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# R-loci and Distributivity: Insights from Czech Sign Language 


#### Abstract

This article describes a distributivity pattern in Czech Sign Language. The pattern is signed via a reduplication at the R-loci and resembles the distributivity behavior of the binominal each that is known in spoken languages. Nevertheless, there are important differences between the sign language reduplication and the spoken language distributivity that is seen in the binominal each; the most significant concerns the range of readings available for the sign language reduplication. We describe the data we gathered, and then formalize them in the Plural Compositional Discourse Representation Theory. The formal framework allows us to analyze the data and explain certain questions which arise from them.


## Keywords

Czech Sign Language, distributivity, reduplication, R-loci, PCDRT, individual and occasional interpretation


#### Abstract

Abstrakt Niniejszy artykuł omawia konstrukcję o interpretacji dystrybutywnej w czeskim języku migowym. Wyraża się ją za pomocą reduplikacji z wykorzystaniem punktów referencjalnych w przestrzeni migowej i przypomina ona dystrybutywne znaczenie dwurzeczownikowego użycia kwantyfikatora każdy, znane z jezzyków mówionych. Niemniej istnieją znaczące różnice pomiędzy reduplikacją w języku migowym a dystrybutywnością w języku mówionym obserwowaną w przypadku tego kwantyfikatora. Najistotniejsza z nich dotyczy zakresu interpretacyjnego reduplikacji w języku migowym. Autorzy omawiają zebrane dane językowe, a następnie dokonują ich opisu formalnego w ramach mnogościowej kompozycyjnej teorii reprezentacji dyskursu (Plural Compositional Discourse


Representation Theory, PCDRT). Stanowi to podstawę dla analizy materiału oraz odpowiedzi na wynikające z niego pytania.

## Słowa kluczowe

czeski język migowy, dystrybutywność, reduplikacja, lokalizacje referencyjne, PCDRT, interpretacja jednostkowa i okazjonalna

## 1. Introduction

In this paper, a specific type of sign repetition in the Czech Sign Language (český znakový jazyk - ČZJ) is analyzed as a structure parallel to the socalled binominal each in spoken language. ${ }^{1}$ Example (1), which is of great significance to our argument, shows that in ČZJ, both numerals and verbs are reduplicated and localized as a morphological marking for distributivity. While reduplicated numerals with various types of distributive readings are not uncommon among spoken and signed languages (Balusu 2006; Cable 2014; Kimmelman 2018; Kuhn 2019; Szabolcsi 2010), our analysis of the ČZJ example (1) demonstrates the connection between the distributive reduplication of a verb and the occasional distributive interpretation of the sentence.
(1) DAUGHTERMYTHEY-THREEDOGTWO $\mathrm{TWO}_{\mathrm{j}} \mathrm{TWO}_{\mathrm{k}} \mathrm{BATHE}_{\mathrm{i}} \mathrm{BATHE}_{\mathrm{j}}$ BATHE $_{\mathrm{k}}$ 'My three daughters bathed two dogs each/each time.'

1. individual distributivity interpretation
2. occasional distributivity interpretation
3. \# cumulative interpretation

We suggest that each instance of the numeral or the verbal repetition at the Reference loci (R-loci; Schlenker 2017) is a realization of the distributive operator. This operator is both anaphoric and selective. The framework of the Plural Compositional Discourse Representation Theory (PCDRT; Brasoveanu 2008; Dotlačil 2012, 2013; a.o.) is used to explain the distributive marking in ČZJ, including the multiple marking. Our analysis of ČZJ data supports the non-standard theories of distributivity, such as the PCDRT or Neo-Davidsonian event distributivity theories (Champollion 2016a, 2016b).

The paper is organized as follows: in Section 2.1 basic observations are considered regarding the types of distributivity markers in natural languages; Section 2.2 discusses the concept of the R-loci in sign languages; Section 2.3 briefly compares two strategies to determine distributive meaning in natural

[^0]languages; Section 3 explains the procedure used to collect the Czech Sign Language data and discusses their relevance within the current theories of distributivity; Section 4 shows how the PCDRT framework can be applied to the data and answers the questions posed by the data, with an initial discussion of the cumulative baseline interpretation of Czech Sign Language sentences, including the plurality denoting expressions; Section 4.2 does not include the distributivity markers, whereas in Sections 4.3 and 4.4 a distributive reading is addressed. Section 5 provides a summary of the research undertaken.

## 2. Morphosyntax of distance distributivity in ČZJ

### 2.1. Markers of distance distributivity

The English each in example (2) and the German jeweils in example (3) are instances of so-called distance distributivity markers (or items). They are distant because syntactically, they are detached from their restrictor (each is detached from girls and jeweils is detached from Mädchen) Additionally, they are interpreted as distributivity markers because, from a semantic perspective, the two dogs (distributed share) are distributed over the girls (sorting key) in such a manner that each girl saw two dogs.
(2) The girls saw two dogs each.
(3) Die Mädchen haben jeweils zwei Hunde gesehen. 'The girls saw two dogs each.'

Employing a distributivity marker in this way to express a relation between the sorting key and the distributive share is usually referred to as the binominal each in the literature, and this topic has received considerable attention from those researching spoken languages (Cable 2014; Balusu 2006; Dotlačil 2012; Champollion 2012; Szabolcsi 2010; Schwarzschild 1996; Landman 2000; Link 1983; Safir and Stowell 1988; among many others). With regards to sign language, a particular type of sign repetition and other related phenomena were reported and thought to contribute a similar (distributive) meaning to the sentence (Kimmelman 2017; Quer 2012; Kuhn 2017 for dependent indefinites).

In example (4) from the ČZJ, the numeral TWO and the verb BATHE are repeated. Each repetition assigns two dogs and a bathing (sub)event to one of the three daughters. ${ }^{2}$

[^1](4) DAUGHTERMYTHEY-THREEDOGTWO $\mathrm{TWO}_{\mathrm{j}} \mathrm{TWO}_{\mathrm{k}} \mathrm{BATHE}_{\mathrm{i}} \mathrm{BATHE}_{\mathrm{j}}$ BATHE $_{\mathrm{k}}$ possible translation: 'My three daughters bathed two dogs each.'

This data superficially resemble examples from certain spoken languages; see Telugu in (5) and Hungarian in (6).
(5) ii pilla-lu renDu renDu kootu-lu-ni cuus-ee-ru these children two two monkeys-Acc saw-3Pl possible translation: 'The children saw two monkeys each.'

Telugu (Balusu 2006, ex. 1)
(6) A gyerekek the children 'The children saw two monkeys each.'
láttak
saw-3Pl
Hungarian (Szabolcsi 2010, ex. 99)

However, there are several differences between the spoken language and the sign language data. The most obvious are the following: i) sign language sign repetition (not necessarily two repetitions) vs. spoken language reduplication (two repetitions); ii) no verbal reduplication in spoken language; iii) overt anaphora in sign language. While the first point is of a more morphophonological nature, the remaining two will be discussed in detail. The third point will be considered initially, before addressing the second point in Sections 4.3 and 4.4.

### 2.2. R-loci in sign language

Kimmelman (2017), while exploring quantification in the Russian sign language (RSL), notes that sign language distributive marking is a combination of sign repetition and spatial distributivity. In other words, the distributive meaning effect arises by placing the repeated signs in specific locations within the signing space. These locations are called Reference loci (R-loci) in a framework which is being developed for sign language research by Schlenker (e.g., Schlenker 2017, 2018; and several other works). Schlenker, building on the work of Lillo-Martin and Klima (1990), describes the R-loci as positions within the signing space that can realize discourse referents / logical variables. The number and exact positions of the R-loci are in principle arbitrary, but once an R-locus is established; it can be retrieved, e.g., by a pronoun; see example (7) from the American sign language (ASL).
(7) IX-1 KNOW BUSH ${ }_{a}$ IX-1 KNOW OBAMA ${ }_{b}$. IX-b SMART BUT IX-a NOT SMART. ${ }^{3}$ 'I know Bush and I know Obama. He [=Obama] is smart but he [=Bush] is not smart.'

