

# In Defence of the Thin Red Line: A Case for Ockhamism

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## ABSTRACT

This paper deals with A.N. Prior's Ockhamism and "the true futurist theory". The introduction contains an outline of the historical background of the theories mainly in medieval theology and logic. In section 2, a formal version of the medieval argument for determinism will be presented without theological references. It will be pointed out that there are two premises used in the argument which are obvious candidates for questioning. In section 3, Prior's Ockhamism will be discussed. The modern criticism of the "the theory of the thin red line" will be presented and evaluated in section 4, and it will be argued that the theory can in fact be defended and that there is after all no strong argument against it.

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## 1. INTRODUCTION

The belief in an indeterministic worldview is closely related to the assumption that there are future contingents, *i.e.* statements which are neither necessary nor impossible. This paper is based on the assumption of an indeterministic worldview (*i.e.* the idea of an open future). It is assumed that statements like "my mother will go to London" and "there will be a sea battle tomorrow" may serve as standard examples of future contingents. The future contingency problem is the question whether such statements have truth-values already today. In other words: Can it be true (or false) today that my mother will be going to London, or that some possible sea-battle will take place tomorrow, given that the future outcome will depend on future decisions freely made by competent women and men in both cases? And if so: How can truth-values be ascribed to statements about such open questions?

During the Middle Ages, several famous logicians discussed the problem of the contingent future in relation to Christian doctrine. According to Christian tradition, divine foreknowledge comprises knowledge of the future choices to be made by men and women. But this assumption apparently gives rise to a straightforward argument from divine foreknowledge to the necessity of the future: if God knows already now which decision I will make tomorrow, then a now-unpreventable truth about my choice tomorrow seems to be given already today. My choice, then, appears to be necessary, not free. Hence, there seems to be no basis for the claim that I have a free choice between genuine alternatives. This conclusion, however,

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violates the idea of human freedom and moral accountability which is normally presupposed in theology.

The future contingency problem does not have to be formulated in terms of theological doctrines. In fact, the medieval discussion regarding the logic of divine foreknowledge is from a formal point of view very close to the modern discussion concerning future contingency, which is mostly formulated in terms of a secular vocabulary. If “known to God” is simply understood as “true”, it is easy to see how, from a formal point of view, the discussion regarding the logic of divine foreknowledge is essentially the same discussion as the modern discussion concerning future contingency. Given that God knows all and only the truths, this understanding of “true” seems to be straightforward.

The argument mentioned above has been presented in several ways during the long history of philosophical logic. It is an argument which can be traced back to Aristotelian and Stoic logic and which was taken up in Scholasticism and reformulated in theological terms. In its medieval form it was briefly sketched by Richard of Lavenham (c. 1380) in the context of his attempt at giving a systematic overview covering all possible responses to the contingency problem. The general structure of the medieval argument can then be represented in a number of steps. In this sequence  $E$  is some event, which may or may not take place tomorrow (e.g. a sea-battle). Non- $E$  is just a state of affairs without  $E$  occurring.  $E$  and non- $E$  are supposed to be mutually exclusive. Moreover, it is assumed throughout that God's knowledge equates plain truth (i.e. ‘God knows that  $p$  if and only if  $p$  is true’):

1. Either  $E$  is going to take place tomorrow or non- $E$  is going to take place tomorrow. (Assumption).
2. If a proposition about the past is true, then it is now necessary, i.e. inescapable or unpreventable. (Assumption).
3. If  $E$  is going to take place tomorrow, then it was true yesterday that  $E$  would take place in two days. (Assumption).
4. If  $E$  is going to take place tomorrow, then it is now necessary that yesterday  $E$  would take place in two days. (Follows from 2. and 3.).
5. If it is now necessary that yesterday  $E$  would take place in two days, then it is now necessary that  $E$  is going to take place tomorrow. (Assumption).
6. If  $E$  is going to take place tomorrow, then  $E$  is necessarily going to take place tomorrow. (Follows from 4. and 5.).
7. If non- $E$  is going to take place tomorrow, then non- $E$  is necessarily going to take place tomorrow. (Follows by the same kind of reasoning as 6.).
8. Either  $E$  is necessarily going to take place tomorrow or non- $E$  is necessarily going to take place tomorrow. (Follows from 1., 6. and 7.).
9. Therefore, what is going to happen tomorrow is necessarily going to happen. And, in consequence, there is no proper freedom of choice. (Follows from 8.).



