Cambridge Handbook of Semantics

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Plurality

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1.1 Introduction

It is easy to indicate what plurality is from a morphological point of view: Plurality is an instance of the inflectional category of number. From a semantic point of view, however, the concept of plurality is much more diffuse. To start with, the general folk linguistic intuition that being plural expresses being *more than one* is inaccurate. For example, while (1-a) suggests that there are multiple stains on the carpet, (1-b) does not suggest that only the carrying of *multiple* guns is illegal, nor does (1-c) suggest it might be the case that there has been a solitary alien walking the earth.

(1) a. There are stains on the carpet.
 b. Carrying guns is illegal in Illinois.
 c. No aliens have ever walked the earth.

Furthermore, plurality may emerge from sentences that lack any plural morphology, such as the first sentence in (2). Despite the morphologically singular *a picture*, this sentence expresses that a multitude of pictures was drawn, each by one of the boys. This emergent plurality becomes apparent in the second sentence in which a plural pronoun refers to the pictures that the boys drew.

(2) Each boy drew a picture. They are hanging on the wall.

It is sometimes thought that, in languages like English (and indeed Indo-European languages, generally), plurality is an essentially nominal phenomenon. Corbett 2000, for instance, uses (3) to point out that the plural verbal form must be an uninterpreted agreement reflex triggered by the (unmarked) plurality of the noun. We cannot for instance use (3) to express that a single sheep drinks from the stream more than once. It has to mean that there were multiple sheep.

(3) The sheep drink from the stream. Corbett (2000)

In other languages verbal number marking does have a semantic substance. For instance, (4), an example from Hausa from Součková (2011), is incompatible with there being a single kick to the table, and typically expresses a quick repetition of kicks. This interpretation is triggered by partial reduplication of the verb stem.

(4) Yaa shùs-shùuri teebùr̃
3SG.MASC REDUP-kick table
'*He kicked the table repeatedly*'

Plurality involving the multiplicity of events and some kind of verbal morphological marking is often referred to as *pluractionality* (Newman, 1980). One difficult question is how (or whether, Corbett 2000) pluractionality differs from aspectual marking or Aktionsart.[a] In this chapter, I will ignore the phenomena of pluractionality and focus on nominal plurality as found in English.[b]

The observation that, in English, plural marking has obvious semantic effects in the nominal, but not in the verbal domain, should however not be mistaken for a deeper semantic conclusion, namely that there is nothing to study beyond plural reference. In fact, it turns out that the most puzzling data involving plurality concern exactly the interaction between plural arguments and verbs. This can be illustrated by the following observation: predicates differ with respect to the entailment patterns they display for plural arguments. Take the *distributivity* entailment pattern in (5).

(5) A predicate "VP" is *distributive* if and only if "X and Y VP" entails "X VP" and "Y VP"

Being wounded is an example of a predicate that clearly supports the entailment scheme in (5), for (6-a) entails both (6-b) and (6-c).

(6) a. Bob and Carl are wounded.

[a] See Wood (2007) for discussion.
[b] See Beck and von Stechow (2007) for a discussion of English adverbials that can be analysed as having apluractional function.

b. Bob is wounded.
c. Carl is wounded.

For other predicates, the pattern in (5) does not hold. Consider, the following examples.

(7) a. Bob and Carl carried a piano up the stairs.
b. Bob carried a piano up the stairs.
c. Carl carried a piano up the stairs.

In a situation in which both (7-b) and (7-c) are true, (7-a) is true too. However, (7-a) is also true in a situation in which Bob and Carl jointly carried the piano up. In that case, it would be false to say (7-b) or (7-c). It turns out then that whilst *to carry a piano up the stairs* is not distributive, it does have the inverse property:

(8) A predicate "VP" is *cumulative* if and only if "X VP" and "Y VP" entails "X and Y VP"

Note that *being wounded*, besides begin distributive, is cumulative, too. It follows from (6-b) and (6-c) that (6-a). Yet other predicates are neither distributive nor cumulative:

(9) a. Bob and Carl are a couple.
b. *Bob is a couple.
c. *Carl is a couple.

In sum, the kind of entailment patterns we can observe for plural arguments depends on the predicate (among other things such as the presence of certain adverbs, floated quantifiers etc., some of which will be discussed below). Plurality will therefore need to be studied in a compositional context. It is this interaction between plural arguments and predicates that is the focus of this chapter. The structure is as follows. In section 1.2, I will introduce the most commonly assumed complication that plurality brings to the logical language, namely the inclusion of individual constants that are not atomic but *sums*. Using such plural individuals, in section 1.3 I turn to distributive entailments, as well as to distributive *readings* of sentences. Section 1.4 turns to a special class of interpretations for sentences with plurality, namely cumulative readings, and the relation this bears to distributivity. Section 1.5 looks at dependency phenomena involving plurals. Finally, in section 1.6, I conclude by briefly touching upon some other issues in the semantics of plurality.

1.2 Sums

Let us consider the simple statement in (10) again.

(10) Bob and Carl are wounded.

