6. Cyclotron resonance of electrons and holes

A remainder of the evolution of semiconductor physics...

Pure crystals of Ge have been obtained and transistor discovered by Bardeen, Brattain, and Shockley at Bell labs in 1947.
Rudimentary bandstructure calculations appeared in early 1950s:

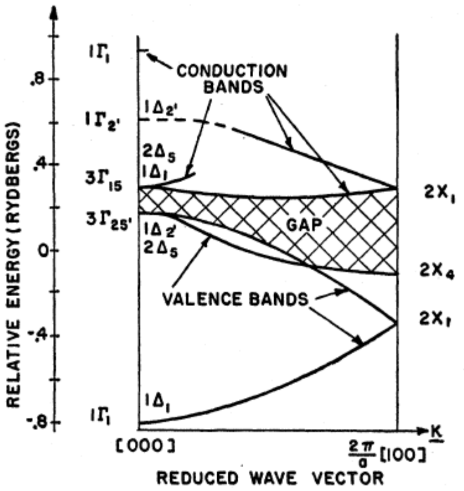

FIG. 1. Energy band structure of the germanium crystal. The present study was restricted to the points $\mathbf{k}=000$ and $k=2\pi(100)/a$. The curves joining the calculated eigenvalues indicate the estimated trends, based on the slopes and curvatures at the end points. The symbols denote degeneracy and symmetry classification. The notation is fully described in references 4 and 5.

Herman and Callaway, Phys. Rev. 1953

Conduction band minimum of Ge located in the Δ direction of BZ.

Let us follow the idea of examining the electron structure of a semiconductor around the minima of conduction bands and maxima of valence bands. Instructive reading in a short paper by Shockley, 1953.

Cyclotron Resonances, Magnetoresistance, and Brillouin Zones in Semiconductors

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ONE of the basic problems in the band theory of solids is to determine the shapes of the energy surfaces in the Brillouin zone. A possible solution is furnished by cyclotron resonances at low temperatures (say 10°K) in weakly doped germanium: For this situation, (1) Maxwell statistics can be used, (2) the interactions of the carriers are unimportant so that a description in terms of single carrier momenta in the Brillouin zone is good, and (3) the collision frequency ν is so much less than $\omega=2\pi f$ for 1.25-cm waves[1] that inertial effects dominate, and the dependence of ν upon position in the Brillouin zone is unimportant.

Some typical situations in which resonance might be observed are illustrated in Fig. 1. The E's and v's vary as $\exp(i\omega t)$ so that the conductivity $nq\mu$ is complex. The standard transverse and longitudinal magnetoresistance configurations are represented in (t) and (l), a combination of (t) and the Hall effect in (tH), and circular polarization in (c).

$$\mu_t(\omega, H)=v_x/E_x, \tag{1}$$

$$\mu_H(\omega, H)=cE_y/HE_x. \tag{2}$$

$$\mu_{tH}(\omega, H)=v_y/E_y=\mu_t(\omega, H)\{1+[H\mu_H(\omega, H)/c]^2\}^{-1}. \tag{3}$$

Some of the resonances may be illustrated by spherical energy surfaces with a single (relaxation) frequency ν and $\omega_H=qH/m^*c$. We find that μ_t, μ_H, and μ_l are independent of H:

$$\mu_t=\mu_H=\mu_l=q/\{m^*(\nu+i\omega)\}, \tag{4}$$

$$\mu_{tH}=q(\nu+i\omega)/\{m^*[(\nu+i\omega)^2+\omega_H^2]\}, \tag{5}$$

$$\mu_c=q[\nu+i(\pm\omega+\omega_H)]/\{m^*[(\nu+i\omega)^2+\omega_H^2]\}. \tag{6}$$

The plus sign in μ_c holds when the field rotation and cyclotron orbit have the same direction. If $\omega>10\nu$, as may be achieved in Ge at low temperatures, the resonance peaks observed at $\omega_H^2=\omega^2-\nu^2$ will be within 1 percent of that proper for m^* alone. If ν is a function of $\mathcal{E}$, then the mobility expressions should be based on finding v_x, v_y as functions of E_x and E_y and averaging over the

