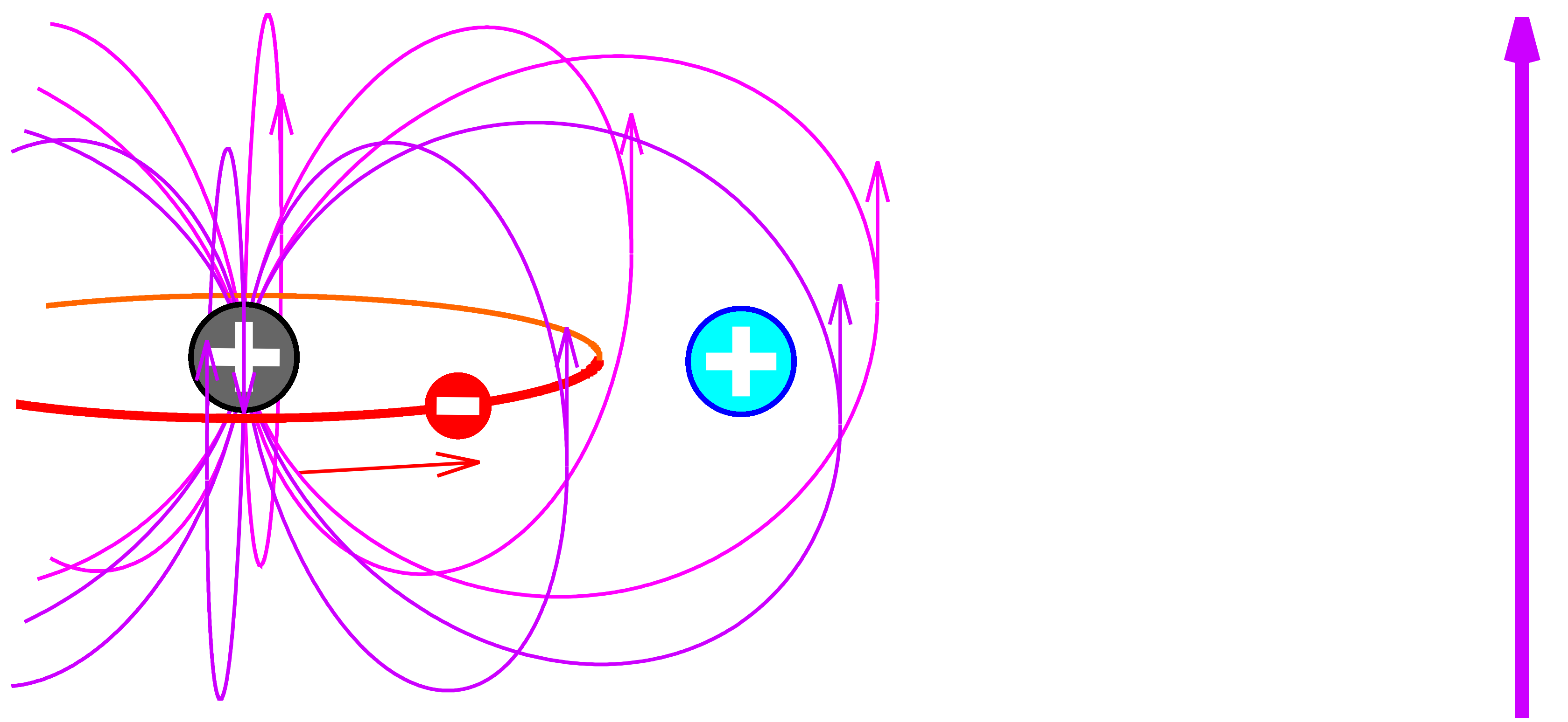
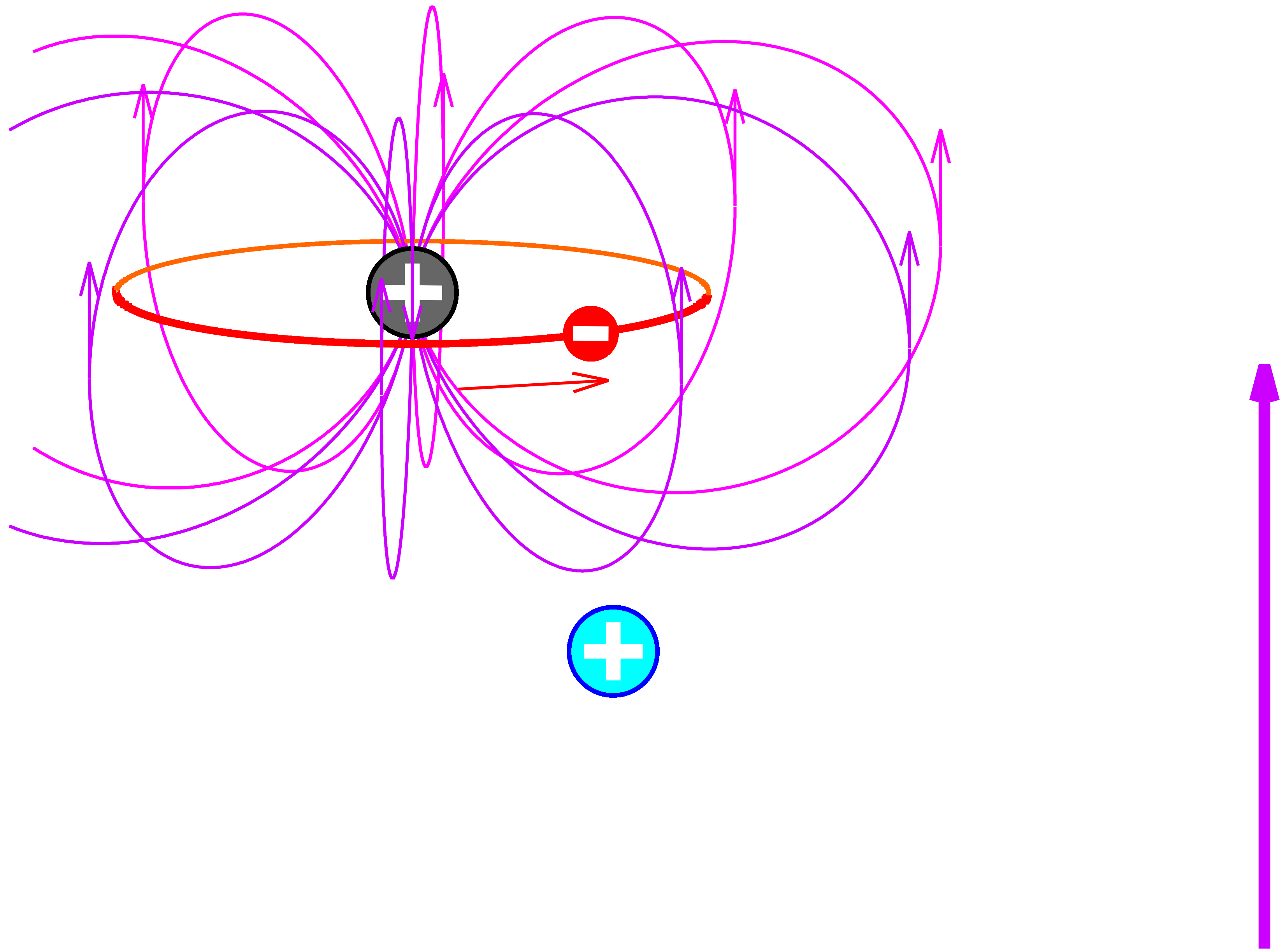
