Příklad animal modelu dle přednášky č. 10

jedince	stádo	laktace	užitkovost
1	1	1	4500
2*	1	1	5000
3	1	2	6500
4	2	2	8000
5*	2	1	7000

Předpokládáme, že naměřená užitkovost krávy je ovlivněna jen stádem, ve kterém je chována, věkem (tj. pořadím laktace) a genotypem (tj. jedincem se svou jedinečnou genetickou výbavou).

modelová rovnice: $y_{ijkl} = S_i + L_j + u_k + e_{ijkl}$

maticový zápis: $\mathbf{y} = \mathbf{Xb} + \mathbf{Zu} + \mathbf{e}$

Odvozená soustava normálních rovnic smíšeného modelu:

$$\begin{bmatrix} \mathbf{X'X} & \mathbf{X'Z} \\ \mathbf{Z'X} & \mathbf{Z'Z} + \mathbf{A}^{-1}K \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{X'y} \\ \mathbf{Z'y} \end{bmatrix}$$

Do okna programu R (www.r-project.cz) vkládáme jednotlivé příkazy (znak > na začátku každého příkazu je automaticky přidáván programem):

```
> y <- matrix(c(4500,5000,6500,8000,7000),5,1)
> y
     [,1]
[1,] 4500
[2,] 5000
[3,] 6500
[4,] 8000
[5,] 7000
> X <- matrix(c(1,1,1,0,0,
+ 0,0,0,1,1,
+ 1,1,0,0,1,
+ 0,0,1,1,0),5,4)
```

```
>X
     [,1] [,2] [,3] [,4]
[1,]    1    0    1    0
[2,]    1    0    1    0
[3,]    1    0    0    1
[4,]    0    1    0    1
[5,]    0    1    1    0
> Z <- diag(1,5)
> Z
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    0    0    0    0
[2,]    0    1    0    0    0
[3,]    0    0    1    0    0
[4,]    0    0    0    1    0
[5,]    0    0    0    0    1
> h2 = 0.25
> K = (1-h2)/h2
> K
[1] 3
> A <- matrix(c(1,0,0,0,0,0,
+ 0,1,0,0,0.25,0.5,
+ 0,0,1,0,0,0,
+ 0,0,0,1,0,0,
+ 0,0.25,0,0,1,0.5,
+ 0,0.5,0,0,0.5,1),6,6)
```

```
> A
   [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    1 0.00    0    0 0.00  0.0
[2,]    0 1.00    0    0 0.25  0.5
[3,]    0 0.00    1    0 0.00  0.0
[4,]    0 0.00    0    1 0.00  0.0
[5,]    0 0.25    0    0 1.00  0.5
[6,]    0 0.50    0    0 0.50  1.0
> XX <- t(X)%*%X
> XX
   [,1] [,2] [,3] [,4]
[1,]    3    0    2    1
[2,]    0    2    1    1
[3,]    2    1    3    0
[4,]    1    1    0    2
> XZ <- t(X)%*%Z
> XZ
   [,1] [,2] [,3] [,4] [,5]
[1,]    1    1    1    0    0
[2,]    0    0    0    1    1
[3,]    1    1    0    0    1
[4,]    0    0    1    1    0
> ZX <- t(Z)%*%X
> ZX
   [,1] [,2] [,3] [,4]
[1,]    1    0    1    0
[2,]    1    0    1    0
[3,]    1    0    0    1
[4,]    0    1    0    1
[5,]    0    1    1    0
```

```
> ZZ <- t(Z)%*%Z
> ZZ
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    0    0    0    0
[2,]    0    1    0    0    0
[3,]    0    0    1    0    0
[4,]    0    0    0    1    0
[5,]    0    0    0    0    1
> Ainv <- solve(A)
> Ainv
     [,1]         [,2] [,3] [,4]         [,5]       [,6]
[1,]    1 0.000000e+00    0    0 0.000000e+00  0.0000000
[2,]    0 1.333333e+00    0    0 5.551115e-17 -0.6666667
[3,]    0 0.000000e+00    1    0 0.000000e+00  0.0000000
[4,]    0 0.000000e+00    0    1 0.000000e+00  0.0000000
[5,]    0 2.960595e-17    0    0 1.333333e+00 -0.6666667
[6,]    0 -6.666667e-01   0    0 -6.666667e-01  1.6666667
> Ak <- Ainv*K
> Ak
     [,1]         [,2] [,3] [,4]         [,5] [,6]
[1,]    3 0.000000e+00    0    0 0.000000e+00    0
[2,]    0 4.000000e+00    0    0 1.665335e-16   -2
[3,]    0 0.000000e+00    3    0 0.000000e+00    0
[4,]    0 0.000000e+00    0    3 0.000000e+00    0
[5,]    0 8.881784e-17    0    0 4.000000e+00   -2
[6,]    0 -2.000000e+00   0    0 -2.000000e+00    5
```

