Homework assignment

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Due date: May 20, 2014 (before the final examination)

Problem 1

You know that $U(x_1, x_2) = x_1 - x_2^{-1}$, prices of goods x_1 and x_2 are p_1 and p_2, and w is the available budget.

1. Find the Marshallian demand functions for both goods and check whether they are normal or inferior, Giffen or non-Giffen. Are both goods complements or substitutes?
2. Compute elasticities of each good with respect to p_1 and p_2. Is any of the goods luxury?
3. Obtain the indirect utility function, derive the cost function and the Hicksian demand.
4. Setup the cost minimization problem and verify that the Hicksian demand in the point above is correct. Compute the income and substitution effects for both goods separately and show that the Slutsky equation holds. How is x_1 different from x_2. Draw IE and SE for both goods in separate graphs.

Problem 2

Let X denote a random variable, then $Var(X) = \mathbb{E}\left[X - \mathbb{E}(X)\right]^2$. Show this can also be written as $Var(X) = \mathbb{E}\left(X^2\right) - \left[\mathbb{E}(X)\right]^2$.

Problem 3

Let $x_1, x_2, ..., x_n$ be a sample from some distribution. The sample variance can be computed as $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$. Show that this is equivalent to $s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right)$.

Problem 4

Revenues reported last week from nine stores franchised by an international clothier are listed in the table below. Based on these figures alone, in what range might the company expect to find the average revenue of all of its stores? That is, compute a 95% confidence interval: $Prob\left(-\alpha_{0.025} \leq \frac{\bar{Y}-\mu}{S/\sqrt{n}} \leq \alpha_{0.025}\right) = 0.95$.

Hint: First, compute $\bar{Y}$ and S from the given sample using the formula in Problem 3. Then, find the critical value, $\alpha_{0.025}$, from the t-statistics table (9 degrees of freedom). Finally, re-arrange the terms in brackets to compute the confidence interval for μ.

Problem 5

Compute the maximum likelihood estimator of θ for the following density functions:

1. $p(k, \theta) = \theta^k (1-\theta)^{1-k}$, $k = \{0, 1\}$, $0 < \theta < 1$. You are given 8 data points: $X_1 = 1$, $X_2 = 0$, $X_3 = 1$, $X_4 = 1$, $X_5 = 0$, $X_6 = 1$, $X_7 = 1$, $X_8 = 0$.
2. $f(y, \theta) = \theta e^{-\theta y}$, $y \geq 0$. You are given four data points: $Y_1 = 8.2$, $Y_2 = 9.1$, $Y_3 = 10.6$, $Y_4 = 4.9$.

store ID	revenue, thousand USD
1	40
2	48
3	60
4	15
5	50
6	80
7	50
8	36
9	16
10	89

3. $f(y, \theta) = \frac{\theta}{2\sqrt{y}} e^{-\theta\sqrt{y}}$. You are given four data points: $Y_1 = 6.2$, $Y_2 = 7$, $Y_3 = 2.5$, $Y_4 = 4.2$.

Problem 6

Suppose that random variables X and Y have a joint density function $f(x, y) = \frac{2}{3}(x + 2y)$, $0 \leq x \leq 1$, $0 \leq y \leq 1$. Find:

1. Find marginal densities, $f(x)$ and $f(y)$.
2. Find $\mathbb{E}$(X + Y).

Problem 7

Suppose that the economy is populated by two consumers: A and B with respective utility functions: $U^A(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$ and $U^B(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$. Endowments are $(e_1^A, e_2^A) = (1, 1)$ and $(e_1^B, e_2^B) = (3, 2)$. Find the vector of competitive allocations and draw your findings in the Edgeworth box.

Problem 8

Production in a capital abundant country, $\frac{K}{L} > 1$, is described by $Y = K^\alpha L^{1-\alpha}$. The country welfare function is $W = rK + wL$, where r and w are the competitive rental and wage rates. The country chooses between two policies: importing capital and importing labor. Rank these two policies.

Problem 9

An object is to be sold on a first-price sealed-bid auction. There are two risk-neutral bidders with private values for that object, v_i, $v_i \sim G[0, \bar{v}]$, where G is a cumulative distribution function.

1. Find agent's expected utility from bidding r when the agent's private value is v.
2. Find the bidding function $b^*(v)$, show that $b^*(v) < v$ and $\frac{\partial b^*(v)}{\partial v} > 0$.

Suppose that the owner of the object considers selling the object on the second-price auction.

1. How does the bidding behavior of bidders change? Draw a graph.
2. Compare the expected revenue in both auctions.