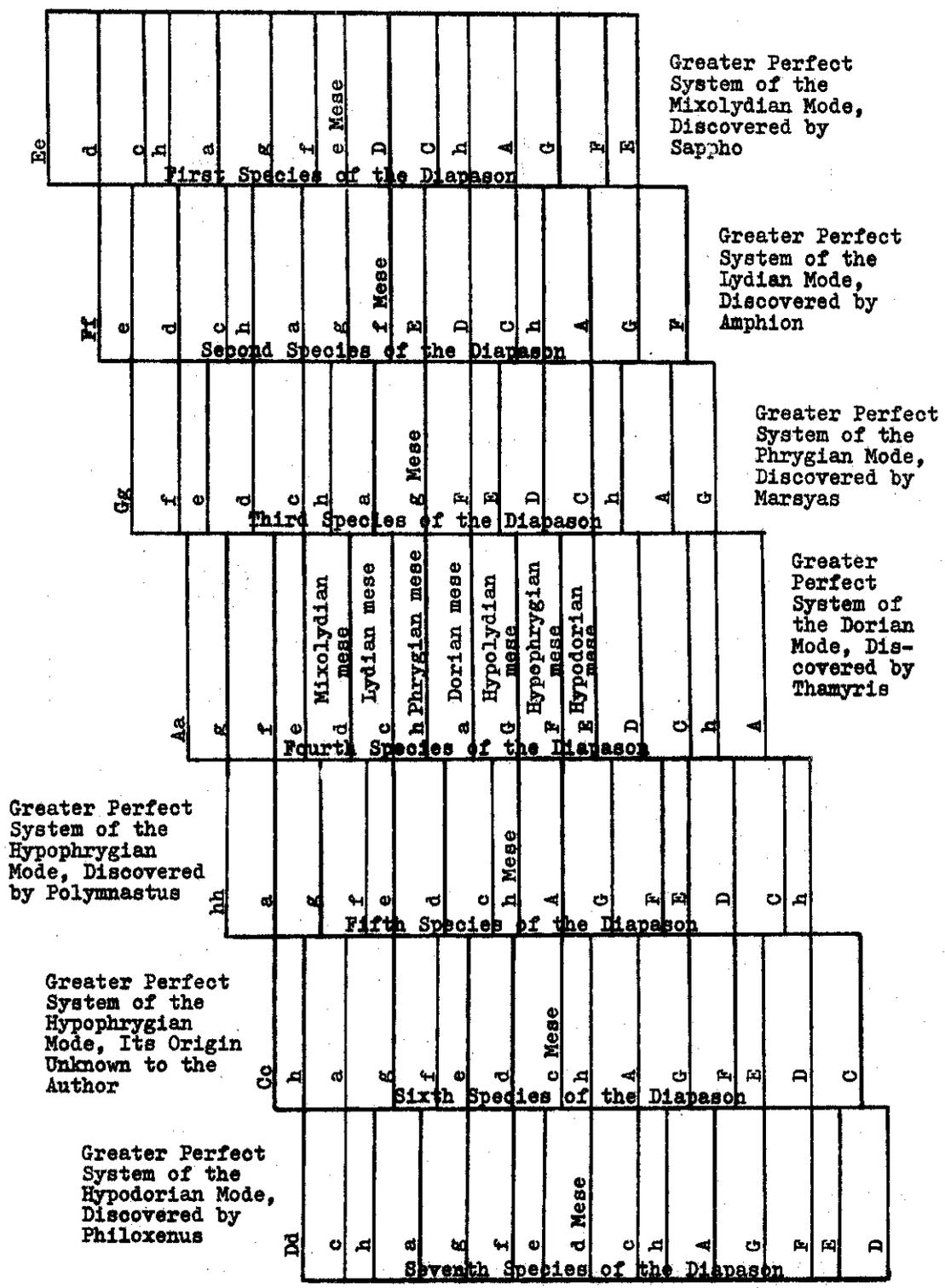
[Boethius] in chapters 9, 10, and 11 of the 2nd [book].

mode by Julius Pollux. See Bethe, ed., Pollucis Onomasticon, I, 220.

⁵²See Bower, Boethius, pp. 121-128.

[DIAGRAM VIII]

DEMONSTRATION OF THE EIGHT MODES, ACCORDING TO THE OPINION OF PTOLEMY



The extremes of their constitution, according to Ptolemy (the conjunct ones as well as the disjunct ones) sought in each genera of harmony no more than twenty-one pitches--as I said--and their differences were twenty-seven [in number]. Thus, the Mixolydian, which was higher than all the others, came to sound a semitone above the Lydian, which in turn came to sound a tone above the Phrygian, and the Dorian sounded a tone below the Phrygian and a fourth above the Hypodorian. Below that, a semitone and a fourth from the Lydian, there was the Hypolydian, and descending a tone below the Hypolydian and a fourth below the Phrygian, there was the Hypophrygian. Finally, the system of the Hypodorian was obtained by lowering that of the Dorian by a fourth, or, actually, [by lowering] that of the Hypophrygian by a tone. These,

[65]

Zarlino⁵³ claims that according to Ptolemy, there were 8 modes, which Ptolemy never said. [Zarlino] even repudiated the 8th mode in chapter 3 of the 4th [book] of his Istitutioni.

⁵³Zarlino, Istitutioni, pp. 299-300.

according to Ptolemy,⁵⁴ prince of mathematics, were the true, legitimate intervals by which the modes of the ancient Greeks were lower or higher than each other and lower or higher than the pitches of which they were sung. Therefore, the description is this. In order that it will be sufficient, I have converted it into the easiest way which I could possibly imagine. One must consider in this, principally, that as the mese and the species of the diapason of each system are different, in the same way the whole order of the scale is different with regard to the steps in ascending and descending from beginning to end by tones and semitones. That trite proverb which they used which says, "We will pass from the Dorian harmony to the Phrygian, that is, from serious matters to foolish ones" not only derived from the opposing nature of the harmonies and tones, but from their contrary way of ascending and descending (with regard to the steps of tones and semitones) from one to the other, because the

[Ptolemy] in chapter 10 of the 2nd [book].

Aristoxenus⁵⁵ in the 2nd [book].

Proverb of the ancient Greek musicians.

⁵⁴See Düring, Ptolemaios, pp. 76-79.

⁵⁵See Macran, Aristoxenus, pp. 187-208.

Phrygian ascends by the same steps which the Dorian does in descending. This can be clearly observed in their examples.

Their constitutions were named by the ancient Greeks, for different reasons, with these three different titles of sound and sense, that is, tones, tropes,

and "ethoi". "Ethoi" [were so called] because they demonstrated the custom; tropes, because the whole system rotated in passing from one to the other; and

tones, with regard to highness and lowness. "Ethoi" demonstrated the mood

because some of them, especially with the example of those who recited them, induced in the listeners important and serious thoughts, and others induced soft, effeminate ones, which happened in listening to them. The system ro-

tated every time one passed from one to the other; for example, in passing from the Dorian to the Phrygian, because those same intervals which were used in the Phrygian in proceeding from low to high were used in the Dorian--as has been said--in moving toward the low range

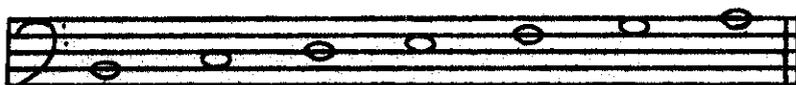
Why tones [modes] were named differently.

Why "ethoi" were so called.

Why tropes were so called.

away from the high. There were also differences of highness and lowness; it happens that the Phrygian was a tone higher than the Dorian, and it [i.e. Phrygian] was found [to be] the same interval lower than the Lydian. We will add to this another consideration; they were called tones and not semitones because the three principal ones, which are more ancient than the others, that is, the Dorian, the Phrygian, and the Lydian just now named, were distant from each other by a tone and acquired their names before the other four had come to be known. It would also not have been inappropriate to have named them tones and not semitones after all seven were in use, because, between the intervals through which their systems proceeded from low to high or high to low, they contained a minor heptachord in the manner which is seen notated here [Figure 110.].

Why tones were so called.



[Figure 110.]

In this [illustration], the semitone is enumerated only twice, and the tone four times, in addition to [which] the [tone] is more natural and nearer to perfection than [the semitone].

Order of
the modes.

Strozzi: I have always understood with regard to scientific, scholarly men, that understanding well how the modes of the ancient musicians stood, how they sang them, and how many they were in number, not to mention the many other cases they considered regarding them, is one of the most difficult things to know and comprehend well; [it is more so] than any other [matter] pertaining to the music of those times. This I now prove through experience and I see that it is so, for, with all your diligence, I am still not entirely capable of understanding what you have told me up to now and shown me with the example; nor am I entirely

satisfied. I hope, however, that it will not be a burden to answer whatever I ask with regard to these, in order that I remain fully appeased. Would you also pardon my importune and perhaps impertinent requests?

Bardi: Speak freely, as always, for I do not intend to fall short, except in that part where my knowledge does not extend.

Strozzi: I understand, for the most part, that the same Ptolemy says that there is a mese particular to each mode. I am now about to tell you that, applying his words to the modern practice. The mese of the Mixolydian is d la sol re; that of the Lydian is c sol fa ut; that of the Phrygian is h mi; that of the Dorian is a la mi re; that of the Hypolydian is G sol re ut; that of the Hypophrygian is F fa ut. Finally, that of the Hypodorian is E la mi.

Bardi: You speak very well, and you are quite correct. You will find the same

[facts] in my description if you will consider it more carefully.

Strozzi: I do not know how to find them without your aid.

Bardi: Then look! When Ptolemy⁵⁶ describes the order of the meses of modes in the manner which you have recited them, he considers all seven in the natural, ordinary system. I come, then, to consider each one of these [modes], in its particular constitution, in the order in which I have stated them to you. You will not find in my example any contradiction between them, because in considering that line which represents the mese of the Mixolydian, which is e la mi, you will find that this mese falls on d la sol re in the ordinary, natural system of the Dorian mode. All the meses of the other systems do the same thing compared to this.

[Ptolemy] in chapter 11 of the 2nd [book].

