Introduction to supergravity 2015: Exercise 2.

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Consider the following action of a scalar field coupled to gravitation

$$S = \int d^4x\sqrt{-g}\left[-\frac{M_P^2}{2}R - \frac{1}{2}\partial_m\phi\partial^m\phi\right]. \tag{1}$$

1. Find the equations of motion for ϕ by extremizing the action

$$\frac{\delta S}{\delta\phi} = 0. \tag{2}$$

   Verify that they are

$$\nabla^m\partial_m\phi = 0. \tag{3}$$

2. Find the Einstein equations

$$G_{mn} = \frac{1}{M_P^2}T_{mn}, \tag{4}$$

   by extremizing the action (1) with respect to the metric

$$\frac{\delta S}{\delta g^{mn}} = 0. \tag{5}$$

   Find T_{mn} and G_{mn}.

3. Using the properties of R_{klmn} calculate

$$\nabla^m G_{mn}. \tag{6}$$

   What does this result imply for $\nabla^m T_{mn}$ on-shell?

4. Show that Minkowski space is a vacuum solution to the Einstein equations. In other words that

$$\langle g_{mn}\rangle = \eta_{mn}, \tag{7}$$

   is a consistent background for the action (1).