Example: (6, 5, 4, 3, 2, 1) is an arithmetical progression because each element is formed by subtracting 1 from the preceding element.

a) 1, 2, 3, 4, 5, 6
b) 2, 4, 8, 16, 32
c) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$
d) x, (x + a), (x + 2a), (x + 3a)
e) 11, 16, 21, 26, 31
f) $x, \frac{x}{a}, \frac{x}{a^2}, \frac{x}{a^3}, \frac{x}{a^4}$

12. Solve these problems:

a) An arithmetical progression begins $\frac{1}{2}, \frac{1}{3}$, ... What are the next two terms?
b) A geometrical progression begins $\frac{1}{2}, \frac{1}{3}$, ... What are the next two terms?

~~Unit 4~~ (2)

Vectors and scalars

A car is travelling north along a road at 60 km h^{-1}. We say that it has a velocity of 60 km h^{-1} north. It has a speed of 60 km h^{-1}. The second quantity, speed, is a scalar quantity, that is, it is a size, or magnitude. The first quantity, velocity, is a vector, that is, it has both magnitude and direction.

The magnitude of a quantity is usually expressed in relation to a standard unit of magnitude. For example, if the mass of a metal cylinder is 2 500 g, then it is 2 500 times the unit of mass, that is, the gramme. If the electrical power of a light bulb is 40 W, then it is 40 times the unit of power the watt. Both of these are scalar quantities.

If a certain town is 150 km from London, then the distance from London to the town is 150 000 times the unit of distance, the metre. Distance, again, is a scalar quantity. Now, if we want to know the exact location of the town, we also need to know the direction, that is, we need the vector quantity displacement, which consists of both distance and direction, 150 km north west.

14. Divide the following quantities into vector or scalar quantities:

speed, mass, displacement, weight, force, acceleration, velocity, distance, volume, temperature, momentum, power

15. Say whether the following statements are true or false. Correct the false statements.

a) The mass of an object is the same as the weight of the object.
b) An ordinary number is a vector quantity.
c) A vector quantity consists of two parts.
d) A position in the Cartesian co-ordinate system may be expressed by a vector.

16. PUZZLE

Comment on this statement:
A train travels 60 km at 30 km h^{-1}, then 60 km at 60 km h^{-1}. Its average speed is therefore 45 km h^{-1}.

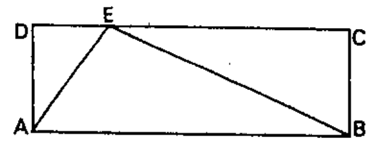
NUCLEUS MATHEMATICS, B. HALL, LONGMAN 1980

Unit 9 Measurement 3
Ratio and Proportion

Section 1 Presentation

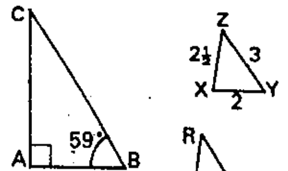
1. Look at these examples:

AB is approximately three times as long as CD.



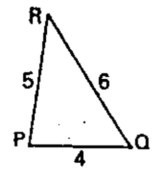
The area of rectangle ABCD is exactly twice as big as that of △ABE.

Now make similar sentences about the following:

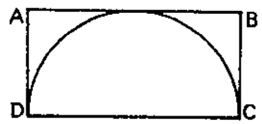


a) $\hat{ABC}$ ...... $\hat{ACB}$.

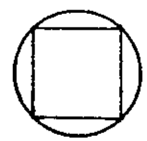
b) PR ...... XZ.



c) △PQR ...... △XYZ.

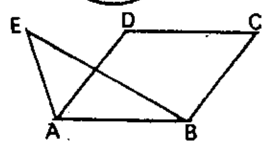


d) ABCD ...... the inscribed semicircle.

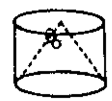


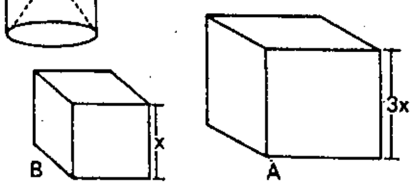
e) ...... any circle ...... the inscribed square.

f) The circumference ...... the diameter.



g) ...... rhombus ABCD ...... ABE.

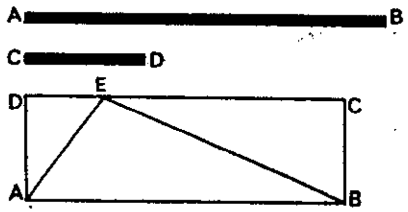




h) ...... cylinder ...... cone with the same base and height.

i) ...... cube A ...... cube B.