[66]

Strozzi: I understand it very well now, but Ptolemy claims, in addition, that the systems, continuing one after the other

⁵⁶See Düring, pp. 79-80.

in the order which you have demonstrated to me, are distant by ditones and semiditones. Instead of these, you have mentioned tones and semitones. I do not know how such a difference can exist between you [and Ptolemy].⁵⁷

Bardi: I made mention of those smaller intervals because they are more necessary to the understanding which I tried to give you of these modes. If you had wanted to strain your intellect a little, you would indeed have found also in my demonstration the ditones and semiditones between one tone and another. You would only have had to consider and compare each particular pitch of one system with that of another, as, for example, the h mi of the Dorian with that of the Phrygian, or the h mi of the Phrygian with that of the Lydian. But

How ditones and semiditones are considered modes.

⁵⁷ See Claude V. Palisca, Girolamo Mei: Letters On Ancient And Modern Music To Vincenzo Galilei and Giovanni Bardi (Rome, 1960), pp. 56-58, for a comprehensive discussion of this discrepancy.

the whole of one compared together with the whole of the other, or even any part of these--let us omit, however, the consideration of the particular names of the pitches according to the usage of this modern practice--is actually not more than a tone or a semitone--as I first said--that is, between those which are connected. The sight of such diversity occurs nowhere else except in the example given you of these [modes], in which the proslambanomenos, the mese, and the extreme nete, and thus all the other pitches of each one have different characters. This problem did not occur in that of Boethius, since the pitches were marked with the same symbols, and proceeded in the same order of steps in one scale as in the other, so to speak. I do not wish to suppress this other observation which one may consider in the demonstration of modes distributed according to the order of Ptolemy. The [fact] is, that those [modes] which are

Other considerations of the author regarding the modes of Ptolemy.

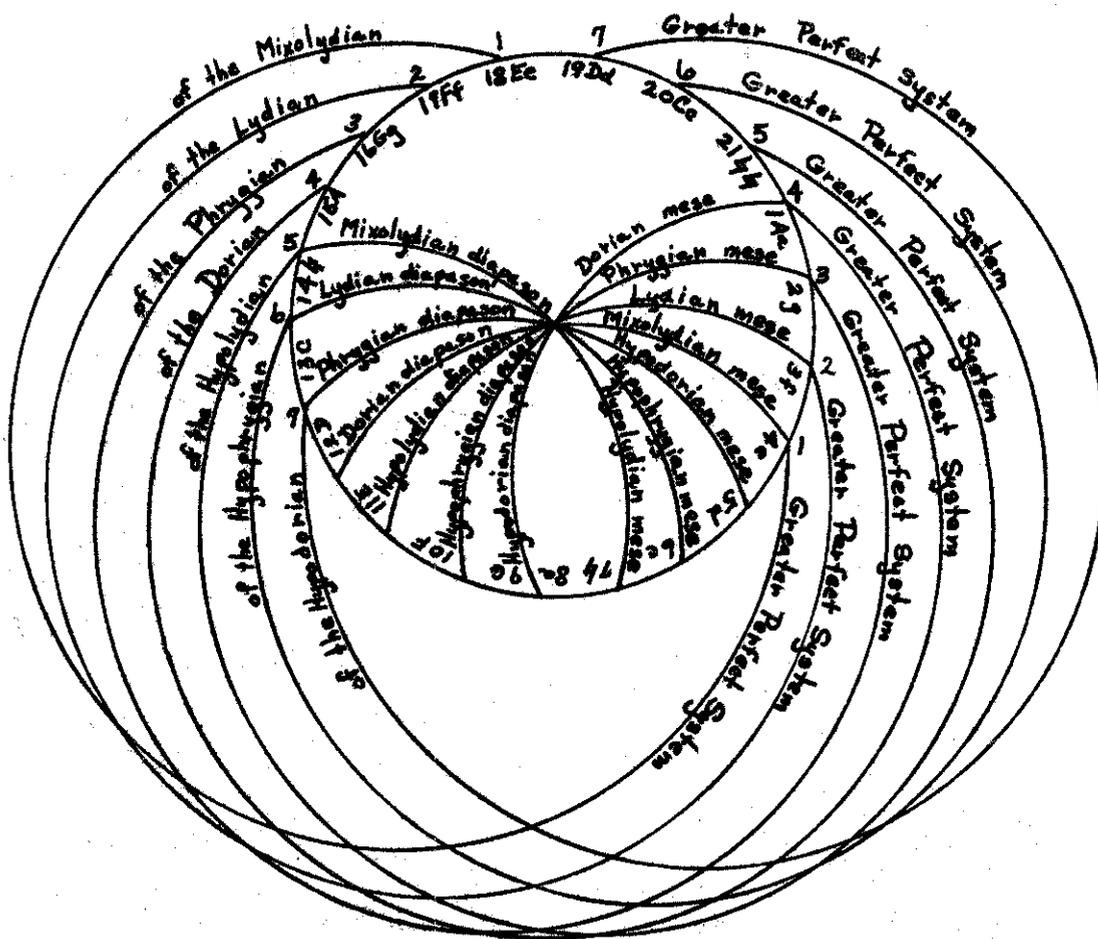
higher than the Dorian are represented to the sight as distant from each other by the same steps in which the notes of its constitution travel when moving away from the nete hyperbolaion by raising c sol fa ut a third. Those [modes] which are below the Dorian proceed by the very same intervals which are found in moving away from the proslambanomenos of the same by descending a third to h mi. This does not occur in the demonstration which Boethius makes. The same is true of the difference of steps and the number of those modes. Whereupon, those [modes] of Ptolemy, when the beginning and the end are connected, are seen going in a perpetual spiral like the celestial spheres, as the present wheel clearly demonstrates [Diagram IX.]. On this [wheel], each similar pitch is enumerated three times. Because of this they have with excellent reason been given the same names as those two circles which in the globe of the world are the limits of the longest and shortest day of the year.

Tropos is a Greek term which is badly explained in that proposition of Zarlino⁵⁸ at the end of the 1st chapter of the 4th part of the Institutioni.

[DIAGRAM IX]

[67]

[A SPHERICAL DEMONSTRATION OF THE
MODES OF PTOLEMY]



Strozzi: This has truly been a most [68] shrewd consideration, but now tell me something else. In considering the mese of each tone in the natural system, still according to the distribution of Ptolemy, I find, particularly, that the mese of the Phrygian mode is below that of the Lydian by a semitone, and that of the Mixolydian is a tone above the Lydian. I am, however, accustomed to believe the opposite of what you have described, that is, that the Lydian was a semitone below the Mixolydian, and the Phrygian was found a tone below the Lydian.

⁵⁸ See Zarlino, Istitutioni, p. 297-298, for a discussion of the distinction between trope, tone, mode, and harmonia. This question has remained a controversial one among leading scholars today. R. P. Winnington-Ingram, Mode In Ancient Greek Music (Cambridge, 1936), p. 75, alludes to the interchangeability of these terms. See D. B. Monro, The Modes of Ancient Greek Music (Oxford, 1894), p. 27. Monro, speaking of the Hyperaeolian and Hyperlydian modes, states that "the word 'harmonia' is used of these keys, and with it, seemingly as an equivalent, the word 'tropes'."

Bardi: This is actually so, It is one of the things which has provided many [people] of our times [with opportunity] for thought and speech, and many volumes have been written about it. It has not been understood, notwithstanding, by these [people], as you will learn at the proper place. Take warning, however. You need to remember what I said above about the meaning of trope, and to strain your fine genius a bit, so that, with the aid of this other consideration, you will understand extremely well. I tell you, therefore, that the species of the diapason of the Dorian mode is that of E la mi; that of the Phrygian is D sol re; that of the Lydian is C fa ut; and that of the Mixolydian is h mi. These are naturally distant from one another by the intervals which I said, but by applying them, then, to the tones which they serve, from that of the Dorian onward, they are transposed

Zarlino⁵⁹ in chapter 8 of the 4th part of his Istitutioni.

⁵⁹Zarlino, Istitutioni, pp. 307-308, contains a discussion of the relative positions of the modes.

up, one by a seventh, one by a fifth, and one by a ditone. On the other hand, the species of diapason of the Hypolydian, the Hypophrygian, and the Hypodorian are transposed down by the same intervals, as it was possible to perceive in the example [Diagram IX.]. The only reasons for this were that some of the former species were underneath the lowest pitch of the diapason of the Dorian mode and some of the latter species were above it (considered in the natural system, however) in addition to the fact that [people] have claimed (and deservedly) that the first species of the diapason serves as the prime of the highest mode and as the final of the lowest one, since it was last to be found among the order and manner of numbering the strings.

I want you now to remember that having to demonstrate a thing which in itself is most difficult, owing to its properties and individualities, which are not only very different but, on the whole, contrary, is most often not only lacking in ease of execution, considering all the circumstances

The species of the diapason of modes are transposed down one time and up another time.

Warning.

pertaining to it which should agree, but difficult, if not impossible. It is not exactly surprising that the example of modes which I have fabricated in order that you might understand [them] does not have in itself such an exquisite clarity that each smallest case is understood with the greatest ease one could possibly desire. The ancient Greek musicians had a character to denote each of their particular pitches. This character signified the proper sound of that pitch without having any need on lines, spaces, or clefs as [do] these of our times.

All of these things are the worst of hindrances which could be removed and largely ameliorated by scholars of the modern music practice with the greatest advantage [to all]. I will tell you how to carry that out easily before I bring our discussion to a close.

Strozzi: This should indeed be a thing which would be desired and adopted by every judicious practical musician, so you should not suppress it in any way.

The names of the musical notes could be greatly improved.

Returning, however, to the business of modes, would you tell me how Ptolemy formed them from two major superparticular consonances?

Bardi: Ptolemy extracted from the Dorian mode, by means of the diatessaron, the highest and lowest modes (which are--as you know--the Mixolydian and the Hypodorian) by raising and lowering them by that particular interval of this Dorian [mode]. Afterwards, ascending a fifth above the lowest, he created the Phrygian and descending a diatessaron below this Phrygian, he formed the Hypophrygian. Ascending a diapente above this [Hypophrygian], he formed the Lydian, and descending a fourth below this [i.e. Lydian], he formed the Hypolydian. Thus, all seven modes were derived by means of two major superparticular intervals.

Strozzi: How will we resolve that tritone which is found between the mese of the Hypophrygian and that of the Phrygian which you say, together with Ptolemy, has been tuned as a fourth? I am referring,

The fourth and fifth are major superparticular consonances. This is how Ptolemy formed the tone by means of two major superparticular intervals.

however, to the way it happens in the ordinary, natural system.