Given the observed distributivity entailment of *being wounded*, it is tempting to analyse (10) as (11). Not only does (11) yield the intuitively correct truth-conditions, it does so without complicating the predicate-logical language. That is, plurality in natural language is *distributed* over several conjuncts of essentially singular predications.

(11) $wounded(b) \wedge wounded(c)$

The discussion on entailment patterns above, however, shows that this kind of analysis is not very promising, for it would fail on predicates that lack distributive entailments. For instance, analysing (12-a) as (12-b) incorrectly predicts that (12-a) is false on a scenario where Bob and Carl collectively carry a piano up the stairs.

(12) a. Bob and Carl carried a piano up the stairs.
b. $[\![\text{carried a piano up the stairs}]\!]\,(b) \wedge$
$[\![\text{carried a piano up the stairs}]\!]\,(c)$

The common solution is to assume that the phrase *Bob and Carl* refers to a plural individual and that this individual is, as a single collective individual, the argument of the predicate. It would not do, however, to just assume that the conjunction Bob and Carl is a name for some plural individual. There must be some connection between the plural individual that *Bob and Carl* refers to and the individual entities Bob and Carl. This is because, ultimately, we want an account of how certain predicates allow for distributivity and/or cumulativity entailments that link statements involving plural arguments to statements involving the singular parts. Here we can just assume that pluralities are to singularities what sets are to their members. That is, if D_e is the set of singular entities, then the set of plural individuals will be the powerset of this domain with the empty set removed: $\wp(D_e)\setminus\emptyset$. We can now say that *Bob and Carl* refers to the smallest set that has Bob and Carl as members. So, for (12-a) we get (13).

(13) $[\![\text{carried a piano up the stairs}]\!]\,(\{b, c\})$

While there exist analyses along this line (Hoeksema (1983), Gillon (1987), Schwarzschild (1996), Winter (2002)), a more common alternative is to use an approach inspired by the work of Link. (See especially Link (1983). See Krifka (1989) and Landman (1989), Landman (1996), Landman (2000) for foundational works building in part on Link.) In Link's approach there is no type-theoretic difference between plural and singular individuals. So, plural individuals are e-type entities just like singular individuals are. However, the domain of entities is structured exactly like the powerset of the set of atomic individuals.

We can set this up as follows. Let D_e be the set of all (i.e. both singular and plural) entities and let $\sqcup$ be a binary operation on elements of D_e, called *summation*. Summation has the following properties:

(14) a. $(\alpha \sqcup \beta) \sqcup \gamma = \alpha \sqcup (\beta \sqcup \gamma)$ associative
b. $\alpha \sqcup \beta = \beta \sqcup \alpha$ commutative
c. $\alpha \sqcup \alpha = \alpha$ idempotent

Given summation, we now define a partial order $\leq$ on D_e:

(15) $\alpha \leq \beta$ if and only if $\alpha \sqcup \beta = \beta$

The idea is that just like $\{b, c\}$ is the smallest set that contains b and c as elements, so is $b \sqcup c$ – i.e. the *sum* of b and c – the smallest entity that has b and c as its parts. This means that $\sqcup$ is a supremum or *join* operator. The desired structure is a particular kind of lattice, namely a complete atomic join semi-lattice (see chapter 6 of Link (1998) as well as Landman (1991) for details). *Complete* since we want the domain of entities to be closed under $\sqcup$; *atomic* since we also want that all atomic parts of sums in the domain are part of the domain themselves. (If we can talk about *Bob and Carl*, then we can also talk about *Bob*.) Finally, it is a *semi*-lattice, since we are only interested in joins, not in meets. An example of a complete atomic join semi-lattice for three atoms a, b and c is given in figure 1.1.

In (16), I give two handy definitions inspired by Link's work. First of all (16-a) says that something is an atomic entity if and only if it only has itself as a part. The operation in (16-b) restricts a set of individuals to the ones that are atomic.

(16) a. $Atom(\alpha)$ if and only $\forall \beta \leq \alpha[\alpha = \beta]$
b. $Atoms(A) = \lambda \alpha . \alpha \in A \wedge Atom(\alpha)$

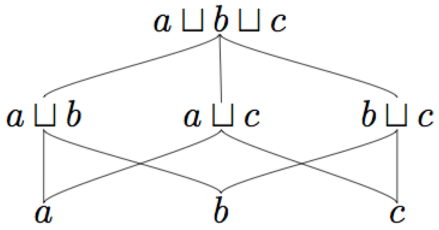

Figure 1.1 A depiction of the complete atomic join semi-lattice with a, b and c as the atomic elements. The arcs represent the $\leq$ relation.

Given these definitions, we can formally express the correspondence between the lattice structure assumed by Link and the powerset structure we were aspiring to:

(17) $\langle D_e, \sqcup \rangle$ is isomorphic to $\langle \wp(Atoms(D_e)) \setminus \emptyset, \cup \rangle$

The operator most borrowed from Link's work is the sum closure operator '*' for sets or one-place predicates:

(18) $^*X$ is the smallest set such that:
$^*X \supseteq X$ and
$\forall x, y \in {^*X} : x \sqcup y \in {^*X}$

Link reasoned that while singular nouns referred to sets of atoms (i.e. elements in $\wp(Atoms(D_e))$), plural nouns refer to sets that include plural individuals. (We will ignore for now the hairy issue of whether or not such plural nouns also contain atomic entities in their extension. See below for discussion.)

