

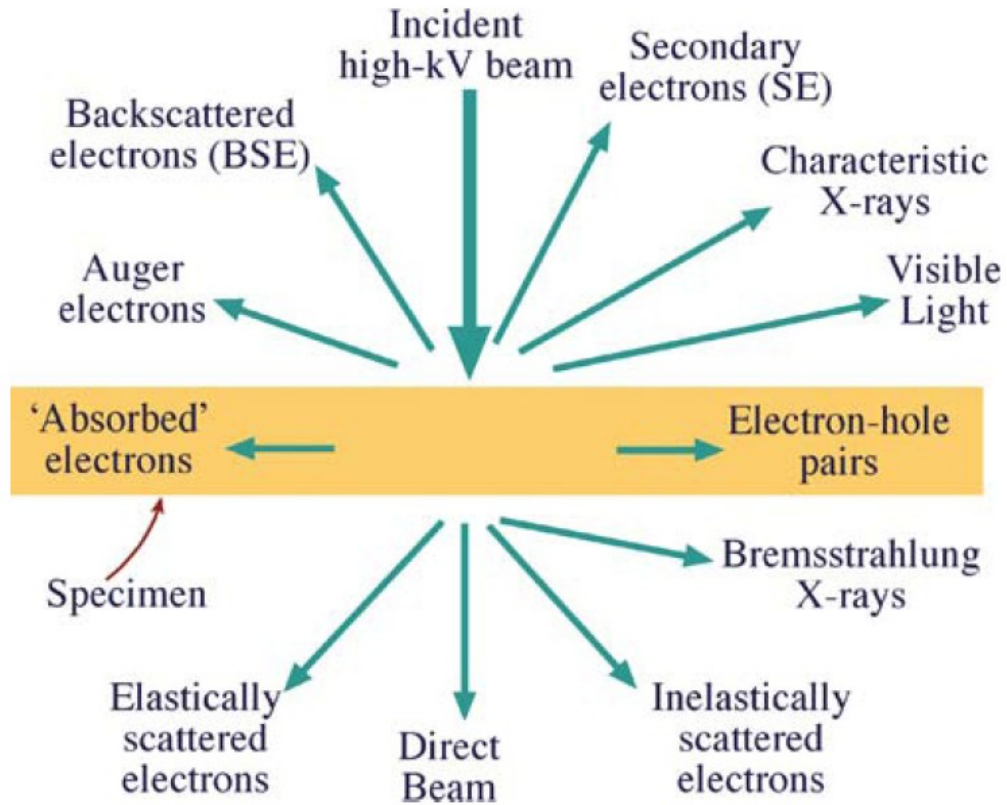
# **C9940: 3-Dimensional Transmission Electron Microscopy**

**Lecture 3: Analysis of electron micrographs**

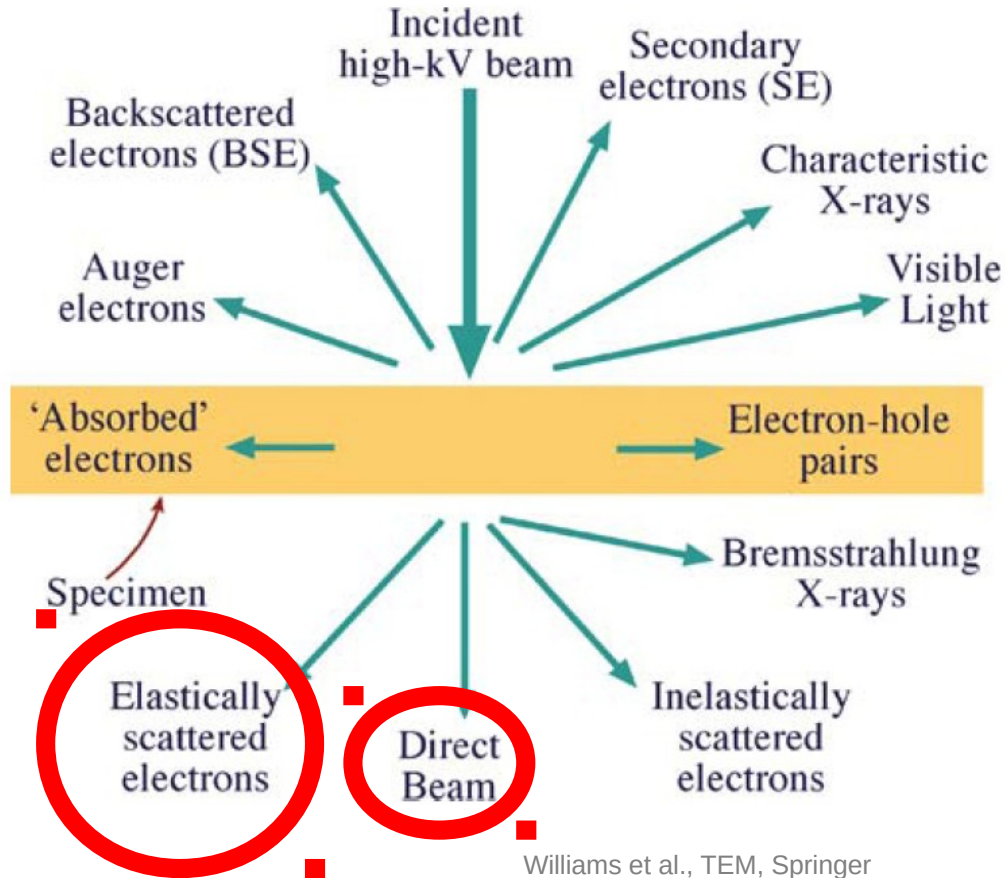
# Content

- interaction of electrons with matter, radiation damage
- data acquisition, image filtering
- projection theorem
- image averaging in 2D
- principal component analysis

# Interaction of electrons with specimen

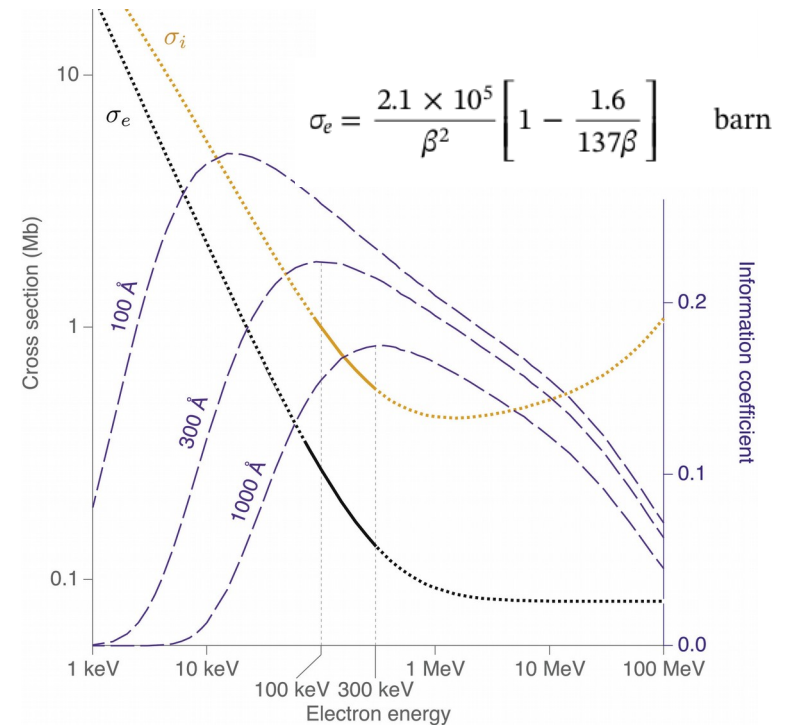


# Interaction of electrons with specimen



**cryo-TEM**

Williams et al., TEM, Springer

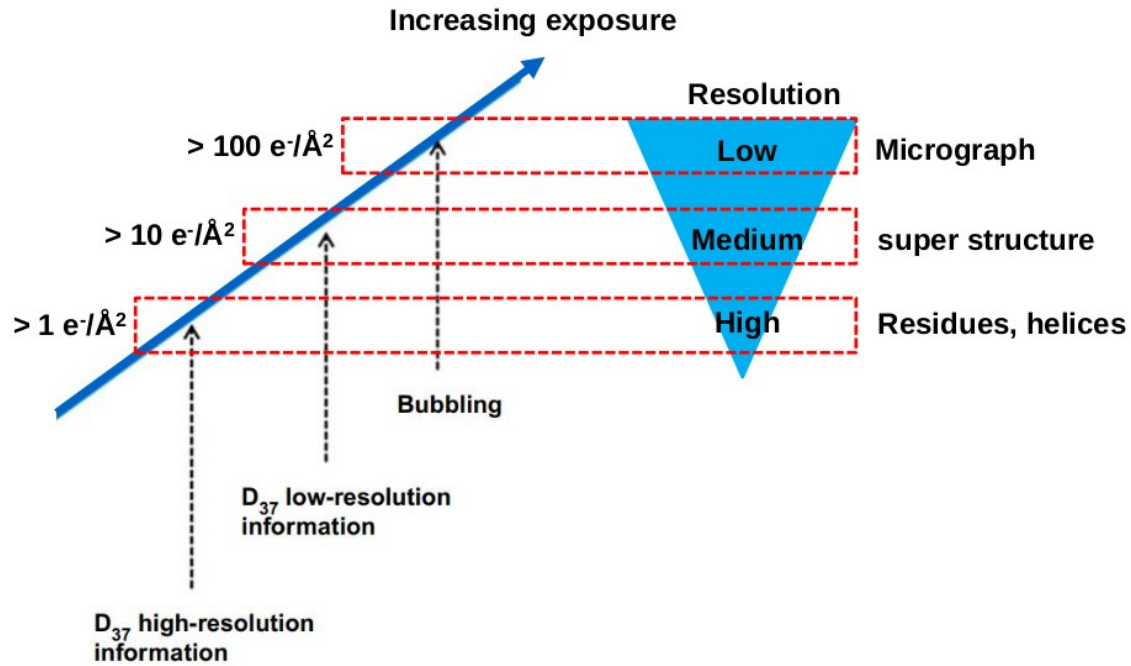


Peet et al., (2019) Ultramicroscopy

mean free path  $\lambda = \frac{1}{\sigma_{\text{total}}} = \frac{A}{N_0 \sigma_{\text{atom}} \rho}$

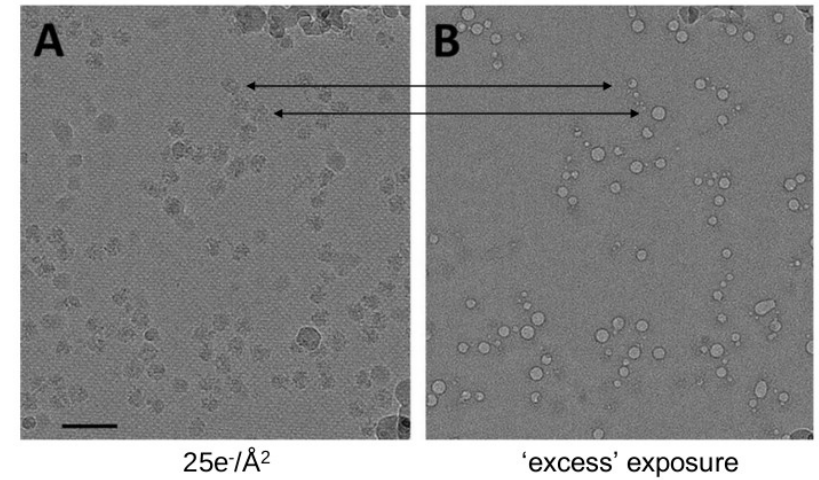
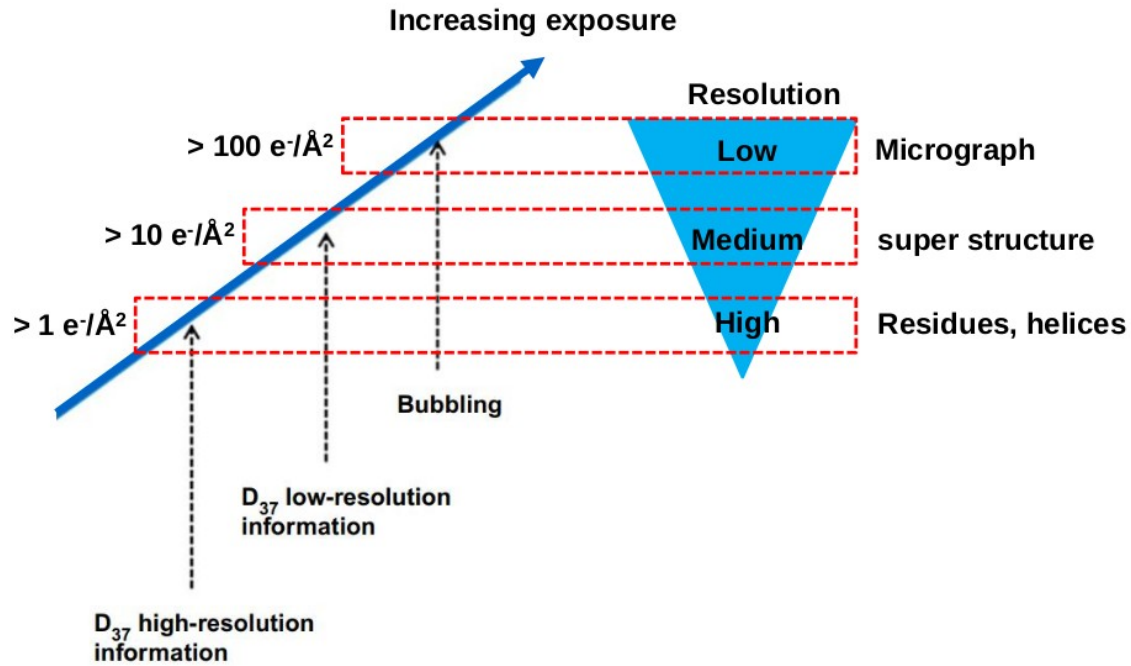
- mean free path of inelastic scattering in vitrified biological specimens: ~395nm

# Radiation damage



Glaser R. (2016), Meth. Enzym.

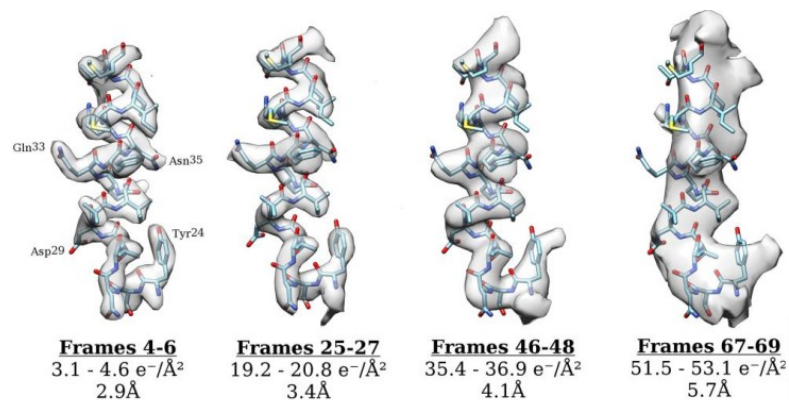
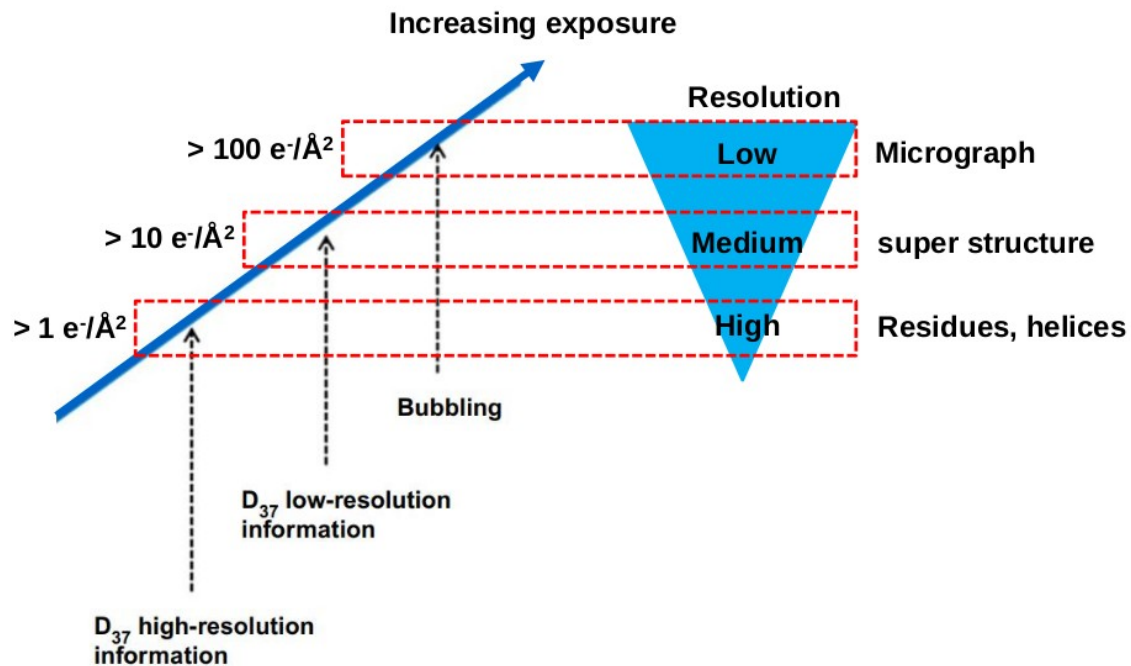
# Radiation damage



Glaser R. (2016), Meth. Enzym.

Glaser R. (2016), Meth. Enzym.