Bardi: Neither Ptolemy nor I said (with regard, however, to its beginnings and endings) that the mese of the Phrygian and [that of] the Hypophrygian were tuned as fourths, but I only said that they harmonized by that interval.

Strozzi: Is it not the same to be in harmony as it is to be in tune?

Bardi: No, Signor. [It is] not the [same] according to Ptolemy, because he says that a consonant is that interval which in penetrating to the ear strikes it inoffensively. Such, according to him, are the diatessarons, which are called symphonies in order to distinguish them. He further said that those which struck the ear, not with offense, but with sweetness, are tuned. These are the diapentes which are called parasones in order to signify them [properly]. The others, then, were those which in being heard, not only struck the sense inoffensively and sweetly, but were so [agreeable] that one could not

Zarlino⁶⁰ in chapter 15 of the 2nd [book].

To be in harmony and to be in tune are not the same [thing].

The meaning of symphonic consonances.

The meaning of parasones.

want for anything more. These were the diapasons, which were called homophonies, or, if you wish, antiphonies. This distinction was also made by Aristotle.⁶²

Strozzi: This has been a new, valuable, and subtle distinction which I had not expected, but tell me, in this case, another thing. Does the same precedence exist between the dissonances when they sound discordantly as exists between the consonances when they harmonize to a greater or lesser degree?

Bardi: Do not doubt it in the least! [In order to prove] that this is true, here is a clear, convenient example. When you have considered it rationally, you will find that the major second and the minor seventh are much less dissonant when the major second has been resolved by the minor third and the minor seventh has been resolved by the major sixth, which does not happen when they are resolved differently. You will find that the

Zarlino⁶¹ in chapter 31 of the 2nd part [of his Institutioni] says that those intervals which are not consonances are necessarily dissonances. The meaning of homophony and antiphony [is mentioned] in many places in the Problems of Harmony. Considerations of the author concerning the qualities of the intervals.

[69]

⁶¹Zarlino, Institutioni, pp. 105-107.

⁶²Aristotle, Problems, 2 vols., translated by W. S. Hett (Cambridge, Massachusetts and London, 1961), I, 378-415 and passim.

hearing is struck less harshly by the tritone and the semidiapente when they are resolved by the minor sixth or by the major third. This had not occurred to the first intervals named, perhaps because of the pleasantness of the union and the contrary motion of the major semitone, both of which effects are created by the parts simultaneously. In the first ones, however, it happened only in that moment when the other [part] proceeded by tone. When that happens to the high part in moving away, it is always much more graceful than it is when this happens to the low part. The contrary occurs when they approach one another, perhaps because of the motion, which becomes languid in the low part and intense in the high part, or because small intervals, in moving away by contrary motion, are more natural in high ranges than in low ones. The opposite effect happens in moving inward, but such an opinion can also result in a person from having

accustomed the ear in such a way. It can also happen since in that place both sevenths have resolved to the sixth and both seconds have resolved to the third, the larger of these [thirds] and the smaller of those [sixths] are less strident since they are nearer to their normal, safe conditions. For the same reason the major sixth should seem harder to us than the minor [sixth] in the passage which is made by that [major sixth] to the octave and by this minor sixth to the fifth, as most of the time happens. I tell you also that when one wishes to come to the diapente from the tritone, it would be much better to make the low part descend while the high part stands fast. Oppositely, if the latter were ascending, the former would stand fast. This happens for no other reason than from having the tritone with the fifth which causes an agreement in that place (that is, the low part), but I do not know how great it is. This agreement

results perhaps because of the slowness of the motion and the softness of the interval between those pitches, because the semitone is always more obvious to the ear when ascending than when descending. In these cases, which have been related above, and for the same reason which has been related there, the tritone is rendered less offensive to the sense than the semidiapente. Perhaps [this is true] because this tritone is found in the midst of two smaller consonances which today are called perfect. Still another reason would be that the semidiapente is able to become a diapente, and the tritone is able to become a diatessaron without altering any note, as one can clearly see and hear in the following example [Example 9.]. This diversity of harmony results in the consonances for no other reason than from the little or great conformity which their extreme sounds have together. On the other hand, the dissonances, being

Why a consonance pleases the ear and a dissonance displeases it.

Dissonant minor
seventh

More greatly disso-
nant major seventh



Dissonant major
second

More greatly disso-
nant minor second



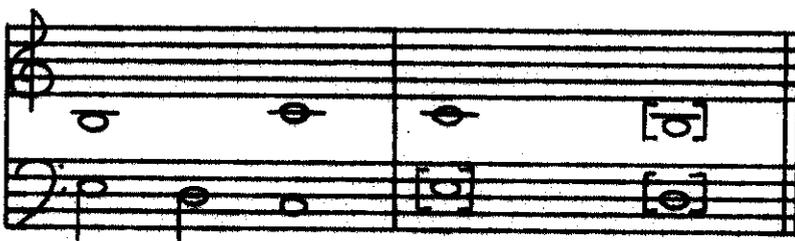
Much less disso-
nant tritone

Less tolerable
semidiapente



Hard major sixth

Less hard, even lan-
quid minor sixth



[Example 9--Dissonant intervals and
their resolutions.]

diverse in form and contrary [to the extreme sounds of these consonances], strike the ear harshly, because if we wish each one of their extreme sounds to be preserved intact in a certain way, and if we do not wish it to yield to the other, they [both] come to strike the sense harshly. The ear is offended even worse by the seventh than by the semidiapente, and less so by the tritone, perhaps because the tritone has the same number of steps as the fourth, and the semidiapente has the same number of steps as the fifth. Because the semidiapente resolves into the major third and the tritone resolves into the minor sixth, [the situation] there is more imperfect. The fourth is less consonant than the fifth, since dissonances are contained by the same quantity of pitches in the diatonic genera. Similarly, that which agrees with the fifth is more dissonant, but that which agrees with the fourth is less dissonant. That is not

surprising, for it happens that when the humors are well proportioned, any smallest case alters them greatly. This does not happen to those which do not agree and unite as well. One perceives this, as I have shown above, in the first two dissonances named, and also in the inconstant distance which is found between them and their adjoining consonances, the sixth and the third, which are generally used to resolve them. In addition, you will find that the minor third, which is between these pitches, and others like them, that is, A re [to] C fa ut have a sad [quality] [70] when descending and a happy [quality] when ascending. You will find that those pitches which are located between A re and C fa ut and between the ones similarly constructed, that is, E la mi and G sol re ut, are the opposite. All these considerations, more in the latter than in the former part, demonstrate their qualities and operate their effects with greater efficacy.

Strozzi: I [now] extract this [item] from among the other most important ones in your discourse which is that the diesis X, when placed in the high part makes the composition happy, and when the b molle is placed in the low part, it makes it sad. But [this happens] when the parts move away from each other, proceeding by contrary motion, and also when the low part is moved toward the high part in a separated and disjunct [manner]. The very same [thing] probably should happen in passing by contrary motion from the minor sixth to the major third and from the minor third to the major sixth. When arranged in an opposite manner their nature should be contrary. These cases should be less easily understood by the sense in proportion to the increase of the number of parts and the low part should be more greatly revealed than the high part or that of the middle.

Bardi: That is true.

Strozzi: You may now continue, if it pleases you, to show me the way to save that tritone which I said was found between the

mese of the Hypophrygian [mode] and that of the Lydian.

Bardi: Our practicing, contemporary contrapuntists, as you know, resolve that [tritone] in their compositions with the minor sixth, and we will resolve it in our compositions thus. Each particular mese needs to be considered in its usual system, and there will be a fourth exactly, and not a tritone.

Strozzi: Even considering them as you now say, we will have a major interval, because the mese of the Hypophrygian mode is c sol fa ut, and that of the Phrygian is g sol re ut, a fifth higher than [the Hypophrygian mese].

Bardi: You have forgotten once again the meaning of trope, and have not--as far as I know--understood, as yet, the status of the most important item of business. The mese of the Phrygian [mode], which is c sol fa ut, although it has the relationship of fifth with that of the Hypophrygian, which is g sol re ut, considered, however, in the manner you say, they [both] need

not be considered thus for this reason, but in the opposite way. Accordingly, the mese of the Phrygian has, by being higher than that of the Hypophrygian, the same amount this c sol fa ut has by being lower and not higher than g sol re ut. This is what Ptolemy told you just now in finding the positions of modes by means of the two larger superparticular consonances, and thus, likewise, it must be understood of the others also, if one wishes the outcome to be similar to the way in which he describes it.

Strozzi: You have produced for me a monumental thought. Clarify another difficulty for me, however.

Bardi: Tell me!

Strozzi: How can it happen, in the Lydian mode, that the pitch of c sol fa ut is--as I have learned--the extreme and the mese of its diapason at the same time?

Bardi: This is the case when it is considered in its usual system and as mese

in that of the Dorian, but in the same constitution this cannot happen in any way.

Strozzi: Why, then, did Ptolemy assign higher species of the diapason to lower modes and lower species of the diapason to higher modes?

Bardi: I showed you above that what he said was not exactly true, but only the contrary. I say, in order to warn you again, that if Ptolemy had, for example, assigned to the Hypodorian mode the first species of the diapason which is contained, as you have learned, between h mi and h mi, and [had assigned] the other species to the other modes in order, [then] this is one of the inconveniences which would result from these. The Mixolydian would become a tone higher than the Lydian. Nonetheless, one reads about this deed that Sappho,⁶³ the illustrious poetess, inventress of this Mixolydian

Sappho, inventress of the Mixolydian harmony.

⁶³Plutarch, On Music, p. 387. Plutarch, citing Aristoxenus, credits Sappho, the poetess of Lesbos (sixth or seventh century B.C.), with the invention of the Mixolydian mode.