(19) $[\![boys]\!] = {^*}[\![boy]\!]$

So, if $[\![boy]\!]$ is the set $\{b, c, d\}$, then $[\![boys]\!]$ is the set $\{b, c, d, b \sqcup c, c \sqcup d, b \sqcup d, b \sqcup c \sqcup d\}$. The example (20-a) is now predicted to be true, by virtue of the fact that the plural individual $b \sqcup c$ is in the pluralised – i.e. *-ed – extension of the noun *boy*, which we could express in a predicate logical language as in (20-b).

(20) a. Bob and Carl are boys.
b. $^*boy(b \sqcup c)$

The question is now how to account for predicates with different entailment patterns. For this, we need to have a closer look at *distributivity.*

1.3 Distributivity

The sentence in (21) is true in a number of quite different scenarios. In a *collective* understanding of the sentence, Bob, Carl and Dirk jointly carry a piano up the stairs. Conversely on a *distributive* understanding Bob carried a piano up the stairs by himself, and so did Carl, and so did Dirk.

(21) Bob, Carl and Dirk carried a piano up the stairs.

The distributive understanding can be isolated by adding a floating distributive quantifier like *each*: that is, *Bob, Carl and Dirk each carried a piano up the stairs* only has the distributive reading. With this in mind, predicates with distributive entailments are always understood distributively, since for (22-a) the adding of *each* in (22-b) seems vacuous.

(22) a. Bob, Carl and Dirk are wounded.
b. Bob, Carl and Dirk are each wounded.

In summary, we get the following overview:

(23)

	entailments			understandings	
	distr	cumul	-	distr	collec
being wounded	yes	yes	-	yes	no
to carry a piano up x	no	yes	-	yes	yes

The stable factor in this table is that all examples allow for cumulative inferences and all examples have distributive understandings. Note that Link's *-operation amounts to enforcing cumulative entailments. That is, if predicates with plural arguments are pluralised using *, then cumulativity follows, due to the following fact:

(24) $P(\alpha) \wedge P(\beta) \Rightarrow {}^*P(\alpha \sqcup \beta)$

The inverse entailment does not follow, for ${}^*P(\alpha \sqcup \beta)$ could be the case without $P(\alpha)$ being the case. Just take a P such that $P(\alpha \sqcup \beta)$ is true but not $P(\alpha)$.

Landman (1996) proposes to account for the difference between predicates like *being wounded* and a predicate like *to carry a piano up the* *stairs* by just assuming two things. Firstly, predicates with plural arguments are plural in the sense that they are pluralised using the *-operation. Second, predicates differ with respect to the kind of entities they have in their (singular) extension. For *being wounded* it suffices to assume that its extension only contains atoms. This captures the intuition that it does not make sense for the property of being wounded to be shared among multiple entities. We can now analyse (22-a) as (25).

(25) $^{*}wounded(b \sqcup c \sqcup d)$

The cumulativity entailment follows from (25). The inverse distributivity entailment follows from the fact that the only way a plurality can end up in the pluralised extension of the predicate is if its atomic parts were in the singular extension, for $wounded(\alpha)$ is only true for atomic α's. Another way of saying this is that there cannot be any collective understandings. All plural arguments are the result of the *-operation.

Adopting a similar strategy for (21) results in (26).

(26) * ⟦carried a piano up the stairs⟧ $(b \sqcup c \sqcup d)$

In Landman's framework, the key to accounting for the properties of (26) is to assume that the extension of predicates like *to carry a piano up the stairs* might or might not include atoms and it might or might not include sums; that this fully depends on the state of affairs. In a distributive scenario, b, c and d occur in the extension as singular individuals. Given the effect of pluralisation, (26) is then true in that scenario. In the collective scenario the extension just contains $b \sqcup c \sqcup d$ as a plurality, and hence (26-a) is also true. There is no distributivity entailment, since we cannot know which scenario made (26-a) true.

Contrary to what my presentation above suggests, it is sometimes assumed that what I have vaguely called *understandings* are in fact different *readings* of an ambiguity.[c] On such accounts, (21) is ambiguous between a reading in which the distributivity entailment holds (the distributive reading) and one in which it does not (the collective one). The latter reading is more basic, while the distributive one is derived using the insertion of a distributivity operator. There exist two main variations on such approaches, depending on whether the operator in question is applied to the verb's argument or to the verb itself. Take, for instance, the following two options.

[c] As far as I can see, no consensus on this topic has been reached. In fact, this topic is surprisingly absent from the recent literature.

