$y' = 2y$

$y=0$ ✓ $\quad y' = 0$

$y = e^x$, $y' = e^x$ ✗

$y = e^{2x}$, $y' = 2e^{2x}$ ✓

$y = Ce^{2x}$, $y' = 2Ce^{2x}$ ✓

$y' = 2y$

$\frac{dy}{dx} = 2y \qquad /:y \neq 0$

! $\int \frac{dy}{y} = \int 2\,dx$

$\ln|y| = 2x + C$

$e^{\ln|y|} = e^{2x+C}$

$|y| = e^{C}\cdot e^{2x} = ke^{2x} \quad k>0$

$y = Ce^{2x}, \; C \neq 0$

$\boxed{y = Ce^{2x}, \; C \in \mathbb{R}}$

$y' = 2y, \; y(0) = 3$

$3 = Ce^{2\cdot 0} = Ce^{0} = C\cdot 1$

$3 = C$

$\boxed{y = 3e^{2x}}$

2) $y'' = \cos x, \; y(0) = 0, \; y'(0) = 1$

$y' = \int \cos x\,dx = \sin x + C_1$

$y = \int (\sin x + C)\,dx = -\cos x + C_1 x + C_2$

$\boxed{y = -\cos x + C_1 x + C_2}$

$0 = -\cos 0 + C_1\cdot 0 + C_2$

$0 = -1 + C_2 \to C_2 = 1$

$1 = \sin 0 + C_1$

$1 = 0 + C_1 \to C_1 = 1$

$\boxed{y = -\cos x + x + 1}$

3) $y' = -2xy, \; y(0) = 2$

$\frac{dy}{dx} = -2xy \qquad /:y \neq 0$

$\int \frac{dy}{y} = \int -2x\,dx$

$\ln|y| = -x^2 + C$

$e^{\ln|y|} = e^{-x^2+C}$

$|y| = Ce^{-x^2}, \; C>0$

Je $y=0$ řešení?

$0 = -2x\cdot 0$ ✓

$\boxed{y = Ce^{-x^2}, \; C \in \mathbb{R}}$

$2 = Ce^{-0^2} \to C = 2$

$\boxed{y = 2e^{-x^2}}$

$$\int \frac{dy}{y} = \int a(x)\,dx$$

$$y = C e^{\int a(x)\,dx}$$

$$\frac{1}{2} y = x + C$$

$$y = 2x + C$$

4)

$$x y y' = 1 - x^2$$

$$y y' = \frac{1-x^2}{x}$$

$$y \frac{dy}{dx} = \frac{1-x^2}{x}$$

$$\int y\,dy = \int \frac{1-x^2}{x}\,dx = \int \left(\frac{1}{x} - x\right) dx$$

$$\frac{1}{2} y^2 = \ln|x| - \frac{x^2}{2} + C$$

$$\boxed{y^2 = 2\ln|x| - x^2 + 2C}$$

$$y = \pm\sqrt{2\ln|x| - x^2 + 2C}$$

5)

$$x + y y' = 0$$

$$y y' = -x$$

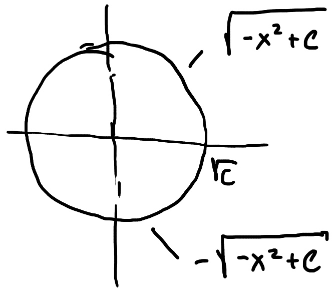
$$\int y\,dy = -\int x\,dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + C$$

$$y^2 = -x^2 + C$$

$$\boxed{x^2 + y^2 = C}$$

$$y = \pm\sqrt{-x^2 + C}$$



Lin. dif. rovnice 1. řádu

$$y' + p(x) y = f(x) \leftarrow \begin{cases} y = C(x) \cdot e^{\int -p(x)\,dx} \\ y' = \ldots \end{cases}$$

$$y' + p(x) y = 0$$

$$\frac{dy}{dx} = -p(x) y$$

$$\int \frac{dy}{y} = -\int p(x)\,dx$$

$$\boxed{y_0 = C e^{\int -p(x)\,dx}}$$

$$C'(x) = \ldots$$

$$y = y_0 + y_p$$

6) $y' - y = x$

$p(x) = -1$
$f(x) = x$

$y' - y = 0$

$y' = y$

$\boxed{y_0 = Ce^{x}}$

$$\begin{cases} y = C(x)\cdot e^{x} \\ y' = C'\cdot e^{x} + C\cdot e^{x} \end{cases}$$

$C'e^{x} + Ce^{x} - Ce^{x} = x$

$C'e^{x} = x$

$C' = \frac{x}{e^{x}}$

$C'(x) = \frac{x}{e^{x}}$

$C(x) = \int x e^{-x} dx = \left| \begin{matrix} u = x & v' = e^{-x} \\ u' = 1 & v = -e^{-x} \end{matrix} \right| =$

$= -xe^{-x} - \int -e^{-x} dx = -xe^{-x} + \int e^{-x} dx =$

$C(x) = -xe^{-x} - e^{-x} + C$

$y = (-xe^{-x} - e^{-x} + C)e^{x}$

$\boxed{y = Ce^{x} - x - 1}$

$\boxed{y_p = -x - 1}$

$y_p' = -1$

$-1 - (-x-1) = x$

$-1 + x + 1 = x$

$x = x$