[mode], since she was unable (according to the opinion of many), being a lady, to sing comfortably her own poems or those of others in the Lydian mode, raised the Lydian system by a semitone, and came to create a new mode which she called Mixolydian. This name almost implied that, because of the proximity of this new mode to the Lydian, they were mixed. We will add to their opinion, as a more solid and firmer one, this other consideration. The fact is that Sappho, being herself, was constrained to this necessity not because she was a woman, but from the very great conformity which the melodies of her poetry had with the effect and nature of that particular harmony. Since she was--as I have said-- a lady of small stature, who possessed only average looks, and whose profession (as the writers tell us) was not very chaste, she was therefore given occasion-- as one sees in some fragments of her poems-- to quarrel and complain most often about

love while she yearned for the beauties of the youth, Phaon.⁶⁴ Besides, [the fact] that the people of Phrygia ordinarily sang their airs a tone higher than the Dorians and a tone lower than the Lydians, is, due to the many verifications by authority, a clear, trite thing. Nonetheless, if anyone wishes to proceed to distribute the species of the diapason in the manner which we last said, the example of it would demonstrate the opposite effect, because, in this order or proceeding, it would not be more than a semitone from the Dorian mode to the Phrygian. Nevertheless, the truth is-- as you have learned--that the Phrygians sang a tone, not a semitone, higher than the Dorians. Although Ptolemy wished the modes to be as they actually had been sung by the ancient musicians, with regard to

⁶⁴See Bekker, Suidae Lexicon, pp. 935-936, with reference to Sappho and Phaon. Her vain devotion to this handsome youth is well known to readers of the ancient classics.

their distance from one another, their order and their locations between the particular pitches, and the proper species [71] of the diapason, neither he nor those who ordered them before him were able to arrange them in a manner superior to the one which has been shown.

Strozzi: I have understood very well. Tell me, however, who was the author of this difference of high and low modes? Why, also, were [these modes] assigned more to one nation than another, or was this customary?

Bardi: Nature was the author of the three original [modes] which are the Dorian, the Phrygian, and the Lydian, because, when one nation or the other uttered words naturally while singing and speaking, or [doing something in between the two], there existed between their sounds (with regard to highness and lowness) the difference which you have learned. It is not the exact difference that was ascribed to it, and afterwards instituted in it by art. This thing is seen and heard by whoever considers well

[what] happens every day in many other provinces, and particularly in those of Italy, because the Lombards generally speak and sing in a deeper tone than the Tuscans do, and the people of Liguria speak in a higher voice than these.⁶⁵

[This is sufficiently clear] without resorting to the Sicilians or more remote nations, and without leaving the confines of our province any more than absolutely necessary. If this, then, results from food, water, air, or climate, we will leave such disputes to the inhabitants [of these places]. It suffices that the same [thing] which occurs today in Italy occurred daily in Asia among the peoples of Lydia, of Phrygia, and of Doris. The same [thing] happened in modulating diatonically, although each interval was afterwards regulated by Pythagoras in the same way Jubal Cain [regulated them]

Tuscans speak with deeper voices than Ligurians and with higher voices than Lombards.

Nature is the inventress of singing diatonically.

⁶⁵ See Vitruvius, The Ten Books on Architecture, translated by Morris Hicky Morgan (Cambridge, 1926), pp. 171-173, with reference to the supposed effect of climate on the voice.

prior to the great flood, to whom the invention of such a practice [i.e. modulation] was afterwards attributed by the unenlightened.

Strozzi: What authority is it that persuades us that the Phrygians sang a tone, rather than another interval, below the Lydians, and that they sang the same interval [i.e. tone] above the Dorian?

Bardi: [We learn this] particularly [from] the description of these modes which Aristoxenus makes, in which he places the Ionian between the Dorian and the Phrygian and the Aeolian between the Phrygian and the Lydian. [We also learn it] from what Ptolemy and Boethius say, in addition to many other authorities who wrote before and after them.

Strozzi: With regard to the beginning and end of the compositions of the ancient musicians, did they have a determined and particular pitch?

Bardi: No, Signor. [This was] because the difference which existed between

one and another of their tones (or modes) consisted principally in the disposition of the strings of varying tension and in the diversity (so to speak) of long and short sounds placed in different order in each system, and not like that of our practicing contemporary contrapuntists who have placed all [their confidence] in the final pitch. Even so, some [of these contrapuntists], in order to appear wiser and more scholarly than the others, add there the arithmetical and harmonic divisions of the diapason. These divisions, with regard to differences of harmony, affection, or tone [mode], have less to do with these [contrapuntists] than you have to do with the kingdom of Peru. According to them, there are twelve different tones (or modes), although they continually compose, play, and sing in the same strange, unknown [mode] which they employ, without exactly noticing it, indiscriminately at weddings and funerals [alike].

Zarlino⁶⁶ in chapter 9 of the 4th part of his Institutioni. Harmonic and arithmetical divisions have no part in the modes of [our] contemporaries.

⁶⁶Zarlino, Institutioni, pp. 308-309.

Strozzi: Where, do we believe, did the application of this particular consideration of arithmetical and harmonic divisions to the tones [modes] of our practicing contemporary contrapuntists have its origin? What do you believe was the factor which led them to believe that such a difference had power to vary one thing from another with regard to the diversity of harmony [melody]?

Bardi: These are indeed two of those things which I have pondered many times, and truthfully, I have never found a way to resolve them entirely.

Strozzi: With all this, it will be gratifying to understand your opinion in order to see if the cause of my doubts concerning this first point is in any way consistent with the way you feel about it, because I consider that they derived such foolishness from that which Boethius⁶⁷ says concerning the initials of tones [modes] and the disposition of the notes of every

⁶⁷Galilei's reference to Boethius IV, 14, is incorrect. It should read Boethius IV, 15. See Bower, Boethius, pp. 275-276.

mode and sound. He says this at the beginning of the fourteenth [sic.] chapter of the fourth book of his [De musica].

"Therefore, those which are called modes result from the species of consonances."

Or perhaps this thing had its origin in the way in which Ptolemy used the diapente and the diatessaron to form the modes which are found between the extremes and that of the middle.

Bardi: These considerations of yours do not displease me [at all], but, in order to make you capable of [understanding] all that which I believe about that matter, it is necessary, first of all, to commit to memory the order in which the eight ecclesiastical modes have been arranged, which have been instituted as follows [Example 10.] among their different species of the diapason.

Bardi: Do not doubt it at all, [for they conform] particularly with those [modes] of the Romans which have been described to us by Boethius. They have, on the other hand, been derived from him, as you will learn, although this is contrary to the opinion of some [musicians].

Zarlino⁶⁹ in chapter 8 of the 4th book of the Institutioni.

Strozzi: Tell me two other things. First of all, who was the author of these particular modes? Secondly, what do we believe induced their composers to call one mode the third rather than the first, or [say that] a certain melody belonged to the second mode rather than to the fourth? For what reason do they not pass beyond the eighth mode?

Bardi: The most ancient reference which I have found to ecclesiastical modes is in the Introduction of Guido d'Arezzo, who flourished during the pontificate of John XX around the year 1020. If I remember correctly, according to what Guido

⁶⁹Zarlino, Institutioni, p. 307-308.

himself shows in the Introduction, Odo⁷⁰ and some others dealt with [modes] a little before him. I have seen these [references] in some extremely ancient books which I have right beside me, and this is all I know to tell you about the origin of modes. Next, there is the question of why is the first [named first] rather than second, or the third [named third] rather than fourth, or [why is one] mode [named one way] and not another? Concerning this, you will learn everything which has been done according to how much my feeble intellect can discern by way of conjecture, although [this will] not [be] without consideration and judgement. There is, however, no book close at hand which speaks of [such things].

The question of authorship of the ecclesiastical modes.

⁷⁰Odo of Cluny (?-942) authored the treatise Enchiridion musices, also known as Dialogus de musica. It contains the first systematic use of letters for pitches in the meaning that was to become standard for the Middle Ages, that is, the entire gamut from A to g, with the addition of the low gamma and the high a'. See Strunk, Source Readings, pp. 103-116.

Strozzi: Who, then, was the author of considering and applying the harmonic and arithmetical divisions to modes in figured songs? [Who was the first] to mention and introduce twelve modes, when only eight had been in use at first?

Bardi: Let us not place under consideration so many controversies at one time, if you please, but let us proceed to decide topic by topic. In order to avoid confusion, let us not examine the second [thing] until the first has been determined. Gaffurio⁷¹ was the author of considering the harmonic and arithmetic divisions in the modes of either plain or measured songs. Glarean⁷² was the author of adding four modes to the eight which were first demonstrated. In that place [Dodecachordon], Glarean

Franchino [Gaffurio was the] author of applying the harmonic and arithmetic divisions to tones. [This is found] in his Practica in the 1st book, chapter 7. Glarean added four modes to the original eight. He re-proved Franchino, who is defended by the author.

⁷¹ See Franchinus Gafurius, The Practica Musicae, translated by Irwin Young (Madison, Milwaukee and London, 1969), pp. 39-45.

⁷² Strunk, Source Readings, pp. 219. Heinrich Glarean (1488-1563), one of the great sixteenth century humanists, is best known to the musical reader as the author of the Dodecachordon, who favored four

severely reproves and attacks Franchino, contrary to every sense of propriety, for two things of no little importance. I do not wish to suppress these matters in any way. Instead, I wish to remove such a stain from the name of Gaffurio with the fewest words possible. Initially, Glarean was surprised, since Gaffurio had had knowledge of the harmonic and arithmetic divisions of the diapason and had considered them, besides, in the eight original modes, that the same consideration had not occurred to him as that which had afterwards dawned upon Glarean himself, with regard to the other four [modes] and bringing [the total number of modes] up to twelve. He then reproved him, largely from ignorance. He admonished, secondly, that [Gaffurio] had not understood the

Glarean⁷³ in the 2nd book at chapter 15 did not understand the matter of modes according to Aristoxenus.

additions to the existing eight ecclesiastical modes. The book exerted much influence on the changing concept of the modal system.

⁷³See Glarean, Dodecachordon, pp. 139-140.

order of modes of Aristoxenus. Both of these calumnies are as unjust as anything one could possibly imagine. The fact that Gaffurio, initially, understood excellently the order of modes of Aristoxenus is known openly from the demonstration which he made of them and by the words with which he describes it. They are, after all, the same as those which Bryennius and Aristides Quintilianus used, according to what I said above in describing the modes conforming to the opinion of Aristoxenus, having drawn them from their writings. Glarean, on the contrary (and others more recent than he) did not understand it, or perhaps--as will be said later--they did not wish to understand it because of their particular interests. This [fact] is clearly manifested here. Glarean claims that the Lydian mode is, according to

Zarlino⁷⁴ in the 8th [chapter] of the 4th part of his Institutioni.

Another error of Glarean. Second book, chapter 9.⁷⁵

Glarean⁷⁶ in book 1, chapters 21 and 22 [and in] book 2, chapter 7.

⁷⁴See Zarlino, Istitutioni, pp. 307-308.

⁷⁵See Glarean, Dodecachordon, I, 125-127.