Richard of Lavenham accepted the validity of this argument, and he pointed out that there are four possible reactions to it.

- a) The argument (including its premises) is accepted, and the doctrine of proper human freedom is rejected, which is clearly equivalent to fatalism (First possibility).
- b) Denial of the doctrine that God knows all truths about the contingent future. (Second possibility).
- c) The claim that in general no truth about the contingent future has yet been decided. (Third possibility).
- d) Rejection of the necessity of the past in general. (Fourth possibility).

Richard of Lavenham himself rejected the first and the second possibility, a) and b), as contrary to the Christian faith. He insisted that there are future contingents and that God knows them. It seems that Richard of Lavenham, like William of Ockham (c. 1285-1349), regarded the Aristotelian approach to propositions concerning the contingent future as being equivalent with the third possibility, c), according to which some propositions about the contingent future are neither determinately true nor determinately false. A number of scholastic logicians favoured this possibility, for instance Peter Aureole (c.1280-1322). Richard of Lavenham, however, rejected this position. He preferred the fourth possibility, d), and he argued that by rejecting the necessity of the past as a general principle the doctrines of free will and God's foreknowledge of the contingent future can be united in a consistent manner. This solution was first formulated by Ockham, although some of its elements can already be found in Anselm of Canterbury (1033-1109). It is also interesting that, much later, Leibniz (1646-1711) worked with a similar idea as a part of his metaphysics. (See Øhrstrøm 1984). The point is that although past events according to Ockham should be regarded as necessary in the sense of being now unpreventable, there are on the other hand true statements in the past tense which should not be regarded as necessary.

The most characteristic feature of Richard of Lavenham's (and William of Ockham's) solution is the concept of 'the true future'. The view is that God possesses certain knowledge not only of the necessary future, but also of the contingent future. This means that, among the possible contingent futures, there must be one which has a special status, namely that it corresponds to the course of events which is really going to happen or take place in the future. This line of thinking may be called 'the medieval solution', even though other approaches certainly existed. Its justification is partly the observation that the notion of 'the true future' is the specifically medieval contribution to the discussion, and partly that leading medieval logicians regarded this solution as the best one. Richard of Lavenham himself called it 'opinio modernorum', i.e., the opinion of the modern people.



A later contribution by the Jesuit Luis Molina (1535-1600) is relevant for a modern interpretation of the fourth possibility, d). Molina's ideas have been thoroughly discussed in (Craig 1988). Molina's special contribution is the idea of (God's) "middle knowledge", which captures the idea of divine foreknowledge without loss of free will in an unusually succinct way: "...the third type [of divine knowledge] is *middle* knowledge, by which, in virtue of the most profound and inscrutable comprehension of each free will, He saw in His own essence what each such will would do with its innate freedom were it to be placed in this or that or indeed in infinitely many orders of things --- even though it would really be able, if it so willed, to do the opposite" (quoted from (Craig, 1988), p. 175). Craig goes on to explain it as follows: "Thus, whereas by His natural knowledge God knows that, say, Peter when placed in a certain set of circumstances *could* either betray Christ or not betray Christ, being free to do either under identical circumstances, by His middle knowledge God knows what Peter *would* do if placed under those circumstances" ((Craig, 1988), p. 175).

Obviously, Richard of Lavenham knew that William of Ockham had discussed the problem of divine foreknowledge and human freedom in his work *Tractatus de praedestinatione et de futuris contingentibus* (see (Øhrstrøm, 1983) and (Tuggy, 1999)). Ockham asserted that God knows the truth or falsity of all future contingents, but he also maintained that human beings can choose between alternative possibilities. In his *Tractatus*, he argued that the doctrines of divine foreknowledge and human freedom are compatible. Richard of Lavenham made a remarkable effort to capture and clearly present the logical features of Ockham's system as opposed to (what was assumed to be) Aristotle's solution (i.e., the third possibility, c)). Richard of Lavenham's classification of the solutions to the problem of future contingents can be translated into a non-theological language: Identifying "truth" with "God's knowledge" in Richard of Lavenham's analysis, the various positions included in his work can be listed in the following way:

1. There are no future contingents i.e. statements about the future are either impossible or necessary.
2. There are future contingents. But no future contingent is true.
3. There are future contingents. But future contingents in general are neither determinately true nor determinately false.
4. There are future contingents, and all future contingents have truth-values ('true' or 'false'), although these truth-values are still unknown to us.