ASL (Schlenker 2017, ex. 5a)
Returning to the ČZJ example (4), repeated here as example (8), the instances of TWO and BATHE are indexed by $\mathrm{i}, \mathrm{j}, \mathrm{k}$ which means that the first instance of BATHE is articulated in the same location (R-locus) as the first instance of TWO; this is repeated analogically in the second and third instance (see Figure 1) ${ }^{4}$.
(8) DAUGHTERMYTHEY-THREEDOGTWO $\mathrm{TWO}_{\mathrm{j}} \mathrm{TWO}_{\mathrm{k}} \mathrm{BATHE}_{\mathrm{i}} \mathrm{BATHE}_{\mathrm{j}} \mathrm{BATHE}_{\mathrm{k}}$ 'My three daughters bathed two dogs each.'


Figure 1: Two repetitions

The three locations correspond to the three daughters - the sorting key (R-locus-i corresponds to daughter A, R-locus-j to daughter B, R-locus-k to daughter C). The numeral TWO is interpreted as the whole DP (two dogs; the distributed share, as in the spoken language in examples (5) and (6). The distributed share is then distributed over the sorting key as follows: two dogs ( $\operatorname{dog} \mathrm{D}$ and $\operatorname{dog} \mathrm{E}$ ) are assigned to daughter A via index i ; two dogs (dog F and $\operatorname{dog} \mathrm{G})$ are assigned to daughter B via index j; two dogs (dog H and $\operatorname{dog} \mathrm{L}$ ) are assigned to daughter C via index k . Similarly, the instances of the verb BATHE assign the (sub)event of washing to the respective daughter-two dogs pairs. This mechanism will be discussed in detail in Sections 4.3 and 4.4. The difference between Schlenker's example (7) and our example (8) lies in the details of the mechanism used for establishing and retrieving the R-loci variables. In Schlenker's example (7), R-locus-a is introduced by BUSH and

[^2]retrieved by a pronoun (pointing to the same location through an index handshape, i.e., IX-a); similarly, for R-locus-b, which is introduced by OBAMA and retrieved through IX-b. In our ČZJ example (8), there is no pointing to an R-loci via pronouns. The R-loci (indexed as i, j, k) are introduced by TWO and interpreted as referring simultaneously to the respective DAUGHTERs. The R-loci are then used by the verb (BATHE).

### 2.3. Licensing of distance distributivity markers

Recently, two closely related phenomena have been discussed in the research focusing on distributive meaning in natural language: pluralizing operators and plurality filters (see overview in Kuhn 2019). The binominal each (as generalized quantifiers with every-type of universal quantifiers) belongs to the first group, and generally is treated as an expression that creates a plurality by summing objects together. The latter group is represented by pluractionals, distributive numerals, and dependent indefinites. Plurality filters are responsible for the operation of a set restriction, resulting in the output of a set of plural objects (Kuhn 2019). The two types of distributive strategies share certain properties: a) atomicity on the sorting key and $b$ ) the required cardinality of a distributed share, see examples (9)-(11) for a comparison.
(9) = example (2) the binominal each:
[The girls] ${ }^{\text {key }}$ saw [two dogs] ${ }^{\text {share }}$ each.
(10) $=$ example (5) distributive numerals:
[pilla-lu] ${ }^{\text {key }}$ renDu-renDu kootu-lu-ni] ${ }^{\text {share }}$ cuus-ee-ru
these children two-distr monkeys-Acc saw-3Pl
'The children saw two monkeys each.'
Telugu (Balusu 2006, ex. 1)
(11) dependent indefinites:
[BOYS THEY-arc-a] ${ }^{\text {key }}$ READ [ONE-arc-a BOOK] ${ }^{\text {share }}$
'The boys read one book each.'
ASL (Kuhn 2017, ex. 16a)
The first crucial distributional contrast consists in the compatibility with regard to universal quantifiers: pluralizing operators are not compatible, as in example (12), whereas plurality filters are, as in example (13). ${ }^{5}$
(12) \# Every boy had one apple each.

[^3]

Telugu (Balusu 2006, ex. 13)
Kuhn (2017) observes that similarly to Telugu or Hungarian, the ASL uses the reduplication of numerals to express a distributive meaning, as in example (14) with a reduplication of the numeral one. In the ASL, the reduplication strategy seems to be semantically equivalent to the use of the arc movement, while articulating the numeral (see example (14) and example (11) above). In Kuhn's seminal approach, examples such as (14) and (15) are instances of dependent indefinites. This is supported, e.g., by the compatibility of the reduplicated or spatially modified (arc) numeral with the distributive universal quantifier as seen in example (15). The agreement in space is glossed by 'a' on EACH and ONE.
(14) ALL BOY LIFT \{ONE-arc/ONE-redupl\} TABLE 'The boys each lifted a table.'

> ASL (Kuhn 2017, ex. 6)
(15) EACH-EACH-a PROFESSOR NOMINATE ONE-redup-a STUDENT
'Each professor nominated one student.'
ASL (Kuhn 2017, ex. 16b)
In this paper, a slightly different kind of data will be presented. The morphological marking in our ČZJ examples (examples (4)/(8)) makes use of several separate R-loci, each having their own referential value. ${ }^{6}$ On the other hand, in the ASL (example (15)), as far as can be understood from the glosses, the spatial agreement between EACH and ONE operates on a single locus (glossed by ' $a$ '). In such a case, it is not relevant, therefore, whether the hand articulating the numeral moves across the area with or without a reduplication.

The arc-movement strategy is also exploited in ČZJ, with the same result as in the ASL. Example (16) in ČZJ acts in parallel to that example (15) in the ASL: the numerals (and verbs) are articulated with the arc movement and agree in space with the preceding quantifier phrase (glossed by 'a'). Example (17) shows the compatibility of an NP marked by the arc movement with a distributive universal quantifier (EACH), which contrasts with example (18) containing an R-loci reduplication structure. Based on this data, it can be claimed that a distributive R-loci reduplication in the ČZJ should not

[^4]be analyzed as an instance of dependent indefinites ${ }^{7}$ since it shows different morphological and distributional properties. ${ }^{8}$
(16) GIRL THEY-arc-a DOG TWO-arc-a BATHE-arc-a 'The girls bathed two dogs each.'

<link to video>
(17) GIRL EACH-arc-a DOG TWO-arc-a BATHE-arc-a
'The girls bathed two dogs each.'

<link to video>
(18) \#/?? DAUGHTER MY THEY-THREE EACH-arc- ${ }_{i \mathrm{ijk}}$ DOG TWO $_{\mathrm{i}}$ TWO $_{\mathrm{j}}$ TWO $_{\mathrm{k}}$ BATHE $_{\mathrm{i}}$ BATHE $_{j}$ BATHE $_{k}$ (intended: 'My three daughters bathed two dogs each.')

## 3. Interpretation of distance distributivity in ČZJ

Languages across the world vary in the specific type of distributivity expressed by their distributive markers. Our aim is to establish readings for the ČZJ data, which are of relevance in the following section.

The English binominal each only allows the distributive operator to use the set of individuals as the sorting key, which leads to an interpretation with an individual distribution. ${ }^{9}$ The German jeweils, on the other hand, is ambiguous. Its distributive operator either takes a set of individuals or a set of subevents as the sorting key, leading to individual distribution (in example (19), to be read as (19b).) or occasional distribution (in (19), to be read as (19c)), respectively. ${ }^{10}$ The crucial observation is the fact that a cumulative reading is ruled out whenever a distributive marker is present (in (19), to be read as (19a)).

[^5](19) Die Mädchen haben jeweils zwei Hunde gesehen.
a. \# cumulative ('The girls saw two dogs.')
b. individual distribution ('The girls saw two dogs each.')
c. occasional distribution ('The girls saw two dogs each time.')

In this section, the availability of the two distributive readings within a variety of ČZJ constructions is explored. Sections 4.3 and 4.4 formally analyze the readings.