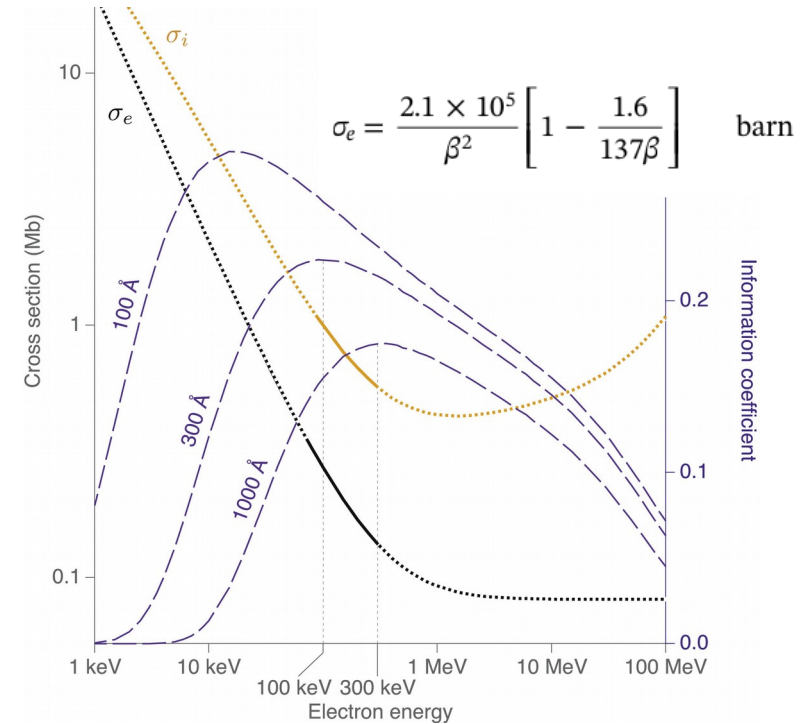
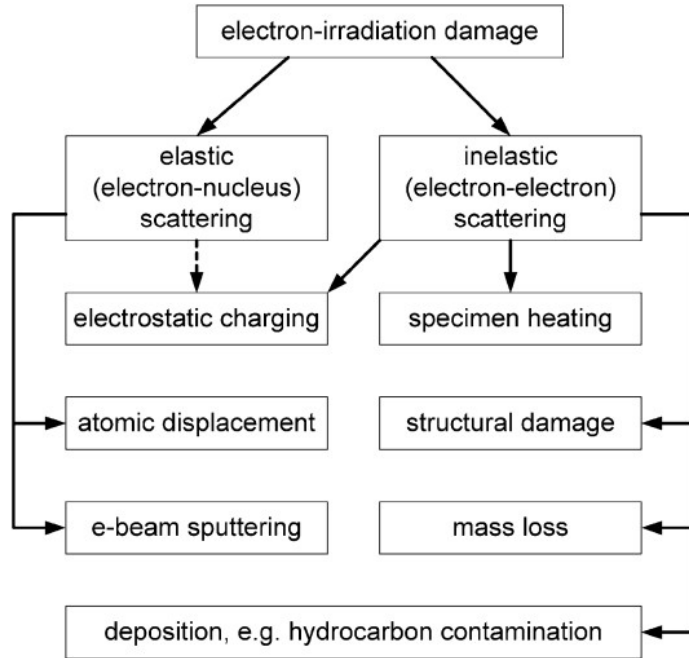
# Radiation damage



Grant. (2015), eLife

Glaser R. (2016), Meth. Enzym.

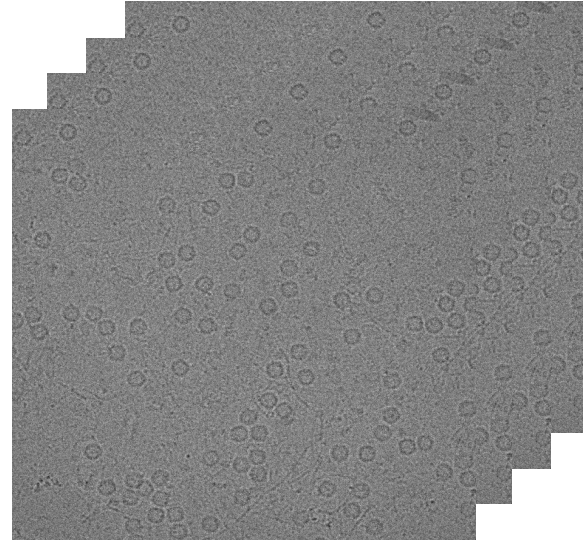
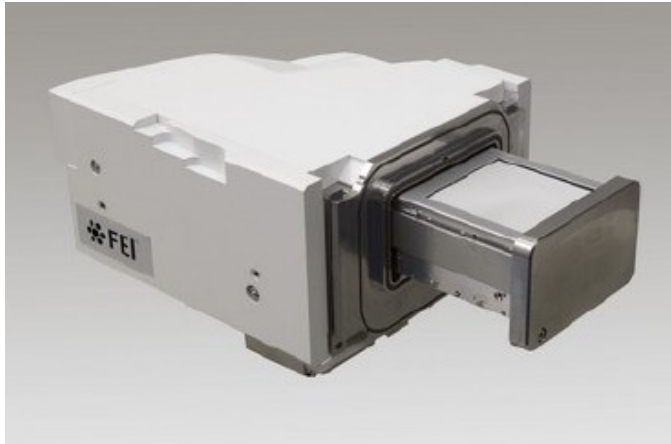
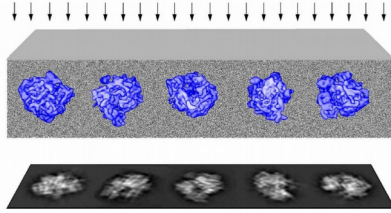
# Interaction of electrons with specimen



Peet et al., (2019) Ultramicroscopy

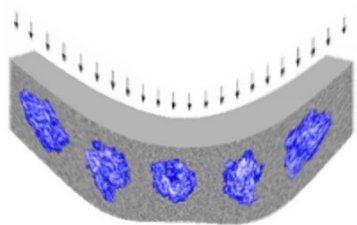
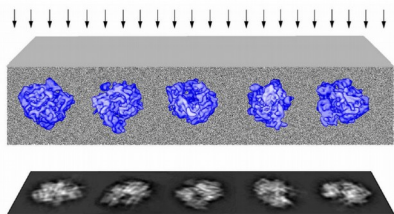


## Data acquisition

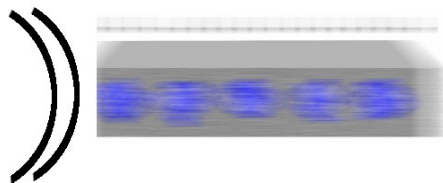


- data from each position on the sample stored as a short movie
- compensation of sample radiation damage
- compensation of the sample motion during exposure

# Data acquisition



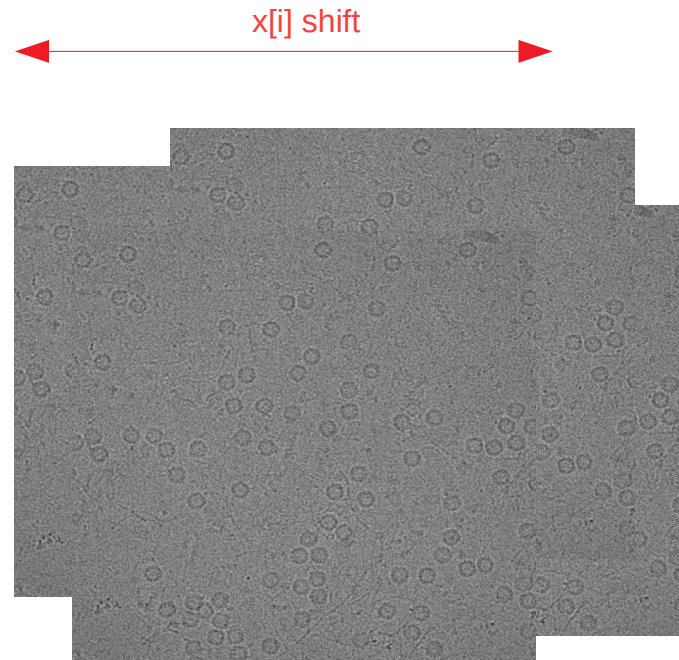
- beam induced motion  
(sample geometry, local)



- drift, vibration  
(external sources, global)



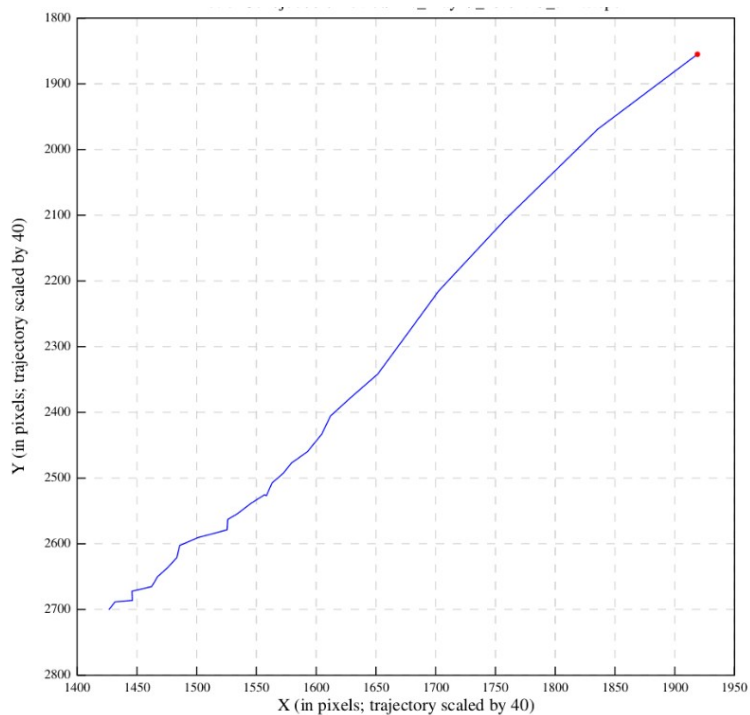
$y[i]$  shift



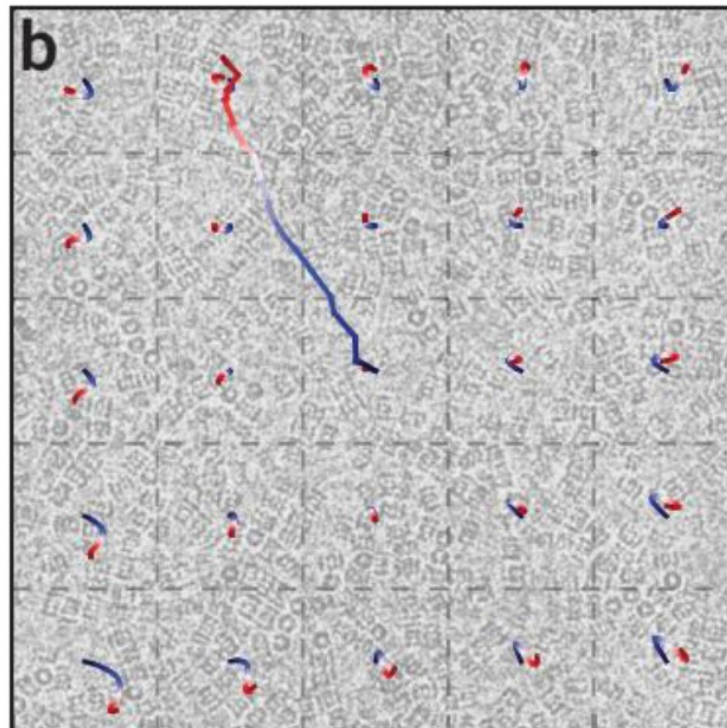
- data from each position on the sample  
stored as a short movie  
- compensation of sample radiation damage  
- compensation of the sample motion  
during exposure

# Data acquisition

- averaging of the movie into single image – increase S/N
- compensation for the global and local motion between the frames – minimize image blur, maximize high-res. Info
- dose-weighting – frame filtering based on acquired radiation damage



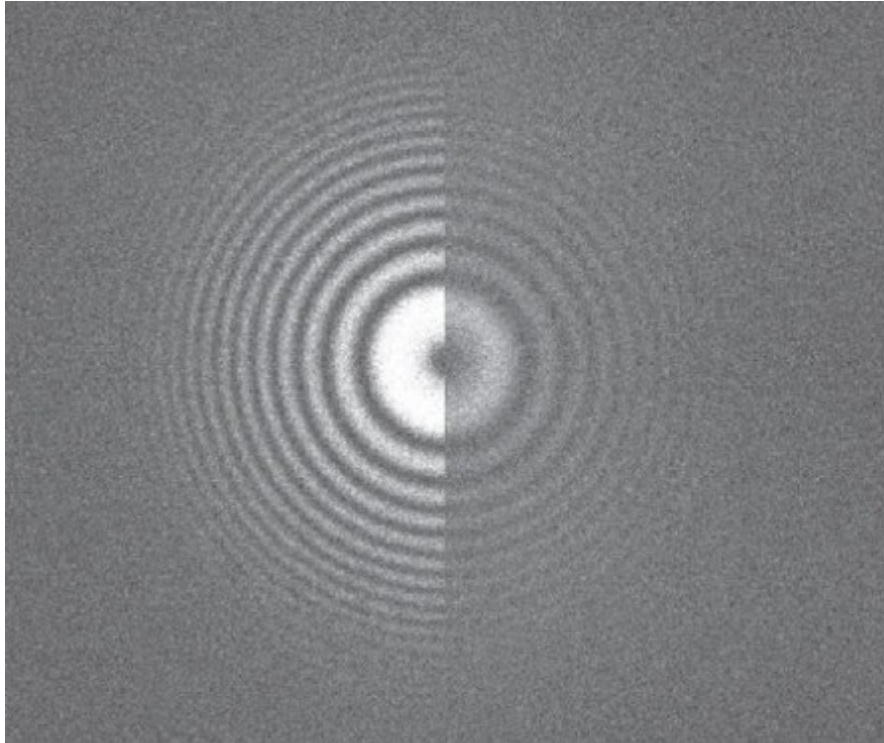
global motion



additional local motion

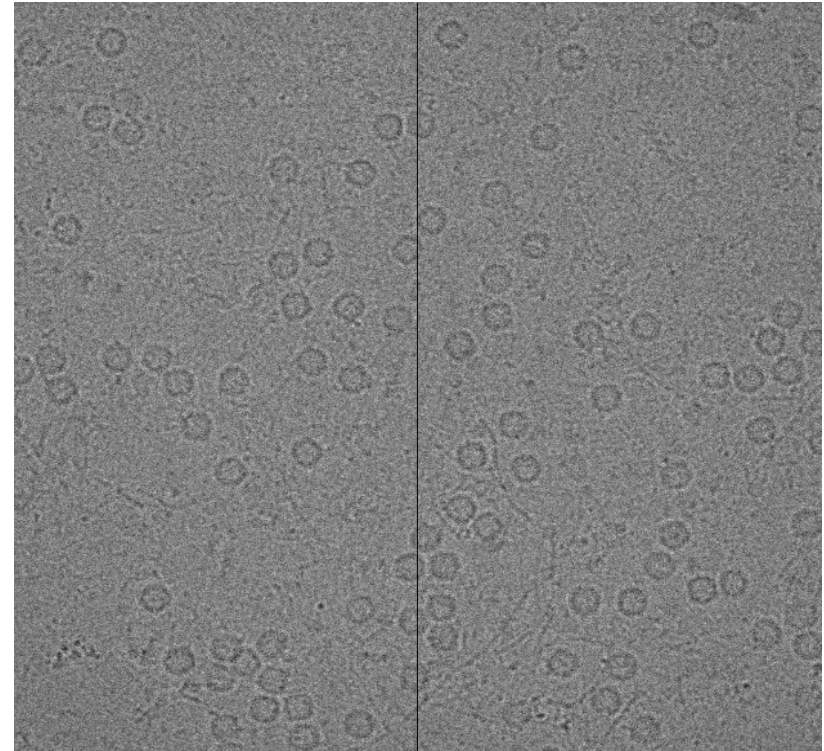
## Data acquisition

- averaging of the movie into single image – increase S/N
- compensation for the global and local motion between the frames – minimize image blur, maximize high-res. Info
- dose-weighting – frame filtering based on acquired radiation damage



aligned image

unaligned image

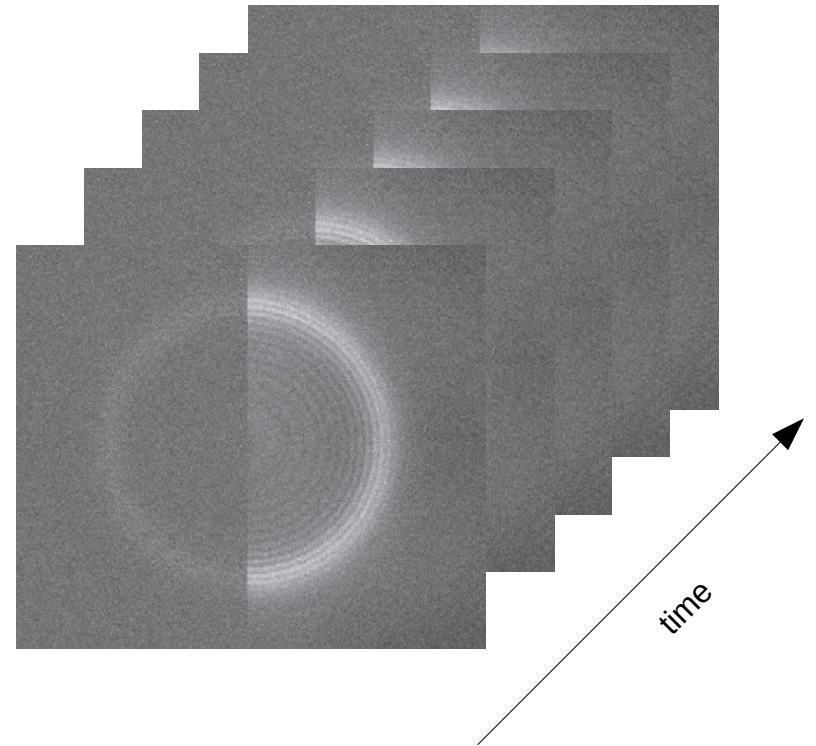
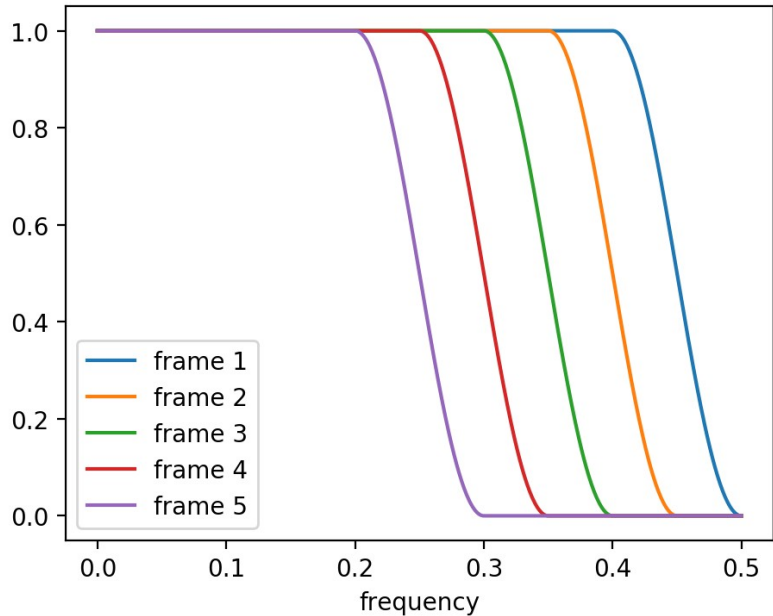


aligned image

unaligned image

## Data acquisition

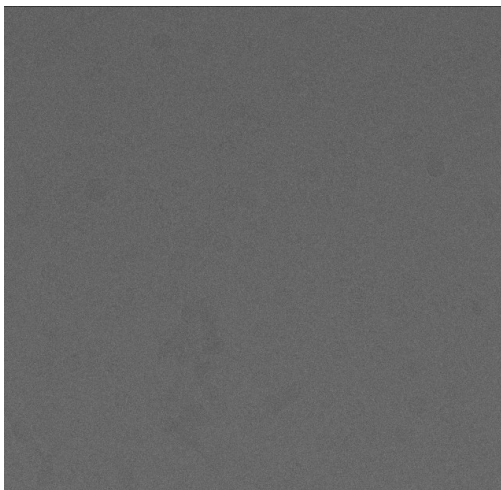
- averaging of the movie into single image – increase S/N
- compensation for the global and local motion between the frames – minimize image blur, maximize high-res. Info
- dose-weighting – frame filtering based on acquired radiation damage



