# Study of glow discharge positive column by electrostatic probes: Double probe

# Contents

1	Symmetric double probe at floating potential	<b>2</b>
<b>2</b>	Operation of double probe	<b>2</b>
	2.1 A) Zero voltage $V_d = 0$	<b>2</b>
	2.2 B) Small negative voltage $V_{\rm d} < 0$	3
	2.3 C) High negative voltage $V_{\rm d} \ll 0$	3
3	Theory of the operation of double probe	3
4	Calculation of plasma parameters from I-V characteristic of double probe	<b>5</b>
	4.1 Electron temperature: method of currents ratio	5
	4.2 Electron temperature: resistance method	5
	4.3 Electron density in plasma	6
	4.4 Ion and electron density in plasma	7
	4.4 Ion and electron density in plasma	1

# Introduction

Diagnostics of positive column of glow discharge using electrostatic Langmuir probes can be used to determine several plasma parameters. There are different constructions of the probe. One of them is double probe which consists of two electrodes of the same sizes with distance sufficient for not overlapping of their sheaths.

There are several advantages in comparison with single Langmuir probe. Double probe system is floating and there is always steep slope around zero voltage in current voltage (I-V) characteristic. Saturated ion current limits total current in the probe circuit therefore probes does not disturb plasma that much.

# 1 Symmetric double probe at floating potential

Double probe system consists of two same probes located in the equipotential surface in the plasma. They are not connected to electrodes and without influence from external potential end up at floating potential  $V_{\rm fl}$ . We can study plasma using double probe by applying small voltage  $V_{\rm d}$  between two probes and measuring current  $I_{\rm d}$  in probe circuit. The schematic representation of double probe is shown in fig. 1. Double probe is suitable for the study of high frequency plasma and plasma decay.



Figure 1: The schematic representation of double probe.

# 2 Operation of double probe

We need to discuss operation regimes for different voltage  $V_{\rm d}$  to understand how the measurement with double probe works. Lets assume same surface areas of the probes, no contact potentials and that they are located in spot with same plasma potential  $V_{\rm p}$ . Voltage between probes  $V_{\rm d}$  does not affect ion current.

### **2.1** A) Zero voltage $V_{\rm d} = 0$

Both probes collect same electron and ion current and are at same floating potential  $V_{\rm fl}$ . The current in the probe circuit  $I_{\rm d} = 0$  because there is not any electromotive force in this circuit. This point is shown as zero of the I-V characteristic in fig. 2. Distribution of the potential at probes is shown in fig. 3a.



Figure 2: Current-voltage characteristic of ideal double probe.

## **2.2** B) Small negative voltage $V_{\rm d} < 0$

Probe potential in reference to plasma is determined by equilibrium condition  $\sum I_{\rm p} + \sum I_{\rm e} = 0$ . Distribution of the potential is shown in fig. 3b. Potential of first probe is close to plasma potential and this probe collects more electrons. Potential of the second probe is lower than floating potential and flow of electrons is decreasing. Electrons from first probe compensate decrease of electron current of the second probe. The sum of currents is zero and system is located in point B of I-V characteristic.

### 2.3 C) High negative voltage $V_{\rm d} \ll 0$

In this case one of the probes collects electron current while the other is strongly negative in respect to plasma potential, therefore electrons can't reach her. Half of the electrons of first probe flow to the second probe trough external circuit. System is described by point C in I-V characteristic in fig. 2. Distribution of the potential is shown in fig. 3c. Further change of the  $V_d$  does not affect probe current due to effect of current flow between probes balancing ion flow at the probes. in case of decreasing  $V_d$ , first probe stays at the value close to the plasma potential, meanwhile second probe becomes more negative. Ion current of the second probe is saturated and probe current in external circuit  $I_d$  remains constant: region X-Y in fig 2.

Total ion current is given by sum of saturated current at first probe  $I_{p1}$  and at second probe  $I_{p2}$  in region X and Y V-I characteristic.