(27) a. $DIST = \lambda\alpha.\lambda P.\forall\beta \leq \alpha[Atom(\beta) \rightarrow P(\beta)]$
b. $DIST = \lambda P.\lambda\alpha.\forall\beta \leq \alpha[Atom(\beta) \rightarrow P(\beta)]$

In a case like (21), insertion of DIST will result in a reading that carries a distributivity entailment: $\forall x \leq b \sqcup c \sqcup d[Atom(x) \rightarrow [\![\text{carried a piano up the stairs}]\!](x)]$. Hence, this reading is exactly what one would get by adding a floating *each* to (21). For cases like (21) the choice between (27-a) and (27-b) is immaterial. It is easy to find examples, however, where these two operators differ in predictions. Following Lasersohn (1995), a sentence like (28-a) illustrates this. Here, it is possible for one part of the conjoined verb phrase to get a collective reading whilst the other part gets a distributive reading. Using (27-b), this is easy to capture, as in (28-b). However, no easy analysis seems to be available using (27-a).

(28) a. Bob and Carl carried a piano up the stairs and then drank a glass of water.
b. $\left[\begin{array}{ll}\lambda x. & [\![\text{carried a piano up the stairs}]\!]\,(x) \wedge \\ & DIST(\,[\![\text{drank a glass of water}]\!]\,)(x)\end{array}\right](b \sqcup c)$

Assuming we should abandon the option in (27-a), we now have two competing accounts of distributivity: one in the form of a freely insertable operator $DIST$, and one as a semantic repercussion of pluralisation using *, as in Landman (1996, 2000). These accounts are closely related. In fact, it is easy to see that the following correspondence holds:

(29) $DIST(A) = {}^*(Atoms(A))$

There exists some evidence against thinking of distributive understandings as separate interpretations that arise through augmenting a logical form with an additional operator. Schwarzschild (1996) uses one of the classic ambiguity tests from Zwicky and Sadock (1975). Whenever an ambiguous term is elided, it has to be resolved to the same interpretation as its antecedent. As an illustration, take the elaborated version of the famous Groucho Marx joke in (30-a). This sentence can mean that the speaker shot an elephant that was wearing pyjamas and that the speaker's sister shot a similar animal or it means that the speaker shot an elephant whilst wearing pyjamas and that his or her sister shot an elephant in similar attire. Crucially, it cannot mean that the speaker shot an elephant that was wearing pyjamas, whilst the sister shot an elephant while she was wearing pyjamas. Schwarzschild now observes that for sentences like (30-b) such a mixed interpretation is perfectly possible. For instance, (30-b) is intuitively true if each of the computers were paid in two instalments and the entire collection of diskettes were collectively paid in two instalments. (See Schwarzschild for a more elaborate scenario that makes this understanding likely.)

(30) a. I shot an elephant wearing pyjamas and my sister did too.
b. The computers were paid in two instalments and the diskettes were too.

While Schwarzschild's observation strongly suggests that we should not really be talking about *readings*, there is some data that seems to indicate that plural sentences at least involve some covert operator (as, incidentally, is the case in Schwarzschild's own account). One reason to believe that this is the case comes from Heim et al. (1991). Consider (31).

(31) Bob and Carl think they will win €1000.

This example has multiple readings depending on how the pronoun *they* is resolved and whether the verb phrases are construed distributively or collectively. Three of those are in (32).

(32) a. Bob and Carl argue that Bob and Carl, as a group, will win €1000.
b. Bob argues that Bob will win €1000 and Carl argues that Carl will win €1000.
c. Bob argues that Bob and Carl, as a group, will win €1000 and Carl argues the same.

In (32-a), the plural pronoun *they* is understood as co-referring with the matrix subject *Bob and Carl*. This is a fully expected reading. However, the reading in (32-b) is not so trivial to account for. In this case, the pronoun is interpreted as a bound variable. But that means that there has to be some quantificational operator that is responsible for distribution over the atoms in the subject.

While these data are usually thought to indicate the presence of a proper distributivity operator akin to the natural language floating *each* in distributive readings, the observations are not incompatible with a theory in which a single operator accounts for a range of understandings, including Landman's use of *. Also, Schwarzschild 1996 offers an analysis that uses a distributivity operator, without predicting that there are multiple readings. In a nutshell, and simplifying drastically, the idea is that pluralities can be partitioned, i.e. divided up into parts and that the distributivity operator quantifies over such parts.[d] A definition of *minimal sum covers* as well as some examples are given in (33), while (34) defines the distributivity operator.

(33) C minimally covers α iff $\alpha \in {}^*C$ and $\neg\exists C' \subset C[\alpha \in {}^*C']$

a. $\{b, c \sqcup d\}$ minimally covers $b \sqcup c \sqcup d$
b. $\{b, c, d\}$ minimally covers $b \sqcup c \sqcup d$
c. $\{b \sqcup c \sqcup d\}$ minimally covers $b \sqcup c \sqcup d$
d. $\{b \sqcup c, c \sqcup d\}$ minimally covers $b \sqcup c \sqcup d$

(34) $DIST_C(P) = \lambda x.\forall y \in C_x[P(y)]$
where C_x is some pragmatically determined minimal cover of x

This one definition covers a range of readings, depending on what kind of cover is pragmatically given. For instance, (35-a) will be interpreted as (35-b).