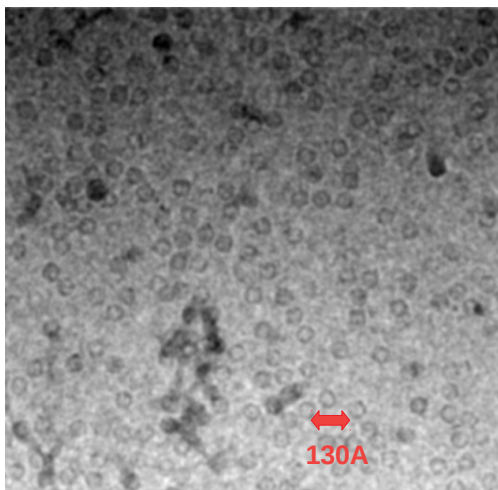
- application of adaptive per-frame low pass filter before averaging

# Image filtering

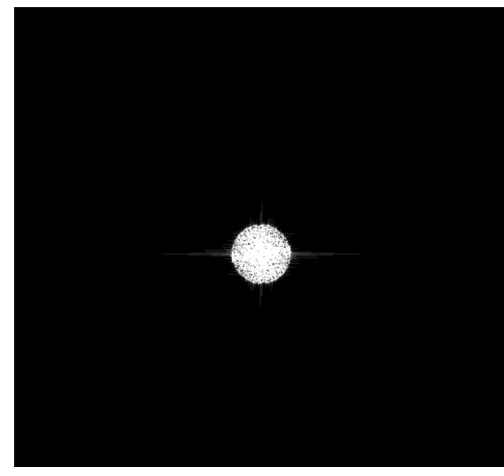
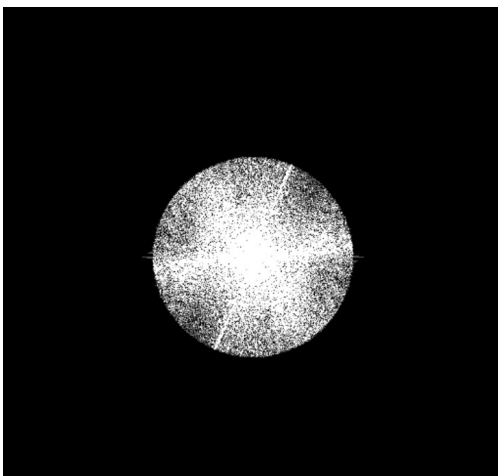
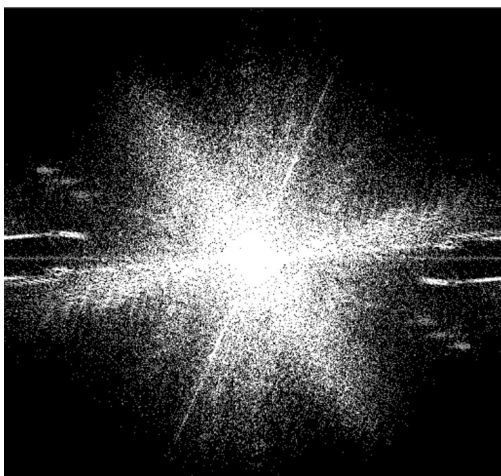
unfiltered image



lowpass filtered (50A)

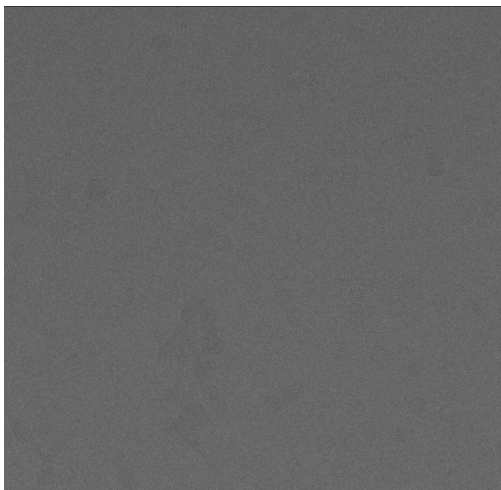


lowpass filtered (250A)

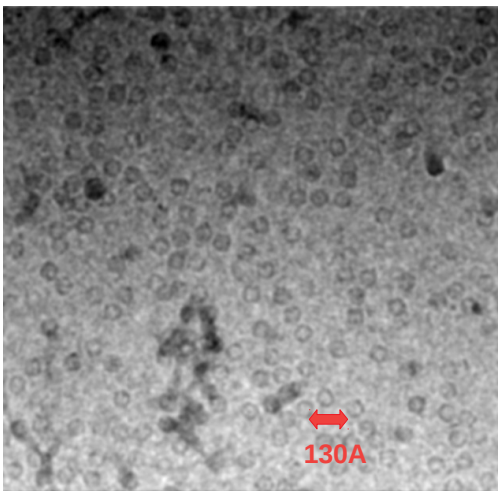


# Image filtering

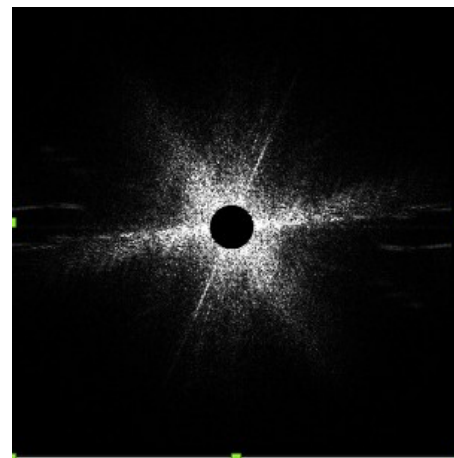
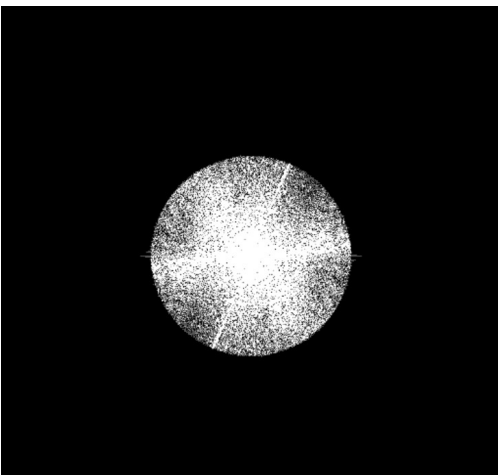
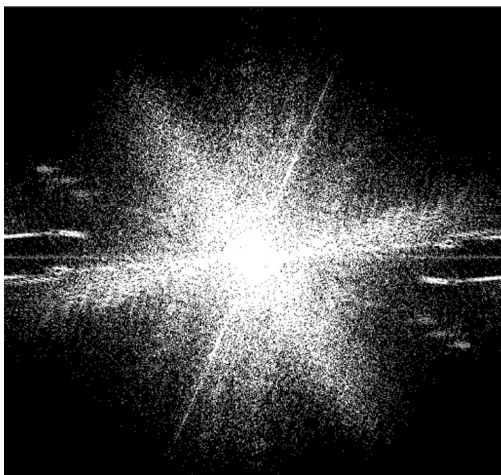
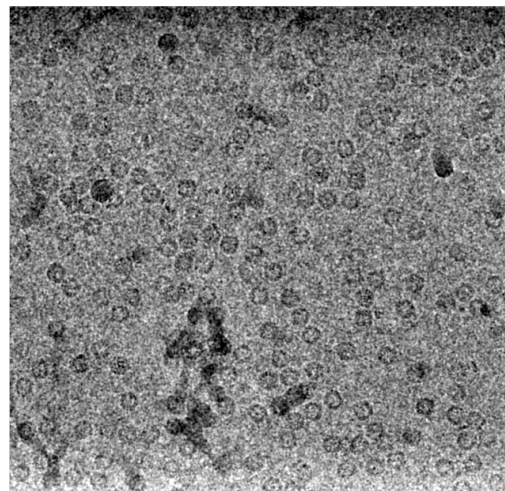
unfiltered image



lowpass filtered (50A)



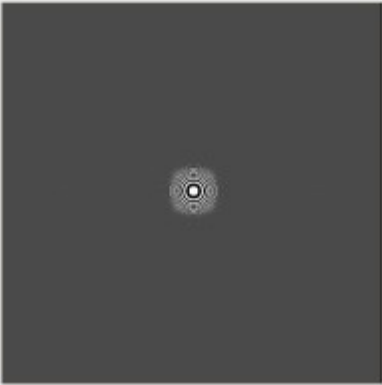
bandpass filtered (1000,10A)



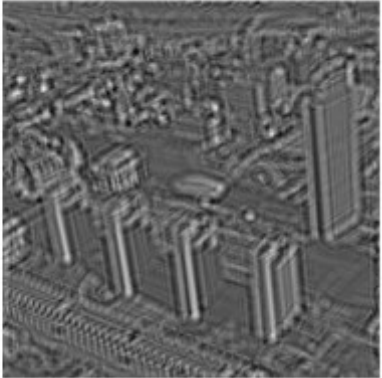
# Image formation



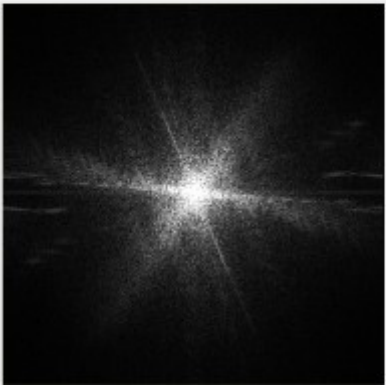
$f(x)$



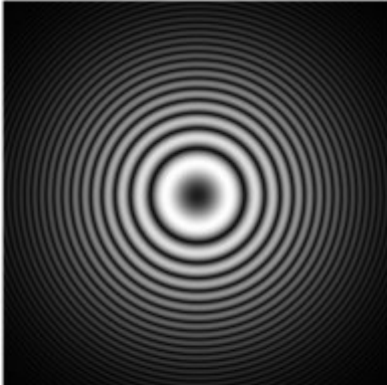
$g(x)$



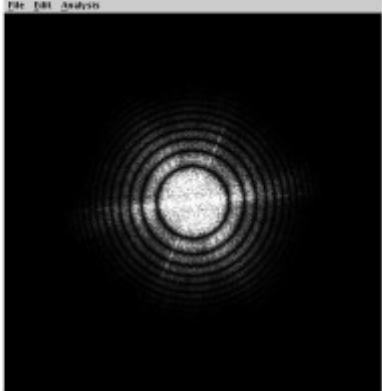
$f(x) \bullet g(x)$



$F(X)$



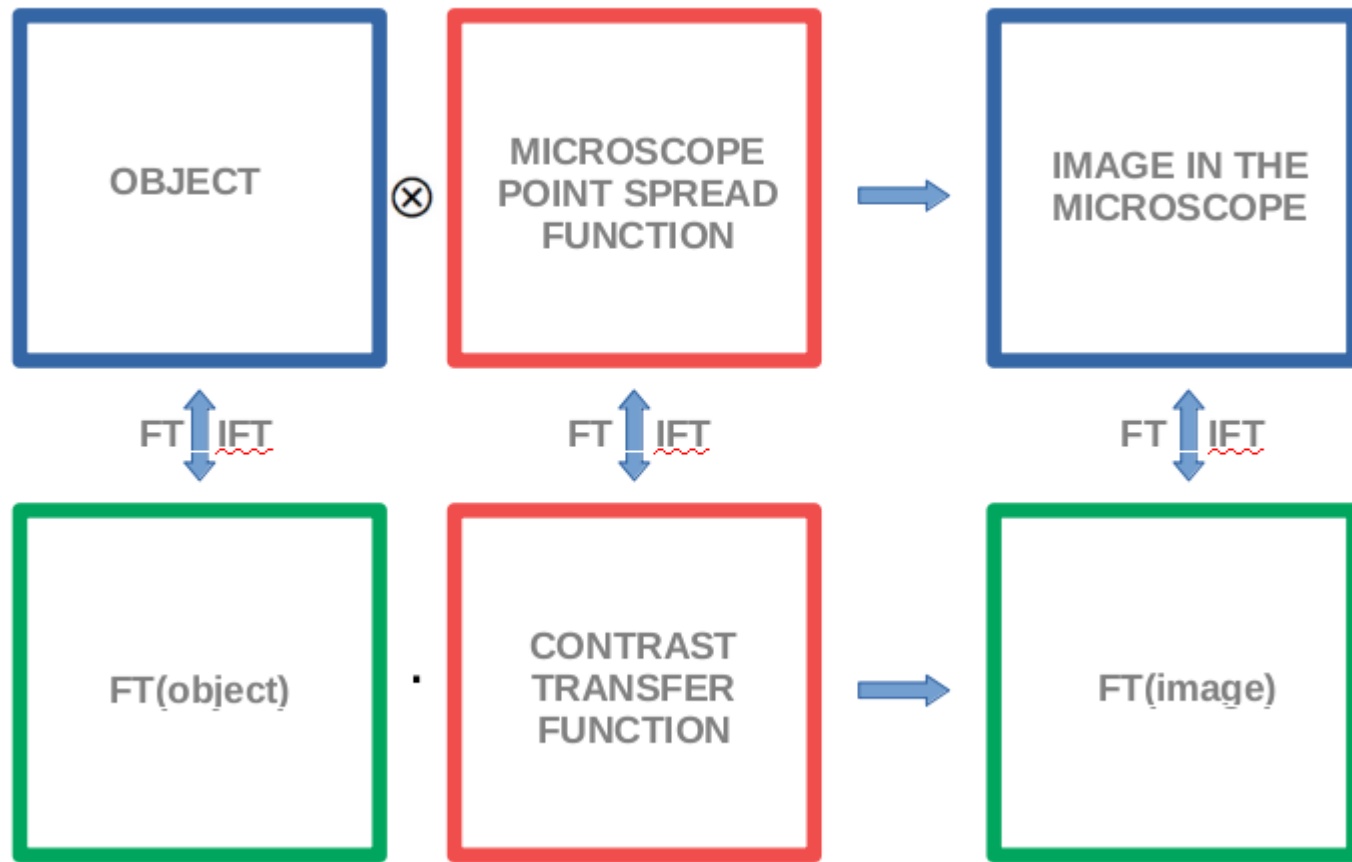
$G(X)$



$F(X) \cdot G(X)$



# Image formation



# Contrast transfer function

$$\text{CTF}(\vec{s}) = -\sqrt{1 - A^2} \cdot \sin(\gamma(\vec{s})) - A \cdot \cos(\gamma(\vec{s}))$$

$$\gamma(\vec{s}) = \gamma(s, \theta) = -\frac{\pi}{2} C_s \lambda^3 s^4 + \pi \lambda z(\theta) s^2$$

A – amplitude contrast

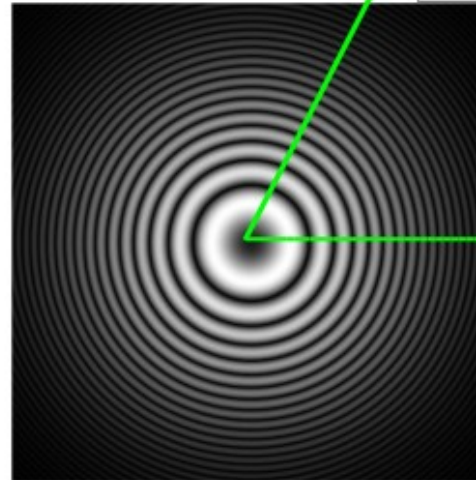
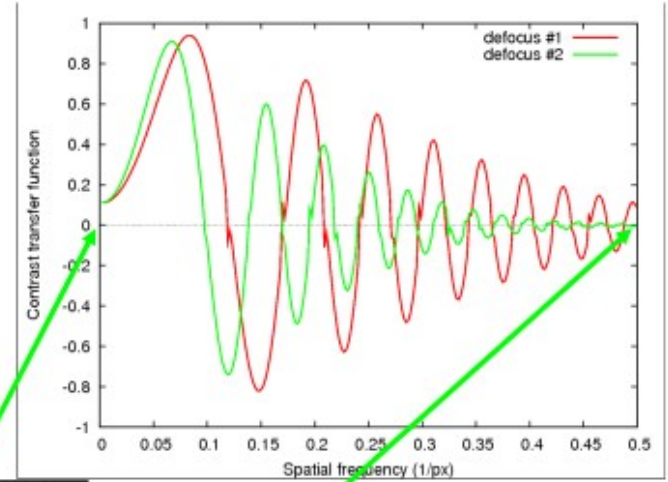
s – spatial frequency

C<sub>s</sub> – spherical aberration

λ – electron wavelength

z – defocus

1D profile



2D power spectrum  
 $G(X)$

# Contrast transfer function

## Envelope function

- Finite source size

$$E_{pc}(k) = \exp[-\pi^2 q^2 (k^3 C_s \lambda^3 - \Delta z k \lambda)^2],$$

- Energy spread (defocus)

$$E_{es}(k) = \exp\left[-\frac{1}{16 \ln 2} \pi^2 \delta z^2 k^4 \lambda^2\right],$$

