C7790 Introduction to Molecular Modelling
TSM Modelling Molecular Structures

Lesson 5
Phenomenological thermodynamics (equilibrium)

JS/2022 Present Form of Teaching: Rev3

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Overview

macroworld **microworld**

states

(thermodynamic properties, G, T,...)

phenomenological thermodynamics

equilibrium (equilibrium constant)

kinetics (rate constant)

free energy

(Gibbs/Helmholtz)

partition function

statistical thermodynamics

microstates

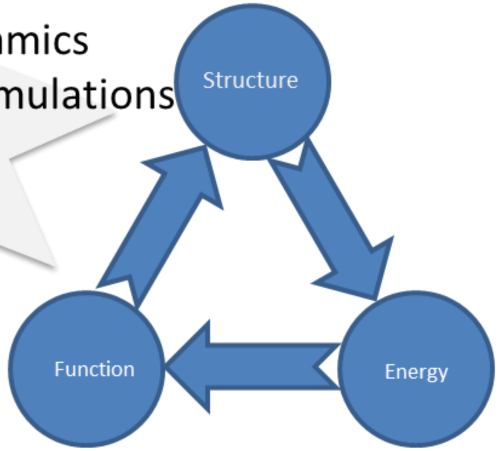
(mechanical properties, E)

Description levels (model chemistry):

- quantum mechanics
  - semiempirical methods
  - ab initio methods
  - post-HF methods
  - DFT methods
- molecular mechanics
- coarse-grained mechanics

Simulations:

- molecular dynamics
- Monte Carlo simulations
- docking
- ...



microstate ≠ microworld

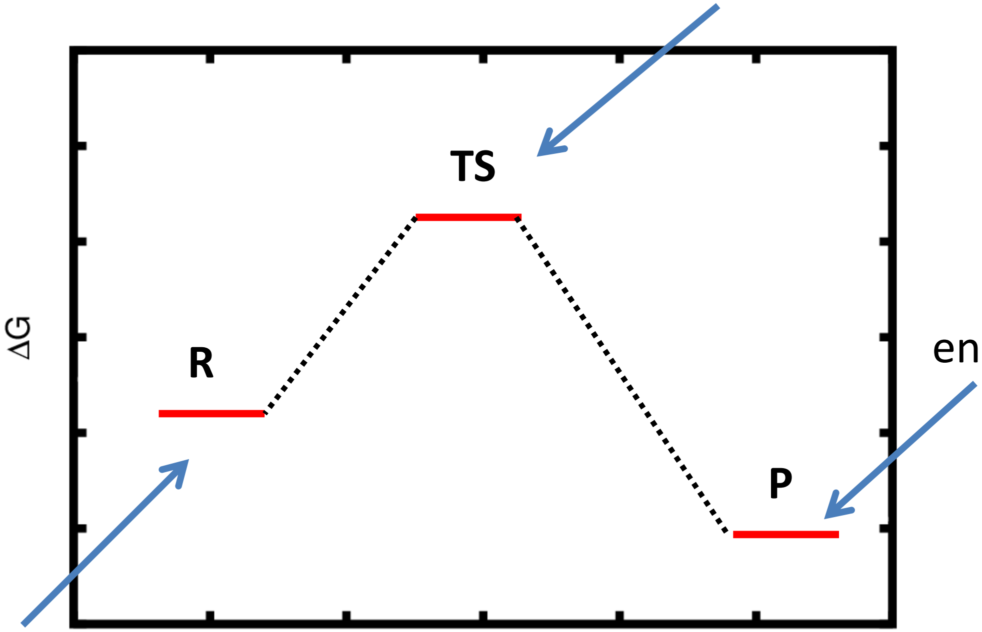
Chemical process

Or what you should already know….

Thermodynamics of chemical process

$$aA + bB \rightleftharpoons cC + dD$$

a, b, c, d - stoichiometric coefficients



activated complex (transition state)

Gibbs (free) energy change

end state (products)