(35) a. These six eggs costs €2.
b. $DIST(\lambda x.cost(x, €2))$(the-6-eggs)

On its most salient reading, the relevant cover would be a cover that does not divide the eggs up, but one which just offers a single cell containing the six eggs. This results in a collective understanding. The distributive reading (where these are particularly expensive eggs) results from taking a cover that divides the six eggs up into the individual eggs such that (35-b) ends up expressing that each egg costs two euros. There are other theoretical possibilities, which are however not so salient for the example in (35-a). For an example like (36), however, the relevant cover would be one in which the shoes are divided up in pairs, since, pragmatically, that is how shoes are normally valuated. (See Gillon (1990) for the first indepth discussion of this type of reading.)

(36) The shoes cost $75. Lasersohn (2006)

So far, I have discussed distributive and collective understandings of plural predication, as well as intermediate ones as in (36). I now turn to yet another understanding, *cumulativity*.

[d] See Dotlacil (2010) for a more advanced integration of Schwarzschild's idea in a sum-based approach to plurality. The idea of using *covers* to account for different understandings of plural statements originates in Gillon (1987).

1.4 Cumulativity

In the discussion of distributivity above, we saw that one approach (basically that of Landman 1996, 2000) involves reducing distributivity to pluralisation of $\langle e, t\rangle$ predicates, using Link's *-operation. Notice that in the examples so far we have always *-ed the VP denotation. This is not the only theoretical option. Consider (37).

(37) Two boys carried three pianos up the stairs.

We could imagine quantifier raising the object and thereby creating a derived predicate 'λx.two boys carried x up the stairs'.[e] Using the *-operation, we would then expect there to exist a distributive reading for the object: there are three pianos such that each of these pianos was carried up the stairs by a potentially different pair of boys. This reading is available to some, but certainly not all speakers of English. While there may be linguistic reasons to limit such object distributivity, it shows that distributivity is tightly linked to scope.

One way to make the relevant scopal relations more visible is by counting the number of entities that are involved in a verifying scenario. To do this reliably, let me change the example to (38). (A crucial difference to (37) is that whilst you can carry the same piano multiple times, a book cannot be written more than once.)

(38) Two Dutch authors wrote three gothic novels.

A collective understanding of (38) is verified by a group of two Dutch authors who collectively wrote three gothic novels. On a distributive reading the verifying situations involve six gothic novels, three for each Dutch author. It is the dependency of the number of books on the number of authors that shows that the subject has scope over the object. Using the number of entities involved as a diagnostic of scope, we can show that there exists another kind of understanding, on top of the distributive and the collective one. This is the so-called *cumulative reading* (Scha (1981)). For instance, (39) (a variation on an example from Landman 2000), is true in a situation in which there were (in total) six ladybird-swallowing frogs and there were (in total) twelve ladybirds being swallowed by frogs.[f]

[e] As would be standard in, say, the textbook account of Heim & Kratzer 1998.
[f] Landman 2000 uses a similar example to argue against the reduction of cumulative readings to collective reading, e.g. Roberts (1987).

(39) Six frogs swallowed twelve ladybirds.

The reported reading cannot be a collective reading, since *swallow* is a predicate with distributive entailments. (It makes no sense to swallow a single entity in a joint effort.) It also clearly is not a distributive reading, since the number of ladybirds is twelve, and not seventy-two.[g]

Similarly, (38) has a cumulative reading in which there were two Dutch authors writing gothic novels and there were three gothic novels written by Dutch authors. Again, this is not the collective reading. Take for instance, a verifying situation like one in which one Dutch author writes a gothic novel by himself and then co-writes two more gothic novels with another Dutch author.

To introduce the popular account of cumulativity, let us go through an elaborate illustration of (38). Assume the following semantics for the subject and the object respectively.

(40) a. $\lambda P.\exists x[\#x = 2 \wedge {}^{*}dutch.author(x) \wedge P(x)]$
b. $\lambda P.\exists y[\#y = 3 \wedge {}^{*}gothic.novel(y) \wedge P(y)]$

Here I use # for the 'cardinality' of a sum, as given by the following definition.

(41) $\#\alpha := |Atoms(\lambda\beta.\beta \leq \alpha)|$

The (subject) distributive reading is now given by pluralising the verb phrase:

(42) $\exists x[\#x = 2 \wedge {}^{*}dutch.author(x) \wedge {}^{*}\lambda x'.\exists y[\#y = 3 \wedge {}^{*}gothic.novel(y) \wedge wrote(x', y)](x)]$

This is not entirely accurate, since this assumes that the object is interpreted collectively. (It is not entirely clear how books can be collectively written.) So, a probably more accurate semantics is a doubly distributive reading, as in (43):

(43) $\exists x[\#x = 2 \wedge {}^{*}dutch.author(x) \wedge {}^{*}\lambda x'.\exists y[\#y = 3 \wedge {}^{*}gothic.novel(y) \wedge {}^{*}\lambda y'.wrote(x', y')(y)](x)]$

The cumulative reading results not from pluralising a derived predicate, but from directly pluralising the binary relation *write* (Krifka 1989,

[g] See Winter 2000 for an attempt at reducing cumulativity to distributivity and dependency and Beck and Sauerland for a criticism of that account.