### 3.1. Semantic methodology

The data reported in this section were obtained through interviews with two Deaf ČZJ signers. The survey was designed as a truth judgment task. The two consultants were provided with videos in ČZJ, and were asked to evaluate both the 'understanding' (grammaticality) and the 'correctness' (truth) of the utterance in the given scenarios. The scenarios for individual distributive, occasional distributive, and cumulative readings were presented via pictures of 3 girls, 2 or 6 dogs, and bathing events; see Figures 2, 3, and $4 .{ }^{11}$ The videos were prepared during earlier sessions with the interpreter, and our target structure was the R-loci reduplication pattern. Some of the predesigned ČZJ utterances were later evaluated as 'unnatural' or 'not understandable' by the consultants. In such cases, the signers were asked to initially correct the ČZJ utterances and then to judge the truth/false value against the contexts. ${ }^{12}$

## 3.2. ČZJ data

As far as it is known, in spoken languages, the most salient interpretation of a sentence with plurality denoting both subject and object (e.g. The girls saw two dogs) is cumulative. It is a non-scopal reading with very weak truth conditions. Our first step, therefore, is to verify this observation in ČZJ. The sentence in example (20) contains a plurality of 'my daughters' (DAUGHTER MY PL) and a plurality of 'dogs' (DOG PL). ${ }^{13}$ The index 'a' on the plural marker (PL) for 'my daughters' represents the placing of the marker on the right side of the signing space. The index ' $b$ ' on the plural marker for

[^6]

[^7]

> Figure 2:

'dogs' signals the articulation on the left side of the signing space. The verb (BATHE) agrees in space with the NP dogs (see the index 'b' on BATHE). Note that the repetition of the verb is not a case of distributive reduplication (as in example (8) above), since the instances of the verb do not bear different R -loci. This particular repetition is analyzed merely as a very broad type of agreement. It reflects the number of dogs in the given context (see Figure 4 above).


Figure 5: Plural
(20) DAUGHTER MY PL ${ }_{a}$ DOG $_{\text {PL }}$ BATHE $_{b}$ BATHE $_{b}$ 'My daughters bathed the dogs.'
d. \# individual
e. \# occasional
f. cumulative

<link to video>
As expected from what is known about spoken languages, the sentence in example (20) can only be accorded a cumulative reading. It avoids both individual and occasional distribution. The sentence was judged as false for the scenarios represented in Figures 2 and 3, and true for the scenario in Figure 4.

Now the sentences that express distributive meanings will be addressed. The sentence in example (21) presents a simultaneous solution for distributivity. ${ }^{14}$ The dominant hand $(\mathrm{DH})$ is given in the first row of the glossed notation and the non-dominant hand (NDH) in the second row. ${ }^{15}$ The straight line represents the holding of the sign articulated by the non-dominant hand (THREE), while the dominant hand continues articulating. The signs DOG and BATHE are two-handed, and the non-dominant hand joins the dominant hand for the articulation (hence in the glosses, the straight line is

[^8]interrupted). See Figure 6 for a clearer idea of the simultaneous construction written in glosses as $\mathrm{TWO}_{\mathrm{i}} \mathrm{TWO}_{\mathrm{j}} \mathrm{TWO}_{\mathrm{k}}$ (daughters) with the hold of THREE (dogs). The simultaneous articulation of the pronoun $\mathrm{THEY}_{\mathrm{i}, \mathrm{j}, \mathrm{k}}$ with a circular movement over the numeral THREE $_{\mathrm{i}, \mathrm{j}, \mathrm{k}}$ is shown in Figure 7.


Figure 6:
THREE with TWO


Figure 7:
THREE with THEY
(21) DH: DAUGHTER THEY ${ }_{\mathrm{i}, \mathrm{j}, \mathrm{k}}$ MY DOG $\mathrm{TWO}_{\mathrm{i}} \mathrm{TWO}_{\mathrm{j}} \mathrm{TWO}_{\mathrm{k}}$ BATHE
'My three daughters bathed two dogs each.'
g. individual
h. \# occasional
i. \# cumulative

<link to video>
The sentence in example (21) was judged as true for the individual distributive scenario (see Figure 2) and as false for both the occasional distributive (see Figure 3) and the cumulative see Figure 4) scenarios.

As for the referential properties, the sign THREE is analyzed as representing the whole NP (three daughters) and, similarly, the sign TWO (two dogs). Additionally, each finger of THREE $_{\mathrm{i}, \mathrm{j}, \mathrm{k}}$ is used for a separate R-locus: the three loci are glossed by the indexes as $\mathrm{i}, \mathrm{j}$ and k . Thus, the distribution of the pluralities of dogs (two dogs) is overtly expressed by the articulation of the numeral TWO on the loci introduced by the NP three daughters (see the indexes on $\mathrm{TWO}_{\mathrm{i}} \mathrm{TWO}_{\mathrm{j}} \mathrm{TWO}_{\mathrm{k}}$ ). Morphologically, example (21) presents a case of distributive marking of the distributed share (two dogs), which is reduplicated and localized. ${ }^{16}$ The anaphoric relation toward the sorting key (three daughters) is realized overtly by the R-loci agreement.

The last ČZJ example presented in this paper is the sentence in example (22) (previously example (4) and example (8)). The example shows multiple distributive markings: the numeral (TWO) representing the NP (two dogs) is

[^9]reduplicated and localized, and consequently the verb (BATHE) is repeated on the same R-loci introduced by the NPs. This is shown in Figure 1, but at this point repeated in Figure 8.


Figure 8: Two repetitions
(22) DAUGHTERMYTHEY-THREEDOGTWO $\mathrm{TWO}_{\mathrm{j}} \mathrm{TWO}_{\mathrm{k}} \mathrm{BATHE}_{\mathrm{i}} \mathrm{BATHE}_{\mathrm{j}}$ BATHE $_{\mathrm{k}}$ 'My three daughters bathed two dogs each/each time.'
j. individual
k. occasional
l. \# cumulative

Our survey showed the ambiguity of example (22), with the sentence judged as true for individual and occasional distributive scenarios. Within our approach, each localized reduplication corresponds to one distributive operator in the syntax. As already mentioned briefly in Section 5, this example shows a non-prototypical use of an R-loci. The loci are introduced and used at the same time. ${ }^{17}$ TWO (interpreted as 'two dogs') is reduplicated and localized (see indexes $\mathrm{i}, \mathrm{j}$ and k ). Although this is the first utterance of the loci in this sentence, they are interpreted without any difficulty as retrieving the referents for daughters: the distributive operator carried by the morphological marking is anaphoric to the sorting key. ${ }^{18}$ The second distributive operator is overtly expressed by the reduplication and localization of BATHE. The same is true for the distributive operator on TWO; it is anaphoric to the sorting key. Thus, it seeks its antecedent in the domain of events (the subevents of bathing). The formal analysis of the readings presented in this section is found in Sections 4.3 and 4.4 below.

[^10]
## 4. Analysis

### 4.1. Two approaches to distributivity

In this section, our assumptions concerning distributivity are introduced. First, the core facts regarding the current standard approach to distributivity in natural languages are presented. The subsequent discussion will address why we consider the standard approach generally to be inadequate, as well as the empirical problems concerning sign languages.

It is necessary to begin with the standard (or most common) treatment of distributivity, as found in the established works of the theories of plurality (see Bennett 1974; Link 1983; Schwarzschild 1996; Winter 2001). Consider example (24a), where the distributive reading is the default reading (each boy is riding his own bike). In a model with three boys ( $a, b, c$ ), the interpretation of the subject DP would be a supremum, $a \sqcup b \sqcup c$ (intuitively the sum of the three boys in a model), which is a sortally inadequate argument for the (by default) distributive predicate ride a bike (in the usual formalization, distributive predicates would accept only atomic arguments: $a, b$ or $c$ but not their sums, $a \sqcup b$, etc.) Almost all the standard theories of distributivity derive the correct reading via a covert distributive operator (DIST), which scopes over the entire VP and requires each atom in the denotation of the subject to distribute itself over the predicate. The more complex cases require the formation of predicates syntactically via $\lambda$-abstraction, and subsequently the DIST scopes over such predicates (see Beck 2000; Beck and Sauerland 2000 for details), although the underlying mechanism of distributing over predicates remains the same.
(23) a. The boys ride a bike.
b. DIST(RIDE A biKe)

As already noted in Section 3, the ČZJ reduplication at an R-loci is definitely a distributivity marker, and from all the gathered evidence, it resembles most closely the binominal each from spoken languages. The incompatibility of the ČZJ R-loci reduplication with a universal quantifier (in example (18)) and its scopal pattern with respect to regular indefinites are strong empirical arguments against its treatment as a dependent indefinite. Moreover, as reduplication happens in the share constituent and not at the key, it behaves syntactically similarly to the binominal each, which is analyzed in Safir and Stowell (1988) as a modifier forming a constituent with the share, as in example (24a). There are several empirical arguments for this constituent structure. First, the binominal each is selective with respect to the share, being compatible only with a bare or non-definite share. Secondly, the binominal each can be fronted with its share (see Safir and Stowell 1987 for details).