Electron flow at the second probe is given by the difference of total current in external circuit and ion current  $I_{p2}$  at the probe as shown in fig. 2.

# 3 Theory of the operation of double probe

We can see generalised diagram of the double probe potential in fig. 4. Potential of the probes are  $V_1$  and  $V_2$  in respect to the plasma.  $V_c$  is constant potential or small difference in plasma potential in probes location. Using Kirhoff Law we can write for total current



Figure 3: Distribution of the potential in double probe system.



Figure 4: Generalised diagram of the potential of double probe.

in probe circuit as:

$$\sum I_{\rm p} = I_{\rm p1} + I_{\rm p2} = I_{\rm e1} + I_{\rm e2} = 0 \tag{1}$$

using Boltzmann relation for electron current we get:

$$\sum I_{\rm p} = S_1 \, j_{01} \, \exp -\frac{e \, V_1}{k \, T_{\rm e}} + S_2 \, j_{02} \, \exp -\frac{e \, V_2}{k \, T_{\rm e}}.\tag{2}$$

The diagram of potential in fig. 4 shows:

$$V_1 + V_c = V_2 + V_d \Rightarrow V_1 = V_2 + V_d - V_c$$
 (3)

Using last two equations we get:

$$\ln\left[\frac{\sum I_{\rm p}}{I_{\rm e2}} - 1\right] = -\frac{e V_{\rm d}}{k T_e} + \ln \sigma, \qquad (4)$$
$$\sigma = \frac{S_1 j_{01}}{S_2 j_{02}} \exp \frac{e V_{\rm c}}{k T_{\rm e}}.$$

where  $j_{01}$  a  $j_{02}$  are electron current densities at probes at potential equal to plasma potential. Electron temperature  $T_{\rm e}$  is given by the slope of straight line  $G = \sum [I_{\rm p}/I_{\rm e2}] - 1$  of the plotted graph of  $\ln G = f(V_{\rm d})$  according to 4.

# 4 Calculation of plasma parameters from I-V characteristic of double probe

#### 4.1 Electron temperature: method of currents ratio

Evaluation of the data obtained by double probe is similar to simple Langmuir probe analysis. From the plot of I-V characteristic is possible to determine current of both probes  $I_{p1}$  and  $I_{p2}$ . It is shown in fig. 5. Then we subtract electron current  $I_{e2}$  and plot  $\ln G = f(V_d)$ . Electron temperature  $T_e$  can be calculated from the slope according to equation 4 in case of Maxwellian velocity distribution. Coefficient  $\sigma$  depends on probe and sheath sizes. If they are same for both probes  $\sigma = 0$ .



Figure 5: Determination of  $R_0$  and G from I-V, currents  $I_{p_1}$  and  $I_{p_2}$  for  $V_d = 0$ .

#### 4.2 Electron temperature: resistance method

Equation 4 can be written as:

$$I_{e2} = \frac{\sum I_{p}}{\sigma \exp\left[-\frac{e V_{d}}{k T_{e}}\right] + 1}.$$
(5)

Derivation of  $I_{e2}$  with respect to  $V_d$ , we get for  $V_d = 0$ 

$$\left. \frac{\mathrm{d}I_{\mathrm{e2}}}{\mathrm{d}V_{\mathrm{d}}} \right|_{V_{\mathrm{d}}=0} = \frac{\sum I_{\mathrm{p}}}{(\sigma+1)^2} \, \frac{\sigma \, e}{k \, T_{\mathrm{e}}}.\tag{6}$$

If we replace  $\frac{dV_d}{dI_{e2}} = dV_d dI_d$ , we get for electron temperature

$$T_{\rm e} = \frac{e\,\sigma}{k\,(1+\sigma)^2} \left[\sum I_{\rm p} \frac{\mathrm{d}V_{\rm d}}{\mathrm{d}I_{\rm d}}\right] \bigg|_{V_{\rm d}=0}.\tag{7}$$

where  $\sigma$  can be calculated using equation 4

$$\sigma = \left[\frac{\sum I_{\rm p}}{I_{\rm e2}} - 1\right] \bigg|_{V_{\rm d}=0}.$$
(8)