1992, Landman 1996, 2000). That is, we need to generalise the *-operator to many-placed predicates. Let X be a set of pairs:[h]

(44) $\langle\alpha,\beta\rangle \sqcup \langle\gamma,\delta\rangle := \langle\alpha \sqcup \gamma, \beta \sqcup \delta\rangle$

Now the *-operator can be generalised to binary relations.

(45) $^*X$ is the smallest set such that $^*X \subseteq X$ and $\forall x, x' \in {}^*X$: $x \sqcup x' \in {}^*X$

The cumulative reading of (38) is now simply (46).

(46) $\exists x[\#x = 2 \wedge {}^*dutch.author(x) \wedge \exists y[\#y = 3 \wedge {}^*gothic.novel(y) \wedge {}^*wrote(x,y)]]$

For instance, if the extension of *wrote* is $\{\langle b, gn1\rangle, \langle b \sqcup c, gn2\rangle, \langle b \sqcup c, gn3\rangle\}$, then **wrote* will contain $\langle b \sqcup c, gn1 \sqcup gn2 \sqcup gn3\rangle$, verifying (38).

There are two well-studied problems with this account. First of all, because the cumulative reading is scopeless, the above account only works for examples with commutative pairs of quantifiers, such as the two existentials in (46). For instance, (47) also has a cumulative reading, but on a standard generalised quantifier approach to modified numerals, as in (48), the quantifiers are not commutative, since they count atoms and thereby distribute over their scope.

(47) In 2011, more than 100 Dutch authors wrote more than 200 gothic novels.

(48) $[\![\text{more than } n]\!](P)(Q)$ = true iff the number of atoms that have both property P and Q exceeds n

Krifka's solution is to analyse numerals as existential quantifiers (as in (40)) and their modifiers as propositional operators on alternative numerical values. So, the interpretation of (47) yields the set of cumulative propositions in (49). The role of the numeral modifiers is now to assert that there is a factual proposition in this set for $n > 100$ and $m > 200$.[i]

[h] More general, for n-ary tuples f, g: $f \sqcup g :=$ that n-ary tuple h, such that $\forall 1 \leq m \leq n : h(m) = f(m) \sqcup g(m)$.

[i] An alternative account is to treat numeral modification as a degree phenomenon. See, for instance, Hackl (2000); Nouwen (2010).

(49) $$\left\{ \begin{array}{l|l} \exists x[\#x = n \wedge {}^*dutch.author(x) \wedge & \\ \exists y[\#y = m \wedge {}^*gothic.novel(y) \wedge & n, m \in \mathbb{N} \\ {}^*wrote(x, y)]] & \end{array} \right\}$$

A second problem with the particular pluralisation account of cumulativity comes from examples where a cumulative reading is combined with a distributive one. Such examples were divised by Schein (1993). The standard example is (50). Crucially, there is cumulativity with respect to the three cash machines and the two new members, but the passwords are distributed over these two.

(50) Three cash machines gave two new members each exactly two passwords.

Since each new member was given two passwords, *two new members* has to have scope over *two passwords*. But that scoping should be independent of the scopeless relation between the cash machines and the members. Schein uses examples like (50) as part of an elaborate argument that a plural semantics should be part of a neo-Davidsonian event semantics, where events have a similar part-whole structure as entities. Partly due to Schein's argument, events play a major role in the dominating approaches to plurality.[j] See, for instance, Kratzer (2007) for recent discussion and Champollion (2010) for a criticism of some of the arguments for the neo-Davidsonian approach. In the interest of brevity, I refrain from discussing events in this chapter.

1.5 Distributivity and dependency

Since, unlike cumulative readings, distributive readings are scopal, transitive distributive sentences may introduce a dependency between the arguments of the verb. For instance, on the distributive reading, (51) expresses a relation between three boys and (at least) three essays.

(51) Three boys wrote an essay.

This dependency becomes grammatically relevant in at least two different ways. First of all, in a number of languages, indefinites that are dependent in the way *an essay* is in the distributive reading of (51) have to be marked as such. (For instance, see Farkas (1997) for Hungarian,

[j] See also Parsons (1990); Lasersohn (1995).

Farkas (2002) for Romanian, Yanovich (2005) for Russian, and Henderson (2011) for the Mayan language Kaqchikel). Dubbed *dependent indefinites* by Farkas, such indefinites come with a constraint that they have to co-vary with some other variable.

(52) A gyerekek hoztak egy-egy könyvet
the children brought a-a book.ACC
'The children brought a book each.'

Another form of dependency involves anaphora.

(53) a. Three students wrote an article.
They sent it to L&P. Krifka (1996)
b. John bought a gift for every girl in his class and asked their desk mates to wrap them. after Brasoveanu (2008)

The example in (53-a) has many readings. The most salient one is the one in which both sentences are distributive. On that reading, the first sentence introduces a dependency between students and articles, which is then accessed by the pronouns in the second sentence. That is, the second sentence can only mean that each student sent *his or her* paper to L&P. Similarly, there is a salient reading for (53-b) in which for each girl, John asked the desk mate of that girl to wrap the gift he bought for *her*.