In this respect, the binominal each is very different both from the determiner each (which is in constituent with the key - see example (24b)) and the floating each which syntactically behaves as a VP modifier: as in example (24c).
(24) a. Two boys [bought [three bikes each]].
b. [Each of the two boys] bought three bikes.
c. Two boys [each bought three bikes].

As was shown in the ČZJ examples (21) and (22), reduplication at the R-loci targets the numeral modifying the share NP and leads to an obligatory individual distributivity reading. The interpretation (the fact that the share R-loci reduplication cannot have an occasional distributivity reading) and the morphosyntactic realization of the ČZJ R-loci reduplication in the pattern demonstrated in example (21), most closely resemble the binominal each. Additionally, it is a widely accepted generalization that if the distributive marker resembles the determiner, it lacks an occasion reading (Zimmermann 2002). The ČZJ parallel is only partial, however, because even if the distribution marking happens on the modifier position (the numeral), the process itself is a repetition, a morphological process, and definitely not a category, as in the case of determiners. Be that as it may, the reduplication happens inside the share projection, which is fully compatible with the syntactic analysis of the binominal each as the share modifier (Safir and Stowell 1988).

The most interesting, yet puzzling, aspect of the data gathered is the pattern demonstrated in example (22): in this example the reduplication targets both the share modifier and the verb, with both reduplications occurring at the same R-loci. The sentence, moreover, has first, the expected individual distributivity reading but, more importantly, also an occasional distributivity reading. Nevertheless, the occasional interpretation seems to be related to the verb reduplication since the share reduplication, as in example (21), allows only an individual distributivity reading. Both the ČZJ consultants judged the individual distributivity reading of example (22) as primary and the occasional reading as possible, but somewhat more difficult to get. Nevertheless, the occasional interpretation is not possible if there is only share reduplication, as in example (21), no matter how much the speakers tried to parse it.

This puzzling, and at the same time intriguing, pattern fails to be understood within the standard theory of distributivity: the DIST operator, as in example (23), predicts only an individual distributivity reading. The double reduplication would be (under standard assumptions) only what could be viewed as a vacuous (DIST(DIST[ ...])), resulting again in an individual distributive reading. Consequently, the standard theory of distributivity apparently under-generates for R-loci reduplication. This failure supports the line of research where instead of the standard DIST formalization, another, and in many cases a dynamic, formalization of distributivity is posited
(see Brasoveanu 2008; Dotlačil 2012, 2013; Dočekal and Šimík 2020). The non-standard approaches to distributivity are able to address more complex cases where the standard theory fails. This line of reasoning will be followed and applied via the PCDRT theory of distributivity to the ČZJ data.

### 4.2. Cumulative interpretation in PCDRT

In this and the following sections, it will be shown how the PCDRT framework is able to include (unlike the standard approaches to distributivity) the unexpected patterns of distributivity as instantiated by the ČZJ data, with the alternative theories of distributivity more successfully addressing the widerranging empirical data than the standard theories. One such recent approach to distributivity can be found in Champollion (2016a, 2016b), who suggests a Neo-Davidsoanian framework for overt and covert distributivity operators across natural languages. It is probable that our analysis can be incorporated into such a framework with similar results, but an in-depth comparison of the PCDRT and Neo-Davidsonian approaches to the data gathered is beyond the main objective of this paper, namely to describe the unexpected ČZJ reduplication data within one coherent semantic framework.

Beginning with the cumulative interpretations of ČZJ within the PCDRT (the most salient interpretation of example (20)) should be discussed: the reading is verified in many situations in which three daughters bathed two dogs, and two dogs were bathed by three daughters, as illustrated in Figure 4, which was used in eliciting the truth judgments from the signers. One information state, that is a set of assignments, with the columns representing the values of the discourse referents, and the rows the assignments to the discourse referents, also called DREFS, which in this case corresponds intuitively to Figure 4, is shown in Table 1. The classical predicate logic has only one assignment function, usually notated as $g$, whereas the PCDRT utilizes sets of assignments ( $j_{1}, j_{2}$ in Table 1). Any additions to the information (the PCDRT belongs to the dynamic frameworks) are represented as incremental changes within the matrix. In this particular case, DREF $u_{1}$ represents the subject, with the values of $u_{2}$ denoting the object reference, and the Drefs ( $u_{1}$ and $u_{2}$ ) are correlated with each other via the predicate. The predicate is moreover represented as an events DREF $e$, which is extension of the standard PCDRT (see Henderson (2012) for formal details of how events are integrated into the PCDRT). The events (bathe ${ }_{1}$ and bathe ${ }_{2}$ ) are related pairs of Drefs, but also represent the event-related meaning of the verb.

Table 1. Information state verifying the cumulative reading of XX-22

| Info state J | $u_{1}$ | $e$ | $u_{2}$ |
| :--- | :--- | :--- | :--- |
| $j_{1}$ | daugher $_{1} \sqcup$ daughter $_{2}$ | bathe $_{1}$ | $\operatorname{dog}_{1}$ |
| $j_{2}$ | daughter $_{3}$ | bathe $_{2}$ | $\operatorname{dog}_{2}$ |

The derivation of truth conditions which are represented as information states, as in Table 1, within the PCDRT proceeds fully compositionally and starts from a syntactic structure. However, because a complete investigation of the ČZJ R-loci reduplication syntax is beyond the scope of this article, it will be assumed that the syntactic structures of the binominal each in spoken languages and the ČZJ R-loci reduplication are at some level of abstraction similar enough (which seems to be supported by the data gathered), so the focus will be on the semantic part of the ČZJ R-loci reduplication. The information state illustrated in Table 1 represents one set of total variable assignments, $H$, which represents contexts. The formulas are interpreted relative to pairs of assignments $\langle G, H\rangle$ and the truth in the PCDRT is a relative notion: a formula $\phi$ is true relative to an input information state $G$ if there is a satisfactory information state $H$, where $\llbracket \phi \rrbracket\langle G, H\rangle=\mathrm{T}$. The PCDRT version found in Brasoveanu (2008) and Dotlačil (2013) is used, as this version builds on the usual Montagovian logical typing approach, but extends it dynamically: type $e$ (the type of individuals) is replaced by type $r$ (the type of discourse referents, e.g. daughter ${ }_{3}$ in Table 1), and because the framework is dynamic, mapping contexts to contexts, the full discourse representation structure is $\langle\langle s t\rangle\langle\langle s t\rangle t\rangle\rangle$, is abbreviated to $\mathbf{t}$. In this system, NPs and verbs are predicates of type $\langle r t\rangle$. If NPs are in the argument position, they can be shifted by an existential closure (EC), or another operator, which will be discussed below, to unary quantifiers of type $\langle\langle r \mathbf{t}\rangle \mathbf{t}\rangle$. The root node of a syntactic composition is of type $\mathbf{t}$, the mapping from the information state $G$ to $H$.

