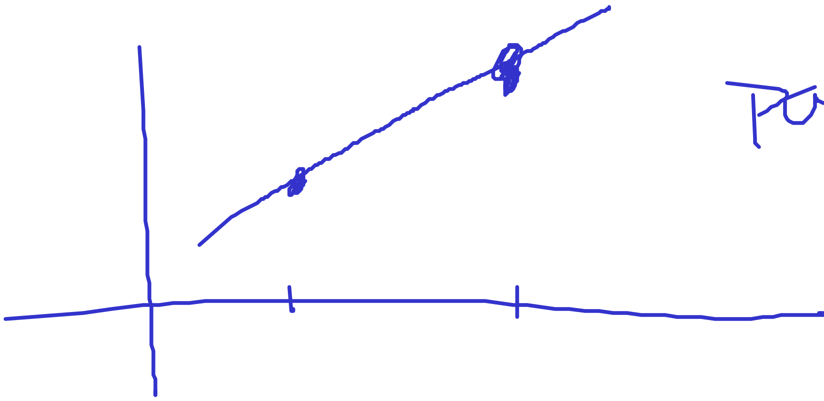
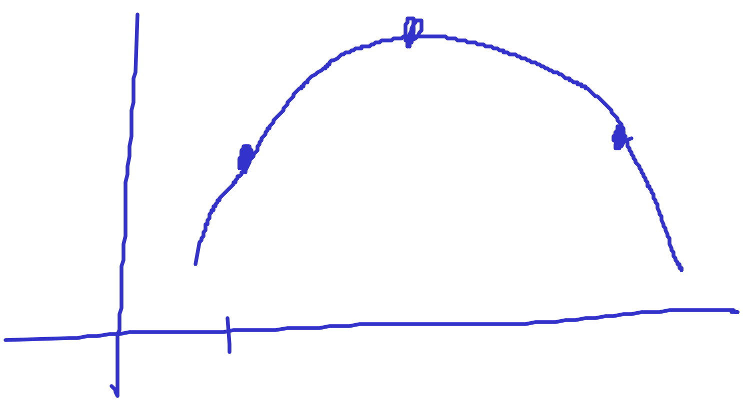
1.2 $P(x)$ co nejmenšího stupně

t. ž.

| $x$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $P(x)$ | 1 | 0 | -1 | 6 |



pol. 1. stupně



pol. 2. stupně

hledáme $P(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$

$a_0, a_1, a_2, a_3 \in \mathbb{R}$

$P(2) = 1 \Rightarrow 8a_3 + 4a_2 + 2a_1 + a_0 = 1$

$P(3) = 0 \Rightarrow 27a_3 + 9a_2 + 3a_1 + a_0 = 0$

$P(4) = -1 \Rightarrow 64a_3 + 16a_2 + 4a_1 + a_0 = -1$

$P(5) = 6 \Rightarrow 125a_3 + 25a_2 + 5a_1 + a_0 = 6$

$$\begin{array}{c}
\begin{array}{cccc} a_0 & a_1 & a_2 & a_3 \end{array} \\
\left(\begin{array}{cccc|c}
1 & 2 & 4 & 8 & 1 \\
1 & 3 & 9 & 27 & 0 \\
1 & 4 & 16 & 64 & -1 \\
1 & 5 & 25 & 125 & 6
\end{array}\right) \sim \ldots
\end{array}$$

Jiné řešení Lagrangeovy interpolační polynomy

| $x_1$ | $x_2$ | ... | $x_n$ |
| --- | --- | --- | --- |
| $y_1$ | $y_2$ | ... | $y_n$ |