⁷⁶Ibid., I, 97-103; I, 114-115.

Aristoxenus, a semitone below the Mixolydian and thus, similarly, that the Hypophrygian is below the Hypolydian by the same interval. Aristozenus, on the contrary, says that there is a tone [in those places]. He also says that the Lydian is a third below the Aeolian, and Aristozenus puts the Aeolian above the Lydian by a semitone. Glarean claims, in addition, that the Dorian is a fourth below the Hypoionian, while Aristozenus says that there is a third [in that place]. Let us say this in conclusion, however, that Glarean distorts all the modes according to Aristozenus, from the Dorian and Phrygian and their plagals, onward. He does all this, moreover, because he wishes to harmonize those [modes] with the ones of today, which is just as possible as making an Ethiopian white. [Neither] Glarean nor any other more recent writer could ever understand how the

Zarlino⁷⁷ in chapter 8, of the 4th part of the Institutioni.

⁷⁷Zarlino, Istitutioni, pp. 307-308.

Lydian mode could be higher than the Phrygian by the space of a tone. It happens that the species of diapason of the Phrygian mode was E la mi (according to the text of Boethius, which these men had brought for authority, seeking--as I have said--to imitate it), and the species of the diapason of the Lydian was F fa ut. These were [both] distant by a semitone, and the same difficulty arose in their plagals, because the species of the diapason of the Hypophrygian was C fa ut. Ignoring this [fact], they proceeded to say (rather than confess that they did not understand the situation) that the texts of Boethius, of Franchino [Gaffurio], of Giorgio Valla,⁷⁸ and of others were incorrect with regard

⁷⁸Galilei is probably alluding to the treatise by Giorgio Valla entitled Musicae, which constitutes five books out of a larger collection of forty-nine called De expetendis et fugiendis rebus (Venice, 1501).

to this topic. I cannot possibly imagine how such a thought could [ever] occur to them. It happens that Aristoxenus, according to the opinion of the above-mentioned interpreters, placed the Aeolian between the Phrygian and the Lydian, and placed the Hypoaeolian between the Hypophrygian and the Hypolydian. Both of these were placed at the distance of a semitone, so that there was necessarily a tone between the Phrygian and the Lydian and a tone between their plagals, as has been said many times. Moreover, Ptolemy places between these [modes] the very same intervals used by Aristoxenus and Boethius, although he attributes to them various species of the diapason which are considered differently and with conditions other than the ones which Boethius imposed. Would it not be a fine thing, however, in this modern practice, to follow the singing of Ancorche col partire of Cypriano [da Rore] in four parts by immediately singing, a tone higher, Donna ch' ornata sete by [73]

the same composer, although the bass of the latter piece begins and ends in F fa ut and the former begins and ends in E la mi, which is the distance of a semitone? Would it not also be a fine thing, following the singing of the same Ancorche col partire, to sing, a semitone lower, the Giustizia immortale of the very same author, also in four parts, although it ends on D sol re, a tone away from E la mi? I am not [at all] certain [about this]. Coming however to the second topic, I say that as much as Franchino [Gaffurio] had considered the four authentics of the eight original modes divided harmonically and the four plagals divided arithmetically, and since such a consideration as that was indeed suited, with nothing added, to make him aware of the other four [modes] which Glarean later added, it is unfair (for the reasons which will be revealed soon) to charge him with ignorance (contrary to the way those men feel) for not having introduced them. It is only right to commend his discretion

and great knowledge, because he found that each one of the seven species of diapason had been occupied with the seven original modes. He considered, in addition, that the highest pitch of the seven, together with the lowest of the second, included all fifteen strings of the Greater [Perfect] System. [He perceived, also] that if others were added above or below, [the system] would exceed its natural limits, the same [pitches] would be repeated, and nothing new would be produced. Ptolemy also had this opinion, although Zarlino⁷⁹ in chapter seven of the fourth part of the Insti-
tutioni harmoniche says the reverse. In addition, he makes a case of this matter, [saying] that it was the cause of making [a] variation between the first and second modes, not on account of the different division of the various species, as Glarean contended, but because the second mode extends a

⁷⁹Zarlino, Istitutioni, pp. 306-307.

diatessaron lower than the first [mode], and thus, it comes to produce a more languid, submissive harmony, which is therefore different in nature from that of the first [mode].

Strozzi: May it not be a burden to you to tell me what moved [Glarean] to consider such a division in the modes, and why the harmonic division of the diapason and diapente pleased the ear more than the arithmetical division.

Bardi: As for the harmonic and arithmetical division of the diapason, I have never been able to find any reason which is worthy to convince me what causes this situation, although many reasons are offered. I not only do not know why the harmonic division is more pleasing than the arithmetical division, but I know even less why the octave is pleasing to us and the seventh is displeasing. I have considered carefully that the same which happens to the hearing with sounds likewise occurs

The reason which led Franchino [Gaffurio] to consider the harmonic and arithmetical division in modes.

The reason why the harmonic division pleases the hearing more than the arithmetical division.

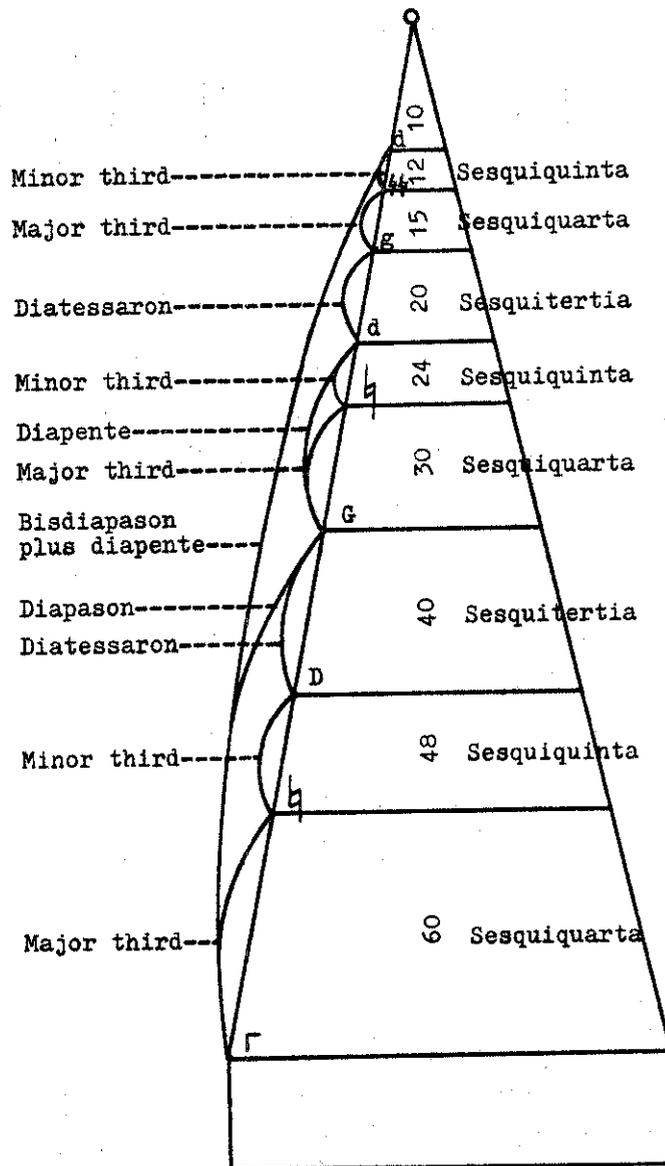
to the sight with visible objects. For example, in regarding a pyramid, the sight [of it] delights us more when the high part of it faces the sky and the obtuse part faces the earth than the opposite, perhaps on account of the proportion and agreement which that solid body has in that particular form and position with our [own] perspective in regarding it from the plane of its base. The same can result from musical intervals with regard to pleasing and displeasing the ear more or less, as you will be able to comprehend easily from the following diagram [Diagram X.]. [You should know that] one interval of a third appears more full of spirit to the sense than the other. Thus, equally, the fifth is more full of spirit above the octave than it is below. Let us also consider that the power of this affection does not consist merely of its location, that is, where the larger part of the interval is underneath and the smaller part is uppermost, as some

[74]

[DIAGRAM X]

[A MIXED DIVISION OF THE EIGHT MODES,
ACCORDING TO GAFFURIO]

Wisdom of
Franchino.



believe; it consists, instead, of the fact that what occupies that place is, at the same time, the nearest to perfection. This particular fact, without any doubt, plays a greater part there than does the other, as you will clearly recognize when you examine, with the usual diligence, the minor tenth divided by an intervening pitch into a low fifth and a high minor sixth, and when you also examine the reverse [i.e. a high fifth and a low minor sixth]. The same will be true in hearing the major sixth separated by an intervening pitch into a low fourth and a high major third, although this particular interval is abhorred today by some persons with delicate hearing, rather than their being pleased by it, and it is widely accepted by modern cithara players. Let me tell you also that those modes according to the modern practice which do not have a fifth below the final note

and a fourth above, for example the fifth and sixth modes, or to say, according to the usage of most contemporary musicians (although it is farther from the truth), the seventh and the octave, their ending is always incomplete and imperfect, and the ear does not remain entirely appeased as [it does] from the other available endings. Coming, however, to deal with the order of modes, I say this. It is clear that in order to distinguish one from the other, the diversity of names which is seen in those [modes] according to the usage of the Greeks is necessary, or else the numbers which are used by the ecclesiastics. These [ecclesiastics] wanted to call a given progression of notes first, rather than third or some other mode. Perhaps this was the reason. They said that the first mode was [exemplified] by that composition which proceeded to modulate among the notes of the species of diapason which served the Dorian mode. They were influenced by nothing else except the fact that that

Whether the manner of numbering the modes is done at random or in order.

particular mode, according to the ancient musicians, was the principal one and the most honorable. This was the only reason that they called such a species of diapason by the name of first.

Strozzi: What, then, motivated Franchino [Gaffurio] afterwards to consider that particular modulation of the first mode [to be] divided harmonically, rather than arithmetically?

Why the first mode is divided more [often] harmonically than arithmetically, and likewise the other authentics.