W. Shockley, Phys. Rev. **90**, 491 (1953).

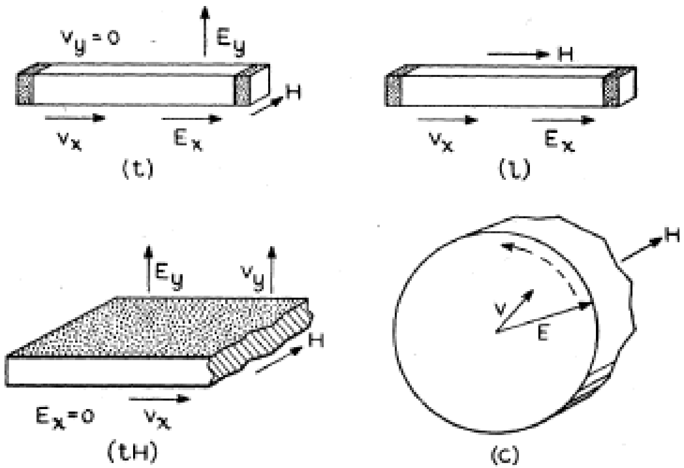


FIG. 1. Some typical situations for observing cyclotron resonances: (*t*), conventional transverse magnetoresistance configuration; (*l*), conventional longitudinal magnetoresistance configuration; (*tH*), a transverse situation in which Hall effect contributes to resistance; (*c*), circularly polarized electric field.

carriers with a weighting factor of $\mathcal{E}$; for a constant mean free path, this will lead to the customary factors of $8/3\pi$, etc. and to a small magnetoresistance in μ_t. This averaging does not affect the conclusion that if $\omega=\omega_H$, μ_c reduces to $\mu_c(0,0)$ for the wave rotating in the cyclotron direction.

Herman's calculations[2] suggest that the energy surface for the conduction band in Ge consists of six ellipsoids of revolution lying on [100] directions with a longitudinal mass m_1 and transverse mass m_2. The valence band is probably triply degenerate with surfaces of three sheets of nonellipsoidal shapes.[3] For these sheets we can define "tubes," each having its characteristic mass m_α and corresponding cyclotron frequency.[4]

For an ellipsoidal energy surface given by

$$\mathcal{E}=(P_x^2/2m_x)+(P_y^2/2m_y)+(P_z^2/2m_z) \qquad (7)$$

and a magnetic field with direction cosines α, β, λ, the effective cyclotron mass is

$$m^*=[m_xm_ym_z/(m_x\alpha^2+m_y\beta^2+m_z\gamma^2)]^{\frac{1}{2}}. \qquad (8)$$

Separate resonances should be observed for each different orientation of ellipsoid to H. Thus, for H parallel to [100] for the six ellipsoids, m^* will equal $(m_1 m_2)^{\frac{1}{2}}$ four times and m_2 twice; there will be no longitudinal resonance. For H parallel to [111], $m^* = m_2[3m_1/(m_1+2m_2)]^{\frac{1}{2}}$ six times, and for (l) we find

$$\mu_l = (2m_1+m_2)[(\nu+i\omega)^2+\omega_H{}^2 9m_1m_2/(2m_1+m_2)(m_1+2m_2)] \div 3m_1m_2(\nu+i\omega)[(\nu+i\omega)^2+\omega_H{}^2]. \quad (9)$$

Similar, but generally more complex, expressions apply to other cases.

Evidently, if the surfaces are ellipsoids, the determination of the resonance field for several conditions will give a unique determination of the mass parameters and hence of the energy surface shapes.

For the triply degenerate surface, a distribution of masses from zero (at the conical contact of the outer surfaces) to infinity will be present. If the inner surface is nearly spherical, a strong isolated resonance will occur. For this and the doubly degenerate case, it appears likely that the predicted resonance behavior will require difficult numerical calculations. However, it also appears probable that a numerical fit based on the three parameters[3] will be unique.

I am indebted to J. K. Galt, C. Herring, H. Suhl, and R. F. Wick for several stimulating discussions.

[1] Marked electron inertia effects with $\omega/\nu \cong 0.2$ at 160°K have been reported for electrons in germanium by T. S. Benedict and W. Shockley, Phys. Rev. **89**, 1152 (1953).

[2] F. Herman, Phys. Rev. **88**, 1210 (1952); F. Herman and J. Callaway, Phys. Rev. **89**, 518 (1953).

[3] W. Shockley, Phys. Rev. **78**, 173 (1950).

