

Komplexní čísla

- Upravte a výsledek zapište v algebraickém tvaru: $\frac{2+i}{2-i} - \frac{i-1}{i} =$
- Vypočítejte: $\overline{(1+i)(3+2i)} =$
- Vypočítejte: $\left| \frac{|4-3i|+i}{3-2i} \right| =$
- Nakresli do Gaussovy roviny obrazy všech komplexních čísel, pro která platí:
 $|z-1+2i| \leq 5$
- Řešte rovnici v \mathbb{C} : $2z + 3\bar{z} = 5 + i$
- Řešte kvadratickou rovnici v \mathbb{C} : $z^2 - (3+2i)z + 5 + i = 0$
- Řešte binomickou rovnici v \mathbb{C} a kořeny zobrazte v Gaussově rovině: $z^4 = -6 - 6\sqrt{3}i$

Řeš.:

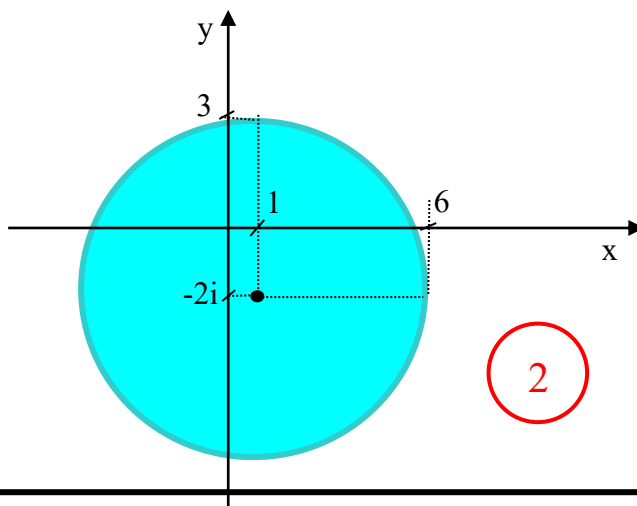
$$1. \frac{2+i}{2-i} \cdot \frac{2+i}{2+i} - \frac{i-1}{i} \cdot \frac{i}{i} = \frac{4+4i+i^2}{5} - \frac{i^2-i}{i^2} = \frac{3+4i}{5} + (-1) - i = \frac{3+4i-5-5i}{5} = \frac{-2-i}{5} = \frac{2}{5} - \frac{i}{5} \quad (2)$$

$$2. \overline{(1+i)(3+2i)} = \overline{(1-i)(3-2i)} = \overline{3+2i^2-5i} = \overline{1-5i} = 1+5i \quad (2)$$

3.

$$\left| \frac{|4-3i|+i}{3-2i} \right| = \left| \frac{\sqrt{4^2+(-3)^2}+i}{3-2i} \right| = \left| \frac{5+i}{3-2i} \cdot \frac{3+2i}{3+2i} \right| = \left| \frac{15+13i+2i^2}{13} \right| = \left| \frac{13+13i}{13} \right| = |1+i| = \sqrt{2} \quad (2)$$

$$4. \begin{cases} |z-1+2i| \leq 5 \\ |z-(1-2i)| \leq 5 \end{cases}$$



$$5. \begin{aligned} 2z + 3\bar{z} &= 5 + i \\ z &= x + yi; \quad \bar{z} = x - yi \\ 2(x + yi) + 3(x - yi) &= 5 + i \\ 2x + 2yi + 3x - 3yi &= 5 + i \\ 5x - yi &= 5 + i \\ 5x &= 5; \quad -y = 1 \\ x &= 1; \quad y = -1 \\ \mathbf{K} &= \{1 - i\} \quad (2) \end{aligned}$$