Bardi: [He was motivated] by this. The mese of the Greater Perfect System separates the species of the diapason of the [same] Dorian mode harmonically, and that was divided arithmetically according to the order of Ptolemy. In this procedure, however, the moderns--as I have said--went imitating Boethius. I say, but not for this [reason], that neither Franchino [Gaffurio], nor Boethius, nor Ptolemy, nor any other more ancient [authority] ever thought or attempted to adapt such a foolish thing to modes, because the unique quality [of these tones]

consisted otherwise, as they have shown, and as I demonstrated even more extensively. Franchino, moreover, was moved to the consideration which you have learned for the alleged reason. Afterwards, since he had been obliged to consider the quality of harmony and melody in such a way, he did not seek to divide it in another manner. From this division of the diapason the modern practical musicians took occasion to call the larger side of the diapente by the name of first species, and they said that smaller side was the first [species] of the diatessaron. In this respect they came to imitate the Greeks rather than the Romans. Whereupon it is openly seen that the modern musicians first sang through h durum, rather than through b molle. Since that species of the diapason was so divided between D sol re and d la sol re, and since the first species of the diatessaron was called re sol--as has been said--when b fa had

H durum was
used before
b molle.

been in the scale of that diapason instead of h mi, the diatessaron in that instance was called mi la. Otherwise, instead of the second species, there would have been the tritone. Their modes, moreover, had been composed and sung originally through h durum rather than through b molle. One finds, according to the usage of our practicing contemporaries, that the first species of diapason is between D sol re and d la sol re, that of the diapente is between D sol re and a la mi re, and that of the diatessaron is between a la mi re and d la sol re. The others proceed moving upward by conjunct steps, that is, through h quadratum. One can argue from this how much those have been deceived who finally have changed, without any reason, the order of modes and the species of the consonances, scheming more closely to approximate the order of modes of the ancients. I do not believe that he [Zarlino] has achieved this [goal] or if

Zarlino⁸⁰ in chapters 10 and 11 of the 4th part of his Institutioni.

⁸⁰Zarlino, Istitutioni, pp. 309-313.

he could [ever] accomplish other than the contrary effect, as much in the location as in the distance, and equally in the quality of the harmony.

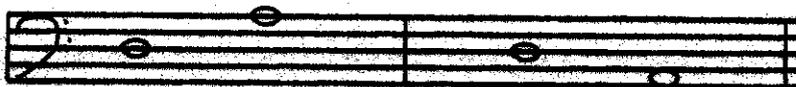
Strozzi: I did not understand before, and I do not understand even now, why it is that no division other than that particular one agreed with the quality of the harmony and the melody of the Dorian mode?

Bardi: This was because the ecclesiastics, having constituted their first mode within the very same species of diapason which served the Dorian mode, and since that [mode]--as you will learn on another occasion--was stable and quiet by nature, without violence, and suited to induce in the minds of the listeners grave and serious thoughts, and customs for strong men, I say that [the Dorian mode] did not require being divided in any other way, since the nature of its melody was what it was between those pitches. It came more greatly to manifest its particular quality when it was applied to words which are suitable to

What agreement the ecclesiastical modes have with those of the Greeks.

it, while, on the contrary, the arithmetical division had made it conform in many respects to that of its plagal. This did not agree, since it was languid, feeble, and timorous, as one can clearly hear by plucking together three strings which have been disposed in such a way.

[75]



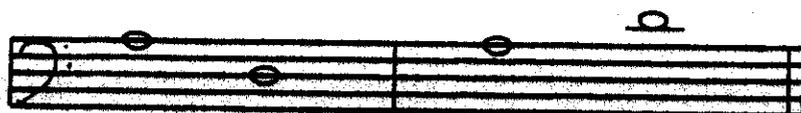
[Figure 111.]

These strings not only have such a property when all three are plucked at the same time, but [it is also inherent in] the simple motion of each one of the intervals separately when it moves away from the mese in order to reach any of the extremes. Thus, similarly, adding to [the Dorian mode] another part in the high register, which proceeds with its proper and natural movement and is consonant with the low register, they are manifested to the sense in the same way. In order [to prove] that that is true,

Arithmetical
division in
the second
mode.

first consider--as has been said--what nature the harmony of the interval has which is found moving away from the mese of the Dorian in order to reach any of the extremes of its diapason, as, for example, descending from a la mi re to D sol re, or ascending from this a la mi re to d la sol re in the same manner.

Harmonic division
of the first mode.



[Figure 112.]

When the motion of that part, being the low one and that which provides the melody to the composition, is accompanied by another in the high register which proceeds with movement suitable to its location, which these two cases [Figure 113.] will demonstrate, you will hear what the harmony will be when each one of these has been sung together with the other, or at the same time.



[Figure 113.]

Strozzi: The melody of each part in itself and the melody which issues from both, or from all three together, when they have been arranged in this particular way, is indeed exactly as you have described. Let us consider more carefully each of these cases in the arithmetical division, if you please.

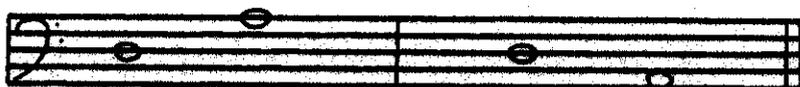
Bardi: We will do it at the proper place. Let us pursue [this present matter] further with regard to the consideration of modes. The ancient musicians, moreover, divided their modes into plagals and authentics, leaving between each pair of these the space of a diatessaron, as you have seen in each one of the demonstrations according to the usage of these ancients. Now thus, also, the ecclesiastics intended for there to be the distance of a diatessaron from the first mode called authentic to the second mode called plagal,

Considerations
of the author
about modes.

and so this came necessarily to occupy the place of the diapason of the Hypodorian mode, that is, always according to the opinion of Boethius, and not that of any other ancient writer. The cause, then, for their diverse divisions stemmed from nowhere else but from wanting the harmony which resulted from these [to be] more consonant than was possible for the nature of modes of the ancients from which they had been derived. Also, as some say, [it was] because the final pitch of the first called authentic and [that] of the second called plagal was common to both. This rule, since it was impertinent, was not entirely approved by the authors of ecclesiastical songs. In this way, the first and second modes were included within the pitches which incorporated the same sides as the first species of the diapason, but with the following difference from these [sides]: since the authentic has its larger segment in the bottom part and its smaller segment in the top part, the plagal, on the other hand, transposing the

species of the diatessaron of its authentic by an octave into the part opposite to the high part, adds [this species] to the low part of the same diapente without moving it out of its place. Thus the plagal comes to reach into this part a fourth more than its authentic, and this [authentic], on the contrary, is extended into the high part more than that [plagal] by a similar interval. This procedure causes the species of the diapason to be changed because what was first [species] in the authentic becomes fifth [species] in the plagal. This has been considered diversely by our practicing contemporaries. Therefore, in that original simplicity of canti firmi where all these conditions had been observed, there resulted between these canti firmi a variety of harmony, and, consequently [a variety] of affection. That resulted mainly because of the small number of pitches which these [canti firmi] contained, because of their diversity, and because of their movements. In order to place such a truth within your grasp,

consider each smallest part of this particular division. [Notice] how different in nature it is from that first [division], not only in sound and movement of one single part (which is what agrees with the low segment when one wishes to move away from D sol re, that is, its mese, [in order] to reach anyone of the extremes of its diapason), but also when it is accompanied by another part placed in the high segment, which proceeds with movement which conforms to the low. Here is an example.



[Figure 114.]

Here is another way you really should consider [Figure 115.]. In addition, in hearing simultaneously the extreme pitches of the simple diapason together with its mese, or even the three parts together, you will find that these--as I have said--have a sad, flat quality. This temperament is not far from that which is attributed to the Hypodorian mode, nor is it at all



[Figure 115.]

surprising that the diversity of sound, with regard to highness and lowness, together with the difference of movement and of interval produces variety of harmony and affection. It happens that nature ordinarily does not produce similar things with contrary ones, nor contrary things by means of the very same quality, but only the opposite. With regard to the consideration of these matters, when the convenience of rhythm and the conformity of conceptions had been added, what force and effect do we believe that that particular melody had afterwards? It is quite certain that it would be suited, as it already was, to incline the spirits of the hearers in whatever direction which the skilled musician pleased. Because some of these things, however, have not been understood, nor considered, nor observed

[76]

by our practicing contemporaries in their compositions, it happens therefore, that the harmonic and arithmetical divisions do not have--as has been said--any part in them.

Strozzi: This is actually so. One can argue from it how much our practicing contemporary contrapuntists have misunderstood their own very famous precept when they have said that the parts of the composition ought to proceed by contrary motion, since they clearly have come to the opposite [viewpoint], that is, the same affection can be expressed with greater efficacy by means of similar [motion] than [it can] with diverse [motion]. Similarly, sadness, together with the other passions

Abuse of the contemporary contrapuntists.

Zarlino⁸¹ in chapter 35 of the 3rd part of his Institutioni.

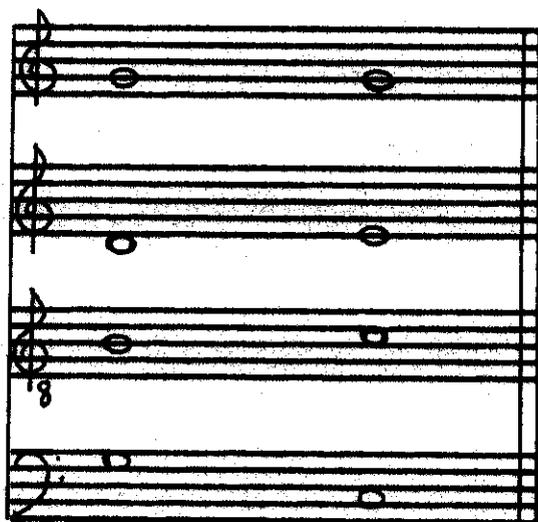
⁸¹Zarlino, The Art of Counterpoint, p. 74. "Harmony is made of opposites or contraries. This applies also to the simultaneous movement of several parts. Whenever possible--and this conforms to ancient practice--when the part on which the counterpoint is written, that is the subject, ascends, the counterpoint should descend, and vice versa. It is not faulty, however, to let them move in the same direction on occasion for the sake of smoother voice movement."

can be caused in the hearer not only with high and low sound and with fast and slow motion, but with the different qualities of the intervals. [This is true] even with the same interval when it is brought toward the low register or toward the high, because the fifth is sad when it is ascending--as you have said--and when it is descending, it is joyful. On the contrary, the fourth is joyful in ascending and sad in descending. One observes [that] the same [thing] happens to the semitone and to the other intervals.