It is interesting that this list of possible solutions largely covers the positions discussed in modern temporal logic in the tradition of A.N. Prior's founding works, mainly his (Prior, 1967). Position 2 in the above list comes close to what Prior called "the Peircean solution". Position 3 bares several resemblances with the solution Prior labelled as "Ockhamism" according to which the truth-value of a future contingent at a moment depends on the histories passing through the moment in question. Position 4 is the position by William of Ockham, Richard of Lavenham and many others. In a modern context, this position has been called "the theory of



the thin red line” or sometimes “the true futurist theory”. This theory has been strongly criticized by several writers. By holding that a future contingent may be true at the present moment, the true futurist differs slightly from a Priorian Ockhamist according to whom the truth-value of a future contingent will depend on the choice of history (or chronicle). In fact, it may be argued that William of Ockham himself was not an Ockhamist in Prior’s sense, but rather a true futurist, since he held that God knows today what is going to happen in the future. About this divine foreknowledge, William of Ockham stated:

... the divine essence is an intuitive cognition that is so perfect, so clear, that it is an evident cognition of all things past and future, so that it knows which part of a contradiction [involving such things] is true and which part is false. ((William of Ockham, 1969), p.50)

However, William of Ockham had to admit that this is not very clear. In fact, he maintained that it is impossible to clearly express the way in which God knows future contingents. He also had to conclude that in general the divine knowledge about the contingent future is inaccessible. God is able to communicate the truth about the future to us, but if God reveals the truth about the future by means of unconditional statements, the future statements cannot be contingent anymore. Hence, God's unconditional foreknowledge regarding future contingents is in principle not revealed, whereas conditionals can be communicated to the prophets. Even so, that part of divine foreknowledge about future contingents, which is not revealed, must also be considered as true according to William of Ockham.

In the following section, a formal version of the above medieval argument for determinism will be presented without theological references. It will be pointed out that there are two premises used in the argument which are obvious candidates for questioning.

## 2. A FORMALISATION OF THE MEDIEVAL ARGUMENT

In the following, I will make use of the branching time semantics and tempo-modal formalism described by Alberto Zanardo elsewhere in this volume. (See also (Burgess, 1980), (Zanardo, 2006) and (Barcellan and Zanardo, 1999).) Time is conceived as a set of *moments*, *TIME*, partially ordered by a earlier-later relation,  $<$ . A linear (*i.e.* totally ordered and maximal) subset of *TIME* is called a *chronicle* or a *history*.

I will, however, make one rather simple extension to Zanardo’s language introducing time units in the tense operators:

$F(x)$ : “in  $x$  time units it will be the case that ...”

$P(x)$ : “ $x$  time units ago it was the case that ...”

I will also make use of the necessity operator,  $\Box$ . It is essential to notice that the necessity at stake here is *historical necessity*. This means that what is not necessary at one moment may



become necessary at another moment. Instead of speaking about what is necessary we might – as already hinted at – talk about what is now *inevitable*, *inescapable*, or *unpreventable*.

The argument may be based on the following five principles, where  $p$  and  $q$  represent arbitrary well-formed statements within the logic:

$$(P1) F(y)p \supset P(x)F(x)F(y)p$$

$$(P2) \Box(P(x)F(x)p \supset p)$$

$$(P3) P(x)p \supset \Box P(x)p$$

$$(P4) (\Box(p \supset q) \wedge \Box p) \supset \Box q$$

$$(P5) F(x)p \vee F(x)\sim p$$

(P5) may be read as a version of the principle of the excluded middle ('tertium non datur'), although it does not take the exact form of  $p \vee \sim p$ , which is usually identified with the principle of the excluded middle. In order to avoid confusion, we shall use the modified name, 'future excluded middle', for (P5).

Regarding (P1–2) it should be noted that these two principles could be deduced if the following equivalence is adopted as an axiom:

$$P(x)F(x)p \equiv p$$

However, this equivalence also entails the theorem  $p \supset P(x)F(x)p$  which is clearly stronger than (P1).

