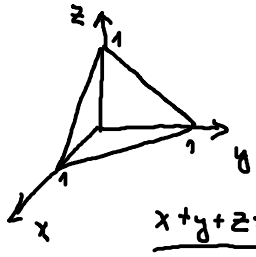
$\iiint_V f(x,y,z)\,dx\,dy\,dz$, $|V| = \iiint_V 1\,dx\,dy\,dz$

$ax+by+cz+d=0$,



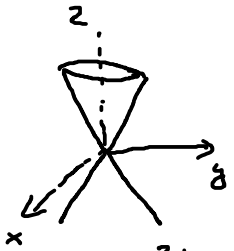
$x+y+z=1$

$z = x^2+y^2$

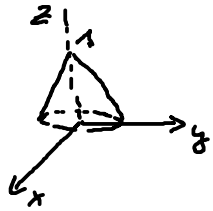
$z = 2-x^2-y^2$

$z^2 = x^2+y^2$

$z = \pm\sqrt{x^2+y^2}$

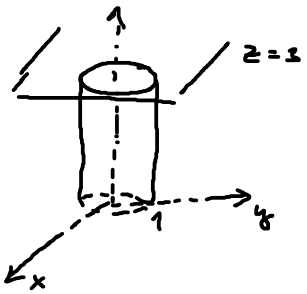


$z = 1-\sqrt{x^2+y^2}$

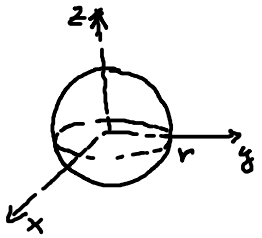


$x^2+y^2 \le 1$

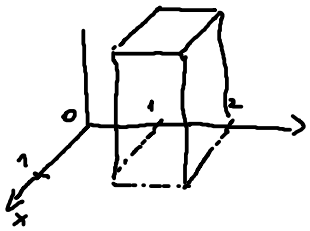
$z \ge 0$, $z \le 3$



$x^2+y^2+z^2 \le r^2$



$\iiint_V xz\,dx\,dy\,dz$, V: $0 \le x \le 1$, $1 \le y \le 2$, $0 \le z \le 2$



$$\int_0^1 \left(\int_1^2 \left(\int_0^2 xz\,dz \right) dy \right) dx = \int_0^1 \left(\int_1^2 \left[x\,\frac{z^2}{2} \right]_0^2 dy \right) dx =$$

$$= \int_0^1 \left(\int_1^2 2x\,dy \right) dx = \int_0^1 \left[2xy \right]_1^2 dx = \int_0^1 (4x-2x)\,dx = \int_0^1 2x\,dx = \left[x^2 \right]_0^1 = 1$$

$$\int_0^2 \left(\int_1^2 \left(\int_0^1 xz\,dx \right) dy \right) dz = \int_0^2 \left(\int_1^2 \left[\frac{x^2}{2} z \right]_0^1 dy \right) dz = \int_0^2 \left(\int_1^2 \frac{1}{2} z\,dy \right) dz =$$

$$= \int_0^2 \left[\frac{1}{2} z y \right]_1^2 dz = \int_0^2 \left(z - \frac{1}{2} z \right) dz = \int_0^2 \frac{1}{2} z\,dz = \left[\frac{z^2}{4} \right]_0^2 = 1$$

$$\int_0^1 x\,dx \cdot \int_1^2 dy \cdot \int_0^2 z\,dz = \left[\frac{x^2}{2} \right]_0^1 \cdot \left[y \right]_1^2 \cdot \left[\frac{z^2}{2} \right]_0^2 = \frac{1}{2} \cdot (2-1) \cdot 2 = 1$$