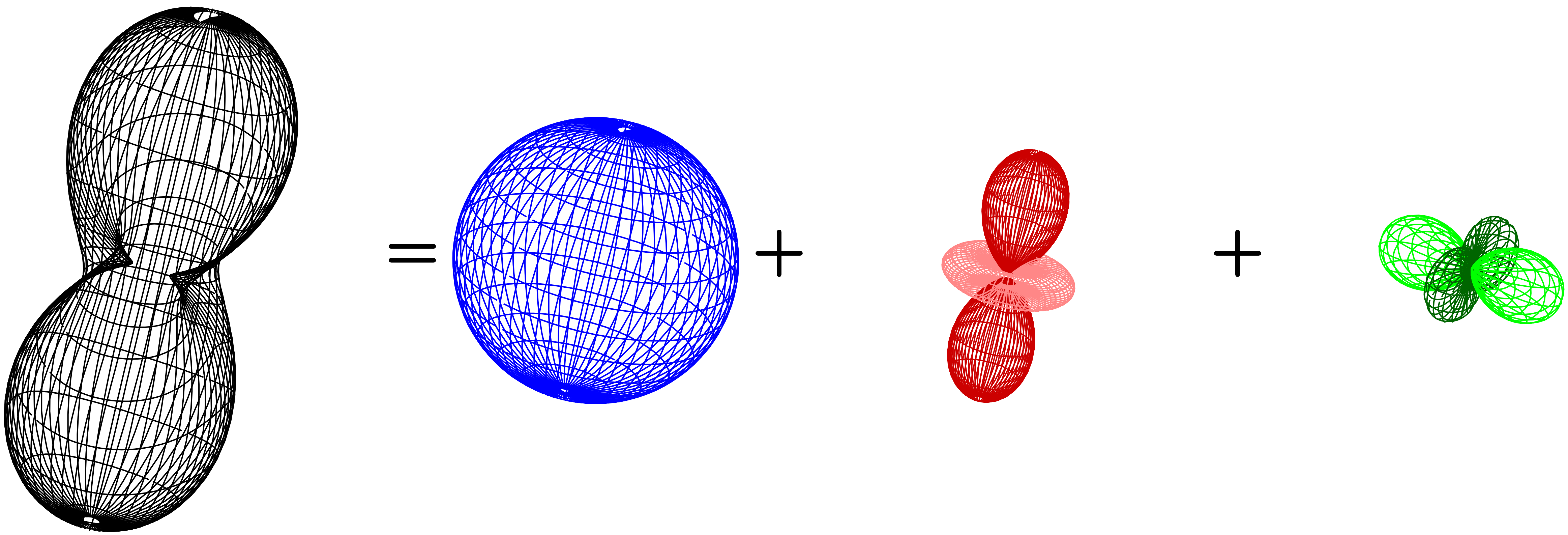
Lecture 2: Relaxation