Let  $q$  stands for some atomic statement such that  $F(y)q$  is a statement about the contingent future. Formally, then, the argument goes as follows:

$$(1) F(y)q \supset P(x)F(x)F(y)q \quad (P1)$$

$$(2) P(x)F(x)F(y)q \supset \Box P(x)F(x)F(y)q \quad (\text{from P3})$$

$$(3) F(y)q \supset \Box P(x)F(x)F(y)q \quad (\text{from 1 \& 2})$$

$$(4) \Box(P(x)F(x)F(y)q \supset F(y)q) \quad (\text{from P2})$$

$$(5) F(y)q \supset \Box F(y)q \quad (\text{from 3, 4, P4})$$

Similarly, it is possible to prove

$$(6) F(y)\sim q \supset \Box F(y)\sim q$$



The second part of the main proof is carried out in the following way:

(7)  $F(y)q \vee F(y)\sim q$  (from P5)

(8)  $\Box F(y)q \wedge \Box F(y)\sim q$  (from 5,6,7)

Remember now that  $q$  may stand for any atomic proposition, including statements about human actions. Therefore, (8) is equivalent to a claim of determinism i.e. it implies a denial of the assumption of human freedom of choice — whatever happens, or fails to do so, does so with necessity. So if one wants to preserve the idea of human freedom as it was conceived by the medieval logicians, at least one of the above principles (P1–P5) has to be rejected.

A.N. Prior constructed two systems showing how that can be done, namely the Peircean system in which P5 is rejected (see also (Burgess, 1980)) and the Ockhamist system in which P3 is rejected. It is well known that each of these systems provides a solution to the future contingency problem. Since Prior, several philosophers have discussed which one of these systems should be accepted, or whether other and more attractive systems dealing with the problem can be constructed.

The rejection of (P5) is very problematic. From a common sense point of view, it seems obvious that one of the propositions  $F(y)q$  and  $F(y)\sim q$  must be true. Let  $q$  stand for my going to the cinema. Clearly, it seems straightforward that if it is false now that I am going to the cinema tomorrow, it must be true now that I am not going to the cinema tomorrow. On the other hand, Prior has convincingly demonstrated that the Peircean system with its denial of (P5) is conceivable. According to the Peircean system, the future should simply be identified with the necessary future. More precisely, to say something about the future is to say something about the necessary future. Although the conflation, or identification, of the future with the necessary future makes the position counter-intuitive, A. N. Prior and many of his followers favoured this possibility. The reason is that Prior strongly believed in free choice and held that this freedom is essential for the understanding of the very notion of future. In his *Some Free Thinking about Time*, Prior pointed out that “if something is the work of a free agent, then it wasn't going to be the case until that agent decided that it was” (Copeland 1996, p.48). According to Prior nobody (not even God) can know what a person will freely choose, before the person has made his or her choice. So whatever could make a statement about a future choice by some free agent true now? From Prior's point of view: nothing. For this reason, Prior held, that such statements must be false. – The key question seems to be whether it makes sense to assume the existence of a truth of a statement, which we, in principle, cannot know to be true. If someone says today that I am going to the cinema tomorrow, and I actually make up my mind tomorrow and decides to go to the cinema, then everybody will probably accept the view that the predictor was right. And if he was right when making his prediction, it seems that we have to accept that there was a truth at that time according to which the prediction was true. John MacFarlane (2003, 2008) has suggested an alternative solution to the problem according to which a statement should be relativised to



both a context of utterance and a context of assessment. The context of utterance is the context in which the speech act is made. But the question is whether such theory solves the problem. If we want to hold on to (P5) and if we also maintain that all future contingents are either true or false, then it seems that we are left with something like Prior's Ockhamism or "the true futurist theory".

### 3. PRIOR'S OCKHAMIST SOLUTION

In *Past, Present and Future* Prior presented his so-called Ockhamist system, which accepts P5 but includes a denial of P3 (see (Prior, 1967), p. 126 ff.). In some ways, it is an attractive system, although it is certainly also possible to criticize the Ockhamist position in various respects — as will be shown in the following.

