Bi7740: Scientific computing

Introductory considerations - additional slides

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```
% Script for Stirling's approximation.
%

s = @(n) sqrt(2*pi.*n).*(n./exp(1)).^n;
%   ^--- check "anonymous functions"!

x = 1:10;
f = factorial(x);
plot(x, (s(x) - f)./f);

return
```

Taylor series for Euler's number

$$\sum_{n=0}^{\infty} \frac{1}{n!} = e$$

For correct 3 decimals, the *tail* (the rest of the sum, not computed) must be upper bounded by 0.0005 (why?)

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{1}{n!} &= \sum_{n=1}^{k} \frac{1}{n!} + \sum_{n=k+1}^{\infty} \frac{1}{n!} \\
&= \sum_{n=1}^{k} \frac{1}{n!} + \frac{1}{(k+1)!} \left[1 + \frac{1}{k+2} + \frac{1}{(k+2)(k+3)} + \dots \right] \\
&< \sum_{n=1}^{k} \frac{1}{n!} + \frac{1}{(k+1)!} \left[1 + \frac{1}{k+1} + \frac{1}{(k+1)^2} + \dots \right] \\
&= \sum_{n=1}^{k} \frac{1}{n!} + \frac{1}{(k+1)!} \left[\frac{1}{1 - \frac{1}{k+1}} \right] = \sum_{n=1}^{k} \frac{1}{n!} + \frac{1}{k \cdot k!}
\end{aligned}$$

```matlab
% real code would check for proper n...
e = @(n) (1 + sum(1 ./ factorial(1:n)));
fprintf('%2s\t%10s\t%10s\n', 'k', 'approx', 'tail');
for k = 1:10
    fprintf('%d\t%10.8f\t%10.8f\n', k, e(k), ...
        1/(k*factorial(k)));
end
```

```
 k      approx        tail
1    2.00000000  1.00000000
2    2.50000000  0.25000000
3    2.66666667  0.05555556
4    2.70833333  0.01041667
5    2.71666667  0.00166667
6    2.71805556  0.00023148   <-----
7    2.71825397  0.00002834
8    2.71827877  0.00000310
9    2.71828153  0.00000031
10   2.71828180  0.00000003
```