We can simplify eq.7 using G:

$$G = \frac{\sigma}{(1+\sigma)} = \frac{I_{\rm e2}}{\sum I_{\rm p}} \tag{9}$$

and by replacing  $\sigma$  using G in eq. 7, we get:

$$T_{\rm e} = \frac{e}{k} \left( G - G^2 \right) \left[ \sum I_{\rm p} \frac{\mathrm{d}V_{\rm d}}{\mathrm{d}I_{\rm d}} \right] \bigg|_{V_{\rm d} = 0} = \frac{e}{k} \left( G - G^2 \right) R_0 \sum I_{\rm p}, \tag{10}$$

where  $R_0$  is so called equivalent resistance of double probe

$$R_0 = \left[\frac{\mathrm{d}V_{\mathrm{d}}}{\mathrm{d}I_{\mathrm{d}}}\right]\Big|_{V_{\mathrm{d}}=0} \tag{11}$$

Electron temperature can be calculated easily from I-V curve using equation 10. We need to determine  $R_0, \sum I_p$  and G first. The slope of the middle part of I-V in point  $V_d = 0$  gives us value of  $R_0$ .

Ion currents  $I_{p1}$  and  $I_{p2}$  in case of  $V_d = 0$  can be determined using asymptotes of saturated currents. We prolong them to y axis. Then we divide distance MN into 5 parts. There is a point  $\alpha$  located in distance equal to  $\frac{1}{5}$  MN from y axis - that is value of  $I_{p1}$  or  $I_{p2}$  for  $V_d = 0$ . This graphical method can is shown in fig. 5. Electron current at second probe is given by:

$$I_{\rm e2} = |I_{\rm p2}| + I_{\rm d} \tag{12}$$

and can be determined directly from I-V according to fig. 5. For calculation of G:

$$G = \left[\frac{I_{\rm e2}}{\sum I_{\rm p}}\right]\Big|_{V_{\rm d}=0} \tag{13}$$

it is important to use  $I_{e2}$  for  $V_d = 0$  and  $\sum I_p = I_{p1}$  and  $I_{p2}$  for  $\alpha$ .

#### 4.3 Electron density in plasma

According to Kirhoff law the current in probe circuit is:

$$\sum I_{\rm p} = I_{\rm p1} + I_{\rm p2} = I_{\rm e1} + I_{\rm e2} = 0 \tag{14}$$

$$I_{i0} + I_{e0} \exp\left[-\frac{e V_1}{k T_e}\right] + I_{i0} + I_{e0} \exp\left[-\frac{e V_2}{k T_e}\right] = 0$$
(15)

$$\left(1 - \exp\left[-\frac{e\,V_1}{k\,T_e}\right]\right) + \left(1 - \exp\left[-\frac{e\,V_2}{k\,T_e}\right]\right) = 0\tag{16}$$

$$-I_{i0} = I_{e0} \tag{17}$$

Current in probe circuit is  $I = I_{p1} = I_{p2}$  and therefore:

$$I = I_{i0} \left( 1 - \exp\left[-\frac{e V_1}{k T_e}\right] \right) = I_{i0} \left( 1 - \exp\left[-\frac{e V_2}{k T_e}\right] \right)$$
(18)

Using:

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1} \tag{19}$$

we can rewrite 18 as:

$$I = I_{\rm i0} \tanh\left(-\frac{e\,V_{\rm d}}{k\,T_{\rm e}}\right) \tag{20}$$

where  $V_{\rm d} = V_2 - V_1$ .

