WITTGENSTEIN’S PHILOSOPHY OF MATHEMATICS

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introduction

- More than half of W.'s writing from 1929 to 1944 is devoted to mathematics.
- His chief contribution: **philosophy of mathematics**.
- **Platonism** (mathematical objects are real)
- **Logicism** (mathematics is reducible to logic)
- Two beliefs that W. maintains:
  - mathematical propositions are **not real propositions**,
  - and that mathematical truth doesn't refer to any **object**.
- W. belongs to no school.
- **His job was to clear the misconceptions**.

early period (tractatus)

- Not a **logicist:** a mathematical identity is not stating a truth at all (it is a rule).
- **tautologies** are not about reality > they do not express truth-value.
- Mathematical ‘pseudo-propositions’ are **equations**: things with same meanings. (They say what they show).
- **equation** and **substitution**.
- We prove : ‘seeing’ that two expressions have the **same meaning**.
- All by symbol **manipulation**.
- W. uses **arithmetical identities** to show that mathematics is simply a **machine** for **tautologies**.

early period (tractatus)

- **numerals** are not names of objects, but labels marking the number of the **iterations** of a **process** of **proposition** derivation. > the natural numbers represent stages in the execution of a **logical operation**.
- The general form of an **operation** $\Omega'(\overline{\eta})$ is $\left[\overline{\xi}, N\left(\overline{\xi}\right)\right]'(\overline{\eta})\left(=\left[\overline{\eta}, \overline{\xi}, N\left(\overline{\xi}\right)\right]\right)$.

early period (tractatus)

- W. introduces $[a, x, O'x]$ as the general **term** of **series of forms**, e.g., $(a, O'a, O'O'a)$; and all the general forms that he introduces in TLP (of an **operation**, a **proposition**, and a **natural number**,) are modeled on this form.
- The first term is the beginning of a series, the second one is an arbitrary term from the list, and O'x is the term that immediately follows x in the series.

early period (tractatus)

- All natural numbers can be generated by **repeated iterations** of the general form of a natural number:

  [0,ξ,ξ+1].
- “**[m]athematics is a method of logic**”.
- general form of a number ([0,ξ,ξ+1]) and the general form of a proposition ($\left[\bar{p}, \bar{\xi}, N\left(\bar{\xi}\right)\right]$) are both the same thing: **general form of a formal operation**.
- $\bar{p}$ stands for all elemental propositions | $\bar{\xi}$ is any set of propositions | $N\left(\bar{\xi}\right)$ stands for the negation of all members of $\bar{\xi}$.

early period (tractatus)

- mathematical inferences are just instances of application of logical rules (by the **symbol** alone) without any observations of states of affairs.

middle period

- ***Differences From the Early Period***
- In the Tractatus, W. was influenced by **Russell** and **Frege**, > **Brouwer**, **Weyl**, **Hilbert**, and **Skolem.**
- The most important difference : rejects quantification over **infinite** domains. So, propositions can no longer be infinite conjunctions and disjunctions : there is no infinity.

middle period

- ***We Invent the Mathematical Machine***
- we **invent** mathematics and we expand it by calculation and proof.
- in mathematics we **use** numbers, we don’t talk about them.
- For example, looking at the schema |||| : arithmetic doesn’t talk about the lines, it merely operates with them.
- mathematics is a **machine** that uses these kinds of symbols for its unfolding.

middle period

- These symbols don’t have any inherent meaning by themselves.
  - “However queer it sounds, the further expansion of an irrational number is a further expansion of mathematics”.
- Any meaning in this regard simply arises from the **process** of doing these kinds of jobs.
- For example, to know the meaning of $2 + 2 = 4$, (process of calculation is what is essential).

middle period

- How a proposition is verified is what it says (meaning).
- Mathematical **symbols** lack **(objective) meaning**, they are not proxy for **(concrete) things** which are their meanings.
- Therefore, in mathematics everything is **algorithm** and nothing is meaning.

middle period

- ***<u>The Infinite</u>***
- The **infinite** is not a **quantity** and mathematical infinite resides in **recursive rules**.
- An **irrational number** is not something that we can find > it's the **rule** itself (same goes for a **geometric line**). > a **never-ending** iteration.
- He adopts the radical position that all expressions that quantify over an **infinite domain** are **meaningless**: (Goldbach's Conjecture, the Twin Prime Conjecture, Euclid's Prime Number Theorem).