[4] W. Shockley, Phys. Rev. **79**, 191 (1950).

The motion of (quasi)electrons and holes in magnetic field will be approximated using parabolic dispersion of bands (i.e., constant effective masses); for simplicity, we will start with the following isotropic band:

$$E(\vec{k}) = \frac{\hbar^2 \left|\vec{k}\right|^2}{2m^*} \quad . \tag{6.1}$$

The classical equation of motion in the static magnetic field of the intensity H and harmonic electric field of the amplitude E_0 and frequency ω reads

$$m^*\left(\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau}\right) = e\left(\vec{E}_0 e^{i\omega t} + \frac{\vec{v}\times\vec{H}}{c}\right) \quad , \tag{6.2}$$

where τ is the average time between successive collisions ("scattering time", counteracting the drift movement due to the external field). Assume the orientation of the magnetic field along z, and linearly polarized electromagnetic wave along x. The harmonic time dependence of the drift velocity is described by the amplitudes along x and y; from Eq. (6.2), they are related by

$$\begin{aligned} m^*\left(i\omega + \frac{1}{\tau}\right)v_x &= eE_x + \frac{ev_y H_z}{c} \quad , \\ m^*\left(i\omega + \frac{1}{\tau}\right)v_y &= -\frac{ev_x H_z}{c} \quad . \end{aligned} \tag{6.3}$$

The amplitude of the velocity in the direction of electric field is

$$v_x = \frac{eE_x}{m^*} \frac{1 + i\omega\tau}{1 + \left(\omega_c^2 - \omega^2\right)\tau^2 + 2i\omega\tau} \quad , \tag{6.4}$$

where

$$\omega_c = \pm \frac{eH_z}{m^* c} = \pm \frac{eB_z}{m^* c} \tag{6.5}$$

is cyclotron frequency. In non-magnetic materials, the magnetic induction (flux density) *B* is nearly equal to the intensity *H*. The alternate sign in Eq. (6.5) selects the direction of rotational movement, which is opposite for electrons and holes.

The above equations are written in the cgs units, frequently used in this context; in SI units, the cyclotron frequency is

$$\omega_c = \pm \frac{eB_z}{m^*} \quad . \tag{6.5)SI}$$

A summary of the typical values of important quantities:
the (microwave) frequencies of 24 GHz band mean

$$\omega_c = 151E9 \text{ rad/s; } T_c = 2\pi/\omega_c = 42 \text{ ps; } \lambda_{vac} = cT_c = 1.3 \text{ cm; } \hbar\omega_c = 0.095 \text{ meV.}$$

The resonant magnetic field intensity is

$$H = 860 \text{ Oe} = 6.8E5 \text{ A/m.}$$

The horizontal component of the field in Brno is about 0.2 Oersted (16 A/m).

We can also estimate the radius, R_c, of the cyclotron orbit. Involved is the mean thermal velocity,

$$\langle v \rangle = \sqrt{\frac{8}{\pi}\frac{kT}{m^*}} \approx 4E6 \text{ cm/s pro } m^* = 0.1m_0 \text{ , } T = 4 \text{ K}$$

which implies

$$R_c = \frac{\langle v \rangle T_c}{\pi} \approx 300 \text{ nm .}$$

Note: the regular cyclotron orbiting is superposed on the chaotic thermal movement.

The drift velocity of charged (quasi)particles is related to a current density; the latter is also of harmonic time dependence, its component along x is

$$j_x(t) = Nev_x e^{i\omega t} = \frac{Ne^2}{m^*}\frac{i\omega + 1/\tau}{\omega_c^2 + \left(i\omega + 1/\tau\right)^2} E_x e^{i\omega t} \quad , \tag{6.6}$$

where N is the carrier density. According Eq. (6.4), the amplitude of the drift velocity is proportional to the amplitude of the electric field; consequently, we can introduce (complex) conductivity as the proportionality factor between the current density and electric field intensity:

$$\sigma = \frac{j_x}{E_x} = \sigma_0 \frac{1 + i\omega\tau}{1 + \left(\omega_c^2 - \omega^2\right)\tau^2 + 2i\omega\tau} \quad , \tag{6.7}$$