6. $z^2 - (3 + 2i)z + 5 + i = 0$

$$z_{1,2} = \frac{3+2i \pm \sqrt{(3+2i)^2 - 4(5+i)}}{2} = \frac{3+2i \pm \sqrt{9+12i-4-20-4i}}{2} = \frac{3+2i \pm \sqrt{-15+8i}}{2}$$

$$\sqrt{-15+8i} = x + yi, \text{ kde } x, y \in \mathbb{R} \quad y = \frac{4}{x} \rightarrow x^2 - \frac{16}{x^2} = -15 \quad x = \pm 1 \rightarrow y = \pm 4$$

$$-15+8i = x^2 - y^2 + 2xyi \quad x^4 + 15x^2 - 16 = 0 \quad \sqrt{-15+8i} = \{\pm 1 \pm 4i\}$$

$$x^2 - y^2 = -15; \quad 2xy = 8 \quad (x^2 + 16)(x^2 - 1) = 0 \quad \text{2}$$

$$z_{1,2} = \frac{3+2i \pm (1+4i)}{2} = \begin{cases} 2 + 3i \\ 1 - i \end{cases}$$

$K = \{2 + 3i; 1 - i\}$ 2

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7. $z^4 = -6 - 6\sqrt{3}i$

Pomocný výpočet: $a = -6 - 6\sqrt{3}i$ do goniometrického tvaru

$$|a| = \sqrt{(-6)^2 + (-6\sqrt{3})^2} = \sqrt{144} = 12; \quad \cos x = -\frac{6}{12} = -\frac{1}{2}; \quad \sin x = \frac{-6\sqrt{3}}{12} = -\frac{\sqrt{3}}{2}$$

$$x = \frac{4\pi}{3} \quad \text{2}$$

Hledám $z = |z|(\cos x + i \sin x)$ tak, aby $z^4 = a$.

Tedy:

$$|z|^4 (\cos 4x + i \sin 4x) = 12 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right)$$

$$|z|^4 = 12 \quad \wedge \quad 4x = \frac{4\pi}{3} + 2k\pi, \quad k \in \{0, 1, 2, 3\}$$

$$|z| = \sqrt[4]{12} \quad \wedge \quad x = \frac{\pi}{3} + \frac{k\pi}{2}, \quad k \in \{0, 1, 2, 3\}$$

Závěr: $z_k = \sqrt[4]{12} \cdot \left(\cos \left(\frac{\pi}{3} + \frac{k\pi}{2} \right) + i \sin \left(\frac{\pi}{3} + \frac{k\pi}{2} \right) \right), \quad k \in \{0, 1, 2, 3\}$

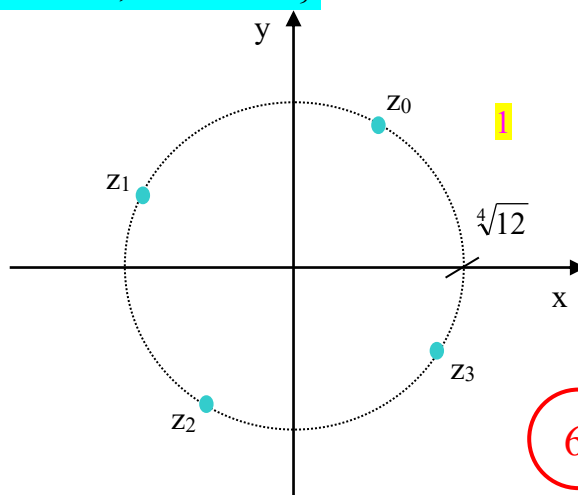
$$K = \bigcup_k \left\{ \sqrt[4]{12} \cdot \left(\cos \left(\frac{\pi}{3} + \frac{k\pi}{2} \right) + i \sin \left(\frac{\pi}{3} + \frac{k\pi}{2} \right) \right), \quad k \in \{0, 1, 2, 3\} \right\}$$

$$z_0 = \sqrt[4]{12} \cdot \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right);$$

$$z_1 = \sqrt[4]{12} \cdot \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right);$$

$$z_2 = \sqrt[4]{12} \cdot \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right);$$

$$z_3 = \sqrt[4]{12} \cdot \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right) \quad \text{3}$$



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Známka:	20 – 18 bodů	1
	17 – 14 bodů	2
	13 – 8 bodů	3
	7 – 4 body	4