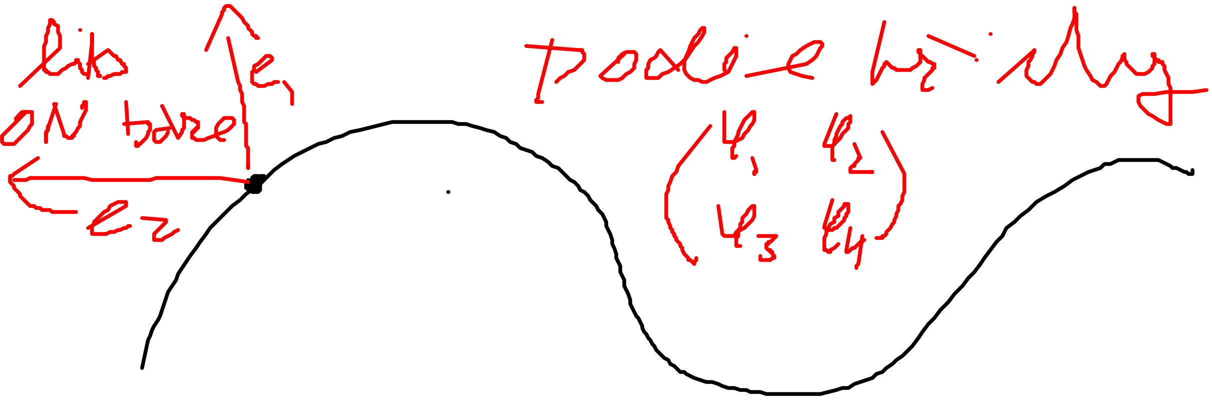
Pozn: $\begin{pmatrix} e_1 \\ e_2 \end{pmatrix}' = \begin{pmatrix} 0 & \varkappa \\ -\varkappa & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$



Ortogonální matice splňují $AA^T = E$

↳ tvoří Lieovu grupu

2.7 Určete délku oblouku křivky

$f(t) = (a(t - \sin t), a(1 - \cos t), 4a \cos \frac{t}{2})$

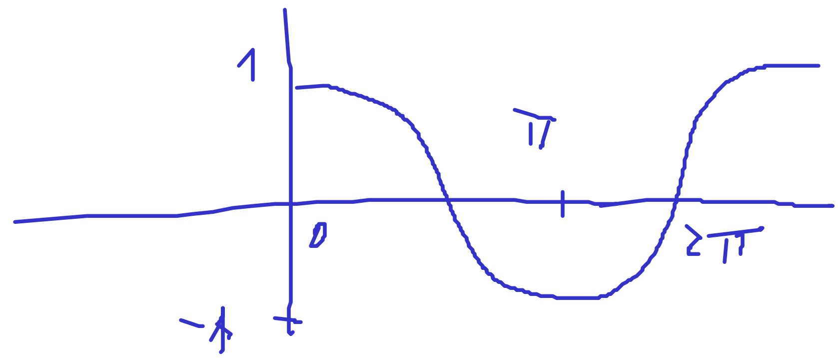
mezi dvěma následujícími průsečíky s rovinou (x,z). $a \neq 0$

- $y = 0$

$1 - \cos t = 0$

$1 = \cos t$

$t = 2k\pi$



$t_0 = 0$

$t_1 = 2\pi$

$$\int_0^{2\pi} a\sqrt{(1-\cos t)^2 + \sin^2 t + 4\sin^2 \frac{t}{2}}\, dt =$$

$f'(t) = a\left(1 - \cos t, \sin t, \overbrace{-4\tfrac{1}{2}}^{-2} \sin\frac{t}{2}\right)$

$$= a\int_0^{2\pi} \sqrt{(1 - 2\cos t + \cos^2 t) + \sin^2 t + 4\sin^2 \frac{t}{2}}\, dt$$

$$= a\int_0^{2\pi} \sqrt{2 - 2\left(\cos^2 \frac{t}{2} - \sin^2 \frac{t}{2}\right) + 4\sin^2 \frac{t}{2}}\, dt$$

