

## HOMEWORK 8

- (1) Prove that the cap product in the following case is defined correctly:

$$\cap : H_n(X, A; R) \otimes H^k(X; R) \rightarrow H_{n-k}(X, A; R)$$

- (2) Prove that  $H^*(S^1 \vee S^1 \vee S^2)$  is not isomorphic to  $H^*(S^1 \times S^1)$ .  
(3) Show that for  $a \in H^k(X; R)$ ,  $b \in H^l(X; R)$ ,  $c \in H_n(X; R)$ , we have

$$(a \cap c) \cap b = a \cap (c \cup d).$$

Deduce that  $H_*(X; R)$  is a right  $H^*(X; R)$  module.

- (4) Prove that closed orientable manifolds of odd dimension have Euler characteristics zero.