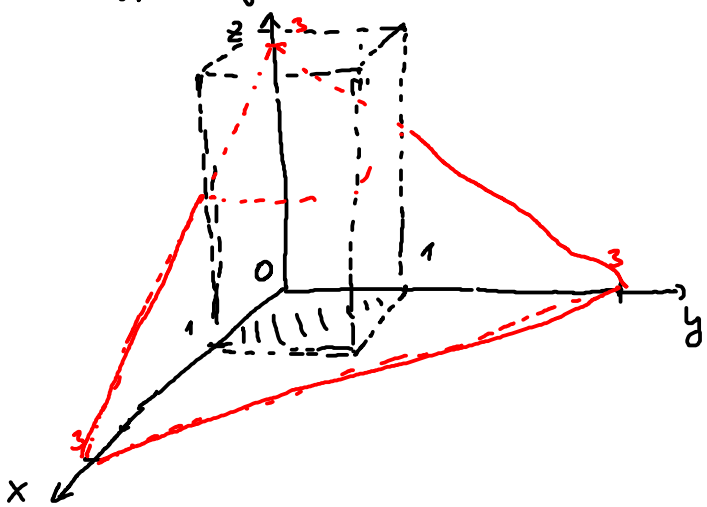
2) $\iiint_V x^2 y z^3 \, dx\,dy\,dz$, V: $0 \le x \le 1$, $0 \le y \le x$, $0 \le z \le xy$

$$\int_0^1 \left(\int_0^x \left(\int_0^{xy} x^2 y z^3 \, dz \right) dy \right) dx = \int_0^1 \left(\int_0^x \left[x^2 y \, \frac{z^4}{4} \right]_0^{xy} dy \right) dx =$$

$$= \int_0^1 \left(\int_0^x x^2 y \, \frac{x^4 y^4}{4} \, dy \right) dx = \int_0^1 \left(\int_0^x \frac{1}{4} x^6 y^5 \, dy \right) dx = \int_0^1 \left[\frac{1}{4} x^6 \, \frac{y^6}{6} \right]_0^x =$$

$$= \int_0^1 \frac{1}{24} x^{12} \, dx = \left[\frac{1}{24} \, \frac{x^{13}}{13} \right]_0^1 = \frac{1}{24} \cdot \frac{1}{13} =$$

3) Vyp. objem tělesa ohraničeného rovinami: $x=0$, $y=0$, $z=0$, $x=1$, $y=1$, $x+y+z=3$, $z \ge 0$



$z = 3-x-y$

$0 \le x \le 1$

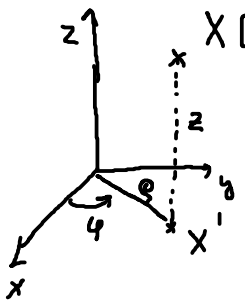
$0 \le y \le 1$

$0 \le z \le 3-x-y$

$$|V| = \iiint_V 1 \, dx\,dy\,dz = \int_0^1 \left(\int_0^1 \left(\int_0^{3-x-y} 1 \, dz \right) dy \right) dx = \int_0^1 \left(\int_0^1 \left[z \right]_0^{3-x-y} dy \right) dx =$$

$$= \int_0^1 \left(\int_0^1 (3-x-y) \, dy \right) dx = \int_0^1 \left[3y - xy - \frac{y^2}{2} \right]_0^1 dx = \int_0^1 \left(3 - x - \frac{1}{2}\right) dx =$$

$$= \int_0^1 \left(\frac{5}{2} - x \right) dx = \left[\frac{5}{2} x - \frac{x^2}{2} \right]_0^1 = \frac{5}{2} - \frac{1}{2} = 2 \text{ j}^3$$



$X[x,y,z] \sim [\varrho, \varphi, z]$

$x = \varrho \cos\varphi$

$y = \varrho \sin\varphi$

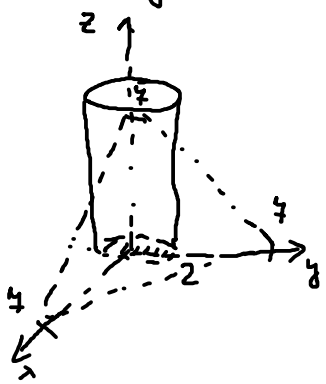
$z = z$

$|J| = \varrho$

$$\iiint_V f(x,y,z)\,dx\,dy\,dz = \int_{\varphi_1}^{\varphi_2} \left(\int_{\varrho_1(\varphi)}^{\varrho_2(\varphi)} \int_{z_1(\varrho,\varphi)}^{z_2(\varrho,\varphi)} f(\varrho\cos\varphi, \varrho\sin\varphi, z)\,\varrho\,dz \right) d\varrho \right) d\varphi$$