2. **Look at these examples:**



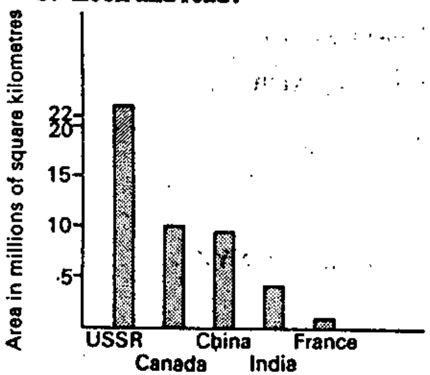
The lengths of AB and CD are in the ratio of approximately 3:1.

The areas of ABCD and ABE are in the ratio of exactly 2:1.

Now make similar sentences about the examples in exercise 1.

Section 2 Development

3. **Look and read:**



This bar graph shows the relative sizes of some countries in millions of square kilometres.

- The USSR is far larger than India.
- India is considerably larger than France.
- Canada is slightly larger than China.

Now compare the other countries in the same way.

4. **Look at the table below showing the heights of the highest mountains in the different continents. Draw a bar graph to illustrate the heights and then compare the heights of different mountains:**

Continent	*Highest Mountain*	*Height (in metres)*
Africa	Kilimanjaro	5963
Antarctica	Vinson Massif	5139
Asia	Everest	8880
Australasia	Wilhelm	4693
Europe	Elbrus	5633
North America	McKinley	6187
South America	Aconcagua	6959

5. **Look and read:**

Variation

$y \propto x$ The ratio between y and x is a constant.
y is *directly proportional* to x.
We say that y *varies directly* as x.

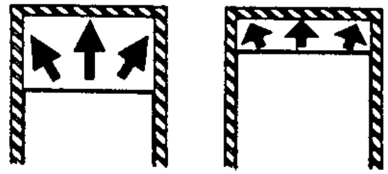
$y \propto \frac{1}{x}$ y is directly proportional to the reciprocal of x.
We say that y is *inversely proportional* to x and that y *varies inversely* as x.

$y \propto xz$ y is directly proportional to the product of x and z.
We say that y is *jointly proportional* to x and z, and that y *varies jointly as* x and z.

Describe the relationship between the following quantities, where k is a constant:

a) $y = \frac{k}{x}$
b) $y = kxz$
c) $y:x = k$
d) $y = kx$
e) $y:\frac{1}{x} = k$

6. **Look and read:**



The volume of a gas is inversely proportional to its pressure.
The smaller the volume, the higher the pressure.

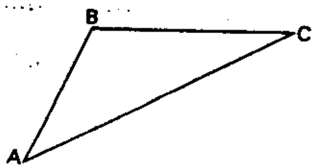
Now make similar sentences about the following:

a) the density and pressure of a gas (for a constant temperature)
b) the volume and temperature of a gas (for a constant pressure)
c) the velocity of a falling body and the time it has been falling
d) acceleration and mass for a constant force
e) the electrical resistance of a wire and its cross-sectional area
f) the circumference of a circle and its diameter
g) the volume of a cylinder and its cross-sectional area and height

Section 3 Reading

7. **Read this:**

The ratio of two quantities is the magnitude of one quantity relative to the other. Division of the quantity a by the quantity b gives the ratio $\frac{a}{b}$, which can also be written as a:b and is read as 'the ratio of a to b'. For example, the ratio of boys to girls in a particular school is 3:2. If the school has 250 pupils, then we can see that $\frac{3}{5}$ of these are boys and $\frac{2}{5}$ girls, i.e, there are 150 boys and 100 girls.



Relative sizes of more than two quantities may be expressed by ratio. For example, the ratio of AB:BC:AC in triangle ABC is 3:4:6. Hence we can see that AC is twice as long as AB. Any triangle which is similar to triangle ABC has sides in exactly the same ratio.

When the ratio of one pair of quantities is equal to the ratio of another pair of quantities, the two pairs are said to be in proportion. If we say that a,b,c,d, are in proportion, we mean that $\frac{a}{b} = \frac{c}{d}$. A property of this proportion is that the reciprocals are also in proportion. Moreover, the ratio of the numerators is equal to the ratio of the denominators.

Say whether the following statements are true or false. Correct the false statements.

a) In a test a student scored 30 out of 100. This gives a ratio of correct answers to incorrect answers of 3:10.
b) The length of the shortest side of a triangle similar to triangle ABC is 12 cm. The other sides are therefore 16 and 24 cm long.
c) Any triangle with sides in the ratio 2:3:6 is a right-angled triangle.
d) We may use ratios to express the relationship between any number of quantities.
e) If a, b, c, d are in proportion, then ad = bc.

8. Complete these exercises:

The last two sentences of the reading passage contain two properties of the proportion $\frac{a}{b} = \frac{c}{d}$. Put these two properties into mathematical form, and number them 1 and 2.
In the calculations which follow, there are four more properties of the proportion. Complete the proof by adding expressions from the lists.