By differentiation of previous equation we get:

$$\left. \frac{\mathrm{d}I}{\mathrm{d}V_{\mathrm{d}}} \right|_{V_{\mathrm{d}}=0} = \frac{e\,I_{\mathrm{i}0}}{2k\,T_{\mathrm{e}}}.\tag{21}$$

Electron temperature can be determined from the slope of I-V in point  $V_d$ . Then we can determine electron density from the equation of saturated ion current:

$$I_{i0} = \alpha n_e A_p v_B \tag{22}$$

$$v_{\rm B} = \sqrt{k T_{\rm e}} \tag{23}$$

$$I_{\rm i0} = 0.61 n_{\rm e} A \sqrt{\frac{k T_{\rm e}}{M}} \tag{24}$$

where M is mass of ions.

#### 4.4 Ion and electron density in plasma

Electron density  $n_{\rm e}$  and ion density  $n_{\rm p}$  in case of  $n_{\rm e} = n_{\rm p}$  can be determined from saturated ion current of the probe. The problem for  $n_{\rm p}$  is that we do not know ion temperature  $T_{\rm p}$ . There are some cases when  $T_{\rm p}$  is equal to gas temperature, such as decaying plasma. Then we can use equation:

$$j_{\rm p} = n_{\rm p} \, e \, \langle v_{\rm e} \rangle. \tag{25}$$

Small error of  $T_{\rm p}$  does not affect calculation of  $n_{\rm p}$  because there is  $T_{\rm p}$  already in equation. Average drift speed of electrons  $\langle v_{\rm e} \rangle$  is given (in case of plasma decay) by flow on ions to probe sheath which is dependent on thermal movement of ions. In case of Maxwellian velocity distribution  $\langle v_{\rm e} \rangle = \frac{1}{4} \langle v_{\rm p} \rangle$ , where  $\langle v_{\rm p} \rangle$  is average ion speed.

Ion density is  $n_{\rm p} = \frac{4 j_{\rm p}}{e \langle v_{\rm p} \rangle}$  and by substituting ion current density  $j_{\rm p} = \frac{I_{\rm p}}{S}$  we get:

$$n_{\rm p} = \frac{4 I_{\rm p}}{S \, e \, \langle v_{\rm p} \rangle},\tag{26}$$

where  $\langle v_{\rm p} \rangle = \left(\frac{8 \, k \, T_{\rm p}}{\pi M}\right)^{1/2}$ .  $T_{\rm p}$  is temperature in Kelvins, M mass of ions,  $I_{\rm p}$  current in Amp. For cylindrical probe is area S bigger than just surface area of the probe and depends on probe potential. There is need to make a correction for  $n_{\rm p}$ .

# 5 Experimental set-up and measurement

Double probe measurements are realised using experimental set-up described in fig. 7.

- Connect double probe according to circuit in fig. 6.
- Discharge tube is evacuated using a rotary vane pump to 3-5 Pa.
- We are using argon as working gas which flows in a continuous flow at rate 80 sccm.
- Pressure is measured by Pirani manometer. Adjust the value of gas flow using flow controller to achieve pressure of 100 Pa.
- Set up the high voltage power source to 500 V and ignite the discharge.
- You can adjust the value of current in the discharge using adjustable current stabiliser next to the power source. Set up the value to 10 mA, 30 mA and 50 mA.
- Measurement is realised by ammeter and voltmeter in probe circuit connected to computer for semi-automated measurements.
- Measure I-V characteristics of double probe for different values of discharge current (10 mA, 30 mA and 50 mA).
- Determine sizes of the probes using photograph of discharge tube if you know its diameter.
- Plot measured I-V characteristics.
- Calculate electron temperature and electron density using mentioned methods.



Figure 6: Scheme of the electrical connections of discharge (above) and double probe (below).

# References

- Tálský A., Janča J. Speciální praktikum z vysokofrekvenční elektroniky a fyziky plazmatu Přírodovědecká fakulta UJEP, Brno 1975.
- [2] Šmíd R. Návod k úloze Měření rozdělovací funkce energie elektronů pomocí vf kompenzované Langmuirovy sondy.



Figure 7: Experimental set-up.