Such data are hard to account for, especially given some other complicating features of anaphora. For instance, dependent indefinites are antecedents of plural, not singular, pronouns, which have maximal reference. In an example like (54), the singular pronoun is infelicitous and the plural pronoun is interpreted as referring to the set of all essay written by a student.

(54) Several students wrote an essay. I will need to grade them / *it before tomorrow.

In contrast to (54), singular anaphoric reference to a dependent indefinite is possible once the pronoun is in the same kind of dependency environment, as shown by the examples in (53).

A powerful account of these facts is offered by the dynamic mechanisms proposed in van den Berg (1996) and subsequent related work

Nouwen (2003, 2007); Brasoveanu (2006, 2008).[k] To get the idea behind these proposals, first consider the simple example in (55).

(55) Two boys were wounded. They are in hospital.

Under a dynamic semantic approach to anaphora), anaphoric relations are captured by assigning antecedent and pronoun the same variable name. (The relevant classic literature is Kamp (1981); Groenendijk and Stokhof (1991); Kamp and Reyle (1993), but see also chapter 4 of this volume for an overview of discourse semantics that includes a discussion of dynamics.) For instance, the first sentence in (55) could correspond to $\exists x[\#x = 2 \wedge {}^*boy(x) \wedge {}^*wounded(x)]$. This sentence is verified in a model in which, say, b and c were wounded, by assigning $b \sqcup c$ to x. Let us write $f[x := \alpha]$ to indicate that the assignment function f assigns α to x, and so the relevant variable assignment for the first sentence in (55) are functions of the form $f[x := b \sqcup c]$. The second sentence may now be represented by ${}^*inhospital(x)$. By interpreting this open proposition with respect to an assignment function that verifies the antecedent sentence, the pronoun is automatically understood to refer to whichever two boys were referred to in the first sentence.

The innovation brought in van den Berg's work is to assume that the assignment of pluralities to variables happens in a much more structured way. So, rather than using the assignment in (56-a), a plurality assigned to a variable, van den Berg uses (56-b), a plurality (set) of assignments to the same variable.[l]

(56) a. $f[x := b \sqcup c]$
b. $\{f_1[x := b], f_2[x := c]\}$

The main reason for doing this is that it opens up a way to represent dependencies created by distributivity. For instance, say that Bob is dating Estelle and Carl is dating Ann. This makes (57) true. In a van den Bergian approach to plural dynamic semantics, the sentence will now introduce the variable assignment in (57-b), rather than the simplistic (57-a).

[k] See Krifka (1996) and Kamp and Reyle (1993) for alternative approaches, involving parametrised sums and a representational rule for recovering antecedents respectively. Nouwen (2003) contains a comparison of Krifka's proposal and distributed assignment. See Nouwen (2007) for a (technical) discussion and critique of Kamp & Reyle's approach.

[l] I see no reason why sets are needed. The same idea could in principle be worked out in a lattice theoretic framework with *sum functions*. However, I follow here the standard move to use sets of functions.

(57) Bob and Carl are dating a girl.

a. $f[x := b \sqcup c; y = e \sqcup a]$
b. $C = \{f_1[x := b; y := e], f_2[x := c; y := a]\}$

The plurality of variable assignment in (57-b) has stored the dependency between boys and girls. In a subsequent sentence, like (58-a), a distributive reading can have access to that dependency. Van den Berg therefore proposes that distributivity is not universal quantification over atoms in plural individuals, but rather over atomic assignment functions in a contextual set of functions, such as C above.[m]

(58) a. They each brought her a flower.
b. $\forall f \in C[f(x)$ brought $f(y)$ a flower]

The same framework also has an elegant way of capturing the constraint on Farkas' dependent indefinites, such as the Hungarian reduplication indefinite in (52). Brasoveanu (2010) proposes that such indefinites are necessarily *evaluation plural*. If C is a set of assignments, let $C(v)$ be the sum given by $^*\lambda\alpha.\exists f \in C[f(v) = \alpha]$, i.e. the projection of all values for v in C into a single sum entity. If a dependent indefinite like a Hungarian *egy-egy* noun phrase is associated to a variable x, it presupposes that $C(x)$ is not atomic. For instance, an example like (52), repeated here in (59), is felicitous in the context in (59-a), but not in (59-b).

(59) A gyerekek hoztak [egy-egy könyvet]$_y$
the children brought a-a book.ACC

a. $C = \{f_1[x := \mathit{child}1; y := \mathit{book}1], f_2[x := \mathit{child}2; y := \mathit{book}2], f_3[x := \mathit{child}3; y := \mathit{book}3]\}$
b. $C = \{f_1[x := \mathit{child}1; y := \mathit{book}1], f_2[x := \mathit{child}2; y := \mathit{book}1], f_3[x := \mathit{child}3; y := \mathit{book}1]\}$

1.6 To conclude

Space limitations have enforced a somewhat narrow focus on plurality in this chapter. There are a number of key issues I have not been able to pay attention to. In the conclusion of this chapter, let me mention two of these.