From properties 2 and 3, we have	But
Adding 1 to each side, we obtain	Therefore
From properties 2 and 5, we have	Given
Similarly	Substituting, we obtain
From properties 2 and 4, we have	

a) $\frac{a}{b} = \frac{c}{d}$ (......)

b) ...... $\frac{a}{b} + 1 = \frac{c}{d} + 1$

c) ...... $\frac{b}{b} = 1$ and $\frac{d}{d} = 1$

d) ...... $\frac{a+b}{b} = \frac{c+d}{d}$ (Property 3)

e) ...... $\frac{a-b}{b} = \frac{c-d}{d}$ (Property 4)

f) ...... $\frac{a+b}{c+d} = \frac{b}{d}$

g) ...... $\frac{a-b}{c-d} = \frac{b}{d}$

h) ...... $\frac{a+b}{c+d} = \frac{a-b}{c-d}$ (Property 5)

i) ...... $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Property 6)

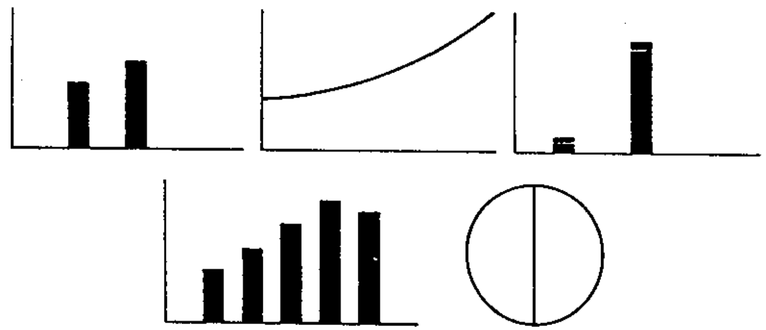
Section 4 Listening

Using percentages in statistics

9. Listen to the passage and write down these words in the order in which you hear them:

passed	respectively	while
whereas	relative	higher
improvement	yesterday	meat

10. Copy the following diagrams, which illustrate the statistics in the five examples:



The five examples given in the passage may be summarised as: (1) bread; (2) wages; (3) exam; (4) restaurant; (5) meat.

a) Put the correct title under each diagram.
b) Label the diagrams as accurately as possible using the figures

11. Each of the examples given in the passage illustrates one of the following general principles. Decide which example illustrates which principle:

a) Two successive increases of x % do not give an increase of 2x %.
b) It is misleading to compare percentages if they are not related to the same quantity.
c) Statistics should be based on a sufficiently large number of examples.
d) Changes in quantities which are percentages should not be expressed in percentages.
e) Choice of starting point for comparisons is important.

Think of other examples which illustrate the same general principles.

12. PUZZLE

A man has a litre of water and a litre of milk. He takes a glassful of the water and adds it to the milk. Then he takes a glassful of this milk-and-water mixture and adds it to the water. Is there more milk in the water than water in the milk, or vice versa?

Unit 9
Using percentages in statistics
Percentages are often used in statistics to represent one quantity relative to another. But it is easy to use percentages in a misleading way. Here are some examples.
1 The price of bread for the last five years has been, respectively, 10p, 14p, 19p, 24p, 21p. The person who buys bread can say that the price of bread has increased by 110 % in five years, whereas the person who sells bread can say that the price of bread has dropped by $12\frac{1}{2}$ % in twelve months.
2 One man's wages go up by 50 %, while another man's only go up by 10 %. But this does not mean that the first man is therefore richer. If the first man's wages were £20 a week, increasing to £30. and the second man's wages were £150 a week, increasing to £165, then the second man has had a larger increase than the first.
3 Last year 40 % of students in a school passed the English exam; this year 50 % passed. We can say either that the results are 10 % better than last year or that there has been a 25 % improvement.
4 A certain restaurant had only two customers yesterday. After the meal, one customer was ill. It is true to say that 50 % of the people who ate at the restaurant were ill.
5 The price of meat has risen by 10 % every year for the last ten years. But the price is not now 100 % higher than ten years ago, but 159 % higher.

Unit C Revision

1. Look and read:

The volume of a right rectangular prism is found by using the formula $l \times h \times w$, where l = length, h = height and w = width.