For any wff  $p$  at any time  $t$  and for any chronicle  $c$  with  $t \in c$ , the valuation function of an Ockhamist model,  $Ock(t,c,p)$  can be defined recursively (given a truth-value for any propositional constant at any moment in *TIME*):

- (a)  $Ock(t,c, p \wedge q) = 1$  iff both  $Ock(t,c,p) = 1$  and  $Ock(t,c,q) = 1$
- (b)  $Ock(t,c, \sim p) = 1$  iff not  $Ock(t,c,p) = 1$
- (c)  $Ock(t,c, Fp) = 1$  iff  $Ock(t',c,p) = 1$  for some  $t' \in c$  with  $t < t'$
- (d)  $Ock(t,c, Pp) = 1$  iff  $Ock(t',c,p) = 1$  for some  $t' \in c$  with  $t' < t$
- (e)  $Ock(t,c, \Box p) = 1$  iff  $Ock(t,c',p) = 1$  for all  $c'$  in  $C(t)$  with  $t \in c'$

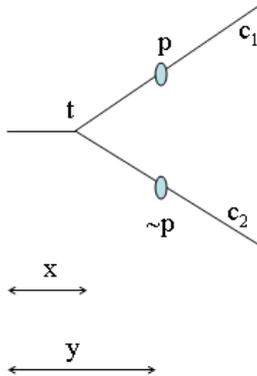
$Ock(t,c,p)$  can be read ' $p$  is true at  $t$  in the chronicle  $c$ '. A formula  $p$  is said to be Ockham-valid if and only if  $Ock(t,c,p)$  for any  $t$  in any  $c$  in any branching time structure,  $(TIME, <, C)$ . It should be noted that (a) – (d) are exactly the same definitions as those used in linear tense-logic.  $C$  is a function from *TIME* into all subsets of chronicles.  $C(t)$  is the set of possible chronicles passing through  $t$ . (In Prior's original formulation of the Ockhamist system all chronicles are regarded as possible. In this case,  $C$  can be constructed from  $(TIME, <)$ , and in consequence there is no need for specifying  $C$  in the structure.)

To obtain a metric version of the Ockhamist system, a duration function has to be added. Let  $dur(t_1, t_2, x)$  stand for the statement ' $t_1$  is  $x$  time units before  $t_2$ '. Using this formalism, (c) and (d) are replaced by:

- (c')  $Ock(t,c, F(x)p) = 1$  iff  $Ock(t',c,p) = 1$  for some  $t' \in c$  with  $dur(t, t', x)$
- (d')  $Ock(t,c, P(x)p) = 1$  iff  $Ock(t',c,p) = 1$  for some  $t' \in c$  with  $dur(t', t, x)$



It can be verified that neither  $P(x)q \supset \Box P(x)q$  nor  $Pq \supset \Box Pq$  are Ockham-valid for all  $q$ . Let for instance  $q$  stand for  $F(y)p$ . It is easy to verify that  $P(x)F(y)p \supset \Box P(x)F(y)p$  will not hold in general in an Ockhamistic branching time model. This may be illustrated using the following diagram, in which it is easily seen that  $Ock(t, c_1, P(x)F(y)p) = 1$ , whereas  $Ock(t, c_1, \Box P(x)F(y)p) = 0$  since  $Ock(t, c_2, P(x)F(y)p) = 0$ .



This does away with P3 in the formal version of the medieval argument discussed above. Still, both formulas,  $P(x)q \supset \Box P(x)q$  and  $Pq \supset \Box Pq$ , will hold if  $q$  does not contain any reference to the future.

If (P3) does not hold in general, one may reject 2 in the informal argument stated in the first section of this paper. According to Ockham, (P3) (that is, its verbal analogue as he could formulate it with the means then available) should only be accepted for statements which are genuinely about the past, i.e., the truth-values of which do not depend on the future. According to this view, (P3) may be denied precisely because the truth of statements like  $P(x)F(x)F(y)q$  has not been settled yet — since they depend on the future.

In this way, one can make a distinction between soft facts and hard facts regarding the past (see (Plantinga 1986)). Following the Ockhamist position, a statement like  $P(x)q$  would correspond to a hard fact, if  $q$ 's truth-value does not depend on the future, whereas statements like  $P(x)F(x)F(y)q$  would represent soft facts. Critics of the Ockhamist position, however, may still say that if  $F(x)F(y)q$  was true  $x$  time units ago, then there must have been something making it true at that time, and that something must have been a 'hard' fact. This clearly makes the distinction between soft and hard facts rather complicated.