/* Abychom mohli spojit matice ZZ(5x5) a AK (6x6) musíme přidat řádek a sloupec nul do matice ZZ, aby vznikla matice o rozměrech 6x6

Stejně musíme upravit i matice XZ (+ 1 sloupec nul) a ZX (+ 1 řádek nul)

Přidáním nul se nic nemění – jen je pak možné spojit tyto submatice do matice levé strany LS */

```
> Z0 <- matrix(c(0,0,0,0,0))
> Z0
    [,1]
[1,]    0
[2,]    0
[3,]    0
[4,]    0
[5,]    0
> ZX0 <- cbind(ZX,Z0)
> ZX0
    [,1] [,2] [,3] [,4] [,5]
[1,]    1    0    1    0    0
[2,]    1    0    1    0    0
[3,]    1    0    0    1    0
[4,]    0    1    0    1    0
[5,]    0    1    1    0    0
> Z0 <- matrix(c(0,0,0,0),1,4)
> Z0
    [,1] [,2] [,3] [,4]
[1,]    0    0    0    0
> ZX0 <- rbind(ZX,Z0)
> ZX0
    [,1] [,2] [,3] [,4]
[1,]    1    0    1    0
[2,]    1    0    1    0
[3,]    1    0    0    1
[4,]    0    1    0    1
[5,]    0    1    1    0
[6,]    0    0    0    0
```

```
> ZZ0 <- matrix(c(0,0,0,0,0),1,5)
> ZZ00 <- matrix(c(0,0,0,0,0,0),6,1)
> ZZa <- rbind(ZZ,ZZ0)
> ZZa
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    0    0    0    0
[2,]    0    1    0    0    0
[3,]    0    0    1    0    0
[4,]    0    0    0    1    0
[5,]    0    0    0    0    1
[6,]    0    0    0    0    0
> ZZZ <- cbind(ZZa,ZZ00)
> ZZZ
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    1    0    0    0    0    0
[2,]    0    1    0    0    0    0
[3,]    0    0    1    0    0    0
[4,]    0    0    0    1    0    0
[5,]    0    0    0    0    1    0
[6,]    0    0    0    0    0    0
> ZZAK <- ZZZ + Ak
> ZZAK
     [,1]         [,2] [,3] [,4]         [,5] [,6]
[1,]    4 0.000000e+00    0    0 0.000000e+00    0
[2,]    0 5.000000e+00    0    0 1.665335e-16   -2
[3,]    0 0.000000e+00    4    0 0.000000e+00    0
[4,]    0 0.000000e+00    0    4 0.000000e+00    0
[5,]    0 8.881784e-17    0    0 5.000000e+00   -2
[6,]    0 -2.000000e+00    0    0 -2.000000e+00    5
```

```
> Zc <- matrix(c(0,0,0,0),4,1)
> LS1 <- cbind(XX,cbind(XZ,Zc))
> LS1
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
[1,]    3    0    2    1    1    1    1    0    0     0
[2,]    0    2    1    1    0    0    0    1    1     0
[3,]    2    1    3    0    1    1    0    0    1     0
[4,]    1    1    0    2    0    0    1    1    0     0
> LS2 <- cbind(ZX0,ZZAK)
> LS2
     [,1] [,2] [,3] [,4] [,5]         [,6] [,7] [,8]         [,9] [,10]
[1,]    1    0    1    0    4 0.000000e+00    0    0 0.000000e+00     0