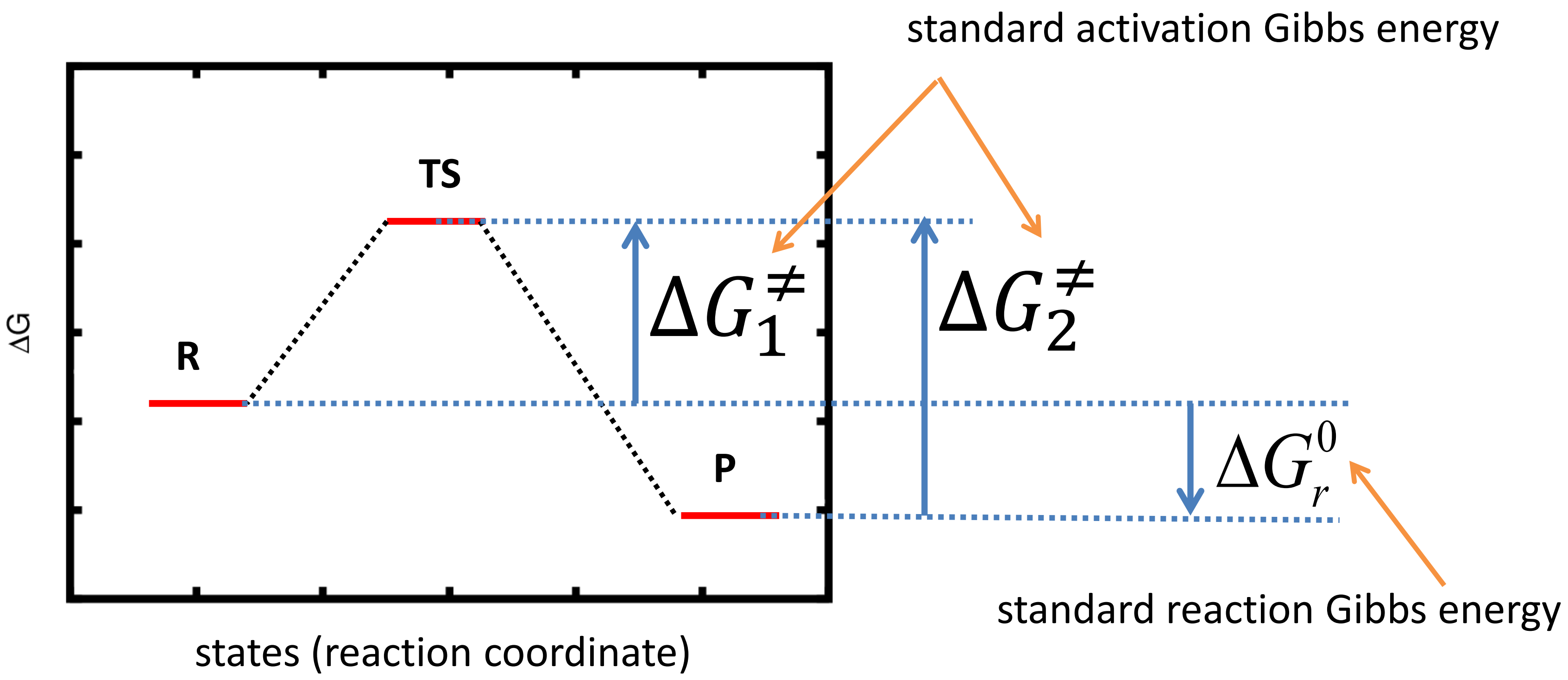
initial state (reactants)

states (reaction coordinate)

Thermodynamics of chemical process

$$aA + bB \underset{2}{\overset{1}{\rightleftharpoons}} cC + dD$$

1 - forward reaction
2 - backward reaction

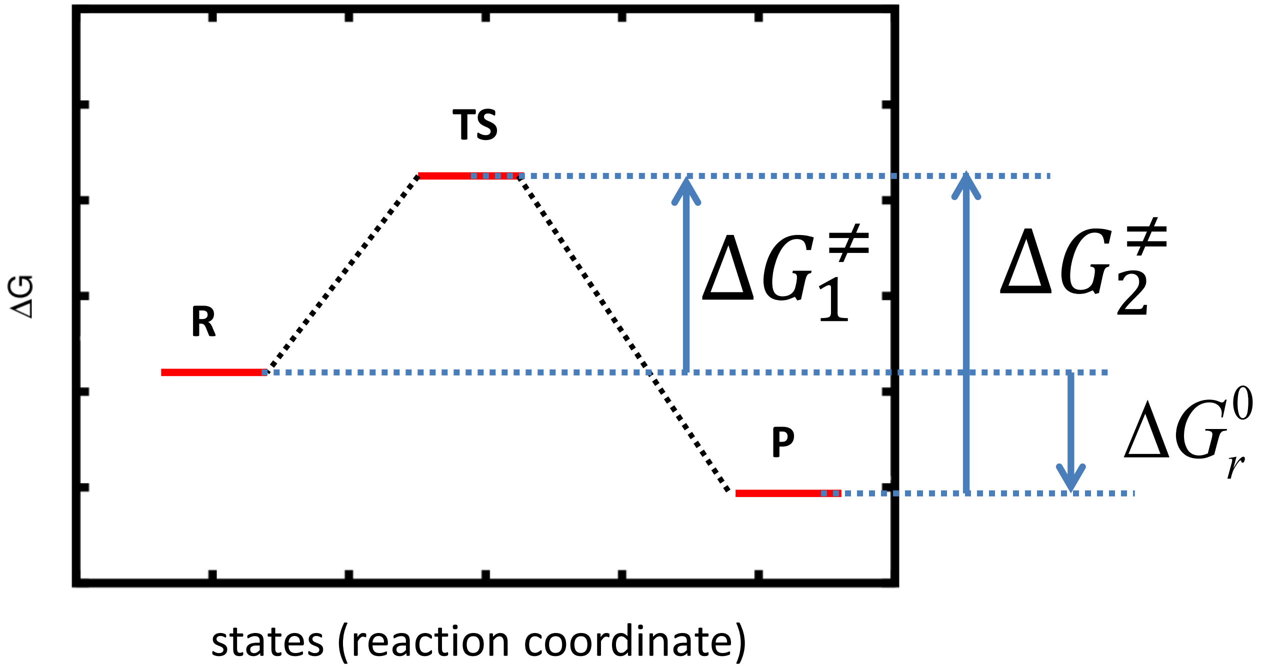


standard activation Gibbs energy

standard reaction Gibbs energy

Thermodynamics of chemical process

$$aA + bB \underset{2}{\overset{1}{\rightleftarrows}} cC + dD$$



Thermodynamic cycle

$$\Delta G_1^{\neq} - \Delta G_2^{\neq} - \Delta G_r^0 = 0 \longrightarrow \Delta G_r^0 = \Delta G_1^{\neq} - \Delta G_2^{\neq}$$

Equilibrium

Or what you should already know….

