Problems Week 9

1. The spacetime position of a particle is given by $\bar{R}(\tau)$ where τ is the proper time. It is observed by an observer with four-velocity $\hat{u}$. The particle's four-velocity is

$$\hat{v} = \gamma(\hat{u} + \bar{v})\,.$$

Give corresponding expressions for the four-acceleration $\bar{A}$ and $\bar{A}^2$.

2. A particle moves so that

$$\frac{d\bar{A}}{d\tau} = \alpha^2 \hat{v}\,,$$

where $\bar{A}$ is the four-acceleration and $\hat{v}$ the four-velocity. Show that $\bar{A}^2 = \alpha^2$. Also show that α is constant along the the particle's worldline. What is the physical interpretation of this fact?

3. Assume that for a particle moving in a 2-plane in spacetime

$$\bar{A}^2 = \alpha^2 = \text{constant}\,.$$

Show that

$$\frac{d\bar{A}}{d\tau} = \alpha^2 \hat{v}\,.$$

This is the converse of the previous problem.