Bardi: I want to tell you even further in this regard, that the diapente [when it is] carried by the voice away from the high part toward the low, or else, on the contrary, between pitches different from the first shown, has a dissimilar nature from the first one, which has already been mentioned. The same is equally true of the diatessaron and every other interval in the disjunct system, for when moving away from G sol re ut with the low

Consideration of the author concerning the nature of the intervals.

part, and descending by a fifth or ascending by a fourth, you will find that such movement has [the properties] of joy, agitation, and (so to speak) virility and naturalness. The same is true of the other parts which are higher than that [low one] which proceed with movement which conforms to it. Here is an example.



[Figure 116.]

I believe, for the stated reason and no other, since the third and the tenth are found to be naturally major between the parts and minor in the [other way of proceeding], and since they must be made this way by means of accidentals, that the

implication in this way of proceeding is that they will turn out in the manner you have understood. This is because Nature produces her effects more vigorously than Art. When the parts which are above the low [part] are made higher, the harmony [concento] should become even more as we have described it, that is, always sound and discreetly [executed]. [To prove] that the low part is actually that which gives the air (when singing in harmony) to the composition, observe, in this last example [Figure 116.], that the conjunct movement of the notes which the contralto part makes when ascending possesses a high degree of virility. On the contrary, when the very same interval is sung in the same manner and between the same notes in the above-mentioned [part], it should have a sad, flat quality as the first one did, provided that one or more parts were added about it which performed the very same function which they did in this last example. The part in question should have

the same [sad, flat quality] when it is transposed an octave higher.

Strozzi: What you say is undoubtedly true, but I am surprised that these things have not been considered before now and put into use by our practicing contemporary contrapuntists. Before you continue with your explanation of the way the other modes proceed, I would like for you to tell me another particular besides the two already mentioned.

Bardi: Tell me what it is.

Strozzi: Which has more part in giving spirit to compositions, the slowness and fastness, or the highness and lowness of the sound? Which one of these two methods is more effective to manifest such compositions, exactly as they are, to the hearing?

Bardi: Each one of these is suitable for its own appropriate function, which is not unlike what lines and colors do in revealing to the sight [both] the beauty and the ugliness of the body. Since in doing that, the lines always have a greater

part than the colors, thus, similarly, in the former [situation], the slowness and fastness have lower or higher sound [respectively]. These lines, without the colors, can indicate to the sight the proportion and disproportion of the body, since the fast and the slow movement of the sound in a single extension [of pitch] can communicate to the ear the air of any composition. There is, however, such a close relationship between these two cases, that one without the other cannot entirely manifest the quality of the air, as the lines without the colors can between those other two. I come, returning to the invention and usage of the ecclesiastical modes, to tell you that the author of these [modes] afterwards constituted the third mode in the same species of diapason which serves the Phrygian harmony. This diapason was considered [to be] divided in the same manner as that which serves the first mode, since it was also one of the authentics and was conformed, for the reasons stated above, to such a division,

Other considerations of the author concerning modes.

and also [was conformed] to the quality of its harmony. The fourth mode, then, since it adjoined the third, they rightfully constituted in that species of diapason which already served the Hypophrygian, assigning to the fifth mode the diapason of the Lydian. For the same reason mentioned with regard to the others, they assigned to the sixth mode the species of the octave which served the Hypolydian. They wanted, in addition, to constitute the seventh [mode] between the pitches which contain the species of the diapason of the Mixolydian, and the eighth and last they assigned to the same species that they assigned to the first [mode]. Because of this similarity, it was called, in imitation of the other plagals, but the name of Hypomixolydian (and not Hypermixolydian as some have dreamed) since Hypo meant "below" and Hyper "above", as you know.

[77]

Franchino⁸² in the first of his Practica in the 7th and in the 11th chapters.

⁸²See Gafurius, Practica musicae, pp. 39-45; 52-54.

Glarean indeed used these words on one occasion. It was called the Hypermixolydian since it was above the Mixolydian, and it is our eighth [mode]. One finds in this description of the eight modes (applied, however, to the modern practice without further consideration) the same discrepancies with regard to the distances of highness and lowness which I mentioned above. [This is] presumably because their authors had not valued any more than they did the other places, the fact that the interval which was found between the Lydian mode and the Mixolydian [mode] of the ancients was the distance of a semitone, and not a tone as one finds between the fifth and seventh ecclesiastical modes. They did not value it because these [ecclesiastical modes] did not have the power to tune together the species of the diapason and the intervals which are found between one and another of the ancient modes. Thus, they have come to omit, although very important, that which they understood less and needed to understand more.

Strozzi: What difference is there, then, between the eighth ecclesiastical mode and the first one, since they occupy the same species of the diapason and the very same pitches?

Bardi: Herein lies the entire importance of the affair. [It is] none other, of course, than the fact that the final pitch and the diversity of the division are considered harmonically in the first mode and arithmetically in the eighth. Now you see, in the modern compositions which are composed and sung in the manner which is customary today, that which has to do with the diversity of the melody and the harmony between one mode and another, the final pitch, and the variety of the division of the species of the diapason.

Strozzi: Since these modes of theirs have been arranged in such a way, it indeed reminds me of the pictures of the singular Hermippus of Athens. This man, in depicting males and females, either because of the antipathy which he

The modes cannot be more than seven [in number].

Hermippus, the Athenian painter.

had naturally for beards and for clothes, or through another of his particular interests, he continually made so many final touches that it was impossible to tell the males from the females, if not for the sex, as if industrious Nature had not formed differences in a thousand other, perceptible ways. The same thing is true of the modes of our contemporary contrapuntists. It is actually impossible in hearing them sung, to tell from the highness and lowness which is the first and which is the second mode, which is the third and which is the fourth, and it is likewise impossible to tell all of these from the other modes. It is only possible to identify each mode from the final pitch, although, in seeing them written, the lower modes are quite often shown higher and the higher modes lower.

Bardi: I want to tell you another thing which I recall in this case, [which is] that, while the ancient musicians, in varying their modes, changed not only the species of the diapason, but the entire

Another abuse of our practicing contemporaries.

system from low to high or, on the contrary, from high to low, those of our times say to vary them by singing the same species of the diapason in the very same intensity and slackness of the system, and not only the neighboring [pitches] but the extreme ones, for example, the unison and the octave. I say, also, that if each of their modes has particular power to move in the hearer a number of different affections as they affirm (although contrary to every precedent), how does it happen that, when they have to put together a sonnet, which has been drawn--in a manner of speaking--from sad things, they will sing its fourth stanza in the second mode and its third stanza in the ninth, or some other way? In addition, what man is so devoid of judgement that he does not notice that, in any of their songs composed in not more than four voices, if the bass sings, for example, within the notes of the diapason which serves the first mode, the tenor will

sing within those of the second, the contralto will sing the Hypermixolydian at the same time the bass the Hyperdorian by means of singing an octave above it, and the soprano will be the same as the tenor? [What men could fail to see] that such confusion and contrary mixing of notes cannot induce any affection in the one who hears it? Each part has by itself what it does not have in any way in this modern practice of counterpoint. It has the specific ability to induce a number of affections in the listener which cannot be induced [in part songs] due to the confusion of one part with another, because as contraries, they impede the natural operations. We come, however, to our matter concerning the addition of the four last modes, and let us leave the foregoing matters as merely statements of opinion. The above-mentioned Glarean, a truly scientific man of great literary prowess, considered that no other factor caused the eighth ecclesiastical mode to be different from

Consideration of Glarean. The addition of four modes to the original eight.

the first [mode] except the final string and the visualization of the harmonic and arithmetic divisions then applied to this new way of composing and singing so many melodies together. He said this, stimulated by the thirst he had to invent such a useless novelty. It was something which he considered most important and necessary, which is openly manifested by the title of his books on music; he calls his work Dodecachordon. One reads these incredible marvels in the [very] first page of this where he eventually claims to have imitated Aristoxenus. We have already demonstrated the conformity which they have together. I say, therefore, that the following is what this Glarean went around pondering:

Since the species of the diapason which serves the first mode has power to form the eighth [mode] with the sole consideration that [the first mode] is divided harmonically and the [eighth mode] is divided arithmetically, why should the same divisions not make a variation of harmony in the other species which contain the other modes?

On the strength of this aspect of reason, [78] although it leaned toward weakness, he added to the eight modes shown, four others, which made the number of twelve. This was the order he maintained in order to accomplish that. He formed the ninth mode from the same species of the diapason which served the second mode, but transposed an octave higher and considered differently divided from its mese. He formed the tenth [mode] from that of the third assigning it and its authentic a la mi re as the final pitch. Since the eleventh mode did not correspond with the fourth [mode] of the division, he formed it from that of the sixth because he considered that capable, causing it to end, together with its plagal, on C sol fa ut. This plagal, he constituted in the species of the diapason which served the seventh mode, observing in both of the foregoing the very same conditions which he made in deriving the ninth mode from the second, omitting--as has been said--the harmonic division of the diapason which serves the

fourth mode, since it is not capable of it, just as that of the fifth [mode] is not capable of the arithmetical division, as one can easily comprehend in this example.

Dorian	Hypo-dorian	Phrygian	Hypo-phrygian	Lydian	Hypo-lydian
1	2	3	4	5	6



Mixo-lydian	Hypomixo-lydian	Aeolian	Hypo-aeolian	Ionian	Hypo-ionian
7	8	9	10	11	12



Description of the twelve modes extracted from the 1st page of the Dodecachordon⁸³ of Glarean and from the 7th and 28th [chapters] of the 2nd book.