$$\begin{pmatrix} \delta_{XX} & 0 & 0 \\ 0 & \delta_{YY} & 0 \\ 0 & 0 & \delta_{ZZ} \end{pmatrix} = \delta_{\mathrm{i}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \delta_{\mathrm{a}} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \delta_{\mathrm{r}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

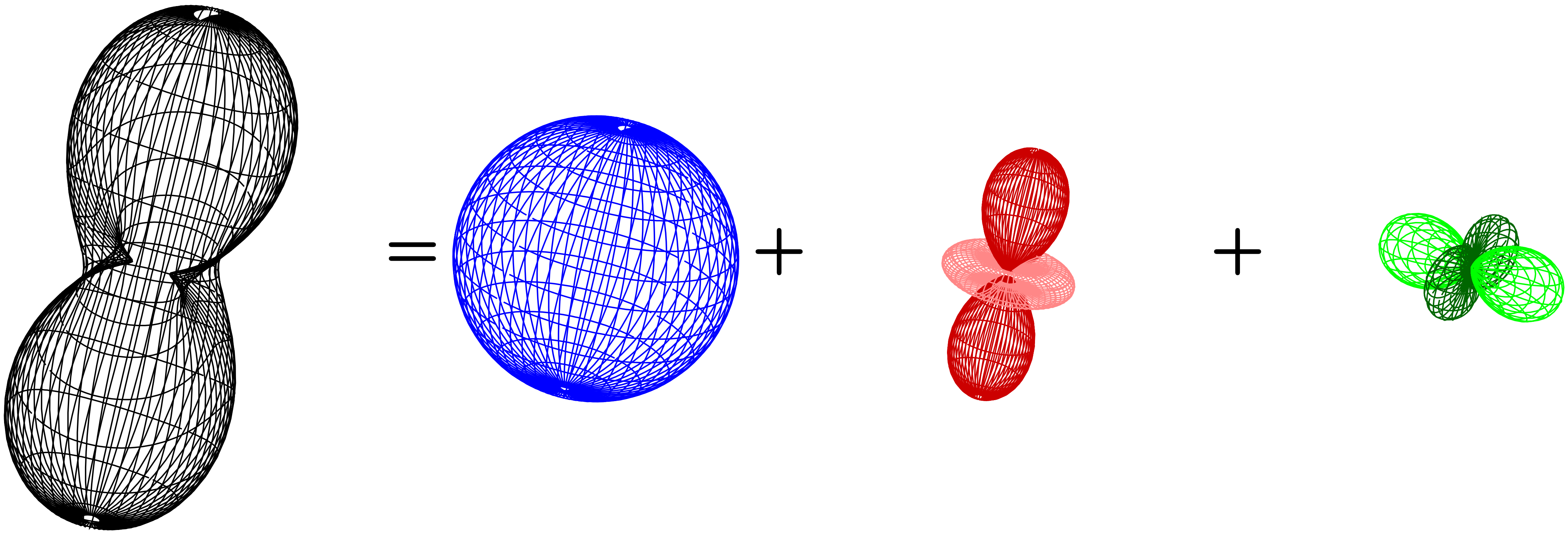


$$\delta_{\mathrm{i}} = \frac{1}{3}\mathrm{Tr}\{\underline{\delta}\} = \frac{1}{3}(\delta_{XX} + \delta_{YY} + \delta_{ZZ})$$