$\rightsquigarrow$ hledáme pol. $P(x)$

t. ž. $P(x_i) = y_i$

Zkusme najít polynomy

$l_i(x)$ t. ž. $l_i(x_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

$1 \leq i, j \leq n$

Pak $P(x) = y_1 \ell_1(x) + \dots + y_n \ell_n(x)$

např. $P(x_1) = y_1 \underbrace{\ell_1(x_1)}_{=1} = y_1$

$$\ell_i(x) = \frac{(x-x_1)\cdots(x-x_{i-1})\cdot(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_1)\cdots(x_i-x_{i-1})\cdot(x_i-x_{i+1})\cdots(x_i-x_n)}$$

$\ell_i(x_j) = 0$ $\quad i \neq j$ $\qquad \ell_i(x_i) = 1$

$\ell_i(x)$ pol. stupně $n-1$

Příklad

| $x_i$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $y_i$ | 1 | 0 | -1 | 6 |

- $$\ell_1(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$

$$= \frac{(x-3)(x-4)(x-5)}{(2-3)(2-4)(2-5)} = \frac{-1}{6}\cdot(x-3)(x-4)(x-5)$$

- $$\ell_2(x) = \frac{(x-2)(x-4)(x-5)}{(3-2)(3-4)(3-5)} = \frac{1}{2}(x-2)(x-4)(x-5)$$

$$\bullet\ l_3(x) = \frac{(x-2)(x-3)(x-5)}{(4-2)(4-3)(4-5)} = -\frac{1}{2}(x-2)(x-3)(x-5)$$

$$\bullet\ l_4(x) = \frac{(x-2)(x-3)(x-4)}{(5-2)(5-3)(5-4)} = \frac{1}{6}(x-2)(x-3)(x-4)$$

Vysledek:

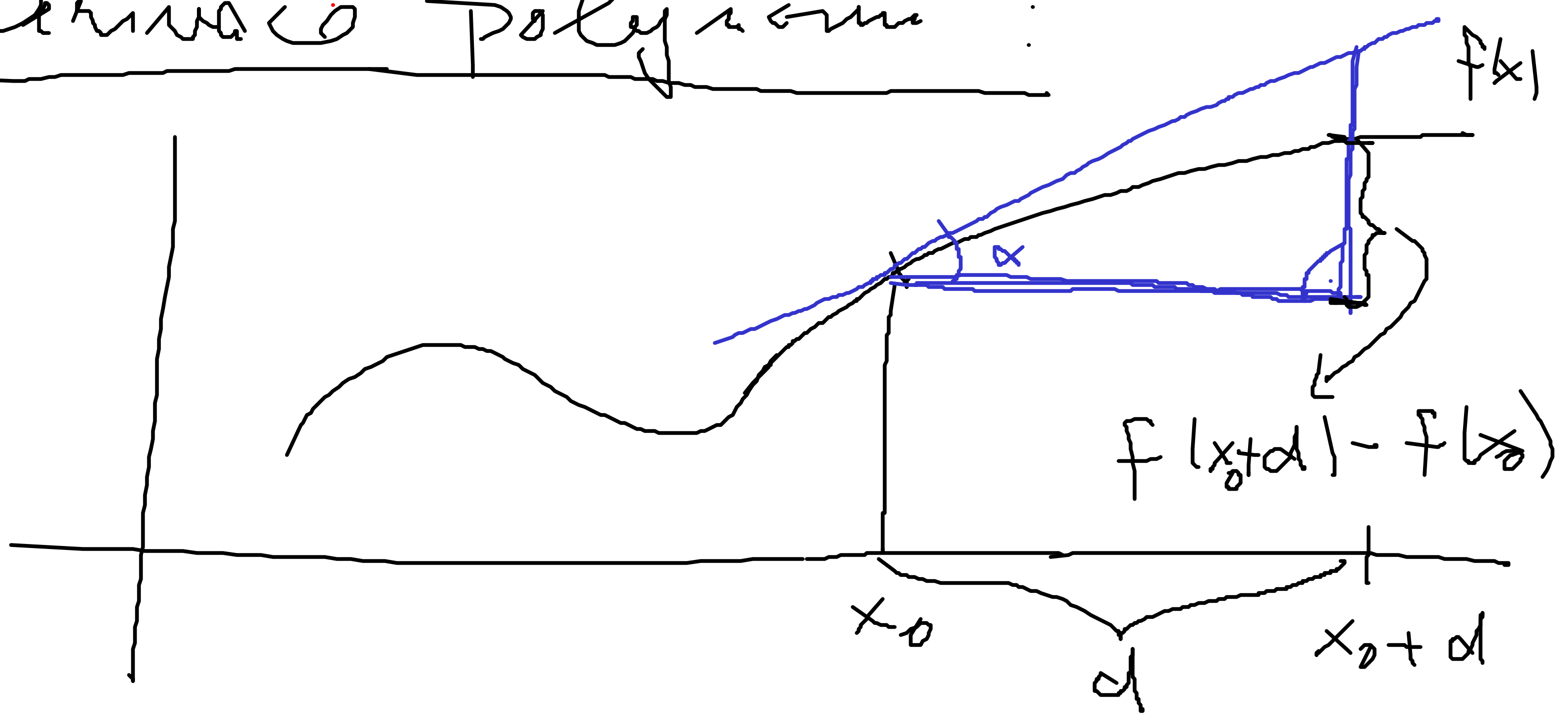
$$p(x) = l_1(x) - l_3(x) + 6\, l_4(x) =$$