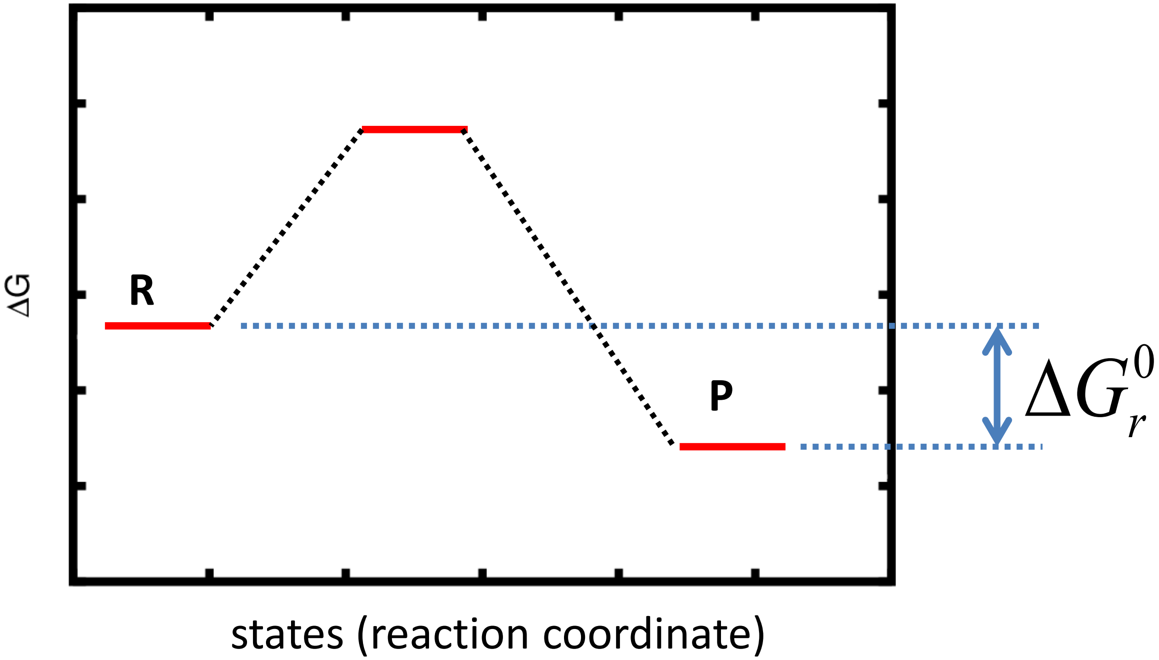
Equilibrium - summary

$$aA + bB \rightleftharpoons cC + dD$$

standard reaction Gibbs energy

$$\Delta G_r^0 = -RT\ln K$$

equilibrium constant



$$K = \frac{\{C\}_r^c\{D\}_r^d}{\{A\}_r^a\{B\}_r^b} \approx \frac{[C]_r^c[D]_r^d}{[A]_r^a[B]_r^b}$$

activities concentration

(r) equilibrium

Equilibrium - summary

$$\mathbf{aA + bB \rightleftharpoons cC + dD}$$

$$K = \frac{\{C\}_r^c \{D\}_r^d}{\{A\}_r^a \{B\}_r^b} \approx \frac{[C]_r^c [D]_r^d}{[A]_r^a [B]_r^b} = \prod_i [X_i]_r^{\nu_i}$$

$$\Delta G_r^0 = -RT \ln K$$

$$\Delta G_r^0 = c\Delta G_{f,C}^0 + d\Delta G_{f,D}^0 - \left(a\Delta G_{f,A}^0 + b\Delta G_{f,B}^0\right) = \sum_i \upsilon_i G_{f,i}^0$$

The standard reaction free energy can be calculated, for example, from standard formation (f) Gibbs energies.

Chemical transformation

$$aA + bB \rightleftharpoons cC + dD$$

Reaction of A and B provides C and D and vice versa. Both processes continues until the rate of both forward and backward reactions is the same and equilibrium is established.

Principle questions:

- **What is the composition of the reaction mixture in equilibrium and how can it be determined?**
- How is it possible to influence the composition of the reaction mixture in equilibrium?