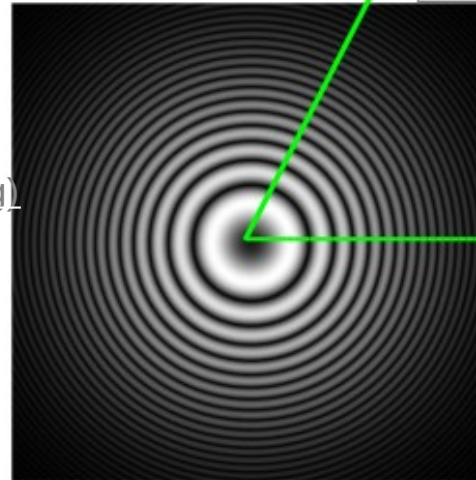
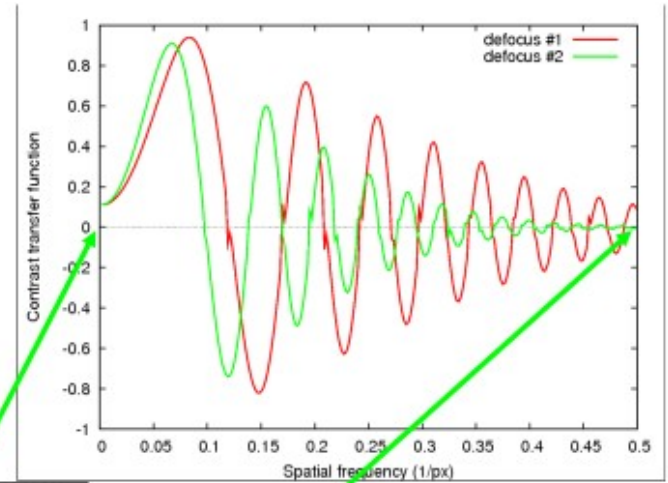
- MTF of the camera

$$E_f(k) = 1/[1 + (k/k_f)^2],$$

- Generic envelope (drift, charging, multiple scattering)

$$E_g(k) = \exp[-(k/k_g)^2],$$

1D profile

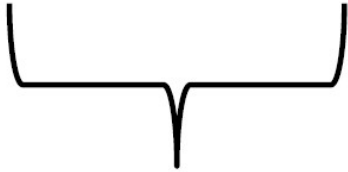


2D power spectrum  
 $G(X)$

# Contrast transfer function

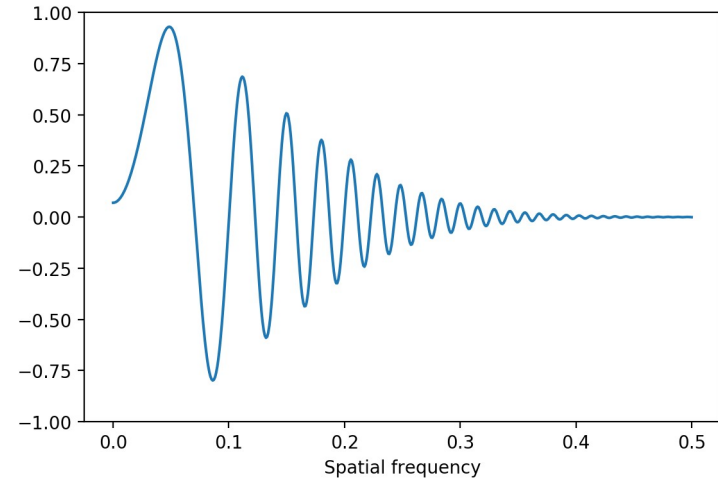
Envelope function

$$I(\mathbf{k}) = E_{pc}(k)E_{es}(k)E_f(k)E_g(k)H(k)\Phi(\mathbf{k}) + N(\mathbf{k}).$$

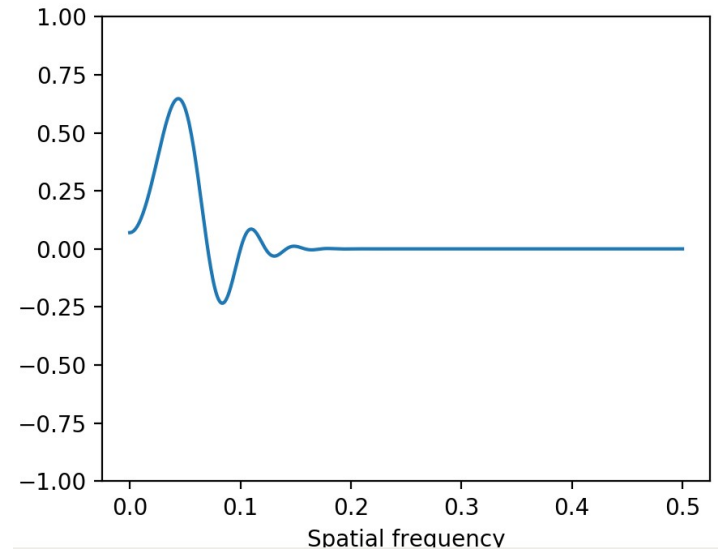


$$e^{-Bk^2}$$

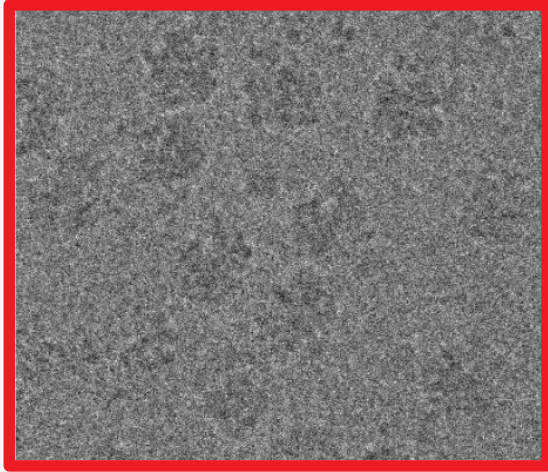
kV=300,ac=0.07,cs=2.7,z=-1,apix=1,B=30



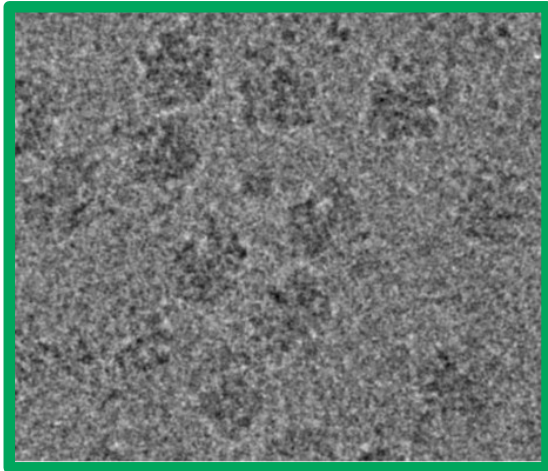
kV=300,ac=0.07,cs=2.7,z=-1,apix=1,B=300



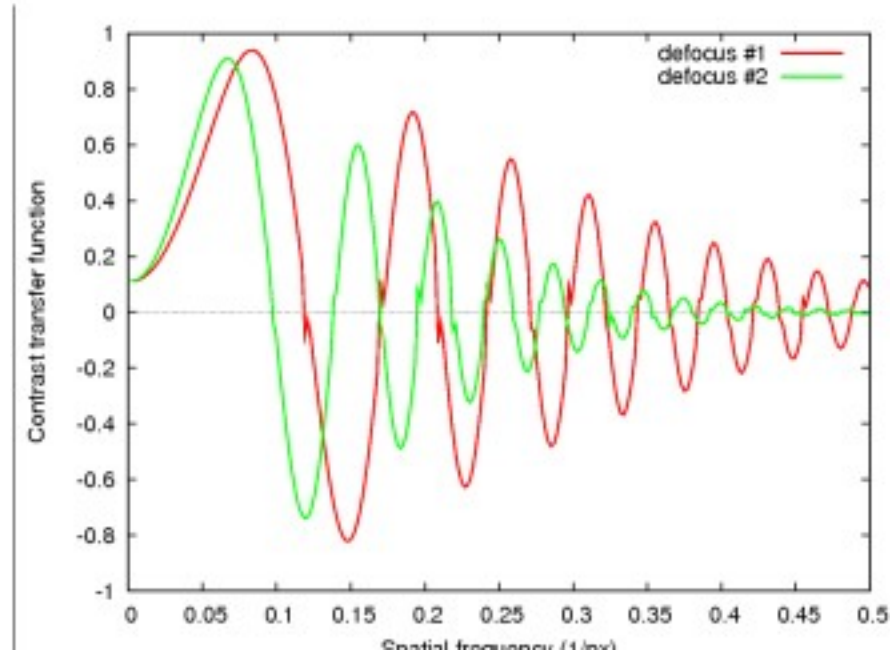
# Contrast transfer function



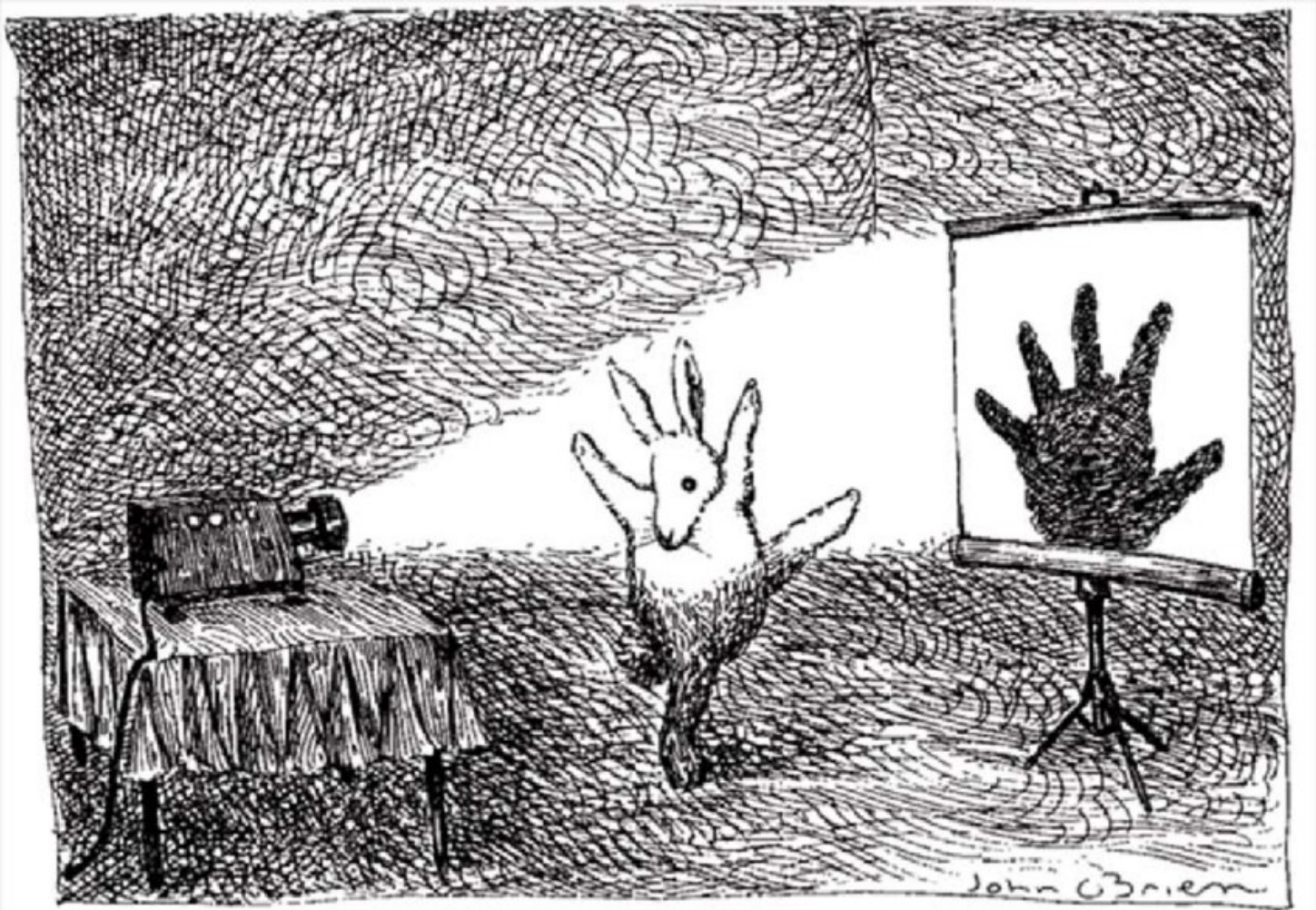
Low defocus



High defocus

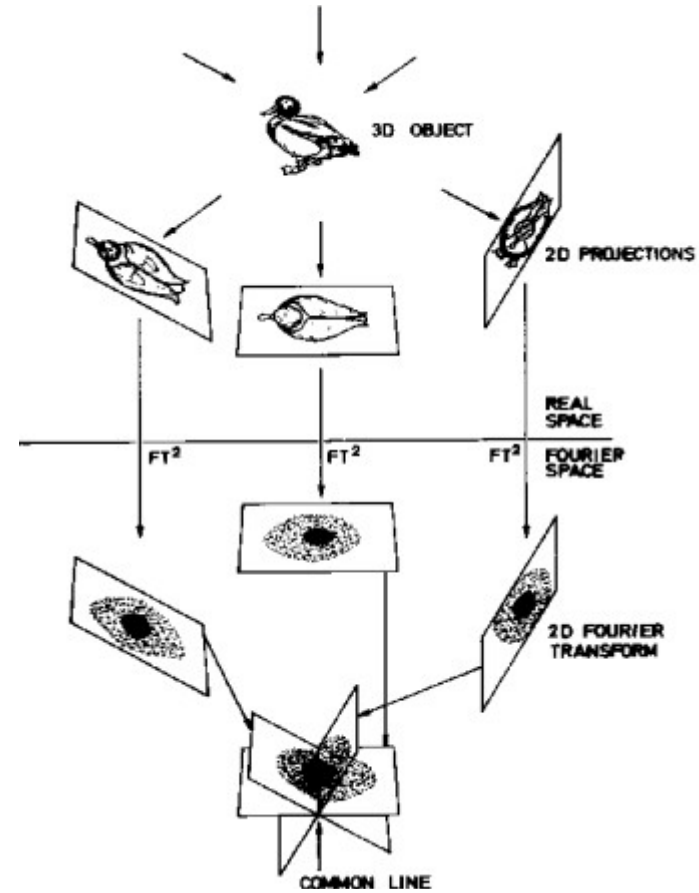
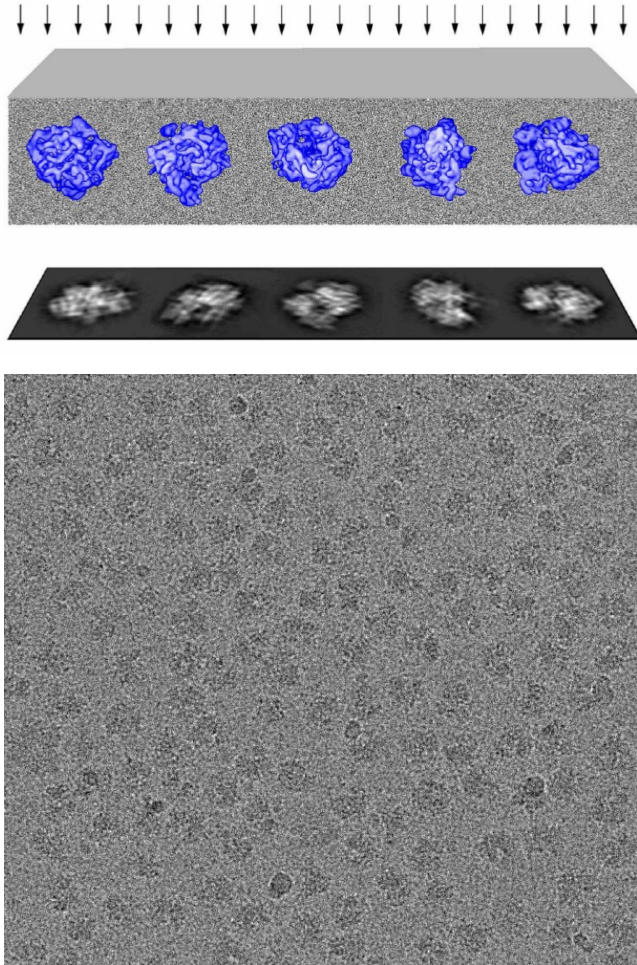


# Projection theorem



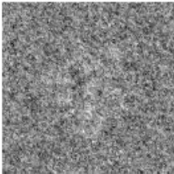
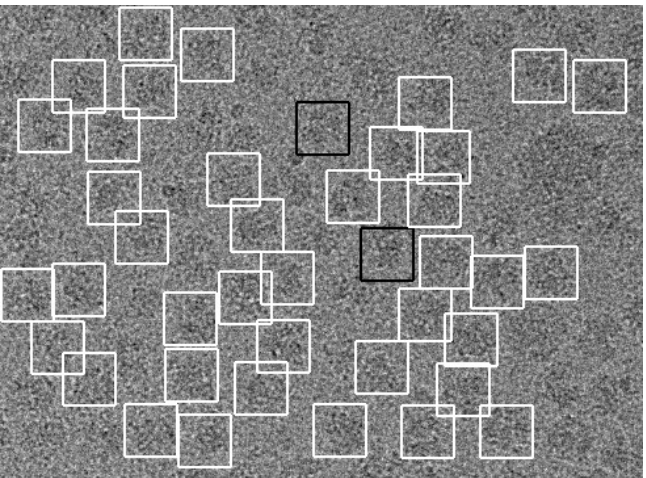
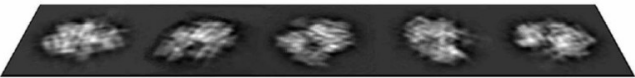
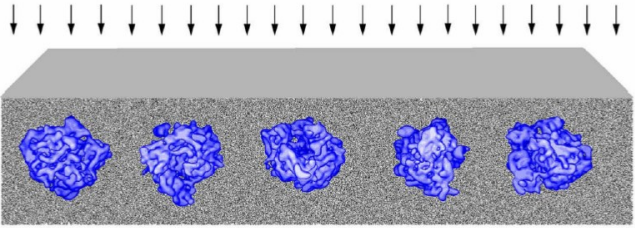
John O'Brien (1991). The New Yorker

# Projection theorem



The 2D Fourier transform of the projection of a 3D density is a central section of the 3D Fourier transform of the density, perpendicular to the direction of projection.

