Symplectic geometry. Program of the exam.

Mechanical system with one degree of freedom. Kinetic, potential, total energy. Conservation of energy. Configuration space. Phase space. First integral. Mechanical system with two degrees of freedom. Potential system. Extremals of a functional. Euler-Lagrange equations. Newton equation as the Euler-Lagrange equations. Geodesic equation as the Euler-Lagrange equations. Legendre transformation. Hamiltonian equations. Conservation low.

Symplectic vector space. Canonical basis. Canonical isomorphism with the dual space.

Symplectic manifold. Oriented manifold. Cohomology of a symplectic manifold. Cotangent bundle as a symplectic manifold. Kählerian manifold as a symplectic manifold.

Skew-gradient. Hamiltonian vector field. Locally Hamiltonian vector field. Lie derivative. Interior product. Cartan formula.

Poisson brackets. Local expression, properties. First integral of a Hamiltonian system. Nöther theorem.

Darboux theorem. Lioville theorem. First integrals of the Kepler problem.