Extent of reaction

The reaction progress can be described by **the extent of reaction,** which considers the stoichiometry of the transformation.

$$\mathrm{aA + bB \rightleftharpoons cC + dD}$$

Extent of reaction ξ is defined as the molar change of a substance in relation to its stoichiometric coefficient:

$$\xi = \frac{\Delta n_i}{\upsilon_i}$$

Sign convention for ν_i

products (end state) - positive value

reactants (initial state) - negative value

Example: initial state: $n_{0,\,A}$ and $n_{0,\,B}$

$$\xi = \frac{n_A - n_{0,A}}{-a} = \frac{n_B - n_{0,B}}{-b} = \frac{n_C}{c} = \frac{n_D}{d}$$

Gibbs energy of reaction mixture

The **Gibbs energy of reaction mixture G** is a function of the composition of the reaction mixture.
At the **constant temperature and pressure,** it is possible to write the Gibbs energy as total differential in the following form:

$$dG(n_1, n_2, \ldots, n_N) = \sum_{i=1}^{N} \left(\frac{\partial G}{\partial n_i} \right)_{p,T,n_j \neq n_i} dn_i$$

N - number of reacting compounds

Derivative of the Gibbs energy with respect to molar amount is a very useful quantity called **chemical potential μ** :

$$\mu_i = \left(\frac{\partial G}{\partial n_i} \right)_{p,T,n_j \neq n_i}$$

n_i is the molar amount of substance *i*

$$dG(n_1, n_2, \ldots, n_N) = \sum_{i=1}^{N} \mu_i dn_i$$

Chemical potential

The chemical potential is a **state function**:

$$\mu_i = \left(\frac{\partial G}{\partial n_i}\right)_{p,T,n_j \neq n_i}$$

Chemical potential expresses the effort of the substance:

- to react with another substance
- to change its status
- to change its spatial distribution

Value of chemical potential:

- is related to the very nature of the substance
- is related to the environment (temperature, pressure, concentration, ...)
- **however, it is not related** to the nature of the substances with which it reacts or is transformed to

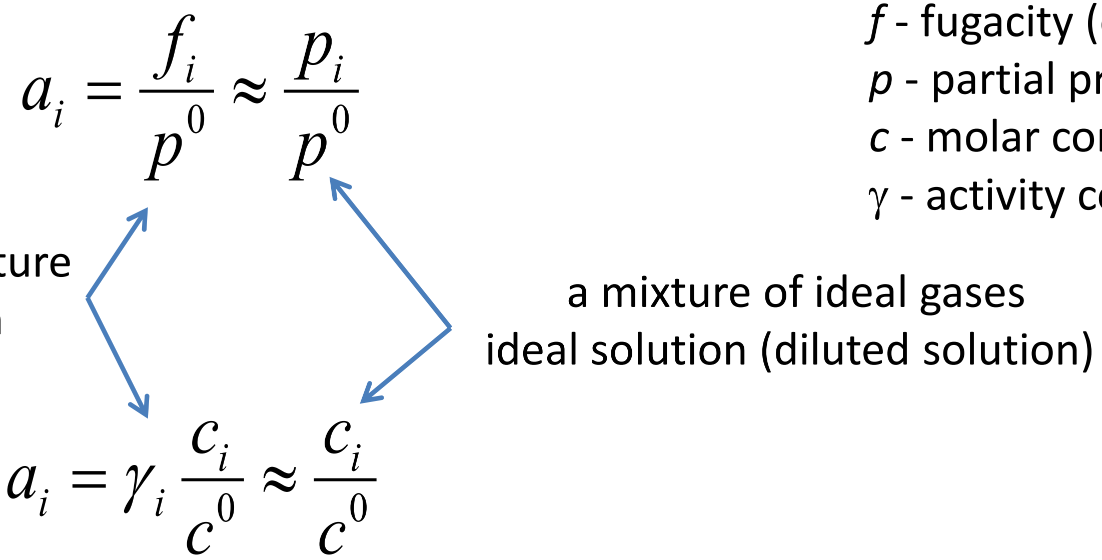
Relationship between chemical potential μ_i and activity a_i *of* substance:

$$\mu_i = \mu_i^0 + RT \ln a_i$$

Activity

Activity expresses the **effective amount** of a substance relative to a standard state.
It is a dimensionless quantity.

f - fugacity (effective pressure)
p - partial pressure
c - molar concentration
γ - activity coefficient



gas mixtures

$$a_i = \frac{f_i}{p^0} \approx \frac{p_i}{p^0}$$

gas mixture
solution

a mixture of ideal gases
ideal solution (diluted solution)

solutions

$$a_i = \gamma_i \frac{c_i}{c^0} \approx \frac{c_i}{c^0}$$

The reason for introducing an activity coefficient (or fugacity) is to maintain a simple relationship between activity and chemical potential. The relationship for chemical potential can therefore be taken as the definition of the activity:

$$\boxed{a_i = e^{\frac{\mu_i - \mu_i^0}{RT}}}$$

Standard chemical potential

Standard chemical potential is the change in Gibbs energy that is associated with the formation of one mole of matter in the standard state.
The change of the Gibbs energy is most often expressed in the form of the **standard formation Gibbs energies**.

$$\mu_i^0 = \Delta G_{f,i}^0$$

Standard formation Gibbs energy is the change of Gibbs energy that corresponds to the formation of one mole of matter from **chemical elements in the standard state**.

Chemical elements in the standard state have zero formation Gibbs energy (this is the definition of the reference state).

