Exam Problems: Physics in Spacetime, Spring 2020

1. Two observers U and V meet at the origin in spacetime. U is moving with relative velocity u with respect to V, with $u > 0$. At the same time V observes a particle passing through the origin with relative velocity v, where $v < u$. U sees the particle moving with relative velocity $-v$. They all move in a 2-plane in spacetime.

   Draw a spacetime diagram to illustrate the situation. Show that

   $$v = \frac{1}{u}\left(1 - \sqrt{1 - u^2}\right) \ .$$

2. Pulsars are (neutron) stars which emit radio pulses at regular intervals. Alice and Bob count pulses from a very distant pulsar in the y-direction. Alice travels in the x-direction with velocity $u = 24/25$ relative to Bob for seven years, then reverses and returns with the same velocity while Bob stays at home. At the end of the trip they have counted the same number of pulses. Show that when Alice returns she has aged by 14 years while Bob has aged by 50.

   Find the angle to the x-axis at which Alice observes the pulsar.

3. Consider a photon-drive rocket, driven by shooting a powerful laser (of fixed power and wavelength) out of its tail. The change in relative velocity between two values of proper time, τ_i and τ_f is v. What is the ratio of the masses M_i/M_f of the rocket at the two times?