[Example 11—Description of the twelve modes.]⁸⁴

⁸³ See Glarean, Dodecachordon, I, 37; I, 114-115; I, 173-175.

usage and to avoid the occasion of being numbered among the deprecators of good, true precepts. They found that each of the fifteen pitches of the Greater Perfect System was already occupied, and having put into effect with their first seven modes each species of the diapason and quite often the other consonances which are called today perfect as well, they added the eighth into the [total] number of these [modes] rather in imitation of Boethius than because they truly knew that it was apt to introduce any new, necessary affection. These things were very well known by Franchino, who had preferred to refute them because of that, rather than form them without any chance of innovation. Since this [eighth mode] had already been accepted by the general public, however, he admitted it, thinking it was just as well. In addition that sort of solo song, since it was sung according to the sound of the pitches among which it was written (as was customary in the beginning when they

The ecclesiastics constructed eight modes in imitation of Boethius.

were introduced) without any alteration, it was not unsuitable as those of today with many voices are, because there is actually a difference of harmony and melody among the pitches of their songs, and a great deal more so when one applies suitable words to them. However, in singing so many melodies together according to this new practice of figured song [canto figurato i.e. polyphonic composition] (so called because of the variety of the singable figures), two modes are too much, let alone eight or a greater number. The reason for this is that any composition put into use always seeks out the same quantity and quality of pitches (speaking of high and low), proceeding in each given part with the very same rhythm with regard to fast and slow movement, because the contrapuntist uses in these [compositions] notes of any value, and rashly employs each interval that suits his pleasure; he never thinks a thing in the world of the conception of

Why measured compositions are so called.

the words. It will be proved, at the proper place, that the diversity and effect of the harmonies and the melodies consist mainly of these cases, so that the modes and the compositions of today necessarily come to be the same in quality, quantity, and form. Thus one comes to have the very same color, taste, and smell (so to speak) as the other. In this case, it appears to me that the example of Philoxenus, a most noble musician, is an effective argument to persuade us how distorted the modes of the modern [musicians] are from those of the ancients. Let me confirm this fact more extensively by telling you this. The said Philoxenus, having once upon a time begun [to play] the dithyramb

There is only one modern mode.

Aristotle⁸⁵ in the 8th [book] of the Politics.

⁸⁵Aristotle, Politics (1342b7-12), Richard McKeon, ed., Basic Works of Aristotle (New York, 1941), p. 1316. "The dithyramb, for example, is acknowledged to be Phrygian, a fact of which the connoisseurs of music offer many proofs, saying, among other things, that Philoxenus, having attempted to compose his 'Mysians' as a dithyramb in the Dorian mode, found it impossible, and fell back by the very nature of things into the more appropriate Phrygian."

in the Dorian mode, was forced by the material of the subject to end it in the adjacent Phrygian mode.

Strozzi: Would you kindly tell me what caused this?

Bardi: In our times, one can hardly satisfy (as far as I know) in a custom of which there is no conception, since no memory has remained of these things. If you want to content yourself, however, by knowing how greatly I value and believe it, I am content to tell you. The dithyramb, according to the ancient Greek poets, was a form of song in praise of Bacchus whose author was Arion.⁸⁶ In the melody of this dithyramb, one imitated the conceptions and customs of excited, tipsy, joyful worshippers of Bacchus, and others like them. It was sung by a chorus, which represented the deeds performed, which all

The meaning of dithyramb according to the ancient Greeks and the identity of its inventor.

⁸⁶ Arion is equally renowned for his calming of the billows and charming of dolphins in order to save his life. See pp. 514-516 below.

[the people] were accustomed to execute (insofar as I believe) in an intense mode. [It happens that] artists sometimes want to have an experience with art, and thus undertake to test it by forcing the limits of their principles. Philoxenus, being one of those [artists], wanted to experiment with something new. Since it did not prove capable of his conception, defeated by its nature, it returned to its normal state. Thus, when he had commenced the dithyramb in the Dorian mode, the harmony of which is quiet--as has been said--and without any violence of affection, and since he abhorred the indecent softness of such material and the allurements of that verse, he was quite willing (skilled as he was) to proceed without the aid of [that particular] harmony in his imitation. [Whereupon] he abandoned it without respect and went on almost violently to the Phrygian mode which was higher than the Dorian, and thus intense in nature and suitable to express the conceptions which

[79]

The reason why Philoxenus, having begun the dithyramb in the Dorian mode, was forced to end it in the Phrygian.

he had at hand. It is not at all surprising that such a conception came to Philoxenus, because he had the nature and constitution to love low, quiet harmonies, even the very lowest ones. This argues that after all the others [had been found], he discovered the Hypodorian, slowest and weakest of them all. This is all I know to tell you in this matter. One can also comprehend from the example of this Philoxenus, how greatly the composing and singing of today is without rule, without method, without order, at random, and according to mere practice. The practical composers do not know anything except that that is a consonant interval and this is a dissonant one, and the modern contrapuntists (notwithstanding this Philoxenus) sing any conception in any mode, having abandoned entirely every observation and law and having been made the prey of the honest will and power of their stratagems, without having considered further any good limitation or rule.

Another abuse
of our practicing
contemporary con-
trapuntists.

Strozzi: Did the ancient musicians, therefore, not use the mixture of one mode with another?

Bardi: It is not certain until [the time of] Sacadas the Argive,⁸⁷ a musician and poet who was extremely famous for three victories which he had in that type of festival called Carneian, which the Spartans made in honor of Apollo's birthday, since he had been the author of proceeding, in the same song, from the first mode into the second, from the second into the third, or otherwise.

Strozzi: Is this that Sacadas who was also inventor of the elegy? Was he also the one in Sparta who discovered that species of song and dance called gymno-paedias,⁸⁹ whose work is particularly remembered by Pindar, among other writers?

Sacadas the Argive, inventor of the mixture of modes.

The meaning of Carneian festivals.

Plutarch⁸⁸ in the 3rd of the Apophthegmata among the orders and customs of the Lacedaemonians.

⁸⁷Plutarch, On Music, p. 371.
"Sacadas of Argos was also a composer of music and of elegiac verse set to music; he was, furthermore, an excellent aulete and is recorded to have won three victories at the Pythian games."

⁸⁸Plutarch, Moralia, 15 vols. (London and New York, 1931), XIV, 373.

⁸⁹Ibid.

Bardi: This is the one.

Strozzi: May it not be a burden to you to tell me what mixture of modes he used, and, afterwards, what were gymno-paedias.

Bardi: I told you all of that after having proved to you that the Hypodorian mode was not only discovered by Philoxenus but [was found] last of all. Since this is an older obligation, I should logically satisfy it first. However, take note! You ought to know that this mode, according to the ancients was called, like all the other [modes], by diverse names, as you can understand in part from that introduction which has been printed both in Greek and Latin under the name of Euclid,⁹⁰ [which says this:] "Among others, some call it Locrian and others call it Hypodorian."

The Hypodorian mode [was] the last to be discovered.

⁹⁰ See Strunk, Source Readings, p. 42. This treatise, once attributed to Euclid, has been found to be the work of Cleonides.

Now Julius Pollux,⁹¹ praising the harmonies, although he does not name the Hypodorian, he says that the Locrian was discovered by Philoxenus. Therefore, we already know, since these two are the very same as the writers tell us, that Philoxenus was its discoverer. The fact that it was the last to come into use, besides the fact that it is, in itself, a very likely thing since it is the eighth mode and last in number, is derived clearly from Bryennius⁹² in the fourth chapter of his third book on music. Having discussed the other higher modes, telling why the Hypodorian was called a low mode by the ancients, since the Hypodorian was lower than it was, he shows why this had resulted. He says that before the Hypodorian came into use, the Hypophrygian was by nature the lowest of the others and therefore

[Julius Pollux]
in the 4th book
of his Onomasticon.
Locrian
and Hypodorian
are the same
mode.

⁹¹See Bethe, I, 220.

⁹²See Wallis, Opera mathematica, III, 481.

it had commonly been called [a low mode]. This name had remained attached to it even though the Hypodorian, which was lower than it in nature, had appeared. Through this discourse, it is clearly ascertained that the Hypodorian was the last of all the eight modes which he describes to be discovered. The gymno-paedias, according to the Spartans, were choruses of young boys who danced together barefoot singing the praises of the gods, and [were] in honor of those Spartans who had fallen dead fighting for their native land during the Therian campaign. These, then, according to what Plutarch tells us, are the mixtures of modes which Sacadas used. They had in his times knowledge of only the three principal species of harmonies, which were, as you know, the Dorian, the Phrygian, and the Lydian, although Athenaeus⁹³ claims, with the testimony of Heracleides of Pontus, that the

In chapter 10
of the 4th book.
Heracleides of
Pontus.

⁹³See Athenaeus, Deipnosophists, VI, 365-367.

Phrygian and the Lydian harmonies, perhaps because they were foreign, were called Aeolian and Ionian, in order to honor the Greeks greatly. This matter does not pass without some trouble from those who accurately examine the names and the order of modes according to Socrates, Plato, and Aristoxenus. Let it be as you wish, however. Since the cards are not all dealt [so to speak] concerning these particular controversies, mainly with regard to modes, let us come to tell how Sacadas used the mixture of these [modes]. He used it mainly in the chorus in this manner. First of all, he set forth (when it suited the quality of the poem, however) the Dorian mutation; after this he set forth the Phrygian, and next the Lydian.

Plutarch's De Musica [deals with] mixtures of tones according to Sacadas.⁹⁴

⁹⁴Plutarch, On Music, p. 371.
 "Thus, there being three systems of tuning in the time of Polymnestus and Sacadas, the Dorian, the Phrygian, and the Lydian, they say that Sacadas composed a strophe in each, and taught the chorus to sing the first in the Dorian, the second in the Phrygian, and the third in the Lydian; and that this nome was called Trimeles because of the modulation."

This rule or law was called tripartite. He adapted this variety of harmony to all the songs with such industry that his work and ability were held in greatest esteem. He accommodated it to the subject of the words with greatest judgement, and expressed the affections, which had been observed in it by the poets, with marvellous art. The practical musicians of today have kept no account of this most important, most principal concern of the art of music, for it pleased God that their errors ended here. There is a much worse situation, however, because all the rules and observations which they have in use are in direct opposition to the adequate expression of the affections and conceptions of any poem. I want to remind you of some of these [rules] at the present [time]. These are the inviolable laws which they have made without alluding to [any] authority or cause. [First, they say] that it is not lawful in any way to have two perfect consonances

[80]

Other abuses
of the practical
musicians of today.

of the same species one after the other. Secondly, when one must proceed to find these [perfect consonances], in moving away from imperfect [consonances] one goes to the nearest [perfect consonance]. Added to this is the respect which one should have in relation to the tritone and the semidiapente. Now the observations of these two precepts alone are sufficient to preclude any affection from ever being expressed. [There are], in addition, other inconveniences which I am reserving to demonstrate to you at a more convenient place and time.

Strozzi: You will certainly require other industry and labor, which was not lacking recently in making me ascertain that the lyre and the cithara of the ancient Greeks and Latins were the same thing, to persuade this new, rather newest, opinion of yours to the general public.

Bardi: I offer to convince you (and any other rational person) of that opinion more clearly than any other thing

which I have mentioned up to now, or
could possibly mention in the future.
If you are willing, however, I want to
reserve it until the proper place.