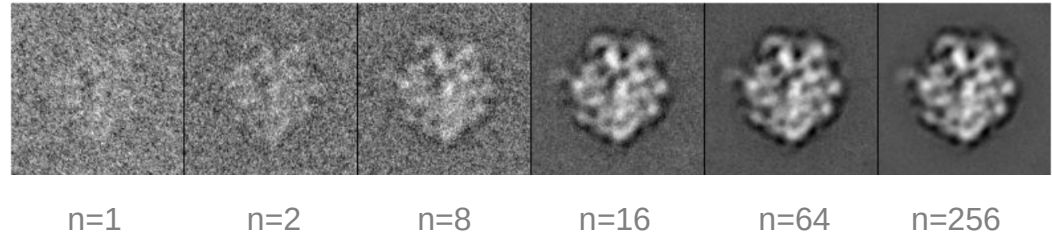
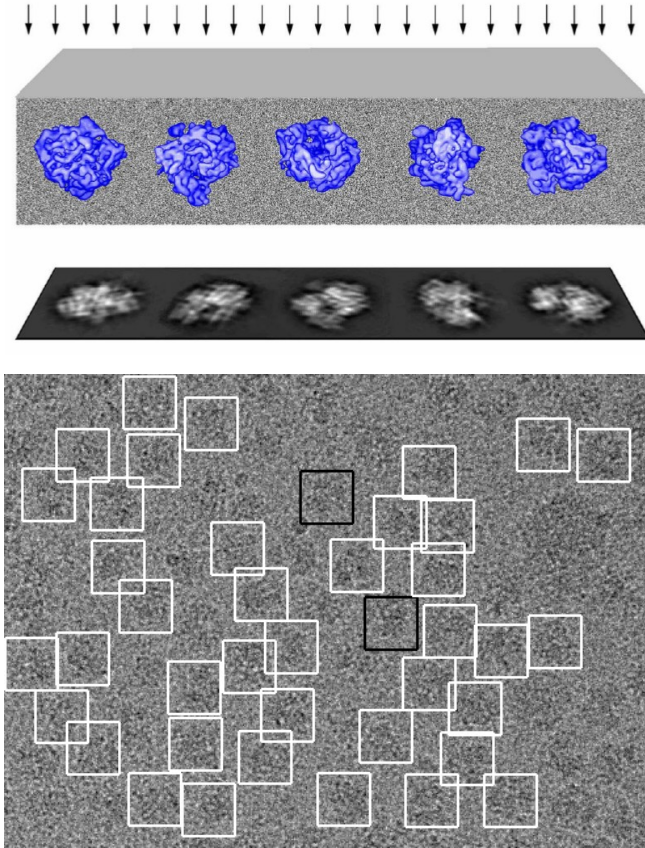
# Particles (regions of interest)



n=1

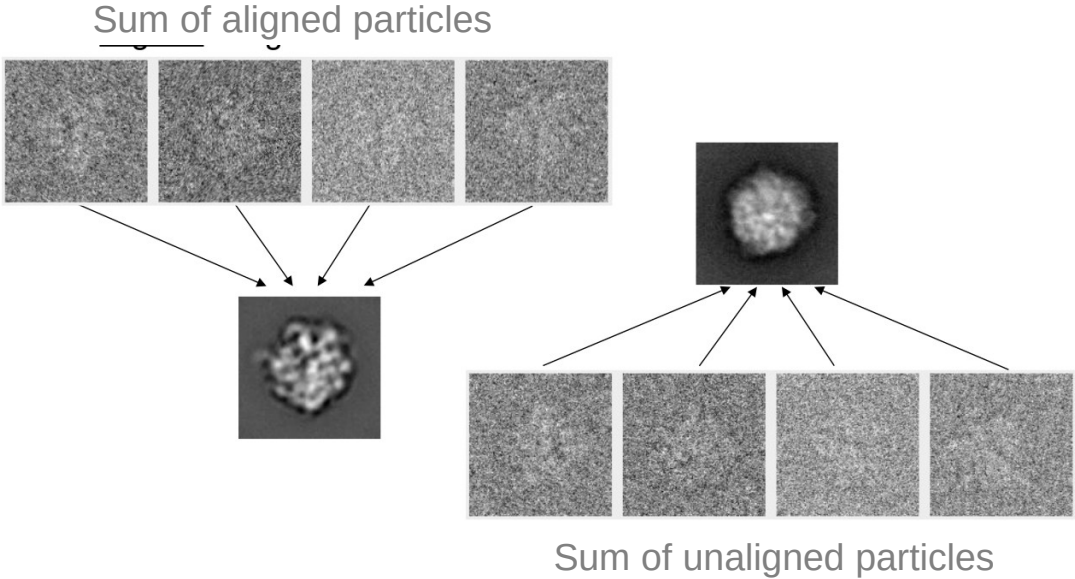
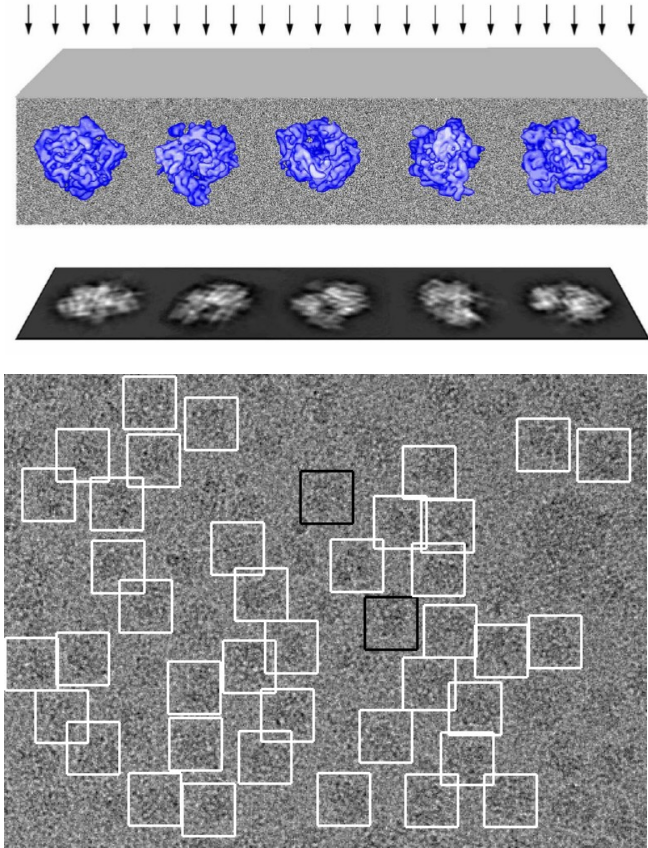


## Particles (regions of interest)

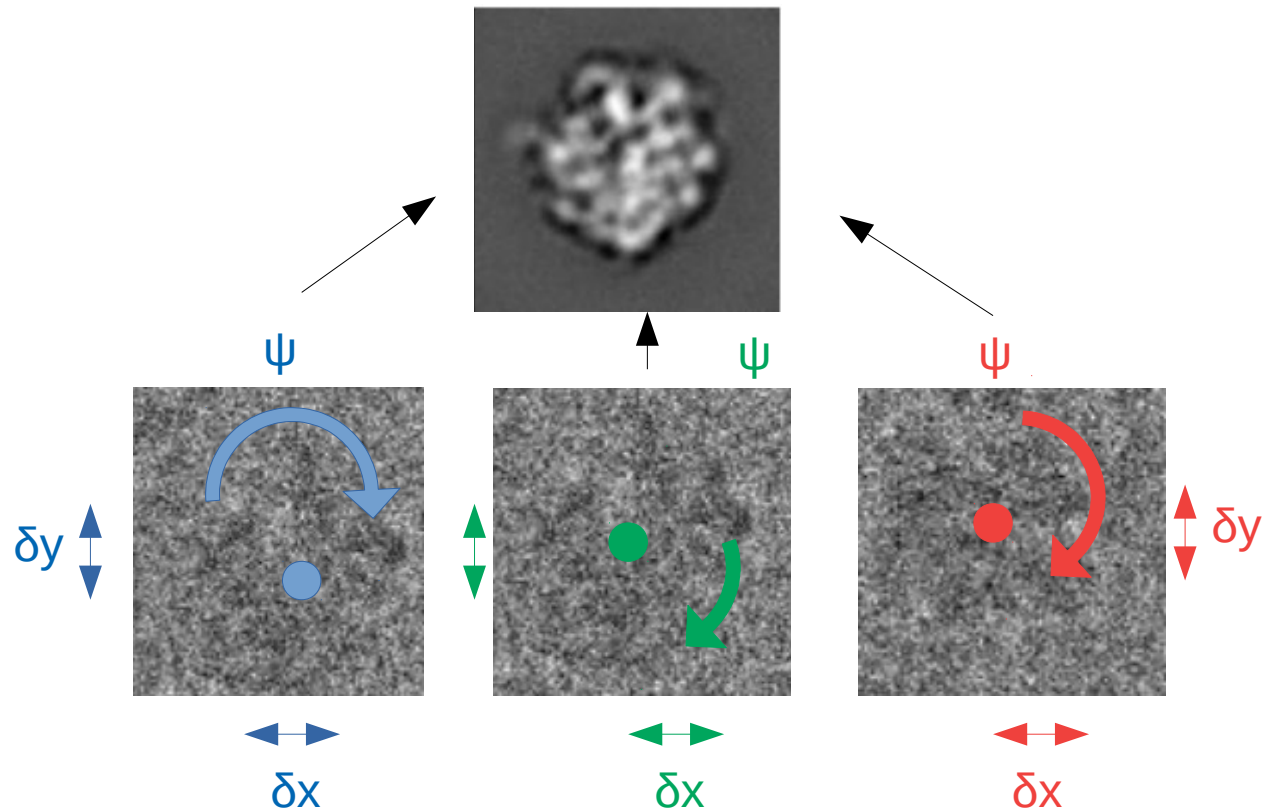
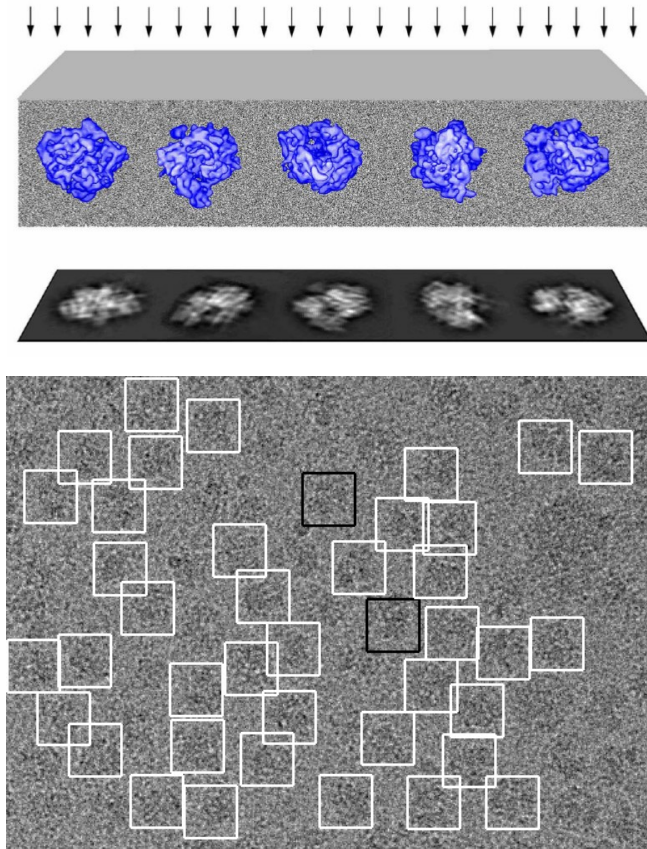


Signal to noise ratio increases with square-root of  $n$

# Image alignment in 2D



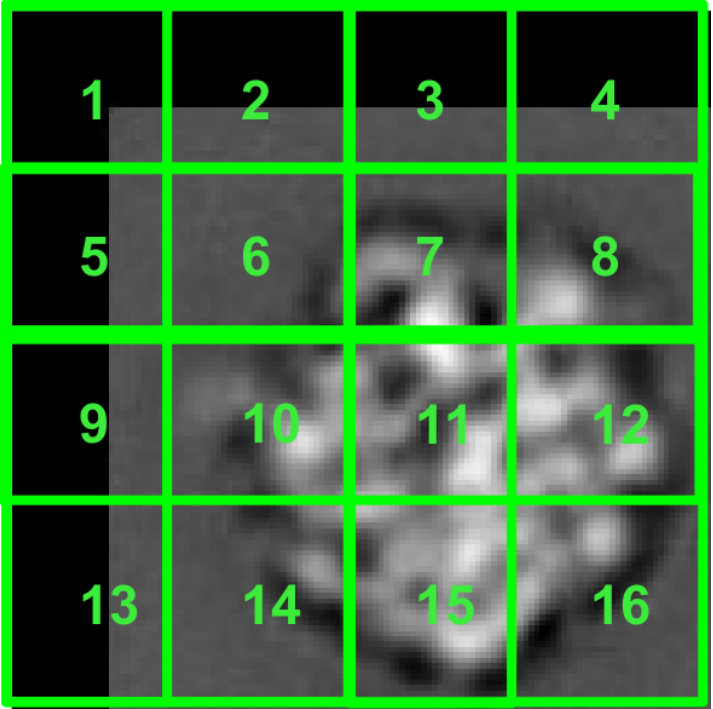
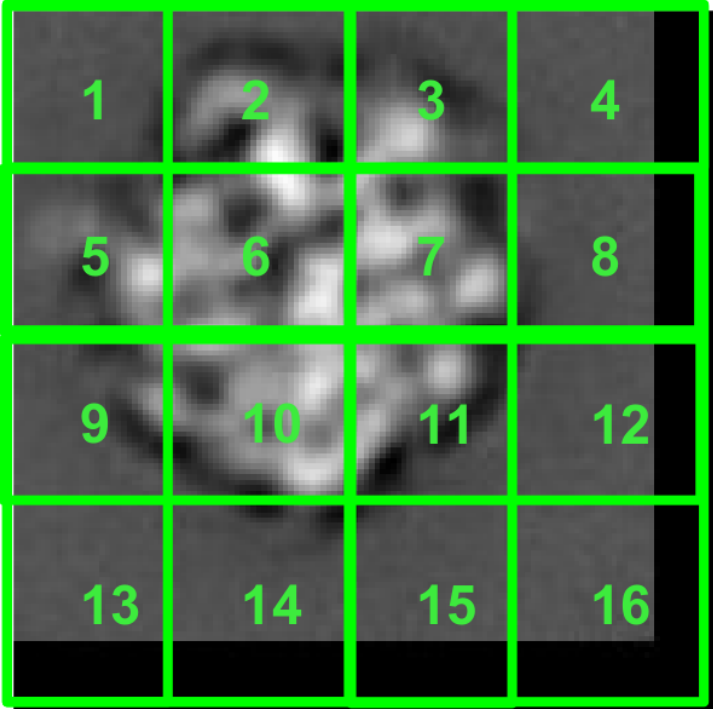
## Image alignment in 2D



In order to align the particles in 2D, we need to determine three parameters:

- two translational
- one rotational (on of the Euler angles)

# Image alignment in 2D



## Cross correlation

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Image  $f$

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Image  $g$

$$\begin{aligned} \text{Unnormalized CCC} = & f_1g_1 + f_2g_2 + f_3g_3 + f_4g_4 + f_5g_5 + f_6g_6 + f_7g_7 + f_8g_8 \\ & + f_9g_9 + f_{10}g_{10} + f_{11}g_{11} + f_{12}g_{12} + f_{13}g_{13} + f_{14}g_{14} + f_{15}g_{15} + f_{16}g_{16} \end{aligned}$$

## Cross correlation

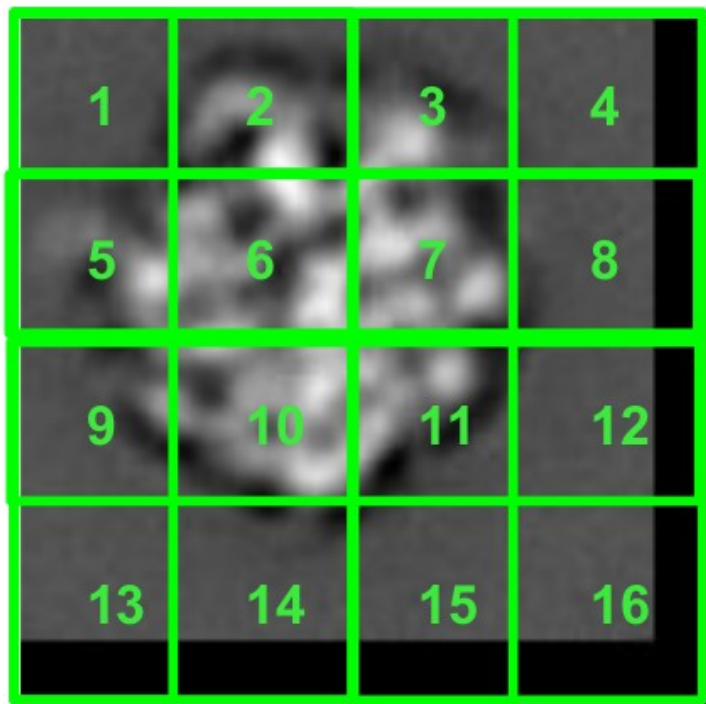


Image  $f$

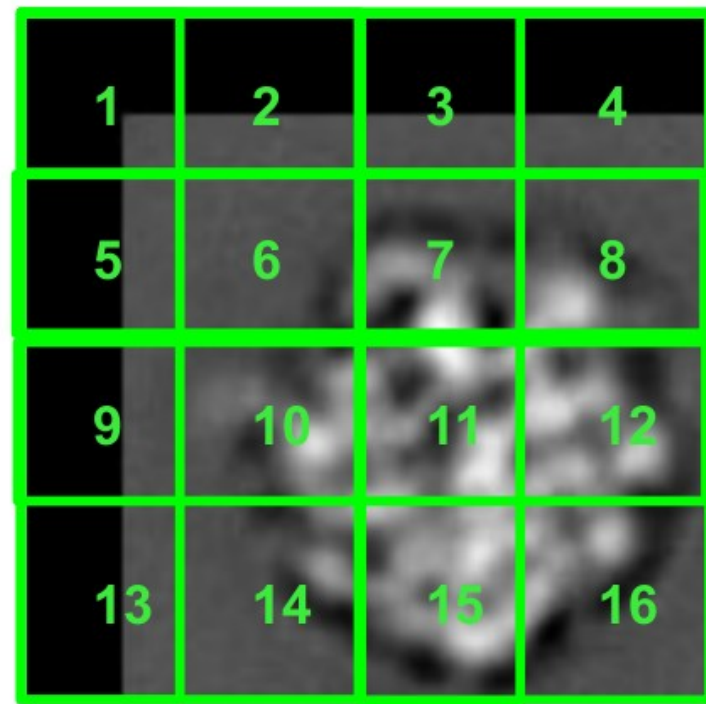


Image  $g$

$$\begin{aligned} \text{Unnormalized CCC} = & f_1g_1 + f_2g_2 + f_3g_3 + f_4g_4 + f_5g_5 + f_6g_6 + f_7g_7 + f_8g_8 \\ & + f_9g_9 + f_{10}g_{10} + f_{11}g_{11} + f_{12}g_{12} + f_{13}g_{13} + f_{14}g_{14} + f_{15}g_{15} + f_{16}g_{16} \end{aligned}$$

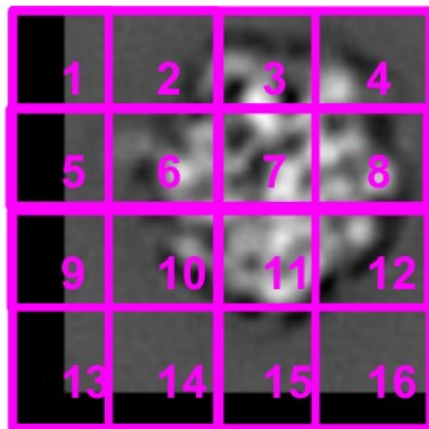
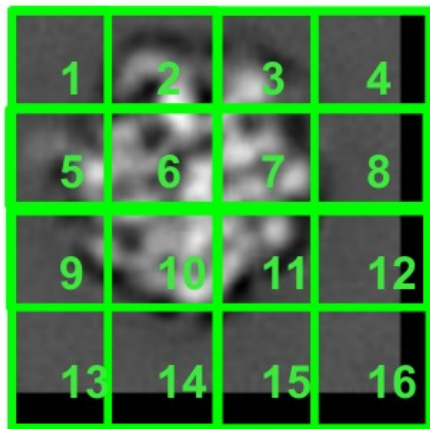
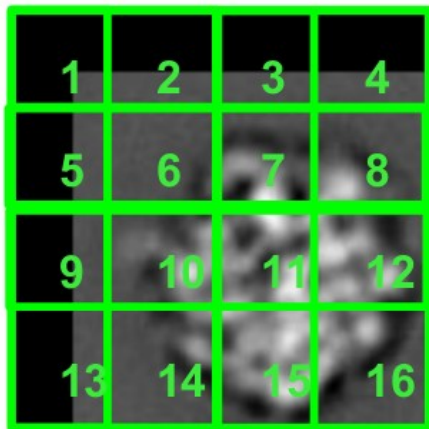
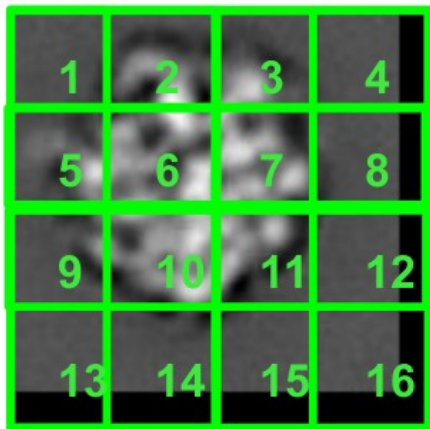
# Cross correlation coefficient