where

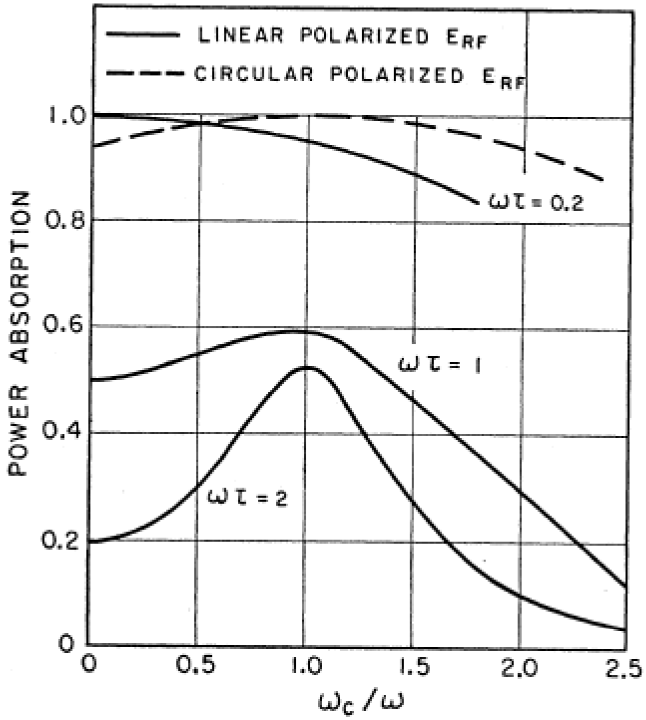
$$\sigma_0 = \frac{Ne^2\tau}{m^*} \tag{6.8}$$

is the dc (ω=0) conductivity.

Energy transferred from the alternating electric field is proportional to the real part of conductivity (it appears at the Joule heat resulting from the damping of the carrier motion, related to the relaxation time τ). Its frequency dependence is described conveniently using dimensionless quantities in the following relation:

$$\frac{\mathrm{Re}\{\sigma\}}{\sigma_0} = \frac{1+r^2+r_c^2}{\left(1-r^2+r_c^2\right)^2+4r^2} \quad , r=\omega\tau,\ r_c=\omega_c\tau. \tag{6.9}$$

Using this result, we can assess the chances of observing the resonant absorption of energy for $\omega \approx \omega_c$, see the following figure.



Frequency dependence of the absorbed energy; from Dresselhaus, Kip, Kittel, Phys. Rev. 1955.

Given the ranges of frequencies ω and ω_c, the pronounced maxima of absorbed energy occur for a small damping of the free carrier movement, i.e., a large relaxation time τ. The latter is proportional to the *mobility* μ, which is a suitable characteristics of the electric transport in semiconductors (remember very broad range of carrier densities N in semiconductors):

$$\sigma_0 = Ne\mu \ , \qquad \mu = \frac{e\tau}{m^*} \ . \tag{6.10}$$

Mobility is the mean drift velocity in the electric field of unit intensity; it is usually measured in the units of (cm/s)/(V/cm) = cm^2/Vs. In order to obtain well resolved cyclotron resonances, the condition $\omega\tau \geq 1$ has to be fulfilled,

$$\mu \geq \frac{e}{\omega m^*} \ . \tag{6.11}$$

Taking the convenient microwave frequencies (e.g., $f = \omega/2\pi \approx$ 24 GHz), rather large values of mobility are required, amplified by the inverse proportionality to the effective mass }usually significantly lower than the free electron mass:

$$\mu \geq 11000 \frac{m_0}{m^*} \ \ \mathrm{cm^2/Vs.} \tag{6.12}$$

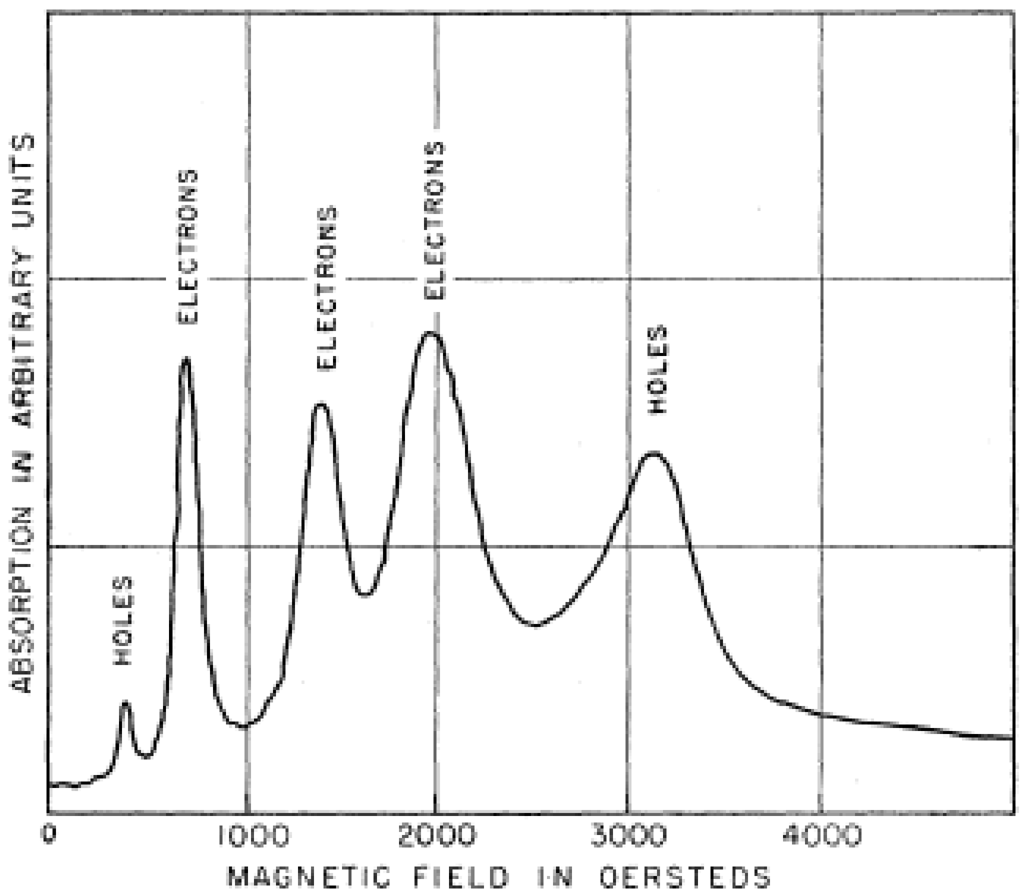
Using the dimensionless quantities of Eq. 6.9, the requirement of observable cyclotron resonance can be also put in the following form,

$$r = r_c \gg 1 \ . \tag{6.13}$$

The real part of conductivity is then equal to the dc value halved. The reason is obvious: we have assumed the linear polarization of the alternating field, which is a superposition of two orthogonal circular polarization states. One of them (that of the same sense of the cyclotron orbit), the other with the opposite phase and no energy dissipation. In the circularly polarized field of the same phase, the resonant absorption would be the same as in the dc field.

In actual cyclotron resonance experiments, the frequencvy of the (microwave) field is kept constant and the cyclotron frequency is variaqble using variable magnetic field.