$$\delta_{\mathrm{a}} = \frac{1}{3}\Delta_{\delta} = \frac{1}{6}\left(2\delta_{ZZ} - (\delta_{XX} + \delta_{YY})\right)$$

$$\delta_{\mathrm{r}} = \frac{1}{3}\eta_{\delta}\Delta_{\delta} = \frac{1}{2}(\delta_{XX} - \delta_{YY})$$

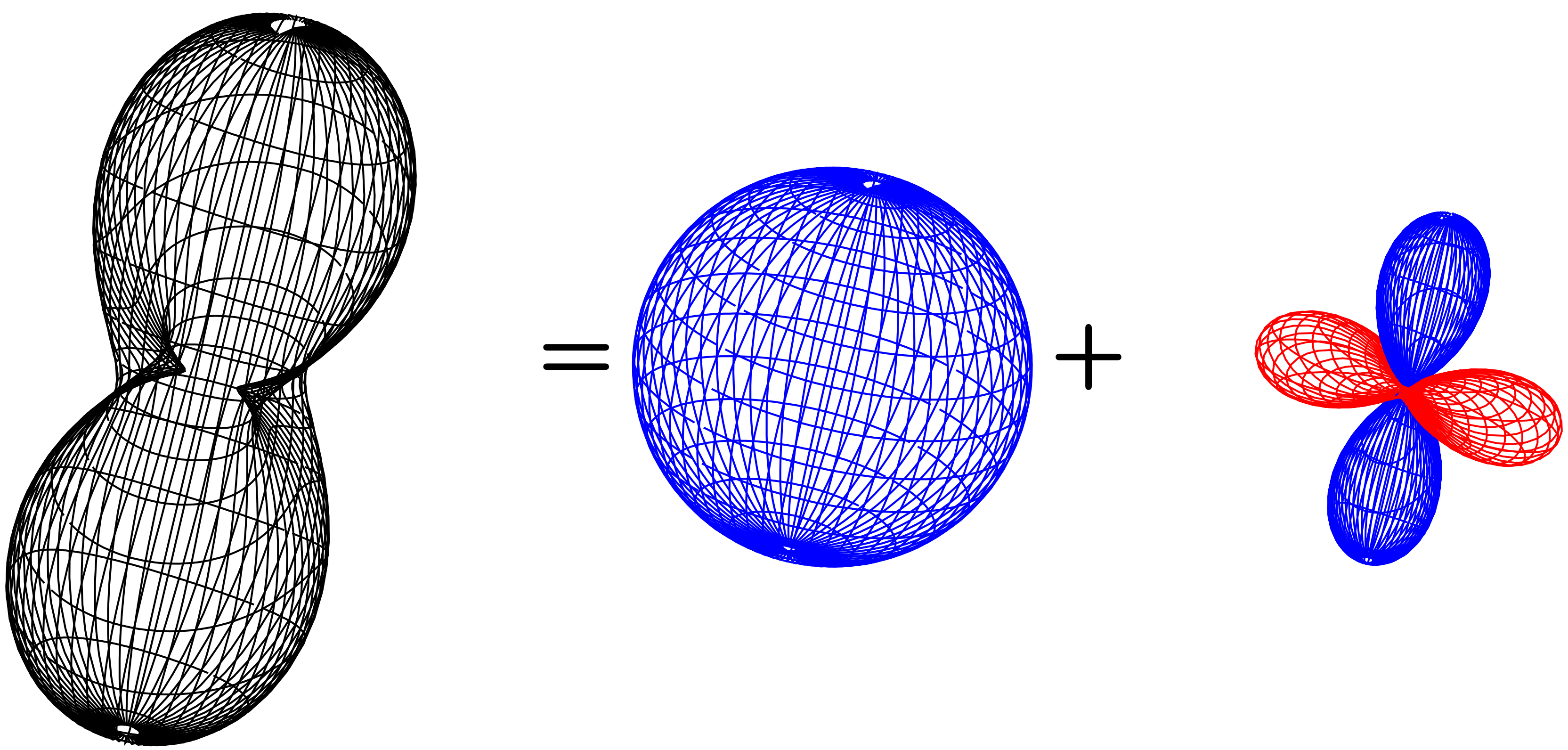
$$\begin{pmatrix} \delta_{XX} & 0 & 0 \\ 0 & \delta_{YY} & 0 \\ 0 & 0 & \delta_{ZZ} \end{pmatrix} = \delta_{\mathrm{i}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \delta_{\mathrm{a}} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \delta_{\mathrm{r}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