Now make similar sentences using this table:

	Volume	Surface Area
Sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$
Cylinder	$\pi r^2 h$	$2\pi r(r+h)$
Cone	$\frac{1}{3}\pi r^2 h$	$\pi r(r+\sqrt{r^2+h^2})$ or $\pi r(r+l)$

2. Look and read:

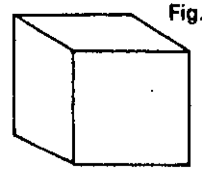
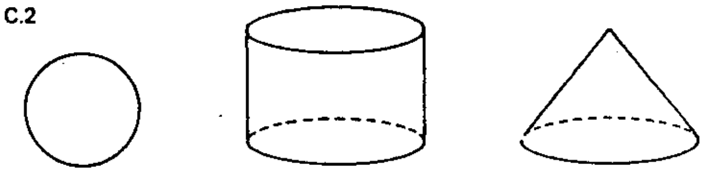


Fig. C.1

The right rectangular prism in figure C.1 is a cube.

Eliminating h and w from the above formula gives volume $= l^3$.

Fig. C.2



The radii of the sphere and of the bases of the cylinder and cone in Figure C.2 are all equal. In addition the height of the cylinder and the cone are both equal to the diameter of the sphere. Therefore h can be eliminated from the above formulae.

Now make correct sentences from the table:

Eliminating h from the formula for the

$$\begin{Bmatrix}\text{volume}\\ \text{surface area}\end{Bmatrix} \text{ of the } \begin{Bmatrix}\text{cylinder}\\ \text{cone}\end{Bmatrix} \text{ gives } \begin{Bmatrix}6\pi r^2\\ 3{\cdot}24\pi r^2\\ \frac{2}{3}\pi r^3\\ 2\pi r^3\end{Bmatrix}$$

Listening lecture 5

http://ocw.mit.edu/OcwWeb/Mathematics/18-02Fall-2007/VideoLectures/detail/embed01.htm

Decide whether the statements are true or false.

1) **Almost all students have heard about vectors before.**
2) **In the first week there will not be many new things to learn.**
3) **If the students have problems with vectors, they can go to instructor´ s house and ask him.**
4) **Vector has both direction and size.**
5) **If we are in the plane, we use x-y-z axis.**
6) **Vector quantity is indicated by an arrow above.**
7) **In the textbooks it is in bold because it is easier to read.**
8) **A vector <j> hat points along the z axis and has length one.**
9) **The notation >a1, a2 is in angular brackets.**
10) **The length of a vector is a scalar quantity.**

What is the nationality of the professor?

Listen again and fill in the missing words

So, let's see, so let's start right away with 1.......... that we will need to see before we can go on to more advanced things. So, hopefully yesterday in 2................., you heard a bit about vectors. How many of you actually knew about vectors before that? OK, that's the vast majority. If you are not one of those people, well, hopefully you'll learn about vectors right now. I'm sorry that the learning 3............. will be a bit steeper for the first week. But hopefully, you'll 4............ fine. If you have trouble with vectors, do go to your recitation instructor's office hours for extra practice if you feel the need to. You will see it's pretty easy.

So, just to remind you, a vector is a 5................. that has both a direction and a magnitude of length. So -- So, concretely the way you draw a vector is by some arrow, like that, OK? And so, it has a length, and it's pointing in some direction. And, so, now, the way that we 6................. things with vectors, typically, as we introduce a coordinate system. So, if we are in the plane, x-y-axis, if we are in space, x-y-z axis. So, usually I will try to draw my x-y-z axis consistently to look like this.

And then, I can represent my vector in 7.................... of its components along the coordinate axis. So, that means when I have this row, I can ask, how much does it go in the x direction? How much does it go in the y direction? How much does it go in the z direction? And, so, let's call this a vector A. So, it's more 8................ When we have a vector quantity, we put an arrow on top to remind us that it's a vector. If it's in the textbook, then sometimes it's in 9............ because it's easier to typeset.

If you've tried in your favorite word processor, bold is easy and vectors are not easy. So, the vector you can try to decompose terms of unit vectors directed along the coordinate axis. So, the convention is there is a vector that we call <i> hat that points along the x axis and has length one. There's a vector called <j> hat that does the same along the y axis, and the <k> hat that does the same along the z axis.

And, so, we can express any vector in terms of its components. So, the other notation is <a1, a2, a3 > between these square brackets. Well, in 10.............. brackets. So, the length of a vector we denote by, if you want, it's the same notation as the absolute value. So, that's going to be a number, as we say, now, a scalar quantity. OK, so, a scalar quantity is a usual numerical quantity as opposed to a vector quantity. And, its direction is sometimes called dir A, and that can be obtained just by 11.............. the vector down to unit length, for example, by dividing it by its length.

So -- Well, there's a lot of notation to be learned. So, for example, if I have two points, P and Q, then I can draw a vector from P to Q. And, that vector is called vector PQ, OK? So, maybe we'll call it A. But, a vector doesn't really have, necessarily, a starting point and an ending point. OK, so if I decide to start here and I go by the same distance in the same direction, this is also vector A. It's the same thing. So, a lot of vectors we'll draw starting at the 12.............., but we don't have to.