Intuitively, sentences in natural language are true in the PCDRT if suitable starting and final contexts can be found in which each part of the sentence meaning can be embedded. The interpretation of the sentences is incremental, and Table 1 represents one of the possible contexts into which it would be natural to embed the meaning of example (20) under its cumulative reading. In more formal terms, example (25a) provides the non-dynamic version of the meaning of example (20), where the relative scope of the individual operators is clear. The dynamic PCDRT version of example (25a) is in example (25b), (as the introduction of the discourse referents is within the square brackets ([ $\left.u_{1}\right],\left[u_{2}\right],[e]$ in example (25b)) The usual operators of the $\theta$-roles are used, which connect the domain of events with the domain of individuals. Both examples (25a) and (25b) specify that there are two drefs, namely $u_{1}$, the daughters, and $u_{2}$, the dogs, that both are pluralities, signed
with the ČZJ marker PL which is signed at the left (index 'b' related to the R -locus b) and the right (index 'a' towards the R-locus a) side of the signing space. Moreover, in example (20) a reduplication of the verb bathe occurred (although this reduplication did not happen at the R-loci), which is reflected as the existential closure of the event variable $e$, a reduplication that is compatible both with the atomic and plural cardinalities of the set of events. In this example, no distributivity occurs, as neither of the $u_{2}$ assignments has the plurality of dogs (in the individual rows representing the assignments $j_{1}$ and $j_{2}$ ). The pluralization of both nominal and verbal drefs results in weak truth conditions modeled in and verified by the information state shown in Table $1 .{ }^{19}$
(25) a
a. $\exists u_{1}\left[\left|u_{1}\right|>1 \wedge\right.$ DAUGHTERS $\left.\left\{u_{1}\right\}\right]$

$$
\left(\exists u_{2}\left[\left|u_{2}\right|>1 \wedge \operatorname{DOGS}\left\{u_{2}\right\}\right]\left(\exists e\left[\operatorname{BATHE}(e) \wedge a g\left(e, u_{1}\right) \wedge \operatorname{th}\left(e, u_{2}\right)\right]\right)\right)
$$

b. $\left[u_{1}\right] \wedge\left|u_{1}\right|>1 \wedge \operatorname{DAUGHTERS}\left\{u_{1}\right\} \wedge\left[u_{2}\right] \wedge\left|u_{2}\right|>1$
$\wedge \operatorname{DOGS}\left\{u_{2}\right\} \wedge[e] \operatorname{BATHE}(e) \wedge \operatorname{ag}\left(e, u_{1}\right) \wedge \operatorname{th}\left(e, u_{2}\right)$

### 4.3. Individual distributivity interpration in PCDRT

In this and the following section, the ways distributivity is employed within the PCDRT will be introduced, and applied to the ČZJ sentences discussed above. It should be recalled that in the ČZJ sentence (example 21) the numeral in the share was reduplicated and so the sentence was judged as true only in the case of individual distributivity, as exemplified in Figure 2. As discussed briefly in Section 3.1, dynamic frameworks (with PCDRT being one of the most influential) do not follow the standard approach to distributivity. Now it is necessary to consider the prerequisites regarding distributivity treatment within the PCDRT. The core ingredient of the dynamic conception of distributivity is the distributivity operator $\delta_{u_{n}}$ (after Nouwen 2003; Berg 1996; among others), which quantifies over information states, but in its domain are only those assignments (represented as rows in table Table 1 as well as all the following tables) in which the anaphoric dref $u_{n}$ (over which $\delta_{u_{n}}$ quantifies) is atomic are located in its domain. The definition of $\delta_{u_{n}}$ is found

[^11]in example (26a) and is a simplified version of Dotlačil's (2012) formalization. Unlike the standard theory of distributivity where DIST has in its domain the whole predicate (in most cases syntactically the VP), quantifies over information states $\delta_{u_{n}}$ ( $\lambda I \lambda \mathcal{F}$ in example (26a)). Next, the distributive operator $\delta_{u_{n}}$ updates the atomic assignments (the atomicity condition: \#( $\left.\mathrm{U} u_{n} I\right)=1$ ) with nuclear scope. The $\delta_{u_{n}}$ operator appears both in the PCDRT formalization of the determiner each (not discussed here), and the formalization of the binominal each, e.g. (26b) (following Dotlačil 2013). The binominal each is anaphoric to its key (notationally as the variable which is $\lambda$-abstracted over and propagated through the semantic composition until it meets its key argument). Crucially it is distributed over its syntactic argument (share $\left.P\left(u_{m}\right)\right)$ and it requires each atom from the key denotation to be paired with the share of the right cardinality. Finally, the distribution is selective and does not scope over the verbal predicate $\left(Q\left(u_{m}\right)\right.$ is not in the scope of $\left.\delta_{u_{n}}\right)$. The distributive operator also introduces a new discourse referent $\left(u_{m}\right)$, so it serves as the existential closure of the property typed share NP. The formalization corresponds well with the ČZJ data: the individual distributivity (e.g. (21)) is signed with the R-loci reduplication inside the share constituent, and the verb remains unpluralized.
\[

$$
\begin{align*}
& \text { a. } \delta_{u_{n}}(D)=\lambda I \lambda J \cdot u_{n} I=u_{n} J \wedge \forall d \in u_{n} I\left(\#\left(\cup u_{n} I\right)=1 \wedge D\left(\left.I\right|_{u_{n}=d}\right)\left(\left.J\right|_{u_{n}=d}\right)\right)  \tag{26}\\
& \text { b. } \llbracket B I N O M-e a c h^{u_{m}} \rrbracket=\lambda v_{r} \lambda P_{\langle r t\rangle} \lambda Q_{\langle r t\rangle} \cdot\left[u_{m} \mid\right] \wedge \delta_{v}\left(P\left(u_{m}\right)\right) \wedge Q\left(u_{m}\right)
\end{align*}
$$
\]

To demonstrate an individual distributivity reading with an information state corresponding to Figure 2, see Table 2. The information state $\mathcal{7}$ in Table 2 is an update of the previous information state $I$ where for each atomic entity in the dref $u_{1}$, there is a corresponding share with the cardinality 2 , as is the case in Table 2.

Table 2. Information state verifying the individual distributive reading of XX-23

| Info state J | $u_{1}$ | $e$ | $u_{2}$ |
| :--- | :--- | :--- | :--- |
| $j_{1}$ | daughter $_{1}$ | bathe $_{1}$ | $\operatorname{dog}_{1} \sqcup \operatorname{dog}_{2}$ |
| $j_{2}$ | daughter $_{2}$ | bathe $_{2}$ | $\operatorname{dog}_{3} \sqcup \operatorname{dog}_{4}$ |
| $j_{3}$ | daughter $_{3}$ | bathe $_{3}$ | $\operatorname{dog}_{5} \sqcup \operatorname{dog}_{6}$ |

The full formalization the meaning of example (21) is in example (27a), with the dynamic version in example (27b): the most important change against the cumulative formalization is, of course, the addition of the distributive operator $\delta_{u_{n}}$. As discussed previously, the distributive operator scopes only over the share, with the predicate (syntactically VP) not within
its scope. Or in other words, it connects the drefs of the key $\left(u_{1}\right)$ with the share $\left(u_{2}\right)$, but it does not directly distribute over the denotation of the verb (of type e). The PCDRT framework predicts that the binominal each is in principle compatible with certain types of collective subjects, as the distributivity that begins with the binominal each takes as its two arguments the drefs of the key and the share, while the application of the verb to the bearers of the subject's $\theta$-role is not within the scope of $\delta$. Dočekal and Šimík (2020) explore the theoretical consequences of this prediction and show that it correctly captures a number of intriguing patterns of collectivity and that the binominal each grammatically appears in one sentence. Kuruncziová (2020) offers experimental support to this local distributivity of PCDRT prediction in Slovak. Further, and a more general, application of the PCDRT framework to cases of reciprocal distributivity can be found in Dotlačil (2013).

$$
\begin{align*}
& \text { a. } \exists u_{1}\left[\left|u_{1}\right|=3 \wedge \operatorname{DAUGHTERS}\left\{u_{1}\right\}\right] \wedge \delta_{u_{1}}\left(\exists u_{2}\left[\left|u_{2}\right|=2 \wedge \operatorname{DOGS}\left\{u_{2}\right\}\right]\right)  \tag{27}\\
& \left(\exists e\left(\text { BATHE }\{e\} \wedge a g\left(e, u_{1}\right) \wedge t h\left(e, u_{2}\right)\right)\right) \\
& \text { b. }\left[u_{1}\right] \wedge\left|u_{1}\right|=3 \wedge \operatorname{DAUGHTERS}\left\{u_{1}\right\} \wedge \delta_{u 1}\left(\left[u_{2}\right] \wedge\left|u_{2}\right|=2 \wedge \operatorname{DOGS}\left\{u_{2}\right\}\right) \\
& \wedge[e] \text { BATHE }\{e\} \wedge \operatorname{ag}\left(e, u_{1}\right) \wedge \operatorname{th}\left(e, u_{2}\right)
\end{align*}
$$