Standard state (IUPAC):

p^0 = 100 kPa

c^0 = 1 mol dm^{-3} = 1 M

The activity of pure substances in condensed phases (solid or liquids) is normally taken as unity (the number 1)

$$a_i = 1$$

Gibbs energy of reaction mixture

$$dG(n_1,n_2,\ldots,n_N)=\sum_{i=1}^{N}\mu_i dn_i$$

It is better to express the Gibbs energy change using the extent of the reaction:

$$\xi=\frac{\Delta n_i}{\upsilon_i}\quad\longrightarrow\quad dn_i=\upsilon_i d\xi\quad\longrightarrow\quad dG=\sum_{i=1}^{N}\mu_i\upsilon_i d\xi$$

$$\boxed{\frac{dG}{d\xi}=\sum_{i=1}^{N}\mu_i\upsilon_i}$$

The chemical potential of the individual substances depends on their effective amount relative to the standard state, i.e., on the composition of the reaction mixture.
The change is therefore proportional to the composition of the reaction mixture and the standard chemical potential of the individual reactants:

$$\frac{dG}{d\xi}=\sum_{i=1}^{N}\mu_i\upsilon_i=\sum_{i=1}^{N}\upsilon_i\mu_i^0+RT\ln\prod_{i=1}^{N}a_i^{\upsilon_i}$$

$$\boxed{\frac{dG}{d\xi}=\Delta G_r^0+RT\ln Q}$$

reaction quotient

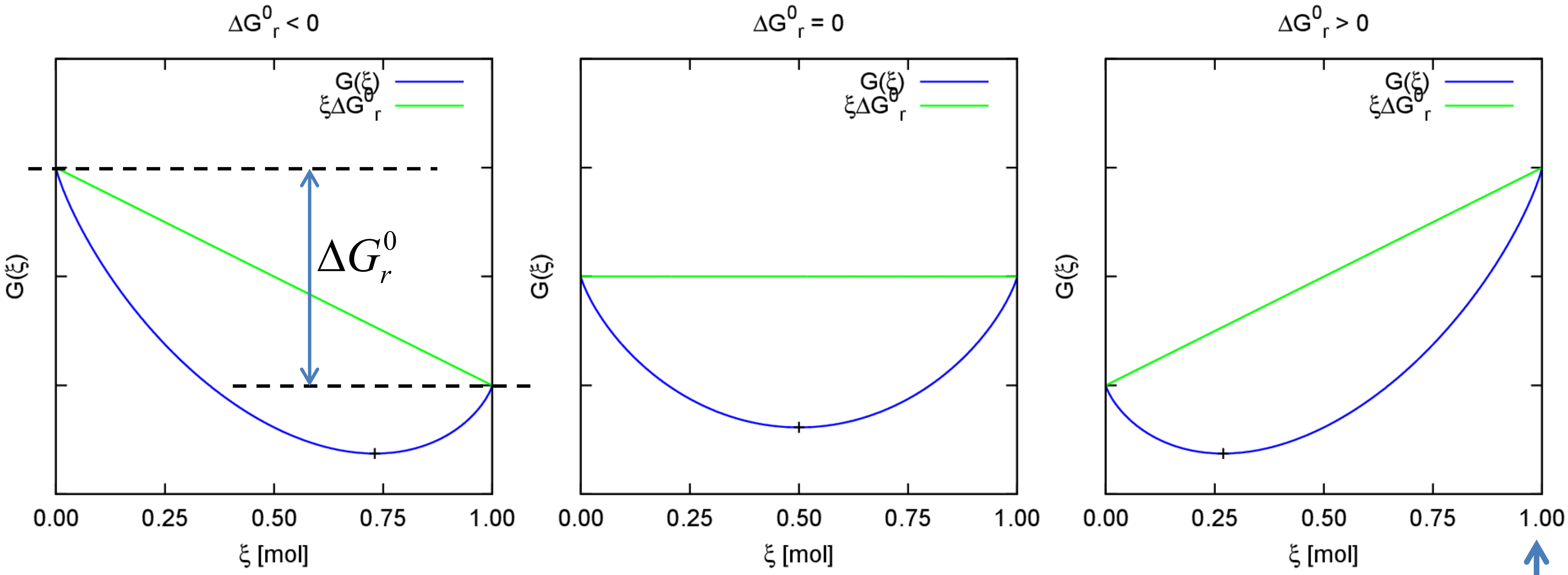
standard Gibbs reaction energy

Change of G during reaction

$$A \rightleftarrows B$$

from integration dG/dξ

$$\boxed{G(\xi)} = \boxed{\xi \Delta G_r^0} + \boxed{RT\left\{\xi \ln \xi - \left(\xi - n_{0,A}\right)\ln\left(n_{0,A} - \xi\right) - n_{0,A}\ln n_{0,A}\right\}} + G_A^0$$