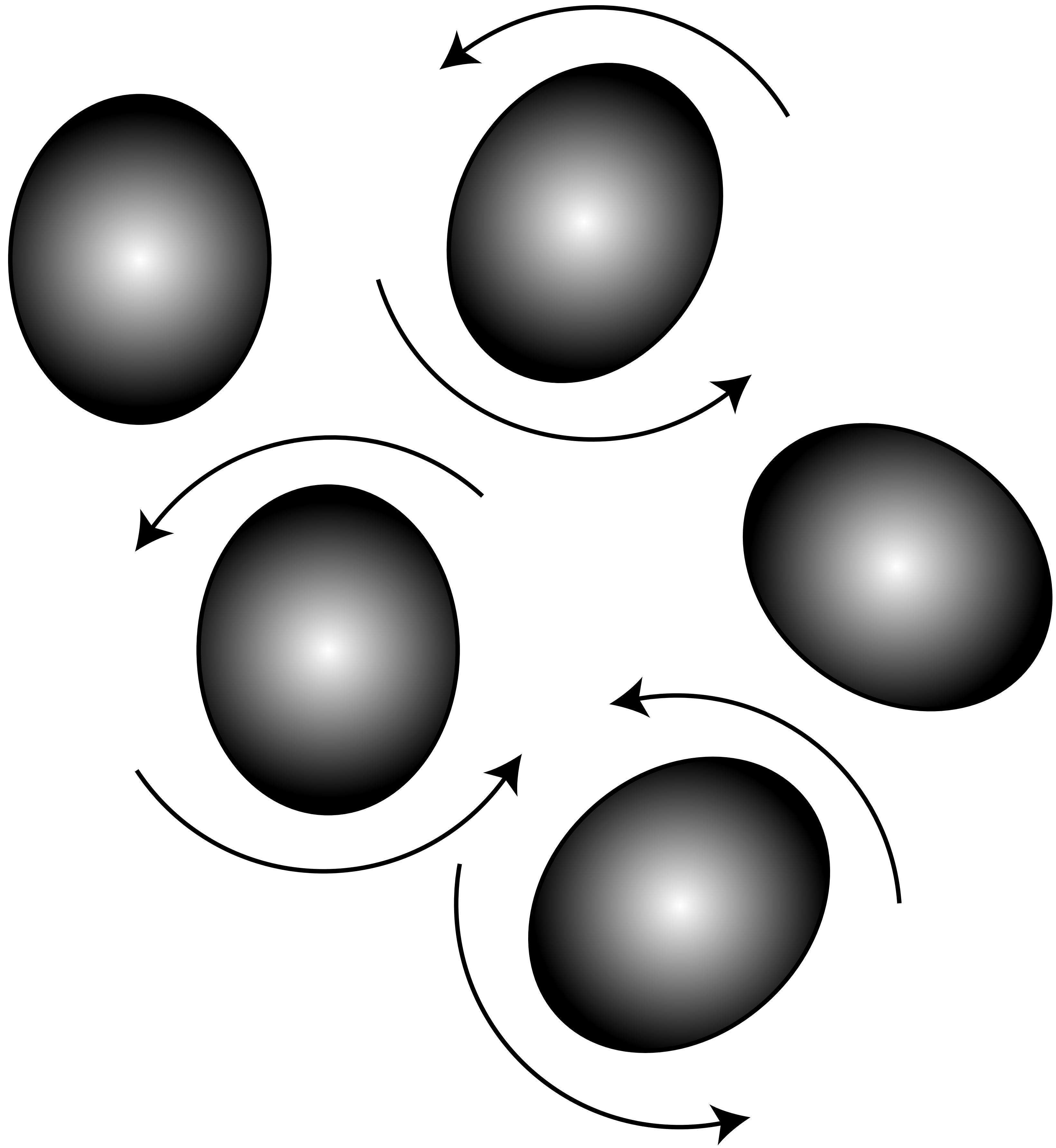
$$\vec{B}_{\mathrm{e}} = \delta_{\mathrm{i}} B_0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \delta_{\mathrm{a}} B_0 \begin{pmatrix} 3 \sin\vartheta \cos\vartheta \cos\varphi \\ 3 \sin\vartheta \cos\vartheta \sin\varphi \\ 3\cos^2\vartheta - 1 \end{pmatrix}$$