### 4.4. Occasional distributivity interpretation in PCDRT

This section describes the most intriguing, but also the most revealing, pattern of the ČZJ R-loci reduplication. Example (22), which is repeated below schematically as example (28), was reported by both consultants as being ambiguous. An individual distributivity reading (the sentence would be true in the same set of situations as example (21) with this reading) was the primary interpretation. However, an occasional distributivity reading was also reported by both signers as a possible, but not the preferred, interpretation, of example (22) (the interpretation is visualized in Figure 3). As both the glosses and Table 3 demonstrate, the latter interpretation was expressed by two R-loci reduplications, the first reduplicating the numeral modifier in the share, with the second reduplication at the R-loci targeting the verb.
(28) DAUGHTERMYTHEY-THREEDOGTWO $\mathrm{TWO}_{\mathrm{j}} \mathrm{TWO}_{\mathrm{k}}$ BATHE $_{\mathrm{i}}$ BATHE $_{\mathrm{j}}$ BATHE $_{\mathrm{k}}$ 'My three daughters bathed two dogs each.'

Before formalizing our ideas about occasional distributivity in ČZJ, a few general remarks are necessary. First of all, since the reduplication occurs to the verb, examples such as $(22) /(28)$ can be categorized under the pluractional agenda. Yet despite the increasing literature on pluractionality (Henderson (2014) for Kaqchikel and Kuhn and Aristodemo (2017) for sign languages, a.o.), pluractional distributivity remains less well-researched and understood
than distributivity in the nominal domain. Thus, despite many remaining caveats, our data from ČZJ and its analysis can be considered rather as another step in the development of an appropriate approach to the pluractionality phenomena in sign languages. ${ }^{20}$

The first important property of pluractionality is the plurality of events requirement. We illustrate it here with an example in example (29a) which demonstrates some of the features usually found in pluractional distributivity, which satisfies the requirement in English. Although English has no pluractional verbal morphology, its lack can be remedied with constructions such as one by one or brick by brick (see Brasoveanu and Henderson 2009 for an analysis of such English pluractionality). The first important property of pluractionality, demonstrated in example (29a) is the plurality of events requirement: example (29b) with a singular argument clearly violates the plurality of events requirement and it is therefore ungrammatical. However, intuitively example (29a) requires not only a plurality of events, but also their temporal sequencing since it would be inappropriate in a situation where all the soldiers were killed simultaneously. The plurality of events and temporal sequencing leads to an incompatibility of pluractionality with certain forms of collective predicates, but the incompatibility of pluractionals with collectivity is not a general phenomenon. See example (30) from COCA (Davies 2009).
(29) a. The soldiers died one by one.
b. *The soldier died one by one.
(30) 14 men in secret CIA custody were gathered one by one from locations across the world. COCA

Returning to ČZJ, in the case of pluractional R-loci reduplication, it is necessary to retain the same type of analysis employed with regard to the numeral R-loci reduplication. Yet it is also clear that the distribution over the atoms in the NP domain would not be possible since this would result in individual distributivity. Consequently, our analysis should be based upon the idea of

[^12](i) STUDENT PL DOG BATHE BATHE BATHE
'The students bathed the dogs.'
group interpretation. Intuitively, as far as example (22) is concerned, it could be claimed that the occasional reading comes from the plurality of events connected with the group denotation of the key, namely, that example (22) was judged as true in the occasional picture in Figure 3 since the signers interpreted the three daughters as a group atom.

This leads to a further issue raised by one of our two anonymous reviewers. As the reviewer correctly remarks, describing pluractional distributivity as involving a collective interpretation (group formation) initially appears to be a somewhat difficult theoretical stance. We agree to some extent: it is clear that some kinds of pluractional distributivity and collectivity are incompatible (but see the discussion regarding examples (29) and (30)). Nevertheless, it is also known that certain sorts of pluractionality and collectivity interact quite well, since the plurality of events and the plurality of participants (the core requirements of pluractionality and collectivity) do not clash. As an example, consider the Kaqchikel example in (31) (cf. Henderson 2014, ex. 100), in this example the pluractional distributivity marker -la' aligns with the collective predicate mol 'group'. Henderson comments on the example as follows: '[it] would be appropriate for describing a situation where I put the beans in a basket one by one' (Henderson 2014: 43). Essentially, the individual events of putting individual beans into a basket do not satisfy the predicate mol 'group,' but this fails to make example (31) ungrammatical. This example exactly mirrors our ČZJ example: the group interpretation of the agents in example (22) means that there is a way to link the subevents of bathing to the individual girls constituting the group atoms, and none of these subevents has to be the whole bathing event. ${ }^{21}$

> (31) X-e'-in-mol-ola' $\begin{aligned} & \text { Xi } \\ & \text { CP-A3p-E1s-group-la' } \\ & \text { 'I grouped the beans individually.' }\end{aligned}$ beans. kinäq.

Now it is necessary to formalize the ideas introduced above. A sample information state corresponding to Figure 3 can be found in Table 3. This information state verifies the occasional reading; the individual distributivity reading would also be verified by an information state similar to that in Table 2, which differs from the information state in Table 3. The most important difference is that in the case of the occasional distributive reading, illustrated

[^13]in Table 3, the key is not atomic at the level of the entities. It is essential to note a collective atom type shift marked with $\uparrow$ (after Landman 2000), a shift which takes as its input the plurality of entities (daughters in our example) and yields a group atom, basically a plurality which from a higher-order perspective resembles an atom. How this is related to the composition and to the individual distributivity reading will now be discussed.

Table 3. Information state verifying the occasional distributivity reading of XX-24

| Info state J | $u_{1}$ | $e$ | $u_{2}$ |
| :--- | :--- | :--- | :--- |
| $j_{1}$ | $\uparrow\left(\right.$ daugher $_{1} \sqcup$ daugher $_{2} \sqcup$ daugher $\left._{3}\right)$ | bathe $_{1}$ | $\operatorname{dog}_{1} \sqcup \operatorname{dog}_{2}$ |
| $j_{2}$ | $\uparrow\left(\right.$ daugher $_{1} \sqcup$ daugher $_{2} \sqcup$ daugher $\left._{3}\right)$ | bathe $_{2}$ | $\operatorname{dog}_{3} \sqcup \operatorname{dog}_{4}$ |
| $j_{3}$ | $\uparrow\left(\right.$ daugher $_{1} \sqcup$ daugher $_{2} \sqcup$ daugher $\left._{3}\right)$ | bathe $_{3}$ | $\operatorname{dog}_{5} \sqcup \operatorname{dog}_{6}$ |

First, it is necessary to discuss how the individual distributivity reading for example (22) is derived. Our null hypothesis is that each R-loci reduplication signals the presence of the distributivity operator (formalized as the PCDRT $\delta$ operator). A question immediately follows: why are examples (21) and (22) (under one reading) synonymous? Let us start with a hypothesis that the syntactic position where the R-loci reduplication is realized is interpreted sortally: if the reduplication happens in the NP (share), the distributivity is related to the entities, whereas if the reduplication is realized in the verb, the signer distributes over the events, rather than the entities, although the key remains the same, as the reduplication happens at an identical R-loci. Consequently, we hypothesize that the individual distributivity reading is formalized as in examples (32-a) and (32-b) (the final dynamic notation). The individual distributivity reading in example (32) has two distributive operators, $\delta_{u_{1}}$, both anaphoric to its key (the dref $u_{1}$ ), both requiring each atom to be a corresponding share. The share in the case of the R-loci reduplicated numeral is the NP, and the distributivity requirement can be viewed as the following: for each atom in the $u_{1}$ (key), there have to be two atoms in the $u_{2}$ (share). This was discussed in the preceding section. But how can the distributivity of the verb be interpreted? We assume in the same way, but possibly with different consequences: in this case, the distribution operator also requires the presence of (for each atom in the $u_{1}$ ) the right share, but because in this instance the share denotes events, the requirement maps each atom in the $u_{1}$ to an event in the $e$, a requirement that is verified in Table 3. Moreover, the verb itself is not pluralized. Consequently, the relationship between entities and events is 1-1. In other words, the two distributive operators collapse into one distributive reading because the distributivity requirement of the share reduplication requires an atomic antecedent for each share (fulfilled),
whereas the reduplication of the verb requires an atomic antecedent for each event (also fulfilled).