Cross-correlation coefficient: 
$$\frac{\sum_{N=1}^{16} f(\vec{x})g(\vec{x})}{\sigma_f \sigma_g}$$

normalization

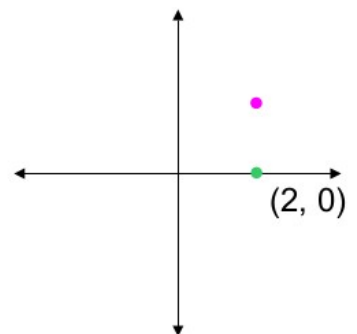
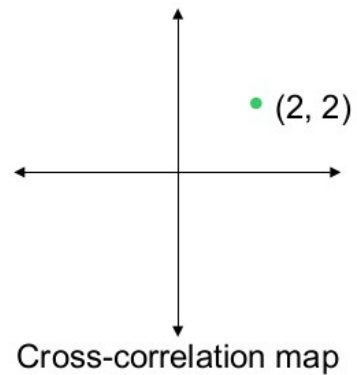
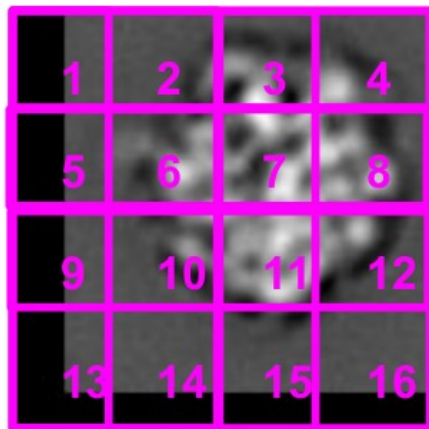
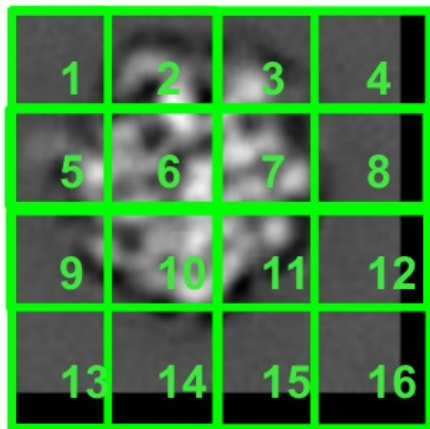
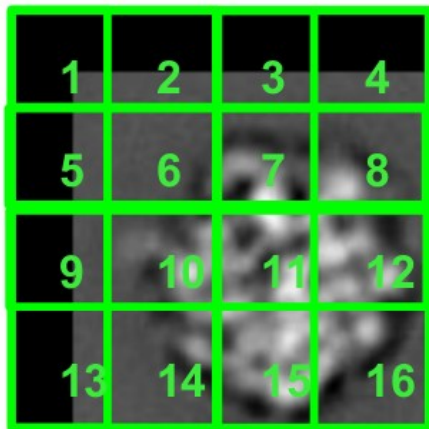
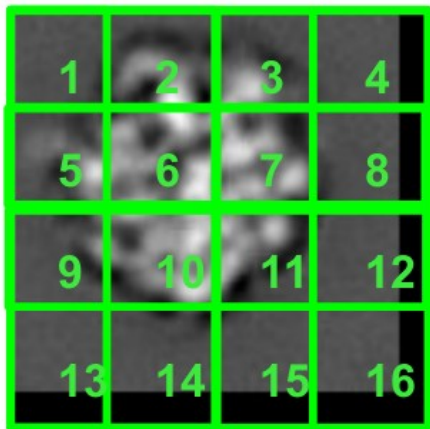
If the alignment is perfect, the cross-correlation coefficient will be 1

## Cross correlation

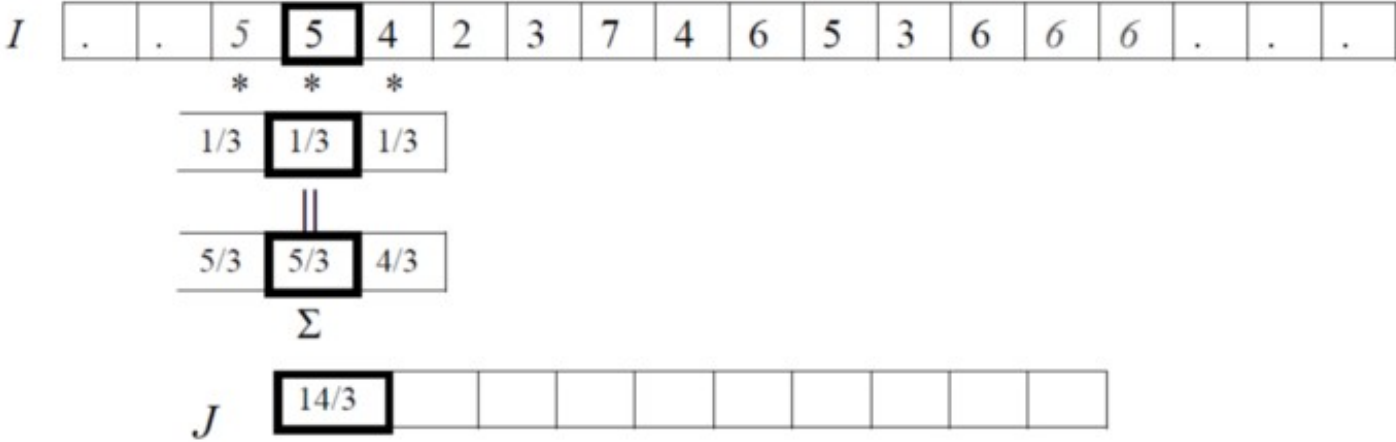




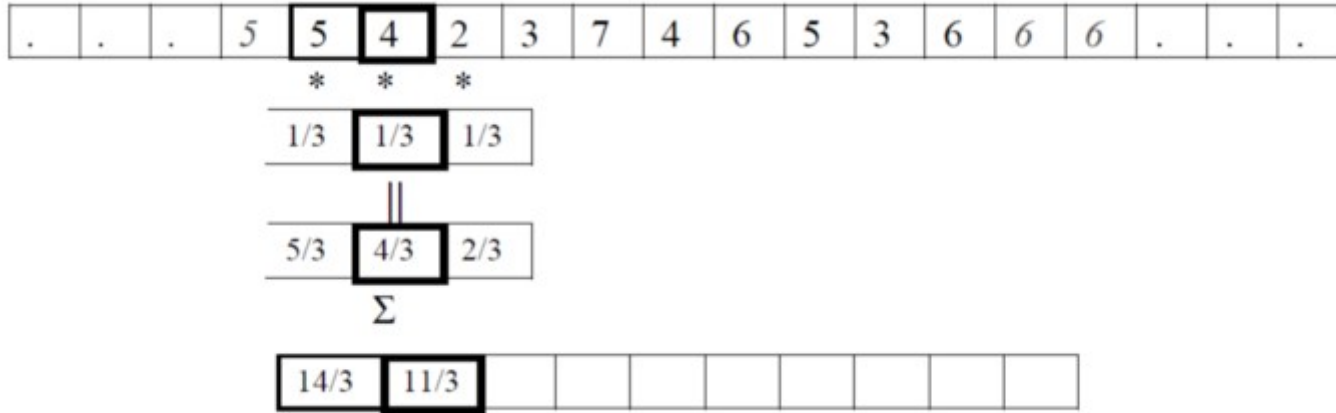
# Cross correlation



# Cross correlation function in 1D



## Cross correlation function in 1D



$$F \circ I(x) = \sum_{i=-N}^N F(i)I(x+i)$$

## Cross correlation function in 2D

$$F \circ I(x, y) = \sum_{j=-N}^N \sum_{i=-N}^N F(i, j) I(x+i, y+j)$$

## Cross correlation function in 2D

$$F \circ I(x, y) = \sum_{j=-N}^N \sum_{i=-N}^N F(i, j) I(x+i, y+j)$$

$$F * I(x, y) = \sum_{j=-N}^N \sum_{i=-N}^N F(i, j) I(x-i, y-j)$$

**Convolution**

## Cross correlation function in 2D

$$F \circ I(x, y) = \sum_{j=-N}^N \sum_{i=-N}^N F(i, j) I(x+i, y+j)$$

$$F * I(x, y) = \sum_{j=-N}^N \sum_{i=-N}^N F(i, j) I(x-i, y-j)$$

**Convolution**

$$\text{FT}(F * I) = \text{FT}(F) \cdot \text{FT}(I)$$

$$\text{FT}(F \circ I) = \text{FT}(F)^* \cdot \text{FT}(I)$$

**Convolution theorem**

# Cross correlation function

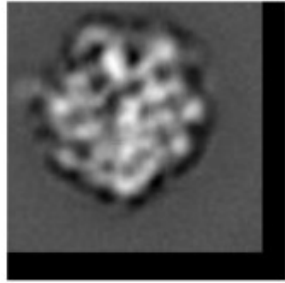


Image  $f(x)$

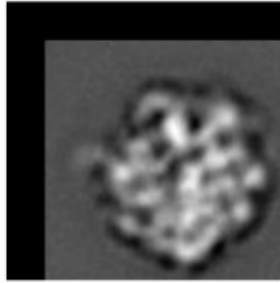
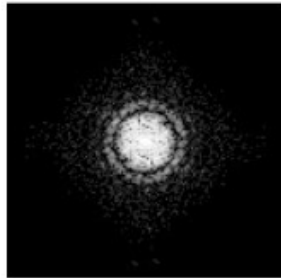
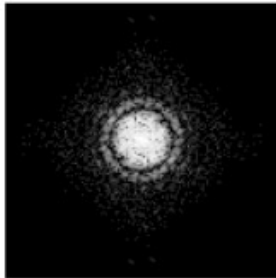


Image  $g(x)$



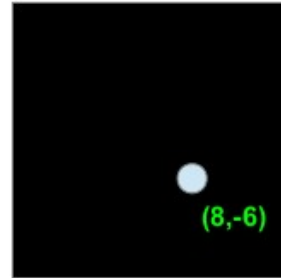
F.T.  $F^*(X)$   
(complex conjugate)

x



F.T.  $G(X)$

=



F.T. (CCF)

## Orientation alignment

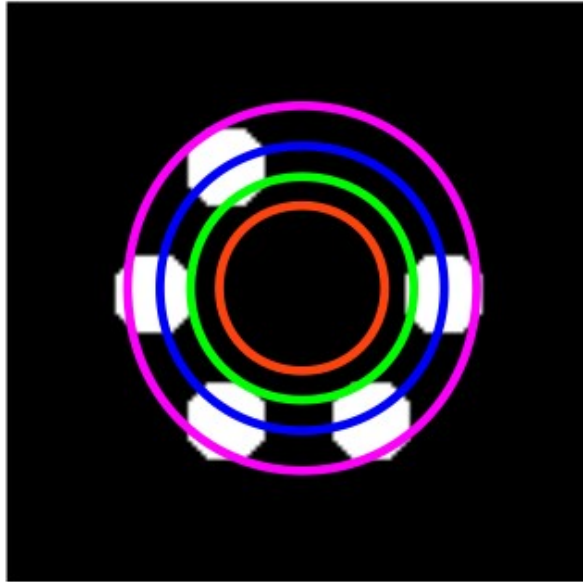


Image 1

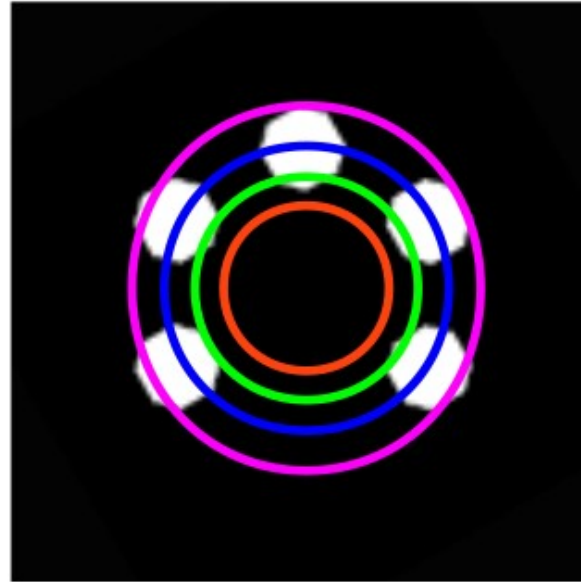


Image 2

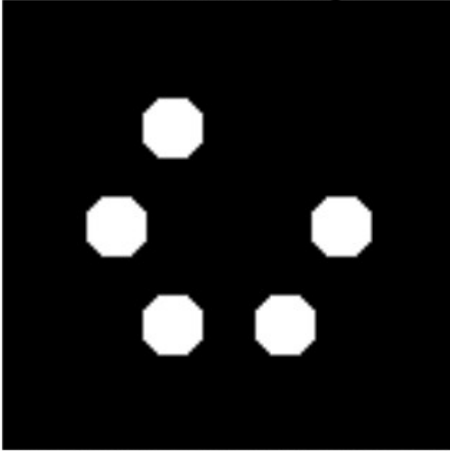
We take a series of rings from each image, unravel them, and compute a series of 1D cross-correlation functions.

Shifts along these unraveled CCFs is equivalent to a rotation in Cartesian space.

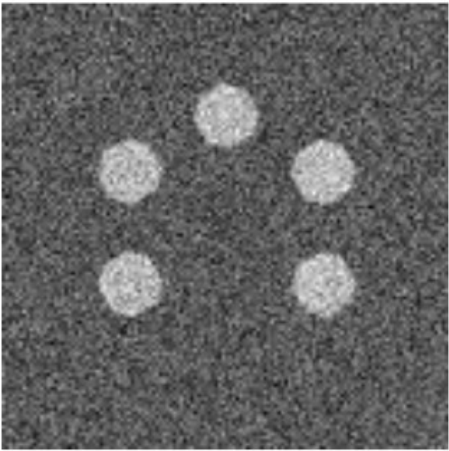
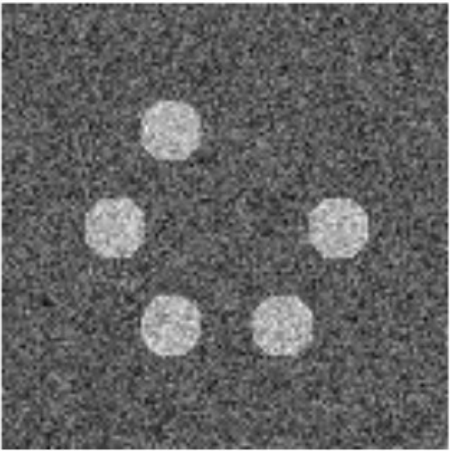


# Orientation alignment

Reference image



Noise added



# Orientation alignment

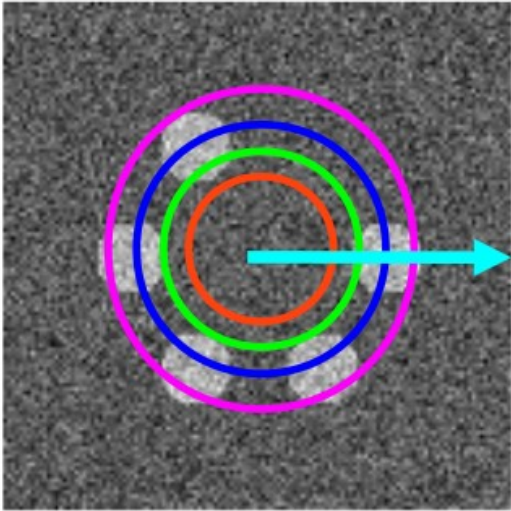


Image 1

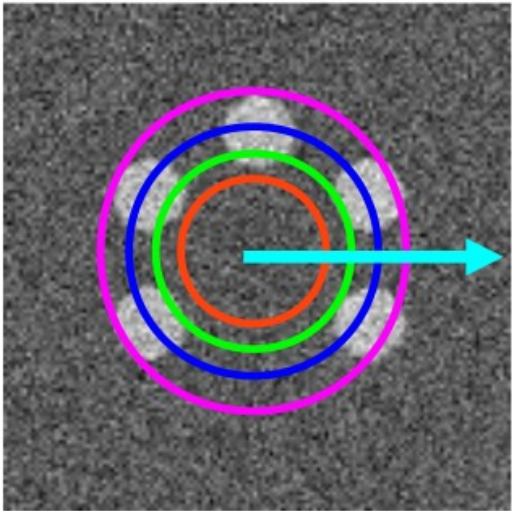
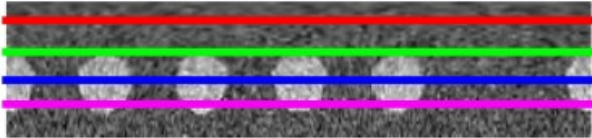
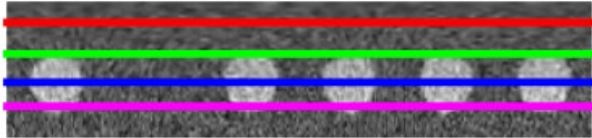


