

HOMEWORK 3

- (1) The short exact sequence of chain complexes

$$0 \longrightarrow A_* \xrightarrow{i} B_* \xrightarrow{j} C_* \longrightarrow 0$$

induces the following sequence of homology groups

$$H_n(A) \xrightarrow{i} H_n(B) \xrightarrow{j} H_n(C) \xrightarrow{\partial} H_{n-1}(A) \xrightarrow{i} \dots$$

Prove that the sequence is exact in the term $H_n(A)$.

- (2) Prove that the sequence above is exact in the term $H_n(C)$.
 (3) Compute the simplicial homology of S^2 . It is a border of the standard 3-simplex i.e.

$$S^2 \sim [\{0, 1, 2\}, \{0, 1, 3\}, \{0, 2, 3\}, \{1, 2, 3\}]$$

- (4) Compute the homology groups of Klein bottle or Real projective plane $\mathbb{R}P^2$.

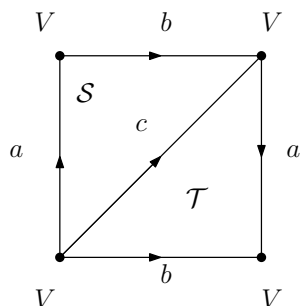


FIGURE 1. Model of Klein bottle.

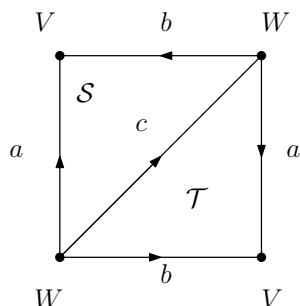


FIGURE 2. Model of Real projective plane