$$
\begin{align*}
\text { a. } \exists u_{1}\left[\left|u_{1}\right|=3 \wedge\right. & \left.\wedge \operatorname{DAUGHTERS}\left\{u_{1}\right\}\right] \wedge  \tag{32}\\
& \left(\exists e\left(B A T H E\{e\} \wedge \operatorname{sig}\left(\exists u_{2}\left[\left|u_{2}\right|=2 \wedge \operatorname{u}\right) \wedge \operatorname{th}\left(e, u_{2}\right)\right)\right)\right. \\
\text { b. }\left[u_{1}\right] \wedge\left|u_{1}\right|= & \left.3 \wedge \operatorname{DAUGHTERS}\left\{u_{2}\right\}\right] \wedge \delta_{u 1}\left(\left[u_{2}\right] \wedge\left|u_{2}\right|=2 \wedge \operatorname{DOGS}\left\{u_{2}\right\}\right) \\
& \wedge[e] \operatorname{BATHE}\{e\} \wedge a g\left(e, u_{1}\right) \wedge t h\left(e, u_{2}\right)
\end{align*}
$$

Now we are in a position to explain the occasional reading. The basic insight with which we wish to engage with our formalization is as follows: if the signer also reduplicates the verb at the R-loci, the hearer's attention is drawn to the events and this de-emphasizes the atomicity of the key, or in other words, the key atoms are permitted to be atomic at a higher-order, by which we mean the domain of group atoms. So essentially, we claim that the occasional reading is a result of the key shifting to a group atom so that the $\delta_{u_{n}}$ quantifies over group atoms (in the sense of Barker 1992; Landman 2000). As in the case of individual distributivity, the meaning of which was discussed in the previous section, the requirement for the atomicity of the key is fulfilled, but in the case of the occasional reading, the atomicity is fulfilled at the level of the group atoms. Therefore, we propose that the occasional reading is formalized as (33a) and (33b) (again the dynamic version). Thus, our solution intuitively builds upon the idea of markedness: if (as in example (21)) the signer reduplicates at the R-loci only in the share, the atomicity requirement is stringent; if the signer reduplicates both the verb and the share at the R-loci (example (22)), the atomicity requirement can be interpreted as, most importantly, requiring a 1-1 mapping between the events and the key atoms, which does not necessarily mean a mapping to additional atoms, but allows the possibility of a mapping to group atoms. For this to proceed, the key has to be shifted to a group atom. ${ }^{22}$ Such covert shifting is (at least at the processing level) costly, so there is some understanding of the individual

[^14]vs. occasional reading preference reported by both consultants. We believe, based upon our analysis, that our work is progressing in the right direction but there are (as always) many questions remaining. One of the more pressing (see also our discussion in footnote 16) concerns the unified treatment of the R-loci reduplication, which (following the ČZJ data) we formalize. It is, of course, possible that further in-depth investigations into the ČZJ reduplication of verbs would strengthen the link to Henderson (2014) and his treatment of pluractionality in terms of partitioning both the events and their arguments.
(33) a. $\exists u_{1}\left[\left|u_{1}\right|=1 \wedge 3-\operatorname{DAUGHTERS}\left\{u_{1}\right\}\right]\left(\exists u_{2} \delta_{u_{1}} u_{1}\left[\left|u_{2}\right|=2 \wedge \operatorname{DOGS}\left\{u_{2}\right\}\right]\right)$ $\left(\exists e \delta_{u_{1}}\left[B A T H E\{e\} \wedge \operatorname{ag}\left(e, u_{1}\right) \wedge t h\left(e, u_{2}\right)\right]\right)$
\[

b. $$
\begin{aligned}
{\left[u_{1}\right] \wedge\left|u_{1}\right|=} & 1 \wedge 3-\operatorname{DAUGHTERS}\left\{u_{1}\right\} \wedge\left\lfloor u_{2} \mid \delta_{u_{1}}\left(\left[\left|u_{2}\right|=2 \wedge \operatorname{DOGS}\left\{u_{2}\right\}\right]\right)\right] \\
& \wedge\left[e \mid \delta_{u_{1}}\left(\left[\operatorname{BATHE}\{e\} \wedge \operatorname{ag}\left(e, u_{1}\right) \wedge \operatorname{th}\left(e, u_{2}\right)\right]\right)\right]
\end{aligned}
$$
\]

## 5. Summary

In this article, the preliminary data from the exploratory work we conducted with Deaf Czech Sign Language signers is reported. We focused on distributivity in ČZJ since the strategies ČZJ employs to communicate distributivity are more familiar than the majority of the distributivity markers we can find in spoken languages. One empirical conclusion we draw from our work concerns the different strategies which are used in ČZJ to signal distributivity. In particular, we paid close attention to the R-loci reduplication of the share and predicate (and contrasted it with the arc movement, which resembles the dependent indefinites in spoken languages) and analyzed this as a subtype of the binominal each construction. This allowed us to distinguish and explain (using the PCDRT framework) the intriguing interpretational patterns connected with the various subtypes of the R-loci reduplication. More generally, our work provides empirical support for the theories (Henderson 2014) and (Champollion 2016a, 2016b) in which participant and occasional distributivity are described as stemming from the same source, but distinguished by the sortal type of the objects the distributivity operators quantify.

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## Appendix

One of the two anonymous reviewers raises a question concerning our analysis of the ČZJ R-loci reduplication as a sign language parallel to the spoken language binominal each and asks whether an analysis (roughly speaking) of the R-loci reduplication as a dependent indefinites strategy would in fact work. We acknowledge that our research into ČZJ is in its initial stages and can be improved in many respects, but we do not believe that the data (at least from what we know) point in the direction of R-loci reduplication as a dependent indefinites strategy.

To support the test with the universal quantifier (which is used as a standard test for the classification of an expression as an operator or a filter), as reported in Section 2.3, after determining the contrast between examples (16) and (17), we conducted a further ČZJ investigation by running a second standard diagnostic test which distinguishes between distributivity operators (such as the spoken binominal each) and dependent indefinites (or pluractional markers). The second standard test concerns the behavior of plain indefinites with respect to distributivity operators and dependent indefinites. Before reporting the ČZJ data, we repeated this widely accepted diagnostic test that distinguishes between distributivity operators and plurality filters (dependent indefinites and pluractional markers). First, we considered the contrast between examples (34a) and (34b) from French Sign Language LSF - (the examples and their interpretation are taken from Kuhn and Aristodemo 2017, ex. 34)). As the judgments suggest, a distributive quantifier in the LSF can interact with the plain indefinite in the expected manner creating an ambiguity - the relative scope is either $\forall>\exists$ (many words) or $\exists>\forall$ (one word). Nevertheless, the situation changes if a plurality filter signed as arcmovement on the index IX is used. Descriptively speaking, this interpretation only permits the wide scope of the indefinite over the plurality filter, $\exists>\forall$. This is intriguing, since $\exists>\forall$ is logically stronger and asymmetrically entails $\forall>\exists$ (the formula $\exists x[P x \wedge \forall y(Q y \rightarrow R x y)] \rightarrow \forall y[Q y \rightarrow \exists x(P x \wedge R x y)]$ is a valid predicate logic tautology). The lack of the many words reading means that $\forall>\exists$ would be true only in limited cases where the set $Q$ would be a singleton, in other words in cases where $\forall>\exists$ is equivalent to the $\exists>\forall$ scope. Be that as it may, the usual conclusion drawn from data such as this (see Henderson 2014; Kuhn and Aristodemo 2017; Kuhn 2019) is that dependent indefinites cannot induce the covariation of a plain indefinite (unlike distributive operators such as the universal quantifier in example (34a)).
(34) a. STUDENT EACH FORGOT ONE WORD. $\checkmark$ many words $\checkmark$ one word 'Each student forgot one word.'

