Q6.9 *Sinh–Gordon equation.* Show that the choice

$$A_{11} = -A_{22} = \frac{\mathrm{i}}{4\zeta}\cosh u, \qquad A_{12} = -A_{21} = -\frac{\mathrm{i}}{4\zeta}\sinh u$$

with $q = r = \frac{1}{2}u_x$ (cf. §6.1.5(b)), gives

$$u_{xt} = \sinh u.$$

Q6.10 *Operator* K. Deduce from equation (6.66) the identity

$$K_-(x,z) = F(x,z) + \int_x^{\infty} K_+(x,y)F(y,z)\,\mathrm{d}y.$$

Q6.11 *Operator identity.* Show that, if

$$\Delta(I + \mathrm{J}_+) = (I + \mathrm{J}_+)\Delta_0 \qquad \text{and} \qquad (I + \mathrm{J}_+)(I + \mathrm{J}_F) = I$$

where Δ_0 and J_F commute, then

$$\Delta(I + \mathrm{J}_-) = (I + \mathrm{J}_-)\Delta_0.$$

Q6.12 *Commuting operators.* Verify that the following pairs of operators

(i) $$\Delta_0^{(1)} = \alpha\frac{\partial}{\partial t} + \sum_{n=1}^{N}\alpha_n\frac{\partial^n}{\partial x^n}, \qquad \Delta_0^{(2)} = \beta\frac{\partial}{\partial y} + \sum_{m=1}^{M}\beta_m\frac{\partial^m}{\partial x^m},$$

where α, α_n, β and β_m are constant scalars;

(ii) $$\Delta_0^{(1)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\left(\mathrm{i}\alpha\frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2}\right), \qquad \Delta_0^{(2)} = \begin{pmatrix} l & 0 \\ 0 & m \end{pmatrix}\frac{\partial}{\partial x},$$

where α, l and m are constants;

(iii) $$\Delta_0^{(1)} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}\alpha\frac{\partial}{\partial t} + \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}\frac{\partial}{\partial x}, \qquad \Delta_0^{(2)} = \begin{pmatrix} \tilde{I} & 0 \\ 0 & 0 \end{pmatrix}\frac{\partial}{\partial x},$$

where α is a constant.

Q6.13 *KdV equation: operators.* Show that, if

$$\Delta_0^{(1)} = \frac{\partial}{\partial t} + 4\frac{\partial^3}{\partial x^3},$$

then $\Delta_0^{(1)}$ and J_F commute provided

$$F_t + 4(F_{xxx} + F_{zzz}) = 0;$$

see equation (6.84).

Q6.14 *Two-dimensional KdV equation.* If

$$\Delta_0^{(1)} = \frac{\partial}{\partial t} + 4\frac{\partial^3}{\partial x^3}, \qquad \Delta_0^{(2)} = \frac{\partial}{\partial y} - \frac{\partial^2}{\partial x^2},$$

and we write

$$\Delta^{(2)} = \Delta_0^{(2)} + u(x,t,y),$$