$$= -\frac{1}{6}(x-3)(x-4)(x-5)$$

$$+ \frac{1}{2}(x-2)(x-3)(x-5)$$

$$+ (x-2)(x-3)(x-4) = \ldots$$

Derivace Polynomu:



$$\operatorname{tg}\alpha = \lim_{d \to 0} \frac{f(x_0+d) - f(x_0)}{d}$$

derivace funkce $f(x)$ v bodě $x_0$

značíme $f'(x_0)$

$$f(x) = a_n x^n$$

$$f'(x) = a_n \lim_{d \to 0} \frac{(x+d)^n - x^n}{d} =$$

$$= a_n \lim_{d \to 0} \frac{\left(x^n + n x^{n-1} d + \binom{n}{2} x^{n-2} d^2 + \dots\right) - x^n}{d}$$

$$= a_n \lim_{d \to 0} \left(n x^{n-1} + (\dots) d + (\dots) d^2 + \dots\right)$$

$$= a_n \cdot n \cdot x^{n-1}$$

Obecně platí: (1.) $(P(x) + Q(x))'$

$$= P'(x) + Q'(x)$$

$(2)\quad (c\,P(x))' = c\,P'(x)$

**1.3** $P(x)$ t.z.

| $x$ | 1 | 2 |
|---|---|---|
| $P(x)$ | 0 | 3 |
| $P'(x)$ | 1 | 3 |

$$P(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$P'(x) = 3a_3 x^2 + 2a_2 x + a_1$$