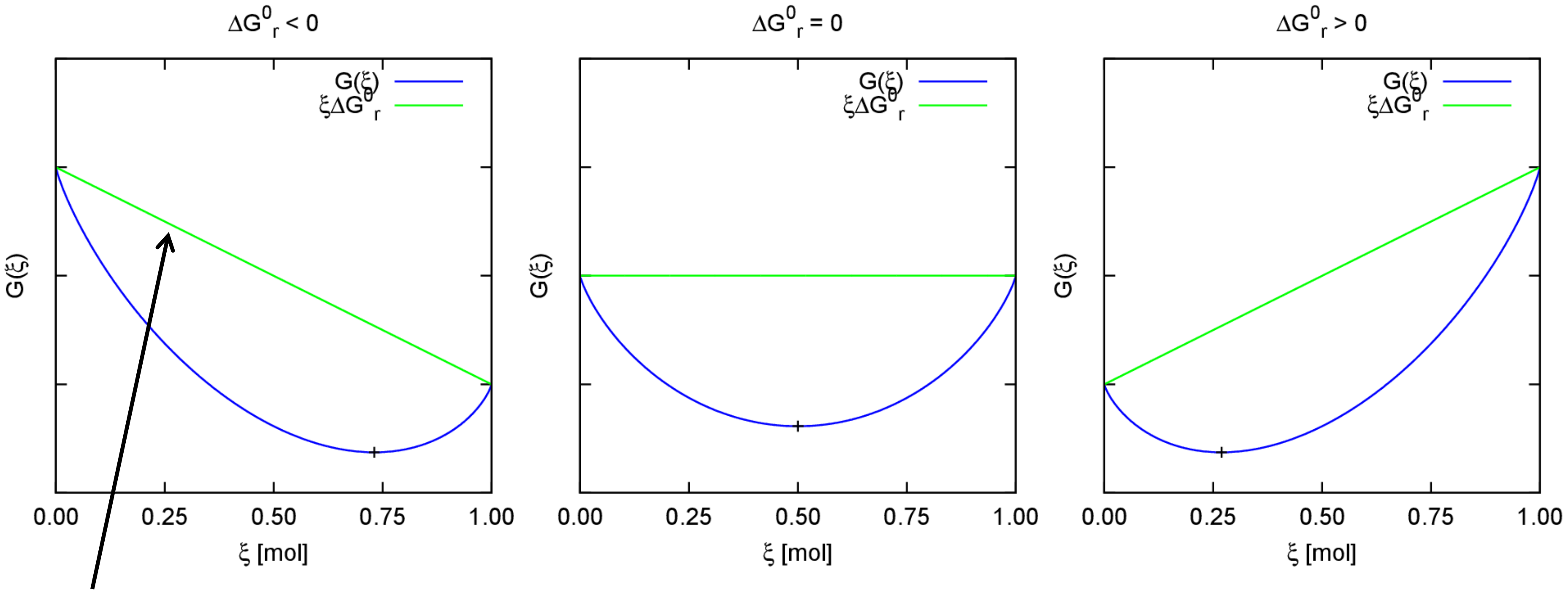


only for a given reaction and $n_{0,A}$ = 1.0 mol

Change of G during reaction

$$\mathbf{A} \rightleftarrows \boldsymbol{B}$$

$$G(\xi) = \xi\Delta G_r^0 + RT\left\{\xi\ln\xi - \left(\xi - n_{0,A}\right)\ln\left(n_{0,A} - \xi\right) - n_{0,A}\ln n_{0,A}\right\} + G_A^0$$