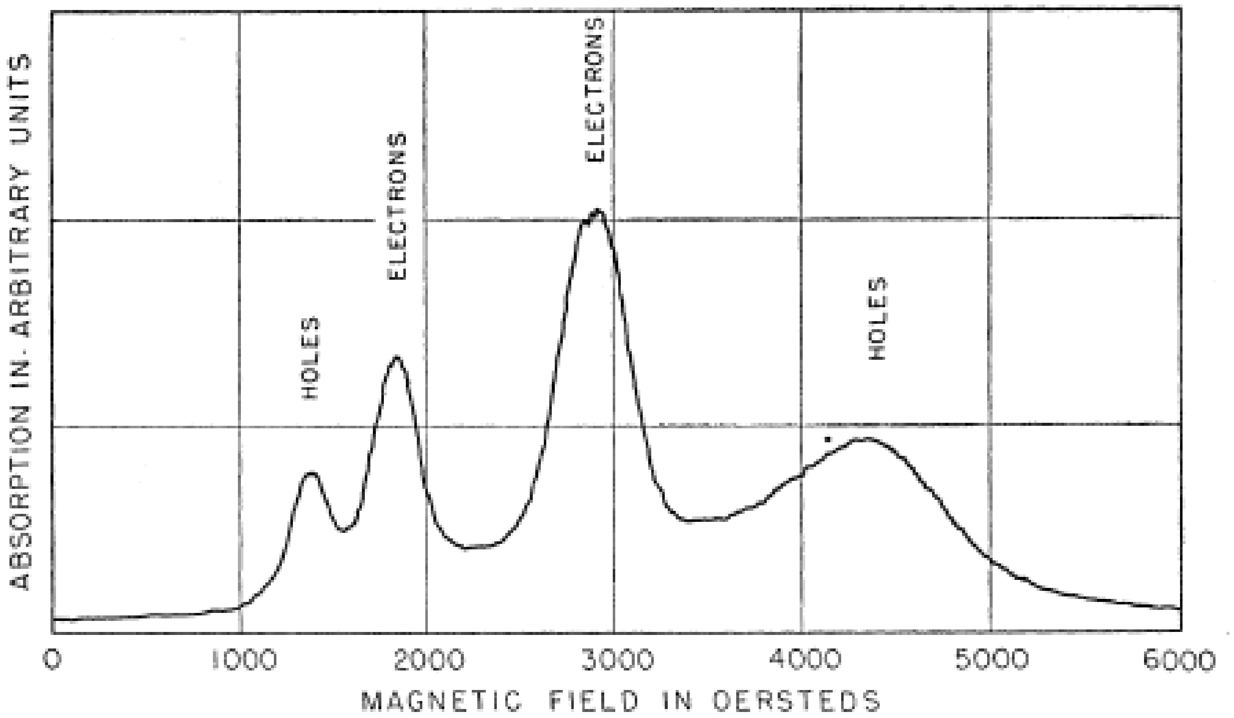
Proper concentration of free carriers is important. It can be produced by optical excitation (chopped light from a tungsten lamp, focused on the sample), which can also produce reference signals for lock-in (phase sensitive) detection of the microwave absorption.



Typical profile of the absorbed energy:

Ge, frequency 24 GHz, temperature 4 K.

From Dresselhaus, Kip, Kittel, Phys. Rev. 1955.

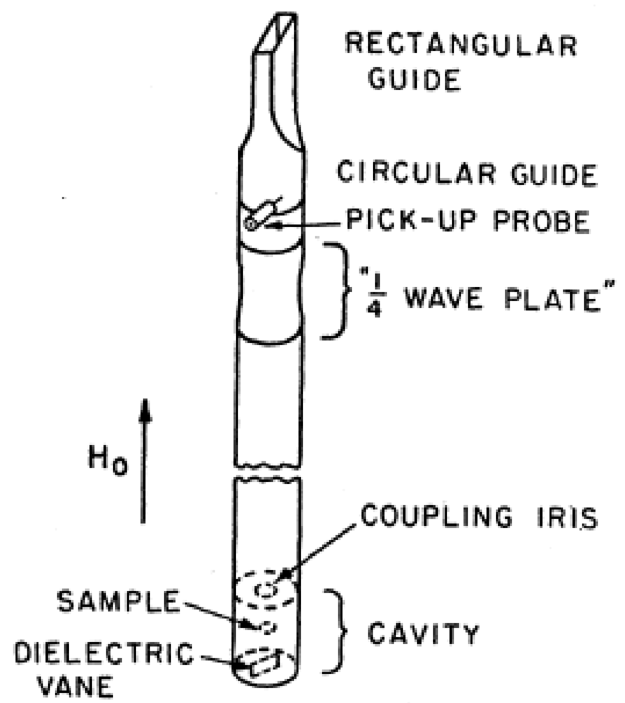


Typical profile of the absorbed energy:

Si, frequency 24 GHz, temperature 4 K.