Image 2

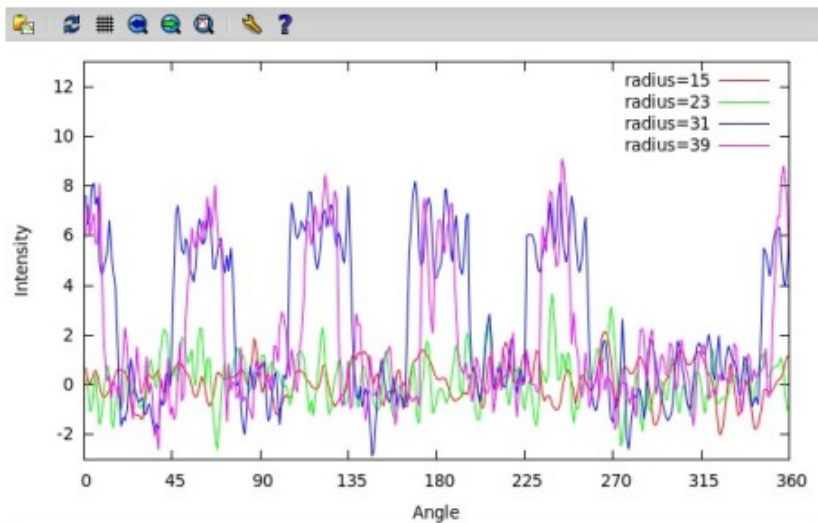
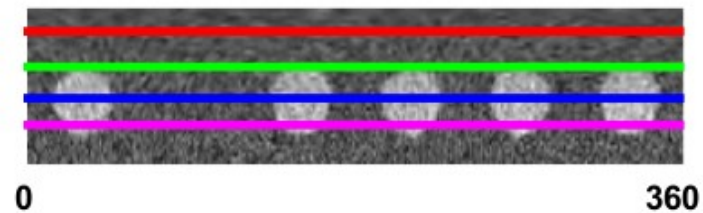
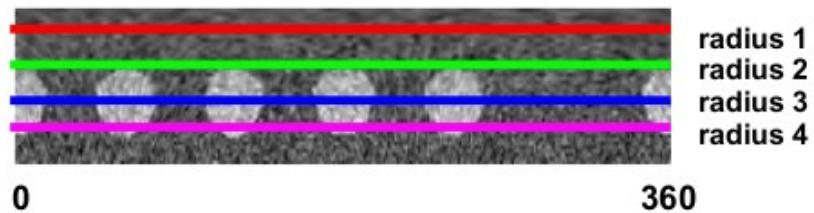


radius 1  
radius 2  
radius 3  
radius 4

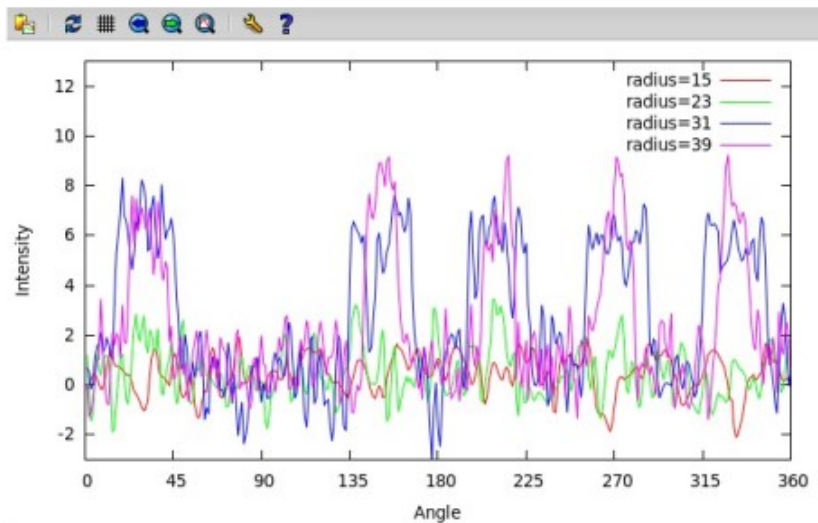


Polar representation

# Orientation alignment



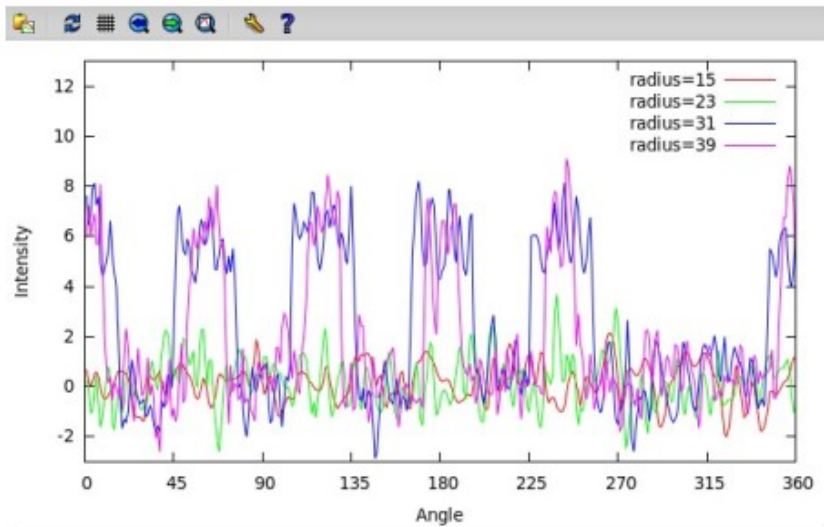
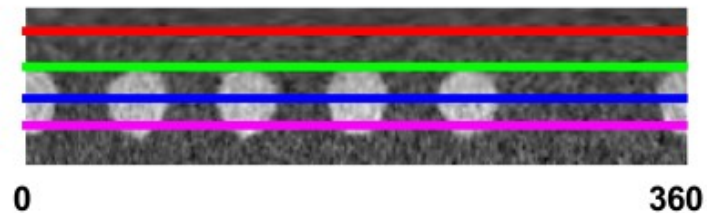
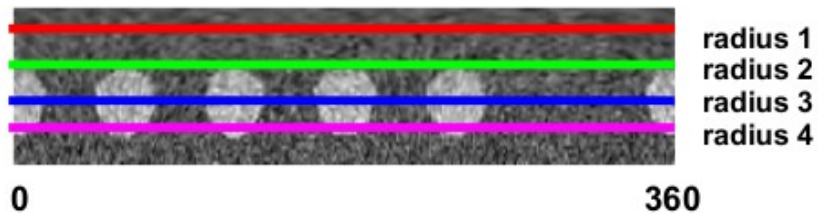
374.951, 4.53721



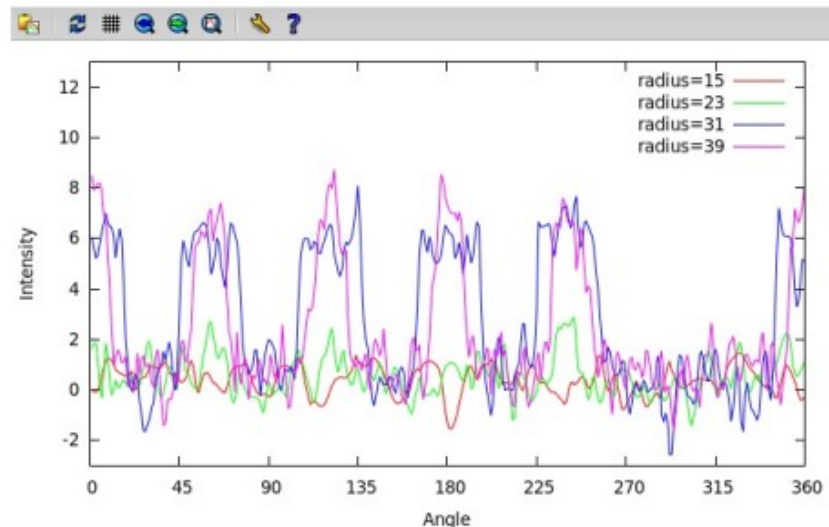
356.141, -2.50024

# Orientation alignment

- after rotation



374.951, 4.53721



372.357, -3.21418

## Image alignment in 2D

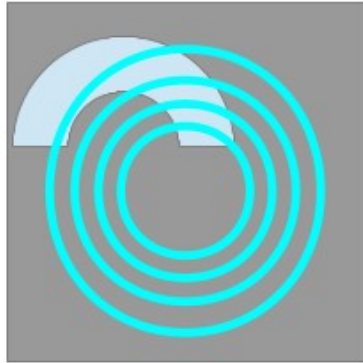


Image 1

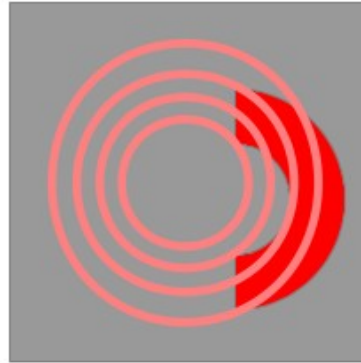
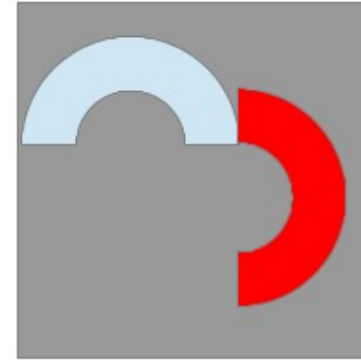


Image 2

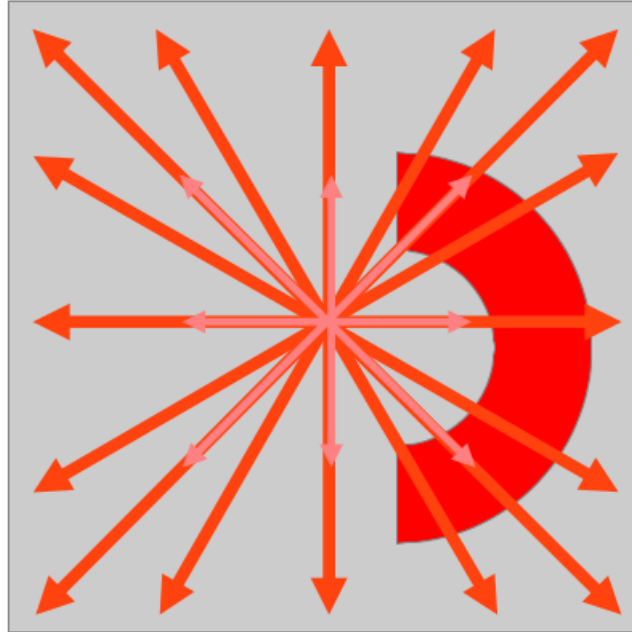


Superimposed

**Translational and orientation alignment are interdependent**

SOLUTION: You try a set of reasonable shifts, and perform separate orientation alignments for each.

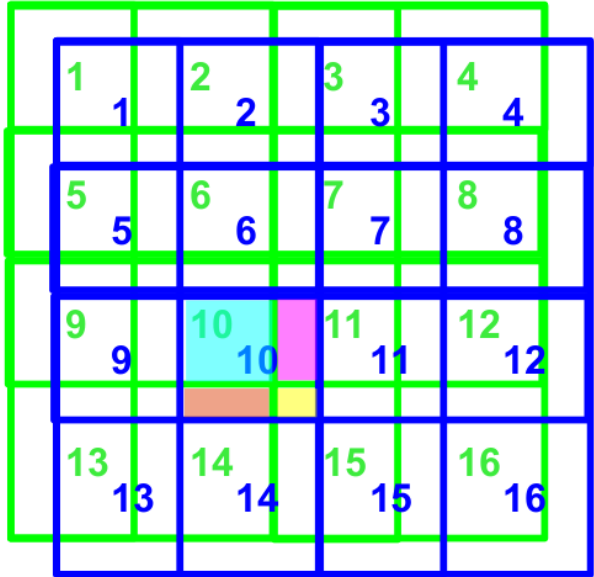
## Image alignment in 2D



Set of all shifts of up to 1 pixel  
Set of all new shifts of up to 2 pixels  
Shifts of  $(0, +/-1, +/-2)$  pixels results in 25 orientation searches.

# Interpolation

Shift



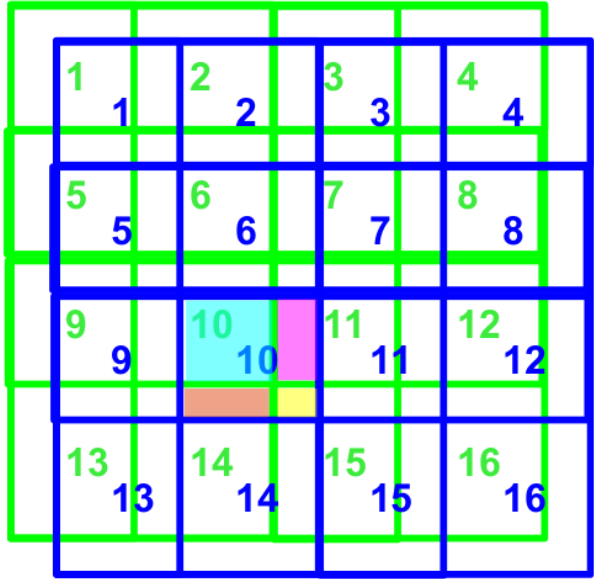
Suppose we shift the image in x & y.  
The new pixels will be weighted averages of the old pixels.  
The more the mix the pixels, the worse the result will be.



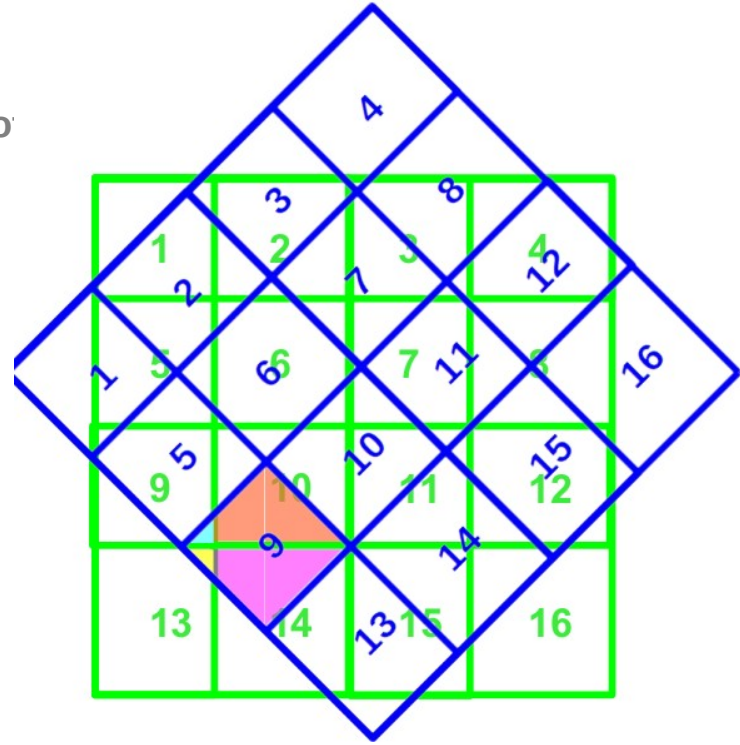


# Interpolation

Shift

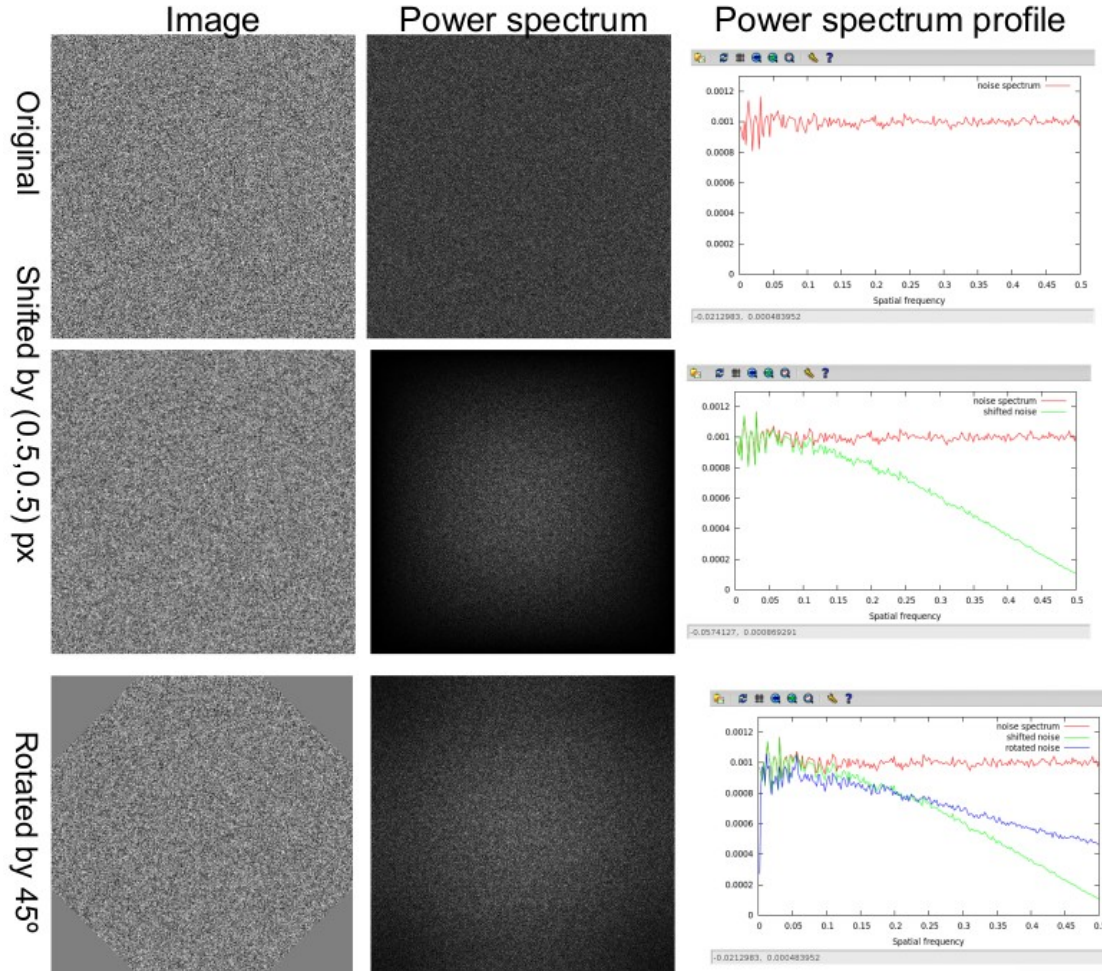


Ro



Suppose we shift the image in x & y.  
The new pixels will be weighted averages of the old pixels.  
The more the mix the pixels, the worse the result will be.

