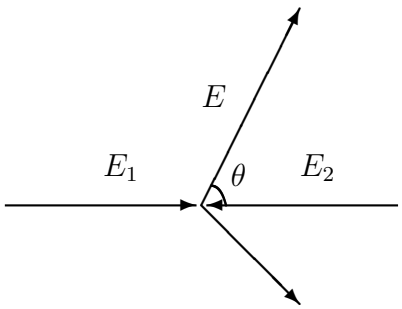
Problems Week 11

1. Photons can scatter against each other even though it happens rarely. An observer sees two photons going towards each other, in his orthogonal space, with energies E_1 and E_2. One of them is scattered an angle θ. The orthogonal space picture is



Calculate its energy as measured by the observer.

2. Consider two parallel null lines and a vector $\bar{r}$ connecting them which is orthogonal to them. Show that *all* vectors connecting the lines are orthogonal to them. Show also that all such vectors have the same length.

3. Protons (p^+) and anti-protons (p^-) have the same mass, m. Protons are accelerated to energy E and then collide with a proton at rest, all with respect to the laboratory. What energy E is needed to form an anti-proton through the process

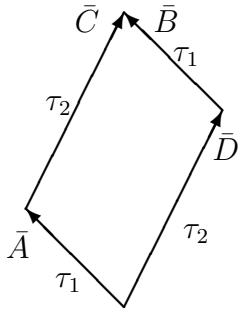
$$p^+ + p^+ \to p^+ + p^+ + p^+ + p^- ?$$

4. A particle travels along a worldline given by $\bar{R} = \bar{R}(\tau)$ where τ is the proper time. Along the whole worldline we have

$$\hat{u} \cdot \bar{A} = 0\,,$$

with $\hat{u}$ a constant four-velocity and $\bar{A}$ the four-acceleration of the particle. Express this in a simple way in terms of the particle's velocity relative to $\hat{u}$.

5. Four spaceships with travel times τ_1, τ_1, τ_2, τ_2 part and meet according to the planar spacetime diagram



Show that opposite worldlines are parallel.

[Hint: Calculate $(\bar{A}+\bar{B})\cdot(\bar{A}-\bar{B})$ and $(\bar{C}+\bar{D})\cdot(\bar{A}-\bar{B})$.]

6. A rocket is propelled by some of its mass being ejected backwards with velocity v relative to the rocket. It moves in a straight line, i.e. the motion is in a 2-plane in spacetime. It is driven until its velocity with respect to the initial state is u. Calculate the ratio of final mass to initial mass of the rocket.

7. Let $\bar{n}$ be a null vector. Show that if $\bar{k}\cdot\bar{n}=0$ then $\bar{k}$ is either space-like or proportional to $\bar{n}$.

8. A light source is moving away from an observer U with velocity v. Another observer V is also moving away with velocity v but perpendicular to the direction of motion of the light source. When the source emits a light signal it is as far from the origin as V is when he receives it.

Calculate the Doppler shift ω_V/ω_0 where ω_0 is the frequency measured by an observer traveling with the light source.

9. Two observers currently at the same location observe a small distant object in their direction of travel. One observes the object to be twice as big as the other. What is their relative velocity.