[m] For the sake of brevity, I am simplifying severely. See Nouwen (2003) for discussion of some complicating factors and for various ways of implementing this idea.

So far, I have mentioned two sources of distributivity. First of all, lexically, some predicates are only compatible with atomic entities in their (singular) extension (*being wounded* was the running example). Second, (in at least some approaches) there is additionally a distributivity operator that quantifies over atoms in a sum. A third source is the presence of an inherently distributive quantifier. That is, some quantifiers are *distributive* in the sense that they quantify over atoms only.

(60) A quantifier Q of type $\langle\langle e,t\rangle,t\rangle$ is distributive if and only if for any P of type $\langle e,t\rangle$: $Q(P) \Leftrightarrow Q(\lambda x.P(x) \wedge Atom(x))$.

An indefinite quantifier like *three girls* or the definite *the three girls* is not distributive. An example like (61) allows for collective as well as distributive construals.

(61) Three girls carried a piano up the stairs.

Things are different for quantifiers like *every girl*, which are distributive.[n] In (62), any carrying of pianos that involves joint action is irrelevant. Only if each girl carried a piano by herself is (62) true.

(62) Every girl carried a piano up the stairs.

Similarly distributive is *most girls*:

(63) Most girls carried a piano up the stairs.

If (63) is true, then it is true of most individual girls that they carried a piano by themselves. If the majority of girls jointly carried a piano up the stairs, whilst a minority stood by and did nothing, (63) is intuitively false.

Quantifiers that are especially interesting from the perspective of plurality are floating quantifiers like *each* and *all*. Whilst *each* enforces a distributive reading in a sentence, as in (64-a), the presence of *all* allows for both distributive and collective understandings. Yet, *all* is incompatible with certain collective predicates, as illustrated in (65).

(64) a. The boys each carried a piano up the stairs.
b. The boys all carried a piano up the stairs.

[n] One complication is that *every* does have non-distributive readings, as in:

(i) It took every/each boy to lift the piano. Beghelli and Stowell (1996)

(65) *The boys are all a good team.

See Brisson (2003) for an analysis of *all* using Schwarzschild's 1996 cover analysis of distributivity and collectivity.

Whilst the semantics of *each* is obviously closely related to (atom-level) distributivity, the main puzzle posed by *each* is of a compositional nature. This is especially the case for so-called *binominal each* (Safir and Stowell (1988)). For instance, in (66) *each* distributes over the atoms in the subject plurality, while it appears to be compositionally related to the object. See Zimmermann (2002) for extensive discussion.

(66) These three Dutch authors wrote five gothic novels each.

Another major issue I have not discussed in this chapter is the interpretation of plural morphology. As I observed at the start, plurals do not generally express *more than one*, for (67) suggests that not even a single alien walked the earth.

(67) No aliens have ever walked the earth.

Krifka 1989 suggests that while plurals are semantically inclusive, that is they consist of both non-atomic and atomic sums, singulars can be taken to necessarily refer to atoms. The idea is now that, under normal circumstances, use of a plural implicates that the more specific contribution a singular would have made was inappropriate, and hence that a non-atomic reference was intended. Cases like (67) are simply cases in which the implicature is somehow suppressed, for instance, because of a downward monotone environment. See Sauerland (2003); Sauerland et al. (2005) and Spector (2007) for detailed analyses along these lines. Such approaches often break with the intuition that the plural form is marked, while the singular form is unmarked and so we would expect that the plural but not the singular contributes some essential meaning ingredient (Horn (1989)).[o] Farkas and de Swart (2010) formulate an account of the semantics of number marking that is in the spirit of Krifka's suggestion but is at the same time faithful to formal markedness in the singular/plural distinction.

Zweig (2008) offers an extensive discussion of the related issue of de-

[o] Most natural languages mark the plural and leave the singular unmarked (e.g. Greenberg (1966), cf. de Swart and Zwarts (2010) for discussion of exceptions).

pendent bare plurals.[p] The most salient reading of (68) is one in which bankers wear one suit at a time, despite the plural marking on the object.

(68) All bankers wear suits.

A typical analysis of (68) is to assume that such sentences are cumulative (Bosveld-deSmet (1998), cf. de Swart (2006)) and that the plural marking on *suits* triggers an interpretation of the object as a multiplicity of suits. Contrary to this, one could assume that this is a distributive reading and that the plural marking on the object is a result of the dependency the object is part of. Here, the issue of the interpretation of plural morphology meets the issues concerning distributive and cumulative entailments that were central to this chapter. Zweig (2008), for instance, argues that although the cumulative analysis of (68) might be on the right track, this will not do for all examples involving dependent bare plurals. This is because of examples like (69).

(69) Most students wrote essays.

There exists a dependent reading for *essays* in (69). But there does not exist a cumulative reading for the whole sentence, since *most* is a distributive quantifier. Such examples illustrate the complexity of the semantics of the plural and stress that plurality is a compositional rather than a referential phenomenon.

[p] Moreover, he discusses a complication that an event-based semantics for plurality brings.

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