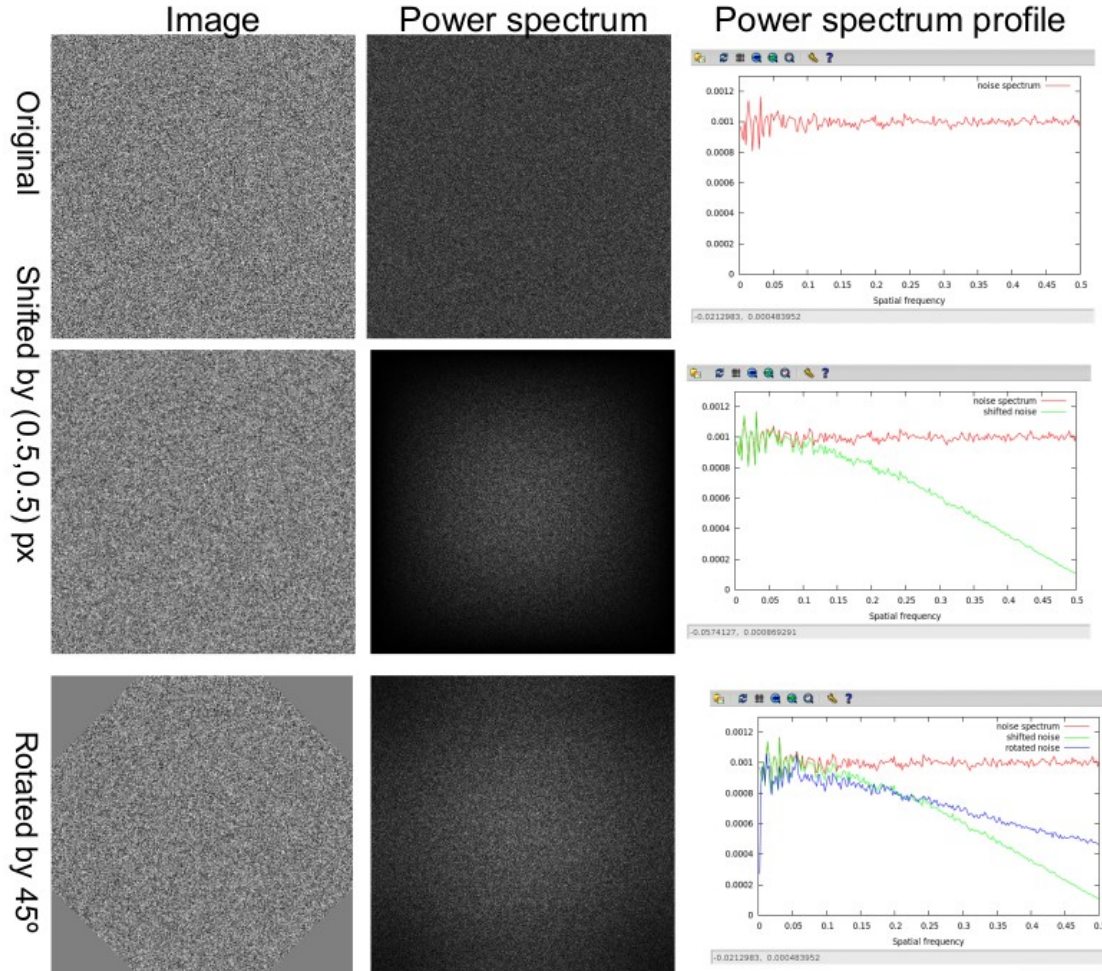
# Interpolation



The Fourier transform of noise is noise

- “White” noise is evenly distributed in Fourier space
- “White” means that each pixel is independent

# Interpolation

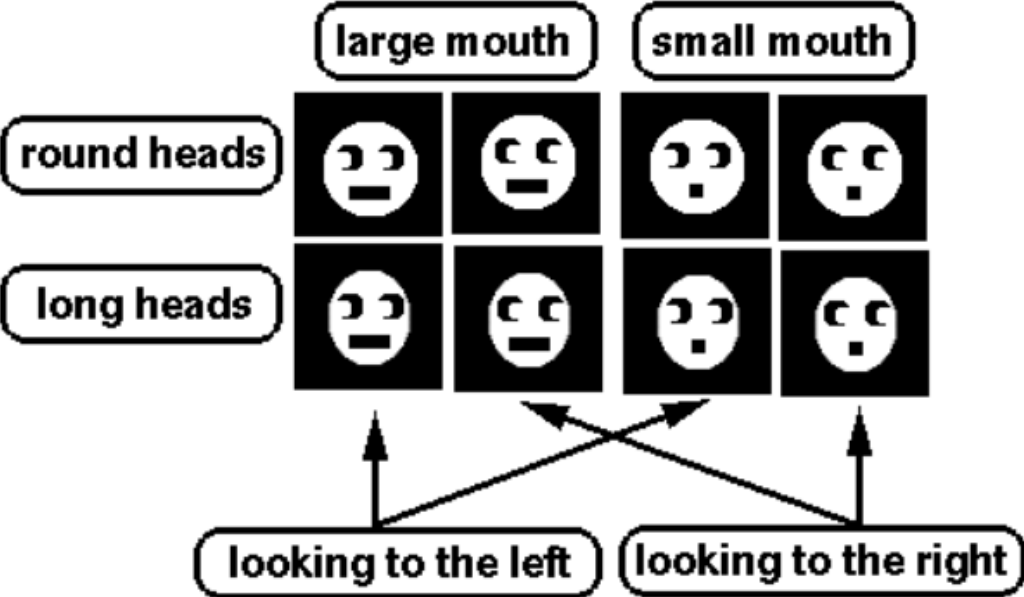


The Fourier transform of noise is noise

- “White” noise is evenly distributed in Fourier space
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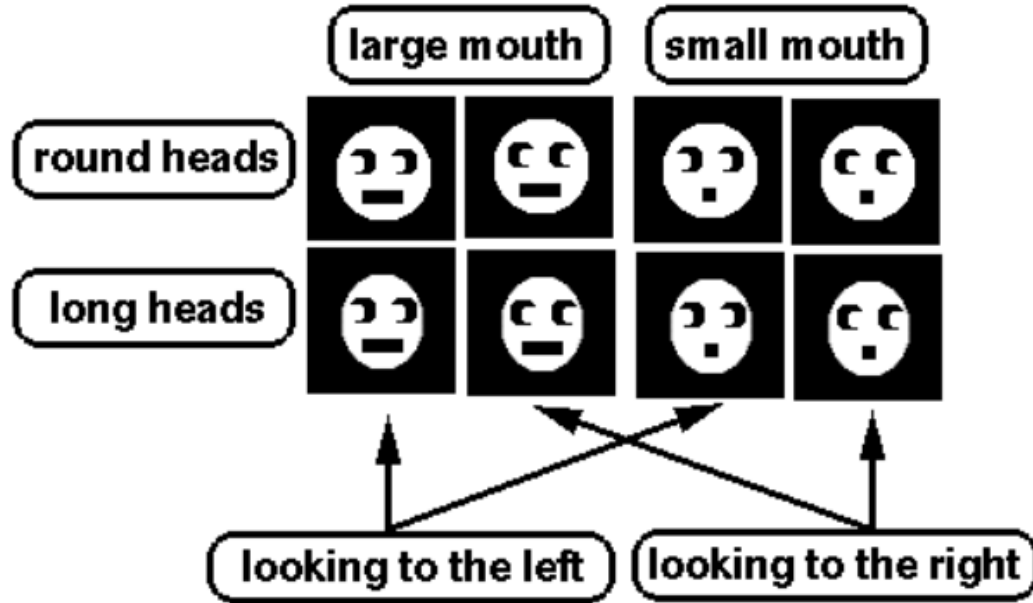
The degradation of the images means that we should minimize the number of interpolations.

# Classification



Frank (1996), J.Microscopy

# Classification



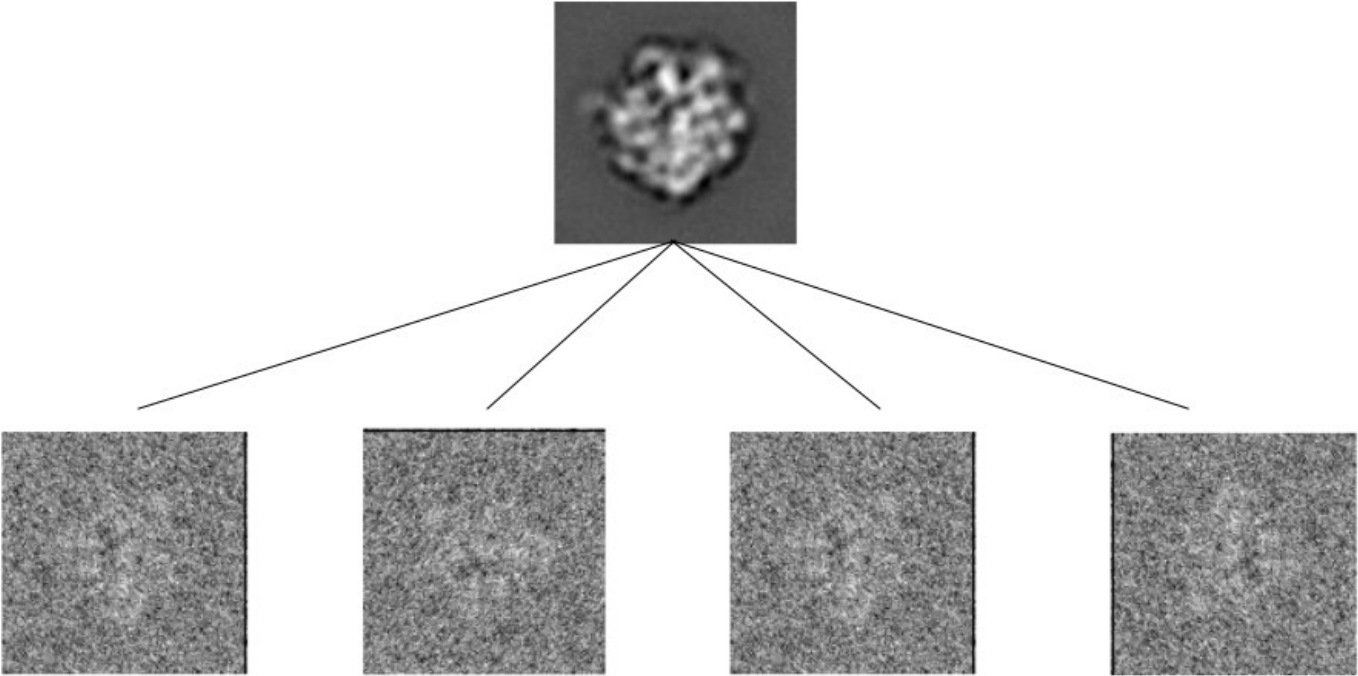
Frank (1996), J. Microscopy

Classification methods are divided into those that are “supervised” and those which are “unsupervised”:

- Supervised: divide or categorize according to similarity with “template” or “reference” (e.g. projection matching)
- Unsupervised: divide according to intrinsic properties (e.g. find classes of projections representing the same view)

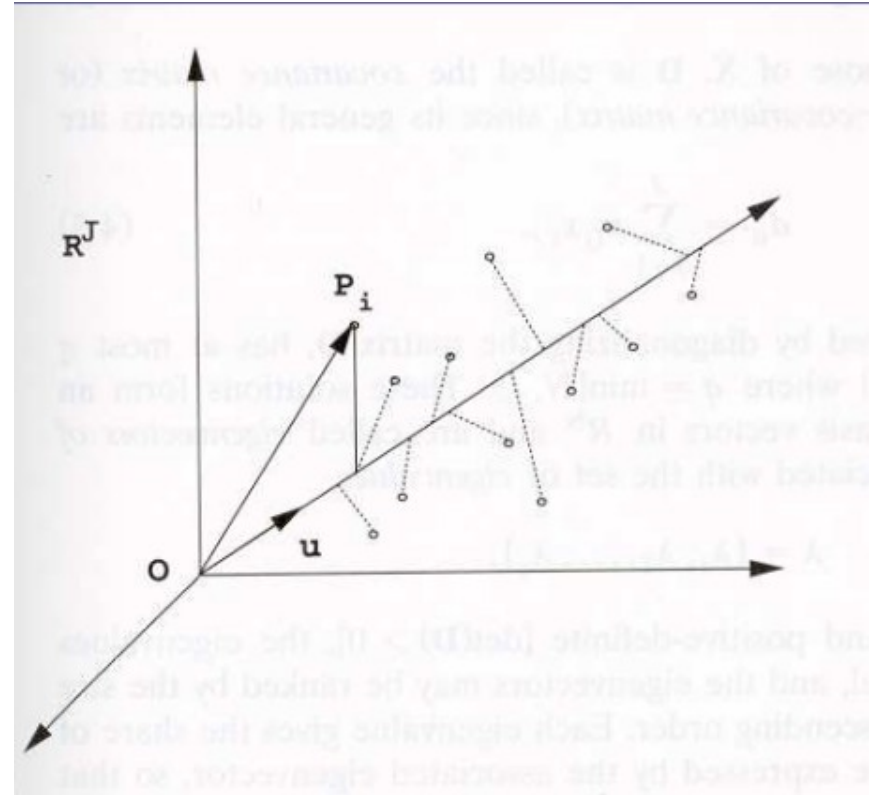
# Classification

Reference based alignment



# Multivariate data analysis (MDA) or multivariate statistical analysis (MSA)

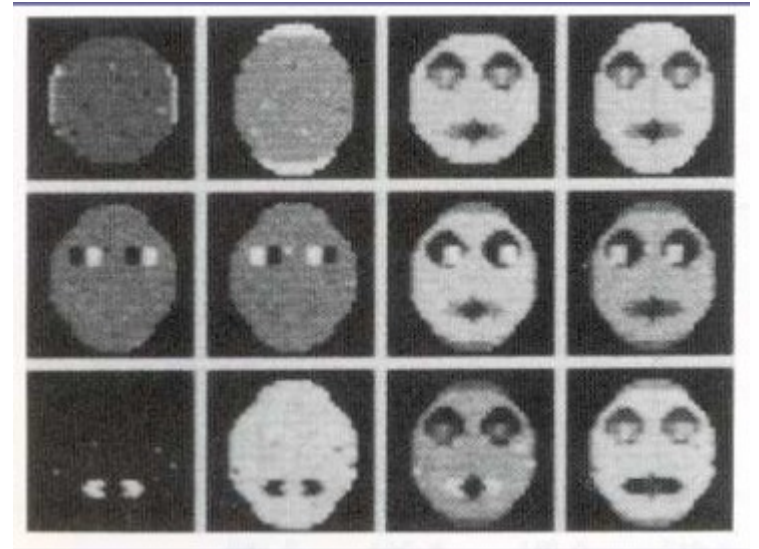
- find new coordinate system tailored to the data
- find a space with reduced dimensionality for the representation of the objects. This greatly simplifies classification.



eigenvectors

# Multivariate data analysis (MDA) or multivariate statistical analysis (MSA)

- find new coordinate system tailored to the data
- find a space with reduced dimensionality for the representation of the objects. This greatly simplifies classification.

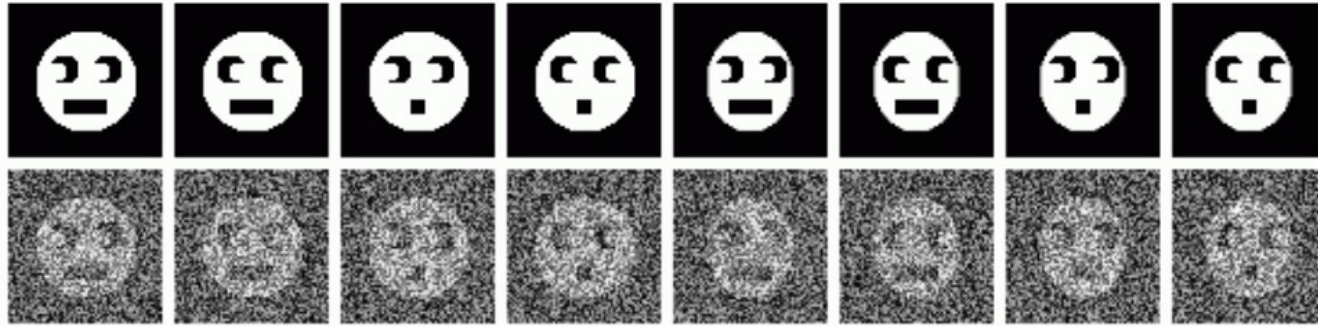


eigenimages



# Principle component analysis (PCA), Correspondence analysis (CA)

8 classes of faces, 64x64 pixels



With noise added

Average:

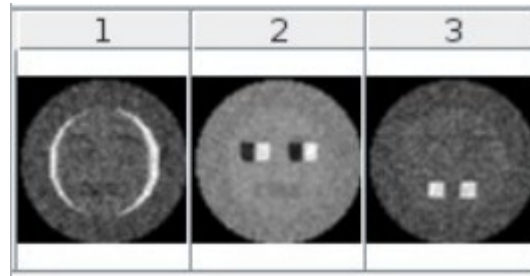
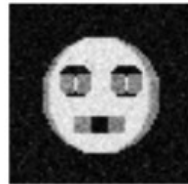


# Principle component analysis (PCA), Correspondence analysis (CA)

For a 4096-pixel image, we will have a 4096x4096 covariance matrix.

Row-reduction of the covariance matrix gives us “eigenvectors.”

- The eigenvectors describe correlated variations in the data.
- The eigenvectors have 4096 elements and can be converted back into images, called “eigenimages.”
- The first eigenvectors will account for the most variation. The later eigenvectors may only describe noise.
- Linear combinations of these images will give us approximations of the classes that make up the data.

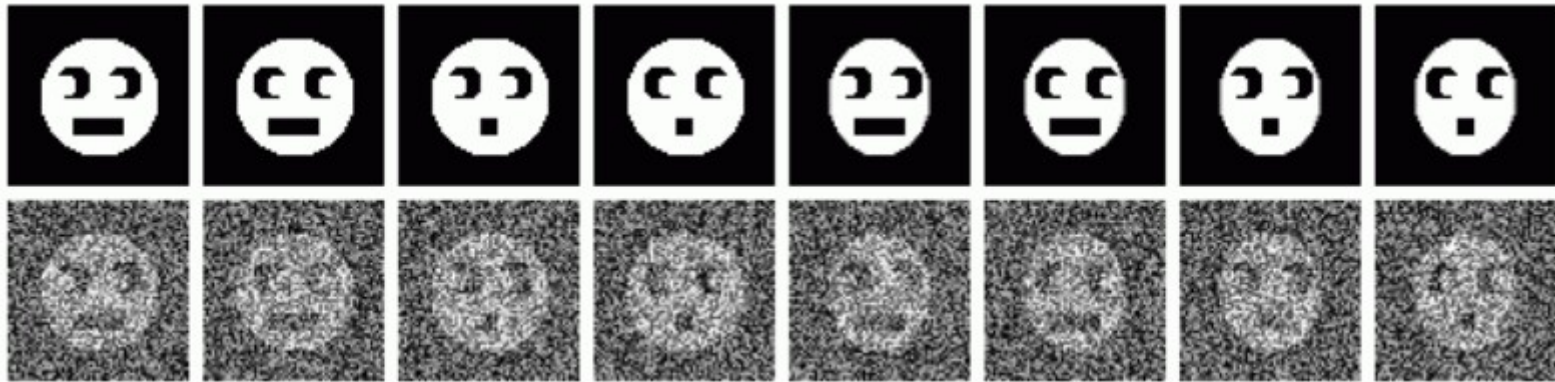


eigenimages

## Principle component analysis (PCA), Correspondence analysis (CA)

$$c_0 \begin{array}{c} \text{Average} \\ \text{Eigenimage \#1} \\ \text{Eigenimage \#2} \\ \text{Eigenimage \#3} \end{array} + c_1 \begin{array}{c} \text{Eigenimage \#1} \\ \text{Eigenimage \#2} \\ \text{Eigenimage \#3} \end{array} + c_2 \begin{array}{c} \text{Eigenimage \#2} \\ \text{Eigenimage \#3} \end{array} + c_3 \begin{array}{c} \text{Eigenimage \#3} \end{array} + \dots$$

Average    Eigenimage #1    Eigenimage #2    Eigenimage #3



Linear combinations of these images will give us approximations of the classes that make up the data.