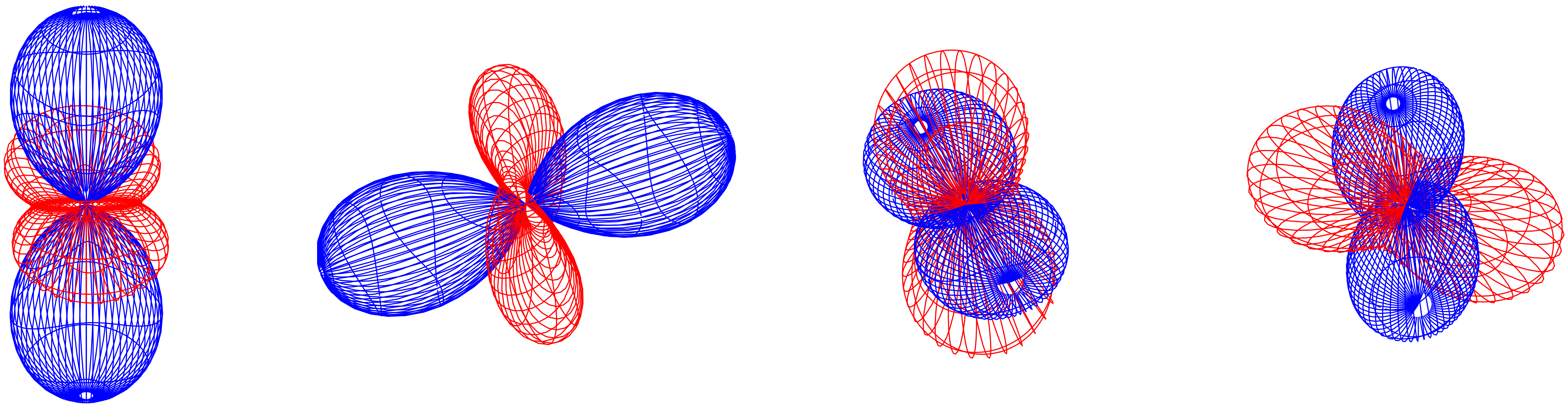
$$+ \; \delta_{\mathrm{r}} B_0 \begin{pmatrix} -(2\cos^2\chi - 1)\sin\vartheta\cos\vartheta\cos\varphi + 2\sin\chi\cos\chi\sin\vartheta\sin\varphi \\ -(2\cos^2\chi - 1)\sin\vartheta\cos\vartheta\sin\varphi - 2\sin\chi\cos\chi\sin\vartheta\cos\varphi \\ +(2\cos^2\chi - 1)\sin^2\vartheta \end{pmatrix}$$

$$\begin{pmatrix} \delta_{XX} & 0 & 0 \\ 0 & \delta_{YY} & 0 \\ 0 & 0 & \delta_{ZZ} \end{pmatrix} = \delta_\mathrm{i} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \delta_\mathrm{a} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \delta_\mathrm{r} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