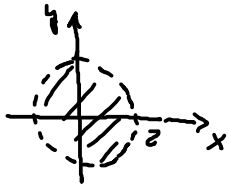
4) Ve válcových souř. popište těleso V:

a) $V: x^2+y^2 \le 4, \quad 0 \le z \le 7-x-y$



$z = 7-x-y$

$x+y+z = 7$



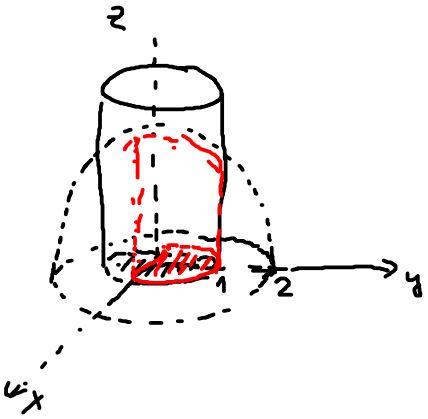
$0 \le \varrho \le 2$

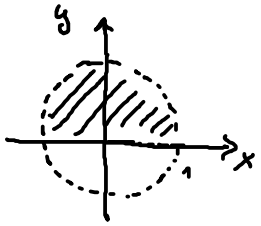
$0 \le \varphi \le 2\pi$

$0 \le z \le 7 - \varrho\cos\varphi - \varrho\sin\varphi$

b) $V: x^2+y^2 \le 1, \quad y \ge 0, \quad z \ge 0, \quad z \le 4 - x^2 - y^2$

$4-(x^2+y^2)$





$z = 4 - x^2 - y^2$

$0 = 4 - x^2 - y^2$

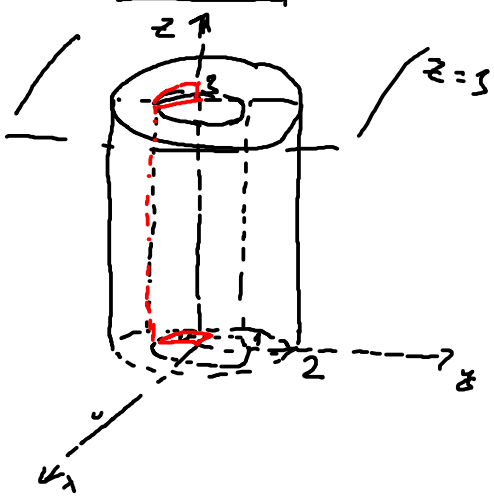
$x^2 + y^2 = 4$

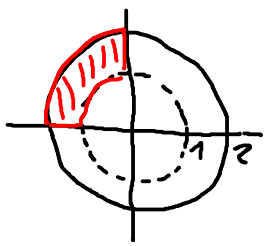
$0 \le \varrho \le 1$

$0 \le \varphi \le \pi$

$0 \le z \le 4 - \varrho^2$

c) $1 \le x^2+y^2 \le 4, \quad 0 \le z \le 3, \quad x \le 0, \quad y \ge 0$



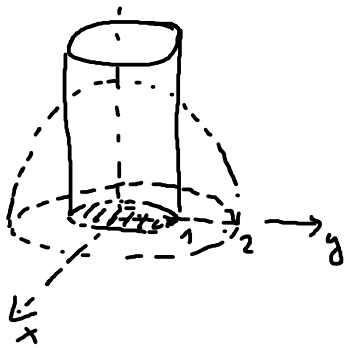


$1 \le \varrho \le 2$

$\frac{\pi}{2} \le \varphi \le \pi$

$0 \le z \le 3$

5) Vyp. objem těl. ohraničeného parab. $0 \le z \le 4 - x^2 - y^2$, pro $x^2 + y^2 \le 1$



$$0 \le \varrho \le 1$$
$$0 \le \varphi \le 2\pi$$
$$0 \le z \le 4 - \varrho^2$$

$$|V| = \int_0^{2\pi} \int_0^1 \left(\int_0^{4-\varrho^2} \varrho \, dz \right) d\varrho \, d\varphi =$$

$$= \int_0^{2\pi} \left(\int_0^1 [\varrho z]_0^{4-\varrho^2} \, d\varrho \right) d\varphi = \int_0^{2\pi} \left(\int_0^1 \varrho (4 - \varrho^2) \, d\varrho \right) d\varphi =$$

$$= \int_0^{2\pi} d\varphi \cdot \int_0^1 (4\varrho - \varrho^3) \, d\varrho = [\varphi]_0^{2\pi} \cdot \left[2\varrho^2 - \frac{\varrho^4}{4} \right]_0^1 = 2\pi \cdot \left(2 - \frac{1}{4} \right) = \frac{7}{2}\pi$$