$\cos t = \cos^2 \frac{t}{2} - \sin^2 \frac{t}{2}$

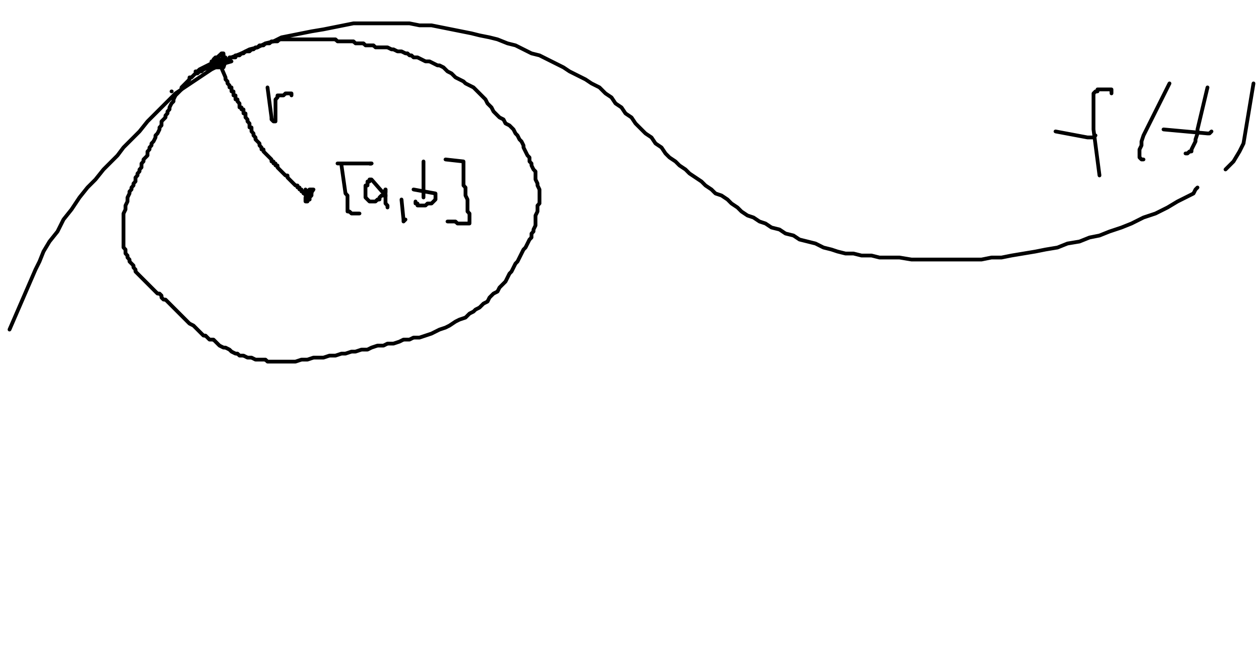
$$= a\int_0^{2\pi}\sqrt{2\sin^2\frac{t}{2}+2\sin^2\frac{t}{2}+4\sin^2\frac{t}{2}}\,dt$$

