Problems Week 12

1. Two light signals follow parallel worldlines, with a vector $\bar{r}$ connecting them being orthogonal to them. Show that an observer who measures them finds that they travel in the same direction. Also show that the spatial distance between them is constant independent of the observer.

2. A plane light wave with wave four-vector $\bar{K}_1$ is incident on a mirror. The reflected wave has wave four-vector $\bar{K}_2$. Calculate the normal vector to the mirror.

Consider the case where incidence/reflection is along the normal direction. Calculate the four-velocity of the mirror in this case.

3. A particle follows the worldline $\bar{R} = \bar{R}(\tau)$ with τ the proper time. At a certain point $\bar{R}_0$ its four-velocity is $\hat{u}$. An unaccelerated observer with this four-velocity measures the particle's velocity $\bar{v}$, acceleration $\bar{a}$ and its derivative $\frac{d\bar{a}}{dt}$. Give $\hat{v}$, $\frac{d\hat{v}}{d\tau}$ and $\frac{d^2\hat{v}}{d\tau^2}$ at $\bar{R}_0$ in terms of $\hat{u}$, $\bar{a}_0$ and $(\frac{d\bar{a}}{dt})_0$.

4. An observer has four-velocity $\hat{t}$. A rod has slope m relative to the x-axis in his orthogonal space, i.e. $y/x = m$ $(z = 0)$. The rod is moving with velocity u in the $\hat{x}$-direction. Another observer moves with velocity v in the $\hat{x}$-direction. Her x-axis can be taken to lie in the $(\hat{t}, \hat{x})$-plane. What is the slope of the rod relative to this axis in her orthogonal space?

5. An observer shoots out two particles with velocity v in perpendicular directions in her orthogonal space. Calculate the two particle's relative velocity.

6. Consider a central elastic collision of a ball of mass m and an object of mass M. Central means all four-velocities lie in a 2-plane and elastic means that the masses are not changed in the collision.

Let v be the velocity of the ball after the collision as seen by an observer who sees the object at rest before the collision. Show that

$$\gamma = \frac{1}{\sqrt{1 - v^2}} \leq \frac{m^2 + M^2}{2mM} .$$

7. Consider the reaction $p^+ + \gamma \rightarrow n^0 + \pi^+$ where $m_{p^+} = m_{n^0} = M$ and $m_{\pi^+} = m$. Calculate the lower limit on the photon's energy for this to happen, as measured by an observer who sees the proton at rest.

8. Protons are bombarded with pions. What energy do the pions need to have in the rest frame of the protons for the reaction

$$\pi^- + p^+ \to \pi^+ + \pi^- + n^0$$

to take place? ($M_{\pi^\pm} = m$ and $M_{p^+} = M_{n^0} = M$)

9. Two particles with masses m_1 and m_2 move on a line in an observer's orthogonal space. She measures their velocities to be u_1 and u_2. The particles collide and form a new particle. Calculate its mass and velocity.