In addition it may be disputed that Prior's Ockhamist system fits the ideas formulated by William of Ockham completely. Although many of Ockham's original ideas are satisfactorily modelled in Prior's Ockhamist system, Prior's system lacks a proper representation of the notion of 'the true future'. This was in fact one of the most basic ideas in Ockham's worldview. Ockham believed that there is truth (or falsity) also of statements about the contingent future, which human beings cannot know, but which God knows. Prior's Ockhamist system cannot be said to include more than the idea of a proposition being true relatively to a moment of time

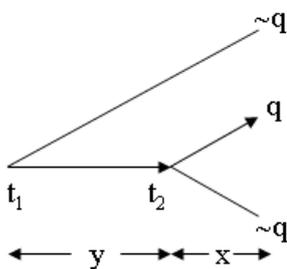


and a chronicle. A proper theory in accordance with William of Ockham's ideas would have to include the idea of a proposition being true relatively to a moment of time (without any specification of a chronicle and of a given selected history). Let us therefore investigate a truth-theory, which includes the idea of a true future in this sense.

The rationality of Ockham's suggestion according to which human beings can (in a very limited sense) influence the past, has been defended by Alvin Plantinga (1986). It should also be mentioned that Ockham's theory as stated above is relevant for the conceptual analysis of the idea of prophecy.

#### 4. THE TRUE FUTURIST THEORY: THE THIN RED LINE

In terms of modern logic and a branching time model, the medieval assumption of the true future can be rendered as meaning that there is a privileged branch at any past, present or future branching point in the model. Consider, for instance, the following model.



In this model,  $F(x)q$  is true at  $t_2$  and  $F(x+y)q$  is true at  $t_1$ , although none of the propositions are necessary, since  $F(x) \sim q$  is possible at  $t_2$ . The reason why  $F(x)q$  is true at  $t_2$  is just that the evaluation of a proposition according to the true futurist theory should be based on the specified branch though  $t_2$  representing 'the future' at  $t_2$  within the model. But what makes the specified branch privileged? Is it merely that it represents what is going to happen? Is there anything in the present situation,  $t_2$ , which makes one branch ontologically special as opposed to the other branches? It might be tempting to refer to some sort of a 'wait-and-see' status of the privileged branch. However, as MacFarlane (2008) has recently argued such a notion very easily leads to confusion. On the other hand, although the true futurist theory does contain some intricate notions, it has not been shown to be inconsistent, and a supporter of the theory may still hold that the theory correctly explains what reality is like. But of course, it should be borne in mind that true futurist theory was introduced exactly to avoid certain counter-intuitive tenets. Therefore, it should be carefully considered which approach ultimately leads to the least problems.

It would of course be fatal for the true futurist theory if it could be demonstrated that it contradicts assumptions, which we should accept for other reasons. Belnap and Green (1994) have argued that there are in fact such fundamental problems related to the true futurist



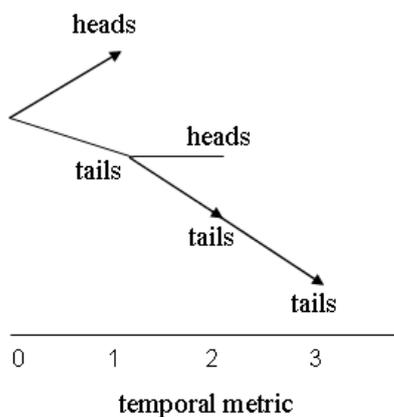
picture. They have tried to demonstrate that the very idea of the true futurist model should be rejected for conceptual reasons — or perhaps even for logical reasons. They have argued that it follows from the true futurist view that it is not sufficient for the model to specify a preferred branch corresponding to the true history (past, present, and future). Belnap and Green argued that in order to maintain a concept of the future, which is “middle ground” between the possible future and the necessary future, it must be assumed that there is a preferred branch at *every* counterfactual moment. They have illustrated their view using the following statement:

“The coin will come up heads. It is possible, though that it will come up tails, and then later it will come up tails again (though at this moment it could come up heads), and then, inevitably, still later it will come up tails yet again.” ((Belnap & Green, 1994), p. 379)

This statement may be represented in terms of tense logic with  $\tau$  representing tails and  $\eta$  heads, respectively:

$$F(1)\eta \wedge \Diamond F(1)(\tau \wedge \Diamond F(1)\eta \wedge F(1)(\tau \wedge \Box F(1)\tau))$$

and in terms of the following branching time structure:



The example shows that if the model is taken seriously, then there must be a function  $TRL$ , which gives the true future for any moment of time,  $t$ . More precisely,  $TRL(t)$  yields the linear past as well as the true future of  $t$ , extended to a maximal set; Belnap and Green call it “the thin red line”. But how can  $TRL(t)$  be specified? Belnap and Green have argued that:

$$(TRL1) \quad t \in TRL(t)$$

should hold in general. Moreover, they have also maintained that:

$$(TRL2) \quad t_1 < t_2 \supset TRL(t_1) = TRL(t_2)$$



should hold for the *TRL*-function. On the other hand, they have argued that the combination of (TRL1) and (TRL2) is inconsistent with the very idea of branching time. The reason is that if (TRL1) and (TRL2) are both accepted, it follows from  $t_1 < t_2$  that  $t_2 \in TRL(t_1)$ , i.e. that all moments of time after  $t_1$  would have to belong to the thin red line through  $t_1$ , which means that there will in fact be no branching at all. However, it is very hard to see why a true futurist would have to accept (TRL2), which seems to be too strong a requirement. Rather than (TRL2), the weaker condition (TRL2') can be employed:

$$(TRL2') (t_1 < t_2 \wedge t_2 \in TRL(t_1)) \supset TRL(t_1) = TRL(t_2)$$

This seems to be much more natural in relation to the notion of a true futurist branching time logic. Belnap has later accepted that (TRL2') is a relevant alternative to (TRL2) (see ((Belnap et al. 2001) p.169).

Belnap and Green have also argued that any such *TRL*-function should give rise to a logic in which the following theorems hold:

$$(T1) PPq \supset Pq$$

$$(T2) FFq \supset Fq$$

$$(T3) q \supset PFq$$

Belnap and Green state no formal semantics, but they seem to assume that the tense operators are interpreted only relatively to a moment of time. This amounts to interpreting tenses using a two-place valuation operator:

$$T(t, Pq) = 1 \text{ iff } \exists t': t' < t \ \& \ T(t', q) = 1$$

$$T(t, Fq) = 1 \text{ iff } \exists t': t < t' \ \& \ t' \in TRL(t) \ \& \ T(t', q) = 1$$

With such a semantics and (TRL2'), (T1) and (T2) are valid. However, with this semantics and (TRL2'), (T3) will not be valid. To see why this is the case, consider a situation with a moment of time  $t$  such that  $t \notin TRL(t')$  for any  $t' < t$ . Assume that  $t$  is the only moment at which  $q$  is true. Then  $PFq$ , hence also  $q \supset PFq$ , will be false at  $t$ .

Even the formula

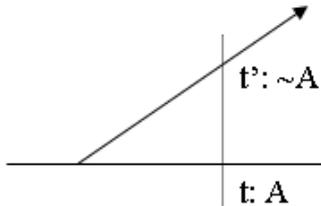
$$(T3') q \supset P(x)F(x)q$$

is false when evaluated with the following semantics:



$$T(t, P(x)q) = 1 \text{ iff } \exists t': \text{dur}(t, t', x) \ \& \ T(t', q) = 1$$

$$T(t, F(x)q) = 1 \text{ iff } \exists t': \text{dur}(t', t, x) \ \& \ t' \in \text{TRL}(t) \ \& \ T(t', q) = 1$$



With this interpretation of the tenses, (T3') becomes invalid as illustrated above. (The vertical line in this diagram represents a set of co-temporaneous moments i.e. what is sometimes called an instant.)

It is, however, possible to ensure the validity of (T3) even if one wants to insist on the assumption of the 'thin red line' by using the system described in (Braüner et al. 2000). Adopting Belnap and Green's basic idea, a function  $TRL$  is defined which to each moment assigns a branch such that the conditions (TRL1) and (TRL2') are satisfied. A novel feature of this semantics is the notion of a (counterfactual) branch with the following property: At any future moment, it coincides with the corresponding thin red line. Given a moment  $t$ , the set  $C(t)$  of such branches is defined as follows:

$$C(t) = \{c \mid t \in c \ \& \ \text{TRL}(t') = c, \text{ for any } t' \in c \text{ with } t < t'\}$$

