

HOMEWORK 4

- (1) Compute the homology groups of $S^1 \vee S^1$.
- (2) Using the Mayer–Vietoris exact sequence, compute the homology groups of the n -hole torus (we cut horizontally)
- (3) Prove the exactness of the long exact sequence for the triple $A \subset B \subset X$ in the map ∂_* given the two long exact sequences

$$\begin{array}{ccccccccccc}
 \cdots & \longrightarrow & H_n(B) & \longrightarrow & H_n(B, A) & \longrightarrow & H_{n-1}(A) & \longrightarrow & H_{n-1}(B) & \longrightarrow & H_{n-1}(B, A) & \longrightarrow & \cdots \\
 & & & & & & & & & & & & \downarrow \cong \\
 \cdots & \longrightarrow & H_n(B) & \longrightarrow & H_n(X) & \longrightarrow & H_n(X, B) & \xrightarrow{\partial_*} & H_{n-1}(B) & \longrightarrow & H_{n-1}(X) & \longrightarrow & \cdots
 \end{array}$$

where $\partial_* : H_n(X, B) \rightarrow H_{n-1}(B, A)$ (right, up, right)

- (4) Let $A \subset X$ Compute the homology groups of CX/A i.e. make a cone and then collapse part of the base (Hint: make cone CA in the other direction).