[2,]    1    0    1    0    0 5.000000e+00    0    0 1.665335e-16    -2
[3,]    1    0    0    1    0 0.000000e+00    4    0 0.000000e+00     0
[4,]    0    1    0    1    0 0.000000e+00    0    4 0.000000e+00     0
[5,]    0    1    1    0    0 8.881784e-17    0    0 5.000000e+00    -2
[6,]    0    0    0    0    0 -2.000000e+00    0    0 -2.000000e+00     5
> LS <- rbind(LS1,LS2)
> LS
      [,1] [,2] [,3] [,4] [,5]         [,6] [,7] [,8]         [,9] [,10]
 [1,]    3    0    2    1    1 1.000000e+00    1    0 0.000000e+00     0
 [2,]    0    2    1    1    0 0.000000e+00    0    1 1.000000e+00     0
 [3,]    2    1    3    0    1 1.000000e+00    0    0 1.000000e+00     0
 [4,]    1    1    0    2    0 0.000000e+00    1    1 0.000000e+00     0
 [5,]    1    0    1    0    4 0.000000e+00    0    0 0.000000e+00     0
 [6,]    1    0    1    0    0 5.000000e+00    0    0 1.665335e-16    -2
 [7,]    1    0    0    1    0 0.000000e+00    4    0 0.000000e+00     0
 [8,]    0    1    0    1    0 0.000000e+00    0    4 0.000000e+00     0
 [9,]    0    1    1    0    0 8.881784e-17    0    0 5.000000e+00    -2
[10,]    0    0    0    0    0 -2.000000e+00    0    0 -2.000000e+00     5
```

```
> Xy <- t(X)%*%y
> Xy
     [,1]
[1,] 16000
[2,] 15000
[3,] 16500
[4,] 14500
> Zy <- t(Z)%*%y
> Zy
     [,1]
[1,] 4500
[2,] 5000
[3,] 6500
[4,] 8000
[5,] 7000
> PS <- rbind(Xy,Zy)
> PS
      [,1]
 [1,] 16000
 [2,] 15000
 [3,] 16500
 [4,] 14500
 [5,]  4500
 [6,]  5000
 [7,]  6500
 [8,]  8000
 [9,]  7000
/* rovněž u matice pravé strany musíme > PS přidat nulu, aby vznikl vektor o 10 řádcích  */
> PS0 <- rbind(PS,0)
```

```
> PS0
      [,1]
 [1,] 16000
 [2,] 15000
 [3,] 16500
 [4,] 14500
 [5,]  4500
 [6,]  5000
 [7,]  6500
 [8,]  8000
 [9,]  7000
[10,]     0
> LSinv <- solve(LS)
Error in solve.default(LS) :
  system is computationally singular: reciprocal condition number = 1.33628e-17
> det <- round(det(LS))
> det
[1] 0
```

/* Protože determinant matice LS je roven nule, je tato matice singulární a nelze ji invertovat -> jedním z řešení je použít zobecněnou inverzi…

Je nutné si nahrát balíček MASS z nabídky: Packages -> Load Packages/

```
> bu <- ginv(LS)%*%PS0
> bu
          [,1]
 [1,] 2302.23138
 [2,] 4229.74702
 [3,] 2547.96760
 [4,] 3984.01080
 [5,]  -87.54974
 [6,]   47.47015
 [7,]   53.43945
 [8,]  -53.43945
 [9,]   61.96703
[10,]   43.77487
```

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