$P(1) = 0 \Rightarrow a_3 + a_2 + a_1 + a_0 = 0$

$P(2) = 3 \Rightarrow 8a_3 + 4a_2 + 2a_1 + a_0 = 3$

$P'(1) = 1 \Rightarrow 3a_3 + 2a_2 + a_1 = 1$

$P'(2) = 3 \Rightarrow 12a_3 + 4a_2 + a_1 = 3$

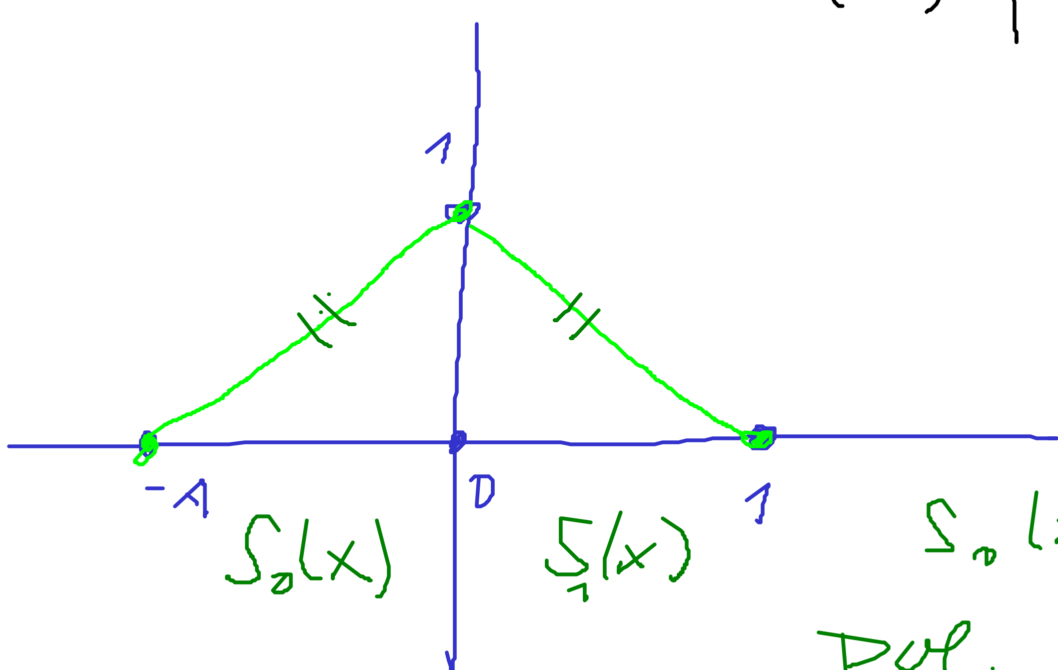
$$\begin{array}{c} \begin{array}{cccc} a_0 & a_1 & a_2 & a_3 \end{array} \\ \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 4 & 8 & 3 \\ 0 & 1 & 2 & 3 & 1 \\ 0 & 1 & 4 & 12 & 3 \end{array}\right) \sim \ldots \end{array}$$

- Jinak: Hermiteovy interpolační polynomy (kubický)

## 1.4 Přirozený splajn $S(x)$

splinyjica

| $x$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $S(x)$ | $0$ | $1$ | $0$ |




$S_0(x)$, $S_1(x)$ pol. 3. stupně

$S_0(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$

$S_1(x) = b_3 x^3 + b_2 x^2 + b_1 x + b_0$

$$\boxed{\begin{array}{ll} S_0(-1) = 0 & S_1(0) = 1 \\ S_0(0) = 1 & S_1(1) = 0 \end{array}}$$

$$\boxed{\begin{array}{l} S_0'(0) = S_1'(0) \\ S_0''(0) = S_1''(0) \end{array}}$$

6 omezení

"Přirozený" → derivace v krajních bodech je nulová

$$\boxed{S_0''(-1) = 0, \quad S_1''(1) = 0}$$

$S_0(-1) = 0 \Rightarrow -a_3 + a_2 - a_1 + a_0 = 0$

$S_0(0) = 1 \Rightarrow a_0 = 1$

$$S_1(0) = 1 \Rightarrow b_0 = 1$$

$$S_1(1) = 0 \Rightarrow b_3 + b_2 + b_1 + b_0 = 0$$

---

$$S_0'(x) = 3a_3x^2 + 2a_2x + a_1$$

$$S_0''(x) = 6a_3x + 2a_2$$

$$S_1'(x) = 3b_3x^2 + 2b_2x + b_1$$

$$S_1''(x) = 6b_3x + 2b_2$$

---

$$S_0'(0) = S_1'(0) \Rightarrow a_1 = b_1$$

$$S_0''(0) = S_1''(0) \Rightarrow 2a_2 = 2b_2$$

---

$$S_0''(-1) = 0 \Rightarrow -6a_3 + 2a_2 = 0$$

$$S_1''(1) = 0 \Rightarrow 6b_3 + 2b_2 = 0$$

Celkem: 8 rovnic o
8 neznámých