## b. STUDENT IX-arc FORGOT-alt ONE WORD. *many words $\checkmark$ one word 'The students forgot (the same) one word.'

With respect to ČZJ, we used the second test introduced in example (34). The sentence in example (35) employs the R-loci reduplication strategy on the whole entity classifier representing a girl (glossed as CL). Each locus (i, j, k) refers to one girl. The sentence is interpreted with a wide scope of the plurality in the subject (different bread for each girl): $\forall>\exists$. This interpretation is more salient than the reverse scope, $\exists>\forall$, but certainly example (35) does not only have the $\exists>\forall$ interpretation reported above as a telltale sign of plurality filters. Based on these judgments, we conclude that the treatment of ČZJ R-loci reduplication as a dependent indefinite is highly improbable and that it is a plurality operator with which we are dealing, a plurality operator that we claim is similar to the spoken binominal each. However, we assume the arc-movement strategy reported in example (16) would lead to a data pattern similar to that of the LSF reported in example (34), because ČZJ employs the dependent indefinite strategy. And in the future, we would like to focus on the contrasts between R-loci reduplication and ČZJ dependent indefinites with respect to their differing scope patterns.
(35) GIRL THREE $C L_{\mathrm{i}} \mathrm{CL}_{\mathrm{j}} \mathrm{CL}_{\mathrm{k}}$ BREAD BUTTER EAT. $\checkmark$ many breads $\checkmark$ one bread 'Each of the three girls ate one bread with butter.'

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[^0]:    ${ }^{1}$ We would like to thank those who attended the Sign Language Syntax and Linguistic Theory workshop at GLOW41 in Budapest, and the two anonymous reviewers of this article for their pertinent comments. We are also grateful to our ČZJ consultants, Mirka Tylová and Ondřej Klofáč, who are Deaf signers exposed to sign language from birth; and to Ivana Kupčíková for interpreting.

[^1]:    ${ }^{2}$ Although there is a terminological difference between reduplication and repetition (see Kimmelman 2018), it is not necessary to discuss this in detail in this paper. Thus, we use them synonymously.

[^2]:    ${ }^{3}$ We maintain the notation of the author: IX is a gloss for an index/pointing sign. IX-1 stands for pointing toward the signer (first person), IX-b for pointing toward R-locus b (the location where OBAMA was signed), and IX-a for pointing toward R-locus a (the location where BUSH was signed). We also follow Schlenker (2017) in treating the pointing signs as pronouns.
    ${ }^{4}$ All figures in the text were created by Hana Strachoňová.

[^3]:    ${ }^{5}$ Some researchers even claim that distributive numerals and dependent indefinites are licensed by the distributive operator carried by the universal quantifier (e.g., Kimmelman 2017).

[^4]:    ${ }^{6}$ Hence reduplication and localization in Kimmelman (2017) for the RSL.

[^5]:    ${ }^{7}$ Although we present the contrast between the ČZJ distributive R-loci reduplication and the ČZJ arc-movement strategy in terms of a plurality operator vs. plurality filter dichotomy, this remains primarily at a descriptive level and uses the theoretical distinction more as a guide, because we are aware there are different approaches to the distinct patterns of data, see Cable (2014) or Law (2022). We would like to thank one of the two anonymous reviewers for raising the importance of this issue.
    ${ }^{8}$ In reality, it is not always easy to determine the exact boundary between the localized reduplication and a pluralizing strategy using continuous movement (such as the arc movement). Schlenker and Lamberton (2019) suggest that these two phenomena should be viewed as a continuum. At the same time, their analysis does not exclude different quantificational and referential properties of the mentioned structures.
    ${ }^{9}$ This reading is called a participant key reading, e.g., in Balusu (2006) or a participantdistributive interpretation in Cable (2014).
    ${ }^{10}$ The occasional distribution is also known as an event-distributive reading, e.g., in Cable (2014).

[^6]:    ${ }^{11}$ Thus, each utterance was assessed once for the grammaticality and three times for its truth value (against the 1 . individual distributive, 2. occasional distributive, and 3. cumulative scenarios).
    ${ }^{12}$ The scenarios were inspired by some of the examples found in Cable (2014).
    ${ }^{13}$ It is beyond the scope of this study to state the exact nature of the expression glossed as PL (we would consider a genuine plural marker, a quantifier, a pronoun, a determiner, and a classifier that locates a plurality in space). The results are currently based on the intuition of the ČZJ users who interpreted the sequence of a noun and PL as a plurality. The visual representation of the sign is given in Figure 5.

[^7]:    Figure 3:
    

[^8]:    ${ }^{14}$ By simultaneous we mean an utterance in which each hand articulates an independent sign, as opposed to one two-handed sign.
    ${ }^{15}$ Our ČZJ consultant is left-handed; thus, in this particular case, the dominant hand is the left hand.

[^9]:    ${ }^{16}$ The crucial role of localization is highlighted in Kimmelman (2017).

[^10]:    ${ }^{17}$ By prototypical use we mean the way loci are described in Schlenker's work (2017, 2018, and related work): the locus is first introduced, and later referred to via indexes.
    ${ }^{18}$ At this point, we have to assume that the DAUGHTER-R-locus matching is realized via a discourse mechanism: we know from the previous discourse that there are three daughters. Thus, it is plausible to match the three repetitions of the NumDP with these three referents. At the present stage of the research on the topic, the exact linguistic nature of R-loci is still unclear, and our proposal for the matching mechanism follows in the main from the intuition of the consultants.

[^11]:    ${ }^{19}$ In a full treatment of distributivity within a framework enriched with events, we would have to disassemble formulas such as example (25) and the subsequent examples in a manner so that events such as e in example (25) would contain the pluralities of subevents in which each sub-event would have its atomic $\theta$-role bearer. In that way, our proposal would satisfy the usual 1-1 mapping requirement between events and $\theta$-role bearers, which is explicit or implicit in all current event-based approaches to pluralities: see Landman (2000) or Henderson (2012). For the sake of clarity, we do not complicate our formulas, but see Champollion (2016a) for such a formalization.

[^12]:    ${ }^{20}$ In this article, we focus on the ČZJ distributivity marking in the nominal domain. And even if obviously we touch upon the pluractional R-loci reduplication, an in-depth study requires a further paper. But we note in passing that the ČZJ R-loci reduplication allows something similar to a purely pluractional reading in (i): in this case, the R-loci reduplication only targets the verb. The consultants' comment on this example in the following way: it would be appropriate in a situation with a plurality of students, a plurality of dogs, and a plurality of bathing events. But it is not necessary for each student to bathe a plurality of dogs; in other words, we are not dealing with an individual distributivity reading here. We are grateful to an anonymous reviewer for reminding us of the importance of this kind of example.

[^13]:    ${ }^{21}$ Implementing this would lead us to the partitioning of events and eventual decomposition of the group atoms (most probably via Landman's $\downarrow$ operator). This would be very much in the spirit of (Henderson 2014) and his analysis of pluractional distributivity in Kaqchikel. We would like to thank one of our anonymous reviewers for highlighting this possibility. This move would also be needed for the reasons discussed in footnote 16, but we wish to address this in the future since gathering and analyzing the ČZJ data related to pluractionality and collectivity would be beyond the scope of the current article.

[^14]:    ${ }^{22}$ One anonymous reviewer raises a question about an alternative and more straightforward formalization of our ideas concerning the occasional reading, and suggests that instead of the distribution over the group atoms, as in example (33), it would be possible for $\delta$ to quantify directly over the events so that the sorting key would be the events instead of the drefs. We agree that this is plausible, but to split the group-atom key idea from the eventkey idea, we would have to construct scenarios in which the plurality of the agents either collaborates or not, and depending on the availability of the occasional reading for each of the scenarios, it would become clear whether the shift to group atoms is necessary or not. We leave this for the future, because although our solution may be more complicated than needed, despite working for the examples investigated, only further empirical research can reveal whether the group-atom strategy is really an unnecessary complication.