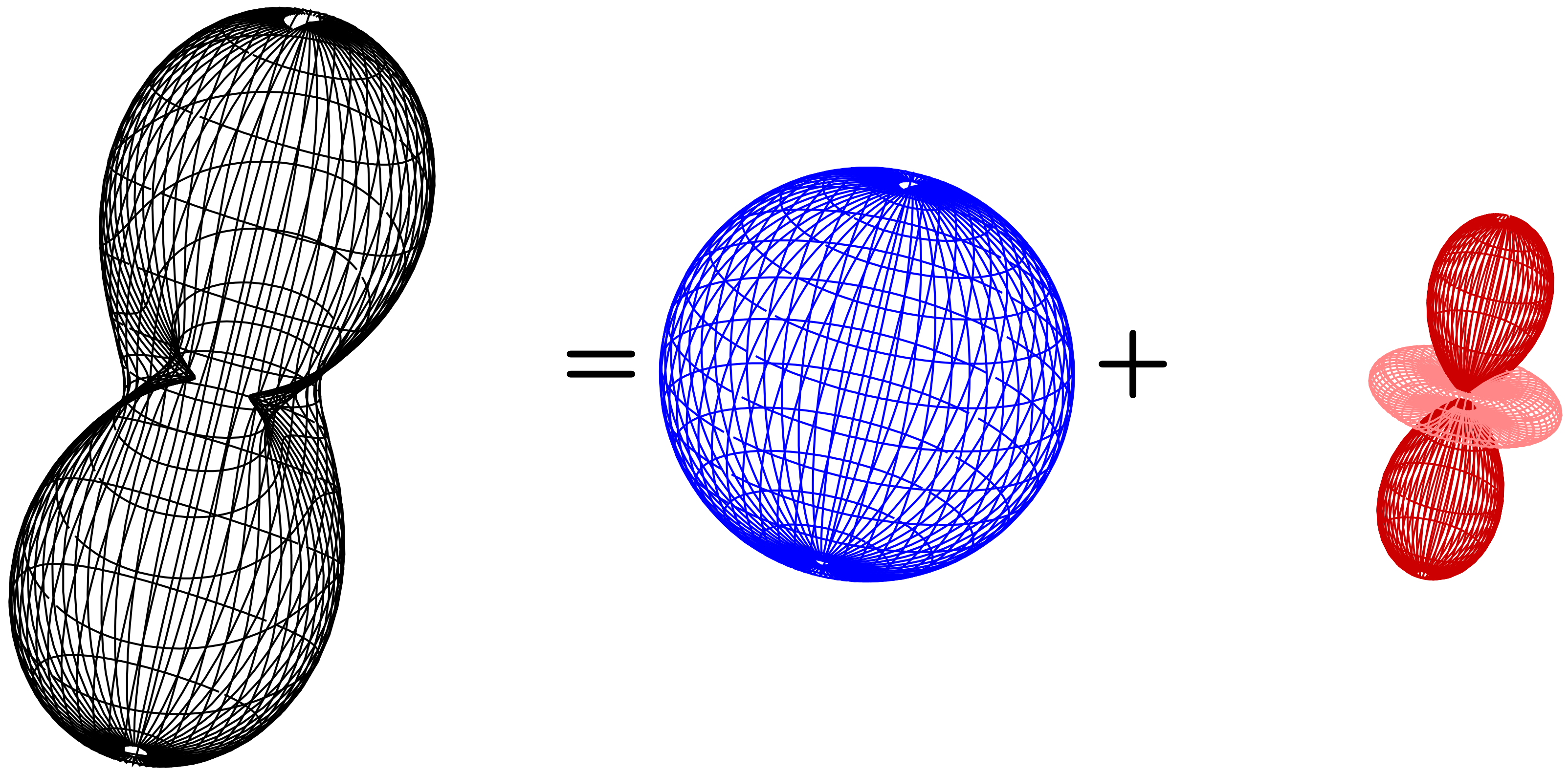


$$\vec{B}_\mathrm{e} = \delta_\mathrm{i} B_0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \delta_\mathrm{a} B_0 \begin{pmatrix} 3\sin\vartheta\cos\vartheta\cos\varphi \\ 3\sin\vartheta\cos\vartheta\sin\varphi \\ 3\cos^2\vartheta - 1 \end{pmatrix}$$
$$+ \delta_\mathrm{r} B_0 \begin{pmatrix} -(2\cos^2\chi - 1)\sin\vartheta\cos\vartheta\cos\varphi + 2\sin\chi\cos\chi\sin\vartheta\sin\varphi \\ -(2\cos^2\chi - 1)\sin\vartheta\cos\vartheta\sin\varphi - 2\sin\chi\cos\chi\sin\vartheta\cos\varphi \\ +(2\cos^2\chi - 1)\sin^2\vartheta \end{pmatrix}$$





$$\begin{pmatrix} \delta_{XX} & 0 & 0 \\ 0 & \delta_{YY} & 0 \\ 0 & 0 & \delta_{ZZ} \end{pmatrix} = \delta_{\mathrm{i}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \delta_{\mathrm{a}} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$



$$\vec{B}_{\mathrm{e}} = \delta_{\mathrm{i}} B_0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \delta_{\mathrm{a}} B_0 \begin{pmatrix} 3 \sin\vartheta \cos\vartheta \cos\varphi \\ 3 \sin\vartheta \cos\vartheta \sin\varphi \\ 3 \cos^2\vartheta - 1 \end{pmatrix}$$

$$R_0 = (\gamma_{B_0}\delta_\mathrm{a})^2 \int_0^\infty \overline{(3\cos^2\vartheta(0) - 1)(3\cos^2\vartheta(t) - 1)}\mathrm{d}t$$

$$= b^2 \int_0^\infty C(t)\mathrm{d}t$$