middle period

- **Proof by induction:** Inductive Base: $P(1)$, Inductive Step: $P(n) \Rightarrow P(n+1)$ therefore we can say $P(m)$.
- W. only accepts induction in a restricted sense: e.g., we have 700 **modus ponens** to prove via the **Inductive Step**, and induction can **save us time**.
- It doesn’t permit us to go on to **infinity**. Inductive Step proxy for a more **direct proof.**
- ***Extensions and Intentions***
- W. thinks that mathematics consists of **extensions** and **intentions** (i.e., rules), so an irrational is only an extension as long as we consider the **numeral** itself e.g., ‘√2’.

late period

- ***Lack of Systemic Theses and Illuminating Effect of Philosophy and the Infinite***
- W.'s later philosophy is its **lack of theses**. (meta-level advice to find the correct account, rather than developing the account.)
- **philosophical light** could get rid of excessive mathematical branches.
- An important **constant** in Wittgenstein's Philosophy of Mathematics (middle and late,) is that he consistently maintains that mathematics is our, **human invention**.
- "**chess** only had to be discovered, it was always there!"
- The later Wittgenstein still rejects the **actual infinite** and infinite mathematical extensions. However there maybe more room for the "**potential infinite**".

late period

- ***Arbitrary Formal Games and Mathematics***
- As we said, mathematical identities like ‘$3 \times 3 = 9$’ are **rules**. Thus, for us, the question is whether they are **arbitrary or not**?
- But, such rules stem from **empirical experiences**, and thus they are not arbitrary. We have mathematics because at some point empirical regularities have “**hardened**” into rules.
- These rules then become a **paradigm**, that experience will be judged against it.
- So, when the **arithmetical rule is formed**, not only it no longer requires empirical experience for its **justification**, but the empirical experience itself will be **justified** by its means.
- mathematical propositions are **non-revisable** in light of empirical investigation. In his own words: “mathematics as such is always measure, not thing measured”.

late period

- ***The Platonism Access Problem***
- To address the **Platonism Access Problem** W. abandons the assumption that **knowledge** is made by a causal interaction with the known objects.
- W. thinks that **knowledge** itself is a **family-resemblance** concept.
- Further, he holds that this type of thinking in which knowledge is made by **causal interaction** with reality is itself based on the **epistemological context** of **natural sciences**.
- So, by '**transferring**' the **methodology** of natural sciences assuming that mathematics has the same kind of reality, we have already **assumed too much** and misled our philosophy.
- Here, what W. rejects is not the possibility to talk about **mathematical objects and truths**, rather it is that we cannot hope for the **same kind of justification** techniques as that of empirical sciences.
- Therefore, we can be free of an 'illegitimate question' which is trying to pinpoint the objects of mathematical discourse.

late period

- ***Constructivism and the Infinite***
- When W. rejects the notion of **actual infinity** (and state the meaninglessness of adherence to infinity for example in the case of disputing the validity of **irrational numbers** or that of **inductive proofs**) or the notion of **existence proofs**, we may label him as a **finitist-constructivist**.
- But if we get rid of infinity we can end up **disputing** large portions of current mathematics, for example, **real analysis**.
- However, W. always held that **philosophy** may not **interfere** with the actual usage of a language and that by philosophical endeavor we may only hope for **describing** a language.

late period

- ***Our Mathematical Practice and Our Nature***
- Understanding the importance of the empirical regularities for mathematics amounts for Wittgenstein to a “**silent revolution**”, in addition to the well-known “**overt revolution**” (the repudiation of the Tractatus).
- Some **intuitionists** (who reject the logical rule of **excluded middle**) such as **Brouwer** see mathematics as a **languageless** activity of mind.
- W. sees mathematics inseparable from the **human language and practice**. The meaning of **calculations** and **proofs** is in our **practices**.
- W. says that we know as much **mathematics** as **God** does.
- There is no **transcendent** justification for what we do in mathematics. For example, **mathematical identities** are **codifications** of contingent but very robust method of correspondence between the collective human **mind** and the **regularities** of the objective world.

late period

- Humans have a sort of a **behavioral agreement** when doing mathematics.
- This type of agreement consists in all of us having, roughly, the same **natural reactions** when presented with the same **'mathematically' related situations** (**arranging**, **sorting**, **recognizing shapes**, and so forth). So:
  - we can be **trained** to have certain similar reactions (neuro-physiological basis) e.g., in doing a **multiplication**,
  - the world itself presents a certain **stability**, many **regular features**.