Note that (TRL1) and (TRL2') together say exactly that  $TRL(t) \in C(t)$ . Also note that  $C(t)$  may contain more branches than just  $TRL(t)$ . This allows for counterfactuality. In this semantic model truth is relative to a moment of time,  $t$ , as well as to a branch belonging to  $C(t)$ . By induction, the valuation operator  $T$  is defined as follows:

$$T(t, c, p) = 1 \quad \text{iff } T(t, p) = 1 \text{ where } p \text{ is a propositional letter}$$

$$T(t, c, p \wedge q) = 1 \text{ iff } T(t, c, p) = 1 \text{ and } T(t, c, q) = 1$$

$$T(t, c, \sim p) = 1 \quad \text{iff not } T(t, c, p) = 1$$

$$T(t, c, Fp) = 1 \quad \text{iff } T(t', c, p) = 1 \text{ for some } t' \in c \text{ with } t < t'$$

$$T(t, c, Pp) = 1 \quad \text{iff } T(t', c, p) = 1 \text{ for some } t' \in c \text{ with } t' < t$$

$$T(t, c, \Box p) = 1 \quad \text{iff } T(t, c', p) = 1 \text{ for all } c' \in C(t)$$

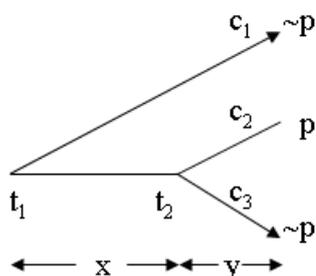
A formula  $p$  is said to be valid if and only if  $p$  is true in any structure  $(TIME, <, T, \text{TRL})$  for any moment of time  $t$  and branch  $c$  such that  $c \in C(t)$ . The tense operators  $P$  and  $F$  are interpreted as usual in Ockhamist semantics. It is straightforward to introduce metrical tense operators. With this semantics, all of the formulas (T1), (T2), and (T3) are valid. This shows that even if we



accept Belnap and Green's view that (T1–T3) should hold in any reasonable true futurist theory, no strong argument against the position has been established, since there is in fact a structure  $(TIME, <, T, TRL)$  according to which (T1–T3) hold. In consequence, Belnap's and Green's analysis does not give rise to any logically inescapable argument against the true futurist position. On the other hand, the possibility operator in the model described in [Braüner et al. 2000] may be somewhat surprising; in the sense that it seems to mean that relatively few (counterfactual) branches are taken into account. In the obvious metrical extension of the system, this invalidates the formula:

$$(T4) \quad F(x)\diamond F(y)p \supset \diamond F(x)F(y)p$$

— which is valid in the usual Ockhamist semantics. The rejection of (T4) may be illustrated with reference to the following model:



Clearly, at  $t_2$  the proposition  $\diamond F(y)p$  holds, since  $C(t_2)=\{c_2, c_3\}$ . This means that  $F(x)\diamond F(y)p$  is true at  $t_1$ . However, the proposition  $\diamond F(x)F(y)p$  is false at  $t_1$ , since  $c_2$  is not included in  $C(t_1)$ . (It is easily verified that  $C(t_1)=\{c_1, c_3\}$ .) This means that (T4) is false in this model at  $t_1$ . This rejection of (T4) amounts to the following idea: Tomorrow some possibilities regarding the following day may emerge even though, today, these possibilities are not available regarding the day after tomorrow. In other words, new possibilities may show up. — However, in order to establish a formal and convincing argument against the true futurist theory, Belnap, Green, and others may of course question the rejection of (T4). However, if insisted that (T1 – T4) should all hold in any acceptable true futurist theory, then in order to have a valid argument, it should first of all be demonstrated that no structure  $(TIME, <, T, TRL)$  can meet the extended requirement. In addition, a convincing philosophical argument should be provided to the effect of showing that (T4) should be included in the set of requirements. As long as no such arguments have been established, the true futurist position must be regarded as a possible answer to the problem of future contingency.



#### 4. CONCLUSION

The medieval analysis of the classical argument about the inconsistency of the doctrines of divine foreknowledge and human freedom, respectively, can be confirmed using modern temporal logic. The modern analysis also reveals the same obvious responses as the medieval analysis. As we have seen, Prior's Peircean solution certainly gives rise to some conceptual problems. If this solution is ruled out for such reasons, and the principle of future excluded middle as well the principle of all future contingents being true or false, then we are left with the true futurist theory (and the idea of the thin red line), unless we want to accept determinism. As we have seen, however, although the true futurist theory has been criticised by several writers, all known arguments against the theory appear to be rather weak. It has been shown that the theory can meet even rather strong requirements. So far, nobody argued convincingly against the theory. On the contrary it seems that the theory can be defended against all attacks so far. For this reason, the true futurist theory (and the idea of the thin red line) should be taken into serious consideration when dealing with the problem of future contingency.

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