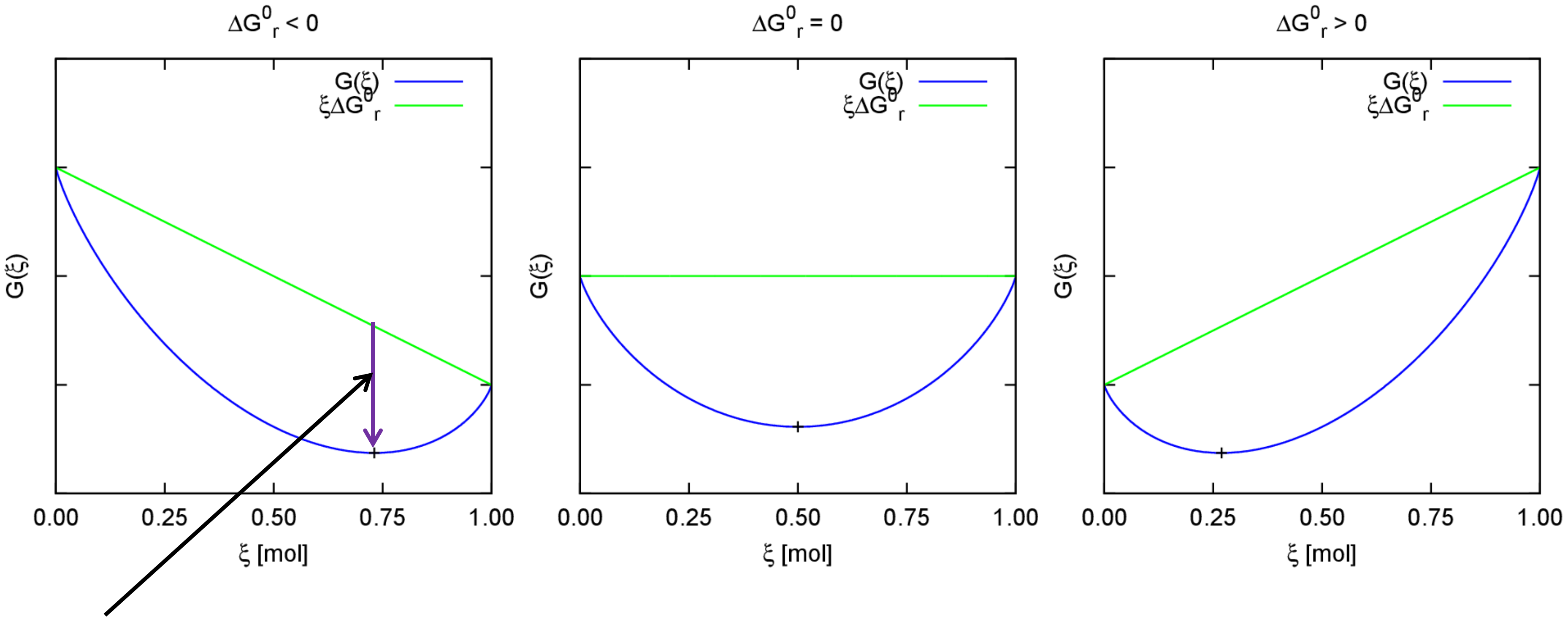


change of Gibbs energy due to the reaction (this is the Gibbs energy of individual substances in the standard state in the amount determined by the extent of the reaction)

Change of G during reaction

$$\mathbf{A} \rightleftharpoons \boldsymbol{B}$$

$$\boxed{G(\xi)} = \boxed{\xi \Delta G_r^0} + \boxed{RT\left\{\xi \ln \xi - \left(\xi - n_{0,A}\right)\ln\left(n_{0,A} - \xi\right) - n_{0,A} \ln n_{0,A}\right\}} + G_A^0$$

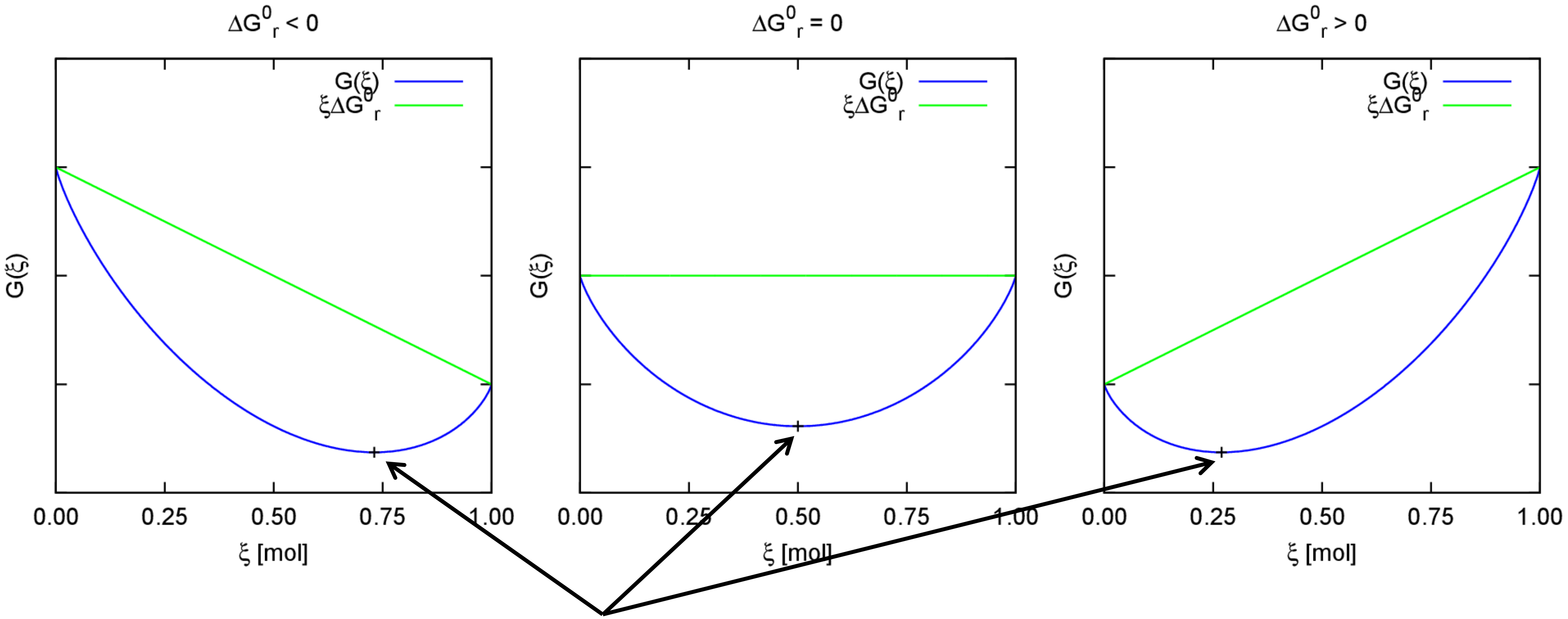


mixing Gibbs energy (Gibbs energy that is released as a result of mixing substances in the standard state in an amount determined by the extent of the reaction)

Change of G during reaction

$$\mathbf{A} \rightleftharpoons \mathbf{B}$$

$$\boxed{G(\xi)} = \boxed{\xi \Delta G_r^0} + \boxed{RT\left\{\xi \ln \xi - \left(\xi - n_{0,A}\right)\ln\left(n_{0,A} - \xi\right) - n_{0,A}\ln n_{0,A}\right\}} + G_A^0$$



local extreme (minimum) determines the composition of the reaction mixture in equilibrium

Qualitative conclusions

- The change in Gibbs energy consists of **two contributions** due to:
  a) reaction
  b) mixing (entropy)
- The change of Gibbs energy from the initial or final state to equilibrium is always negative, so it is a spontaneous process. Even if the standard Gibbs reaction energy is zero or positive.
- **There is only one local extreme** (minimum) of reaction Gibbs energy and it corresponds to the equilibrium state.

