Introduction to supergravity 2015: Exercise 1.

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Here we treat the supergravity multiplet as a theory invariant under the global supersymmetry, therefore the supersymmetric parameter is global

$$\xi_\alpha = \text{fermionic global parameter (no space-time dependence).} \tag{1}$$

This is possible in the linearized level. The supersymmetry transformations of the linearized free supergravity multiplet are

$$\begin{aligned}
\delta_\xi \psi_{m\alpha} &= \frac{1}{2}\partial_n h_{km}\left(\sigma^k_{\alpha\dot\rho}\bar\sigma^{n\dot\rho\rho} - \sigma^n_{\alpha\dot\rho}\bar\sigma^{k\dot\rho\rho}\right)\xi_\rho, &(2)\\
\delta_\xi h_{mk} &= -\frac{i}{2}\bar\psi_{m\dot\alpha}\bar\sigma^{\dot\alpha\alpha}_k\xi_\alpha - \frac{i}{2}\bar\psi_{k\dot\alpha}\bar\sigma^{\dot\alpha\alpha}_m\xi_\alpha \\
&\quad + \frac{i}{2}\bar\xi_{\dot\alpha}\bar\sigma^{\dot\alpha\alpha}_k\psi_{m\alpha} + \frac{i}{2}\bar\xi_{\dot\alpha}\bar\sigma^{\dot\alpha\alpha}_m\psi_{k\alpha}. &(3)
\end{aligned}$$

The conventions can be found in the book of Wess and Bagger [1].

1. Find $\delta_\xi\bar\psi_{m\dot\alpha}$. This can be found by the definition $\delta_\xi\bar\psi_{m\dot\alpha} = (\delta_\xi\psi_{m\alpha})^*$.

2. Calculate $[\delta_\xi, \delta_\eta]\, h_{mn}$. This will have the form

   $$[\delta_\xi, \delta_\eta]\, h_{mn} = B^r\partial_r h_{mn} + \partial_m\Gamma_n + \partial_n\Gamma_m. \tag{4}$$

   Find B_r and Γ_n and explain what is their physical meaning.

3. Calculate $[\delta_\xi, \delta_\eta]\, \psi_{m\alpha}$. This will have the form

   $$[\delta_\xi, \delta_\eta]\, \psi_{m\alpha} = C^r\partial_r\psi_{m\alpha} + \partial_m G_\alpha + \tilde r_{m\alpha}, \tag{5}$$

   where $\tilde r_{m\alpha}$ will vanish on-shell. Find C_r, G_α and $\tilde r_{m\alpha}$ and explain what is their physical meaning.

4. Off-shell the graviton (h_{mn}) has 6 degrees of freedom while the gravitino ($\psi_{m\alpha}$) has 12 off-shell degreed of freedom. How does this relate to the fact that we need to use the equations of motions to close the supersymmetry algebra?

References

[1] J. Wess and J. Bagger, "Supersymmetry and supergravity," Princeton, USA: Univ. Pr. (1992) 259 p