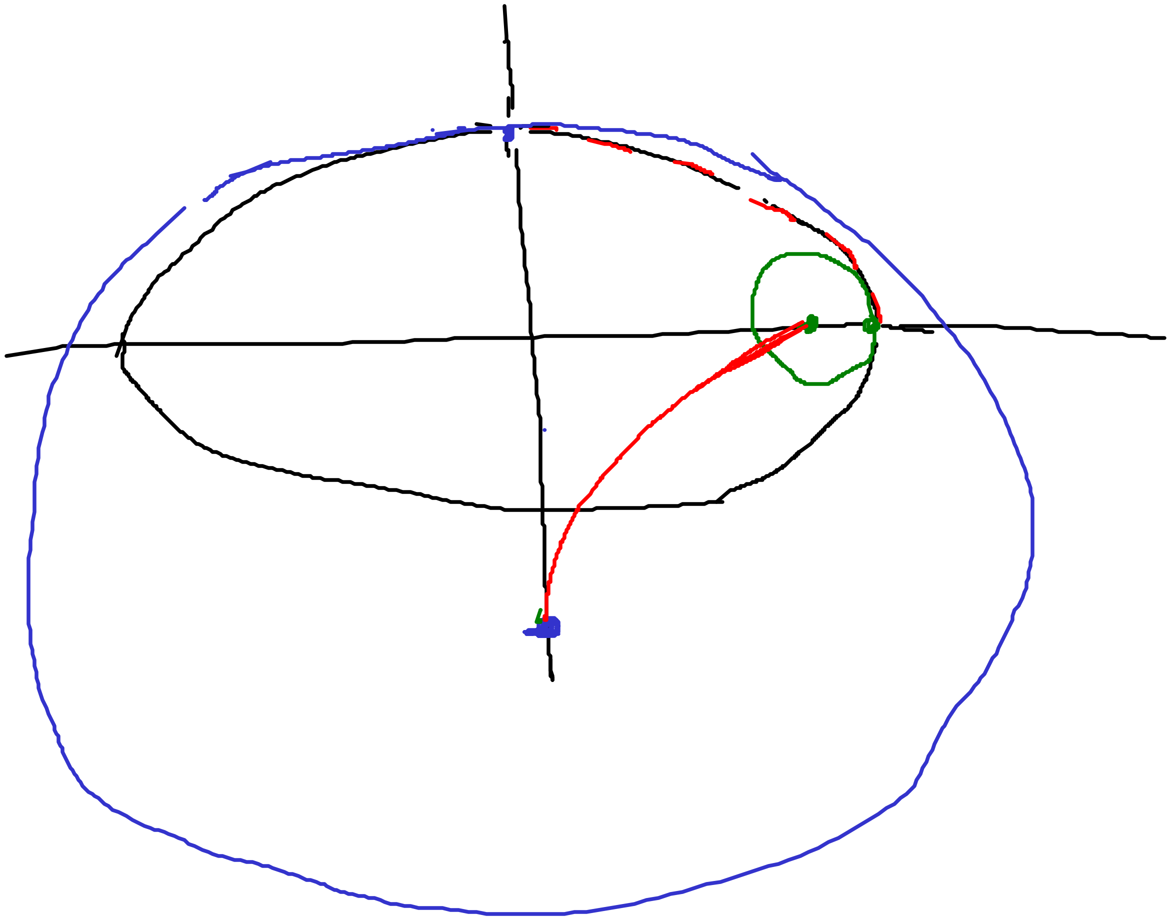
$$= 2\sqrt{2}a\int_0^{2\pi}\sqrt{\sin^2\frac{t}{2}}\,dt = 2\sqrt{2}a\int_0^{2\pi}\sin\frac{t}{2}\,dt$$

$$= 2\sqrt{2}a\left[-2\cos\frac{t}{2}\right]_0^{2\pi} =$$

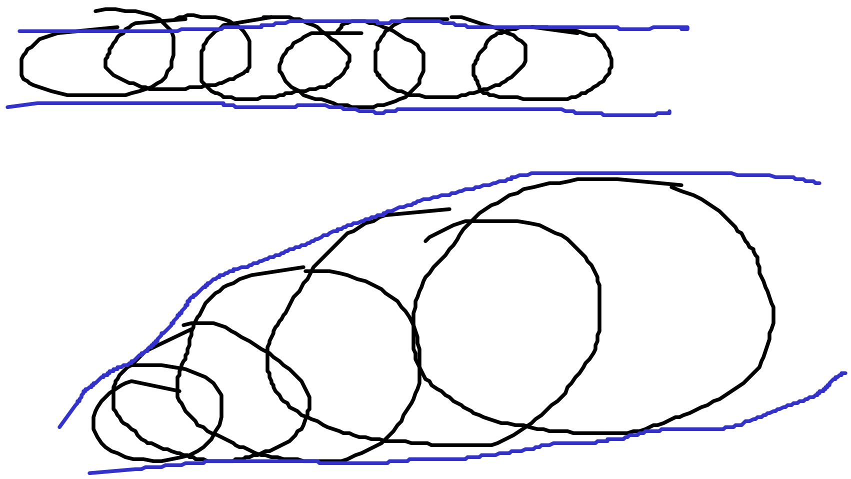
$$= 2\sqrt{2}a\left(-2(-1)-(-2)1\right)$$

$$= 2\sqrt{2}a\cdot(2+2) = 8\sqrt{2}a$$





Soustava kružnic



obálka soustavy kružnic