Finding the extreme

At the local extreme, the derivative of the function takes zero value:

$$\frac{dG}{d\xi} = \Delta G_r^0 + RT\ln Q_r = 0$$

$$\boxed{\Delta G_r^0 = -RT\ln Q_r = -RT\ln K}$$

Equilibrium constant K is a dimensionless quantity that corresponds to the reaction quotient in the equilibrium state. Equilibrium constant **depends only on the nature of the reaction, the temperature and the definition of the standard state, but it does not depend on the initial composition of the reaction mixture.**

$$K = \prod_{i=1}^{N} a_{r,i}^{\upsilon_i}$$

Sign convention for ν_{and}

end state - positive value
default state - negative value

at equilibrium (r)

Example

$$K = \prod_{i=1}^{N} a_{r,i}^{\upsilon_i}$$

$$\boldsymbol{a}\mathbf{A} + \boldsymbol{b}\mathbf{B} \rightleftharpoons \boldsymbol{c}\mathbf{C} + \boldsymbol{d}\mathbf{D}$$

$$K = \underbrace{a_{r,A}^{-a} a_{r,B}^{-b} a_{r,C}^{c} a_{r,D}^{d} = \frac{a_{r,C}^{c} a_{r,D}^{d}}{a_{r,A}^{a} a_{r,B}^{b}} = \frac{\{C\}_r^c \{D\}_r^d}{\{A\}_r^a \{B\}_r^b}}_{\text{dimensionless}} \approx \frac{[C]_r^c [D]_r^d}{[A]_r^a [B]_r^b}$$

!!! it has a size !!!

K dimension is (mol dm^{-3})^n (or M^n), where *n* is the sum of stoichiometric coefficients

This follows from the definition of the standard state for solution.

at equilibrium (r)

Conclusion

- At the given temperature and definition of the standard state, the **equilibrium constant** is determined only by the **standard reaction Gibbs energy**:

$$\boxed{\Delta G_r^0 = -RT\ln K}$$

- The standard reaction Gibbs energy corresponds to the conversion of the initial state to the final state, which is a hypothetical process that does not actually occur.
- When equilibrium is established either from the initial or final state, the change of reaction Gibbs energy is always negative, regardless of whether the standard reaction Gibbs energy is zero or positive.
- Thus, the reactions always proceed spontaneously from the initial or final state to equilibrium.

$$\Delta G_r^0 <> \Delta G(\xi)$$

it determines composition of the reaction mixture but it does not say anything about spontaneity of the reaction

it determines spontaneity of the process

ξ is the extent of reaction (≠ reaction coordinate)

Recommended Literature

- Atkins, P. W. *Physical Chemistry*, 5. ed., repr. (with correct.).; Oxford Univ. Press: Oxford, 1994.
- Dill, K. A.; Bromberg, S. *Molecular Driving Forces: Statistical Thermodynamics in Biology, Chemistry, Physics, and Nanoscience*, 2nd ed.; Garland Science: London ; New York, 2011.

Homework

Homework

1. What is pH of acetic acid solution with $c_0(CH_3COOH)=10^{-3}$ M?

2. Determine the equilibrium composition of the reaction mixture under standard conditions for the reaction below, provided that the standard reaction Gibbs energy is 0.5; 1.0; 2.5; 5.0 and 10.0 kcal/mol. The starting amount of substance A is 0.001 mol. The volume of the reaction mixture, which is constant during the reaction, is 1 liter. Next, determine the extent of the reaction and the ratio of the concentrations of substance B to substance A.

$$\mathbf{A} \rightleftharpoons \mathbf{B}$$

Homework - results

1. $c_0=10^{-3}$ M (pK_a = 4.756), **pH=3.9**

2. Solution for constant volume 1L:

T=	298.15	K		**ΔG_r^0**	**K**	**-logK**	**[A]**	**[B]**	**x**	**[B]/[A]**
c0=	0.001	M		(kcal/mol)			(M)	(M)	(mol)	
				0.50	4.300E-01	0.4	6.993E-04	3.007E-04	3.007E-04	4.300E-01
				1.00	1.849E-01	0.7	8.439E-04	1.561E-04	1.561E-04	1.849E-01
				2.50	1.470E-02	1.8	9.855E-04	1.449E-05	1.449E-05	1.470E-02
				5.00	2.162E-04	3.7	9.998E-04	2.161E-07	2.161E-07	2.162E-04
				10.00	4.672E-08	7.3	1.000E-03	4.672E-11	4.672E-11	4.672E-08