Introduction to supergravity 2015: Exercise 7.

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Our superspace conventions are found in [1]. The components of the chiral superfield

$$\bar{\mathcal{D}}_{\dot{\alpha}}\Phi = 0 \tag{1}$$

are defined as

$$\begin{aligned}
\Phi | &= A \\
\frac{1}{\sqrt{2}} \mathcal{D}_{\alpha} \Phi | &= \chi_{\alpha} \\
-\frac{1}{4} \mathcal{D}^{2} \Phi | &= F.
\end{aligned} \tag{2}$$

Calculate

$$\mathcal{D}^{\alpha}\mathcal{D}_{\alpha}\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}}\bar{\Phi}|. \tag{3}$$

You will need to make use of

$$\left(\mathcal{D}_{C}\mathcal{D}_{B} - (-1)^{|B||C|}\mathcal{D}_{B}\mathcal{D}_{C}\right)V^{A} = -\mathcal{T}_{CB}^{D}\mathcal{D}_{D}V^{A} + (-1)^{|D|(|B|+|C|)}V^{D}\mathcal{R}_{CBD}^{A}.$$
(4)

References

[1] J. Wess and J. Bagger, "Supersymmetry and supergravity," Princeton, USA: Univ. Pr. (1992) 259 p