$$\overline{\left(\frac{3}{2}\cos^2\vartheta(0) - \frac{1}{2}\right)\left(\frac{3}{2}\cos^2\vartheta(t) - \frac{1}{2}\right)} = \frac{1}{5}e^{-t/\tau_\mathrm{c}}\mathrm{d}t = \frac{1}{5}\mathrm{e}^{-6D^\mathrm{rot}t}$$

$$D^\mathrm{rot} = \frac{k_\mathrm{B}T}{8\pi\eta(T)r^3}$$

$$R_0 = \frac{1}{5}b^2 \int_0^\infty \mathrm{e}^{-t/\tau_\mathrm{c}}\mathrm{d}t = \frac{1}{5}b^2\tau_\mathrm{c} = \frac{1}{5}b^2\frac{1}{6D^\mathrm{rot}}$$

$$R_1 = 3\left(\gamma B_0 \delta_{\mathrm{a}}\right)^2 \left(\frac{1}{2} J(\omega_0) + \frac{1}{2} J(-\omega_0)\right) \approx 3\left(\gamma B_0 \delta_{\mathrm{a}}\right)^2 J(\omega_0)$$

$$J(\omega_0) = \int_{-\infty}^{\infty} \overline{\left(\frac{3}{2}\cos^2(\theta(0)) - \frac{1}{2}\right)\left(\frac{3}{2}\cos^2(\theta(t)) - \frac{1}{2}\right)} \cos(\omega_0 t)$$

$$R_2 = 2\left(\gamma B_0 \delta_{\mathrm{a}}\right)^2 J(0) + \frac{3}{2}\left(\gamma B_0 \delta_{\mathrm{a}}\right)^2 J(\omega_0)$$

$$\begin{aligned} R_1 &= \frac{3}{4} b^2 J(\omega_0) \\ R_2 &= \frac{1}{2} b^2 J(0) + \frac{3}{8} b^2 J(\omega_0) \end{aligned}$$

BLOCH EQUATIONS:

$$\frac{\mathrm{d}M_x}{\mathrm{d}t} = -R_2 M_x - \Omega M_y + \omega_1 \sin\varphi M_z$$
$$\frac{\mathrm{d}M_y}{\mathrm{d}t} = +\Omega M_x - R_2 M_y - \omega_1 \cos\varphi M_z$$
$$\frac{\mathrm{d}M_z}{\mathrm{d}t} = -\omega_1 \sin\varphi M_x + \omega_1 \cos\varphi M_y - R_1 (M_z - M_z^{\mathrm{eq}})$$