late period

- Mathematics is **objective** (based on regularities), but its objects are not **transcendent**. This is very clever because it avoids the problems of both **Platonism** and **subjectivism**.
- For W. A **list of strokes** is not an abstract list, it is **concrete**—it is concretely expressed on a medium (ink on paper). For example, ||| can be thought as a **paradigm** for the number 3. Further, the meaning of these strokes, arises in our practices.

late period

- Numerals **outside** mathematics are being used transitively, they derive their meaning from **paradigmatic samples**. Numerals **within** mathematics express **internal relations** between different samples.
- In PI, W. notes that any process can be made out to **accord with some rule**, and thus no process could be determined by a rule (the **rule-following paradox**).
- This is because it's impossible to make a list that contains all the rules. But W. argues that there is something related to our **nature**, and how the mind works that make us able to **follow rules** as it is intended.
- The **behavioral agreement** is not simply an agreement of **opinion**. > the **whole natural procedure** that leads to an agreement about a matter at hand. It is a **'consensus of action'**. E.g., if we are to run a **simulation…**
- In W.'s own words these agreements are "**not agreement[s] in opinions but in form of life**."

late period

- ***<u>Proof As the Meaning of a Mathematical Proposition</u>***
- Meaning of mathematical proposition is its **proof**.
- E.g., Goldbach's Conjecture that is justified **heuristically** (brute force or statistical methods) implies a different meaning from when it is rigorously proven.
- A mathematical **proof** of a proposition shows the **internal relations** of the proposition to a system of mathematical rules.
- A proof is like a **cinematographic picture**.
- W. holds that a completely **analyzed** mathematical proposition (i.e. when all the internal relations are depicted) is its own **proof**.
- If the meaning of an arithmetical generalization is given by its proof, then none of us understands **Goldbach's conjecture**. But then how could anyone try to find one? But W. mentions that a new problem (a senseless conjecture) can **stimulate** mathematicians to try to find some internal relations, and there is nothing wrong with this.

late period

- ***Archives***
- W. thinks that we have mathematics because of the act of **archiving**. These archives are the **standards** based on which we will expand mathematics.
- What transforms a proof (which at first is only an experiment) into a picture is the act of archiving the actual (typed down) proof. This is how a proof turns into a **paradigm**.

late period

- ***<u>Consistency</u>***
- Regarding **consistency**, W. thinks that mathematicians are mistaken in being so much **concerned** about the proof of the consistency of **axioms**. “I have the feeling that if there were a contradiction in the axioms of a system it wouldn't be such a great misfortune. Nothing easier than to remove it.”
- The idea that a **contradiction** is not harmful per se is one Wittgenstein made repeatedly. In his discussion of **Gödel's incompleteness theorems**, for instance, he says: “Is there harm in the contradiction that arises when someone says: ‘I am lying.—So I am not lying.—So I am lying.—etc.’? I mean: does it make our language less usable if in this case, according to the ordinary rules, a proposition yields its contradictory, and vice versa?—the proposition itself is unusable, and these inferences equally.... Such a contradiction is of interest only because it has tormented people.”
- But, Wittgenstein seems to have been laboring under the **misconception** that we can **repair** a contradictory system simply by **refusing** to draw any conclusions from a contradiction. But regarding contradiction he never really developed this material into a stable account.

conclusion

- What will distinguish the mathematicians of the future from those of today will really be a greater sensitivity [...] people will then be more intent on absolute clarity than on the discovery of new games.
- Philosophical clarity will have the same effect on the growth of mathematics as sunlight has on the growth of potato shoots. (In a dark cellar they grow yards long.)

conclusion

- A mathematician is bound to be horrified by my mathematical comments, since he has always been trained to avoid indulging in thoughts and doubts of the kind I develop. […] he has acquired a revulsion from them as infantile. That is to say, I trot out all the problems that a child learning arithmetic, etc., finds difficult […] I say to those repressed doubts […] demand clarification!

further reading

- Tractatus Logico-Philosophicus (§6.2 - §6.36311)
- Philosophical Investigations (§185 - §238)
- Remarks on the Foundations of Mathematics (§1 - §5)

references

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thank you