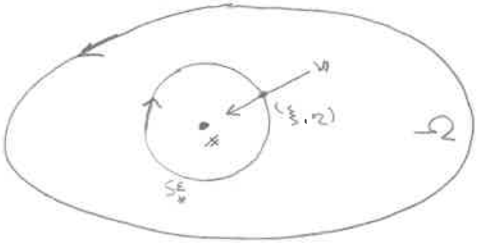
$\Omega \subseteq \mathbb{R}^2$ oblast, $u : \overline{\Omega} \to \mathbb{R}$ spojitá, na Ω^0 dvakrát diferencovatelná

$\mathbf{x} = (x,y) \in \Omega^0$, $K^r_{\mathbf{x}} = \{(\xi,\eta) \in \mathbb{R}^2 : (\xi - x)^2 + (\eta - y)^2 \le r^2\}$ ... kruh se středem x a poloměrem r

$S^r_{\mathbf{x}} = \{(\xi,\eta) \in \mathbb{R}^2 : (\xi - x)^2 + (\eta - y)^2 = r^2\} = \partial K^r_{\mathbf{x}}$ ... kružnice se středem x a poloměrem r

$\varepsilon > 0$, $\varepsilon < 1$ libovolné, že $K^\varepsilon_{\mathbf{x}} \subseteq \Omega^0$



$(\xi,\eta) \in S^\varepsilon_{\mathbf{x}}$: $\nu = \nu(\xi,\eta) = \frac{1}{\varepsilon}(\xi - x, \eta - y)$

$$v(\mathbf{x},\cdot) = v(x,y,\xi,\eta) = \ln \frac{1}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} = -\frac{1}{2}\ln\left((x-\xi)^2 + (y-\eta)^2\right)$$

$$\frac{\partial v(x,y,\xi,\eta)}{\partial \xi} = -\frac{1}{2}\,\frac{2(x-\xi)\cdot(-1)}{(x-\xi)^2+(y-\eta)^2} = \frac{x-\xi}{(x-\xi)^2+(y-\eta)^2} = \frac{x-\xi}{\varepsilon^2}\,, \quad \frac{\partial v(x,y,\xi,\eta)}{\partial \eta} = \frac{y-\eta}{\varepsilon^2}$$

$$\frac{\partial v(x,y,\xi,\eta)}{\partial \nu(\xi,\eta)} = \frac{1}{\varepsilon}(\xi - x, \eta - y)\cdot\frac{1}{\varepsilon^2}(x-\xi, y-\eta) = -\frac{1}{\varepsilon^3}\left((x-\xi)^2 + (y-\eta)^2\right) = -\frac{\varepsilon^2}{\varepsilon^3} = -\frac{1}{\varepsilon}$$

$$\int_{S^\varepsilon_{\mathbf{x}}} u\,\frac{\partial v(\mathbf{x},\cdot)}{\partial \nu}\,dS = -\frac{1}{\varepsilon}\int_{S^\varepsilon_{\mathbf{x}}} u\,dS \underset{\uparrow}{=} -\frac{1}{\varepsilon}\,2\pi\varepsilon\, u(\tilde{\xi},\tilde{\eta}) = -2\pi\, u(\tilde{\xi},\tilde{\eta})\,, \quad (\tilde{\xi},\tilde{\eta}) \in S^\varepsilon_{\mathbf{x}}$$

věta o střední hodnotě

$$\left|\int_{S^\varepsilon_{\mathbf{x}}} v(\mathbf{x},\cdot)\frac{\partial u}{\partial \nu}\,dS\right| \le \left|-\frac{1}{2}\ln\varepsilon^2\right| \int_{S^\varepsilon_{\mathbf{x}}}\left|\frac{\partial u}{\partial \nu}\,dS\right| \le (-\ln\varepsilon)\,2\pi\varepsilon\,\max\left\{\left|\frac{\partial u(\xi,\eta)}{\partial \nu}\right| : (\xi,\eta) \in S^\varepsilon_{\mathbf{x}}\right\} = \text{const}\ \varepsilon\ln\varepsilon$$

2. Greenův vzorec:
$$\int_{\Omega \setminus K^\varepsilon_{\mathbf{x}}}\left(u\,\Delta v(\mathbf{x},\cdot) - v(\mathbf{x},\cdot)\,\Delta u\right)dV = \int_{\partial\Omega}\left(u\,\frac{\partial v(\mathbf{x},\cdot)}{\partial \nu} - v(\mathbf{x},\cdot)\frac{\partial u}{\partial \nu}\right)dS - \int_{S^\varepsilon_{\mathbf{x}}}\left(u\,\frac{\partial v(\mathbf{x},\cdot)}{\partial \nu} - v(\mathbf{x},\cdot)\frac{\partial u}{\partial \nu}\right)dS$$

$$\underbrace{\int_{\Omega \setminus K^\varepsilon_{\mathbf{x}}} u\,\Delta v(\mathbf{x},\cdot)\,dV}_{=\,0} - \int_{\Omega \setminus K^\varepsilon_{\mathbf{x}}} v(\mathbf{x},\cdot)\,\Delta u\,dV = \int_{\partial\Omega}\left(u\,\frac{\partial v(\mathbf{x},\cdot)}{\partial \nu} - v(\mathbf{x},\cdot)\frac{\partial u}{\partial \nu}\right)dS + 2\pi\,u(\tilde{\xi},\tilde{\eta}) - C\,\varepsilon\ln\varepsilon$$

$$- \int_{\Omega \setminus K^\varepsilon_{\mathbf{x}}} v(\mathbf{x},\cdot)\,\Delta u\,dV \xrightarrow{\varepsilon \to 0} -\int_{\Omega} v(\mathbf{x},\cdot)\,\Delta u\,dV$$

$$2\pi\,u(\tilde{\xi},\tilde{\eta}) \xrightarrow{\varepsilon \to 0} 2\pi\,u(x,y)\,, \qquad C\,\varepsilon\ln\varepsilon \xrightarrow{\varepsilon \to 0} 0$$

(ve smysl [illegible] hodnot)