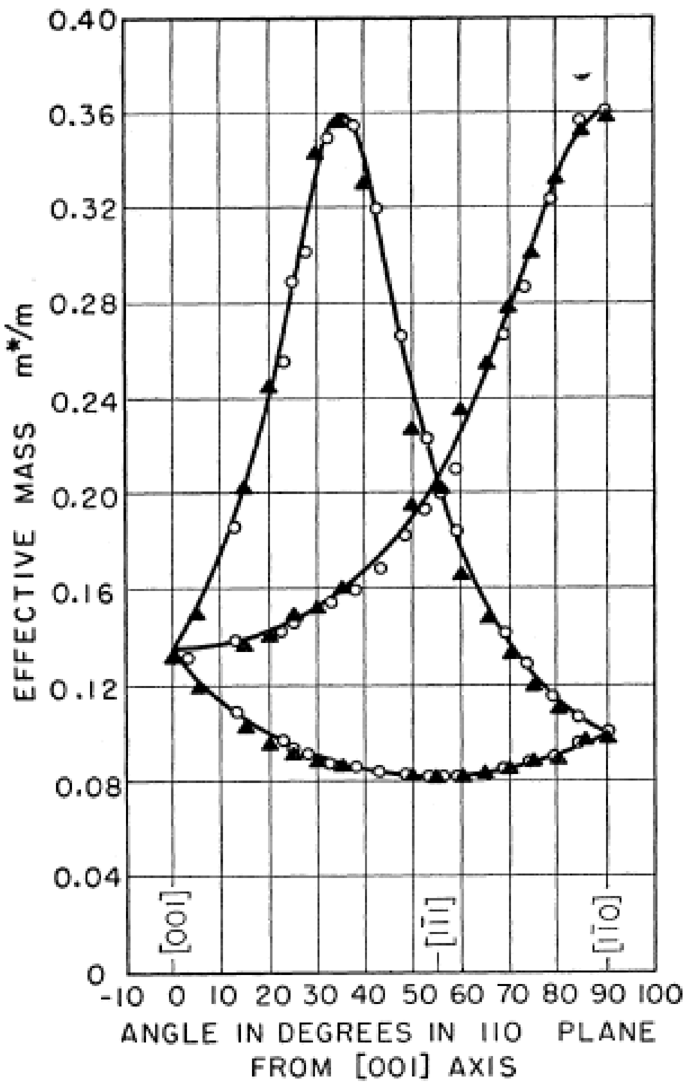
Static magnetic field in the (100) plane, inclined 30 deg towards (100).

From Dresselhaus, Kip, Kittel, Phys. Rev. 1955.

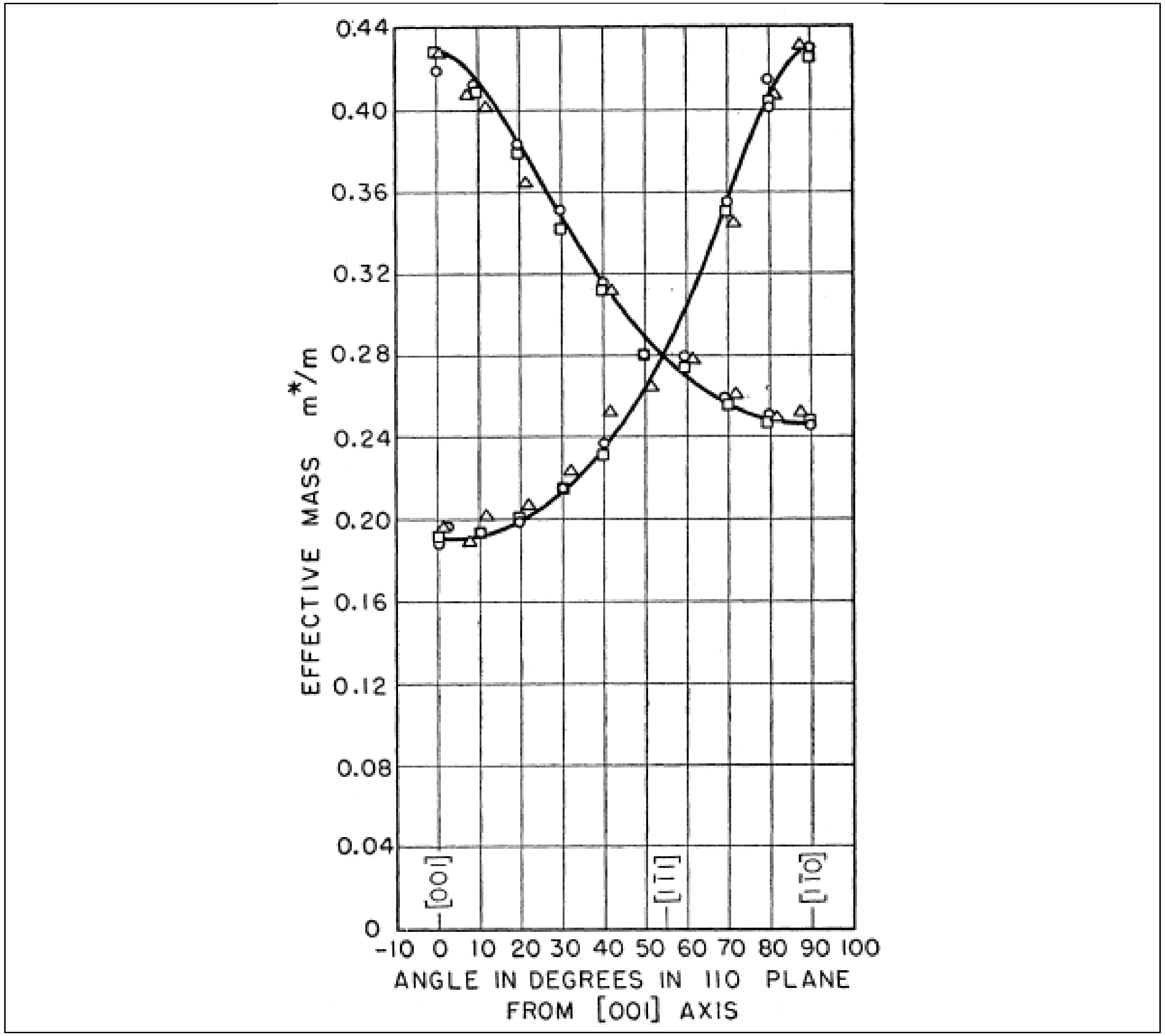


Experimental setup for measurements with circularly polarized microwave field; from Dresselhaus, Kip, Kittel, Phys. Rev. 1955.

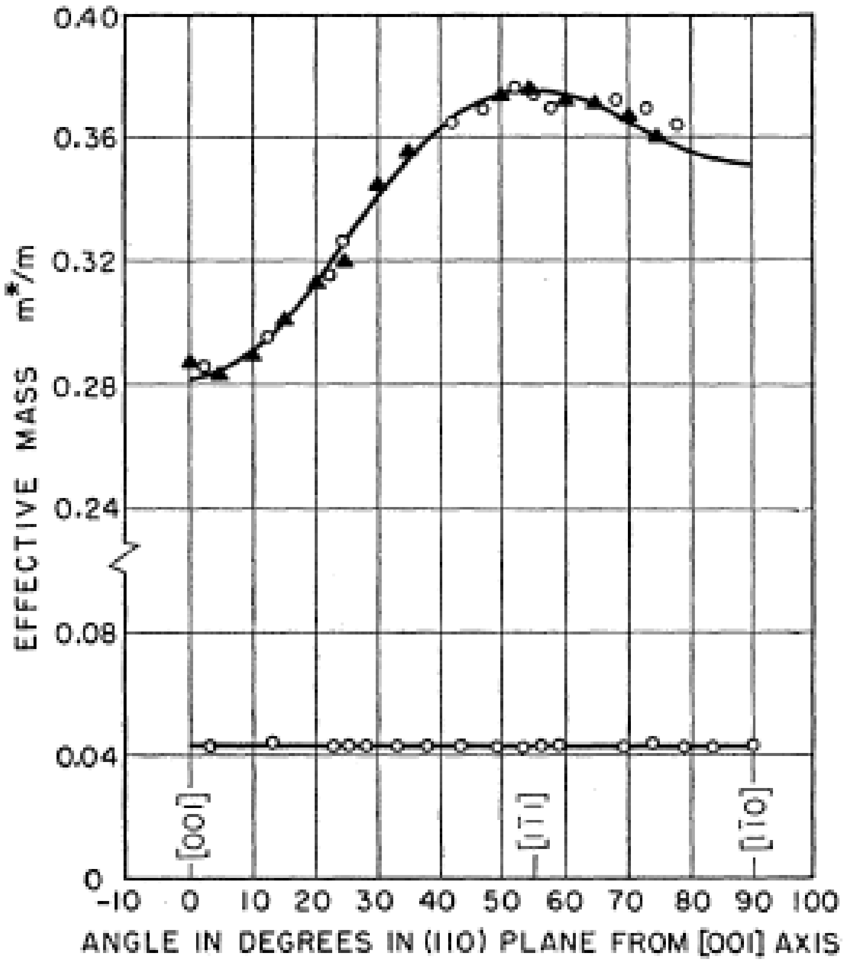
Changing orientation of single-crystalline sample allows studies of the directional dependence of effective masses.



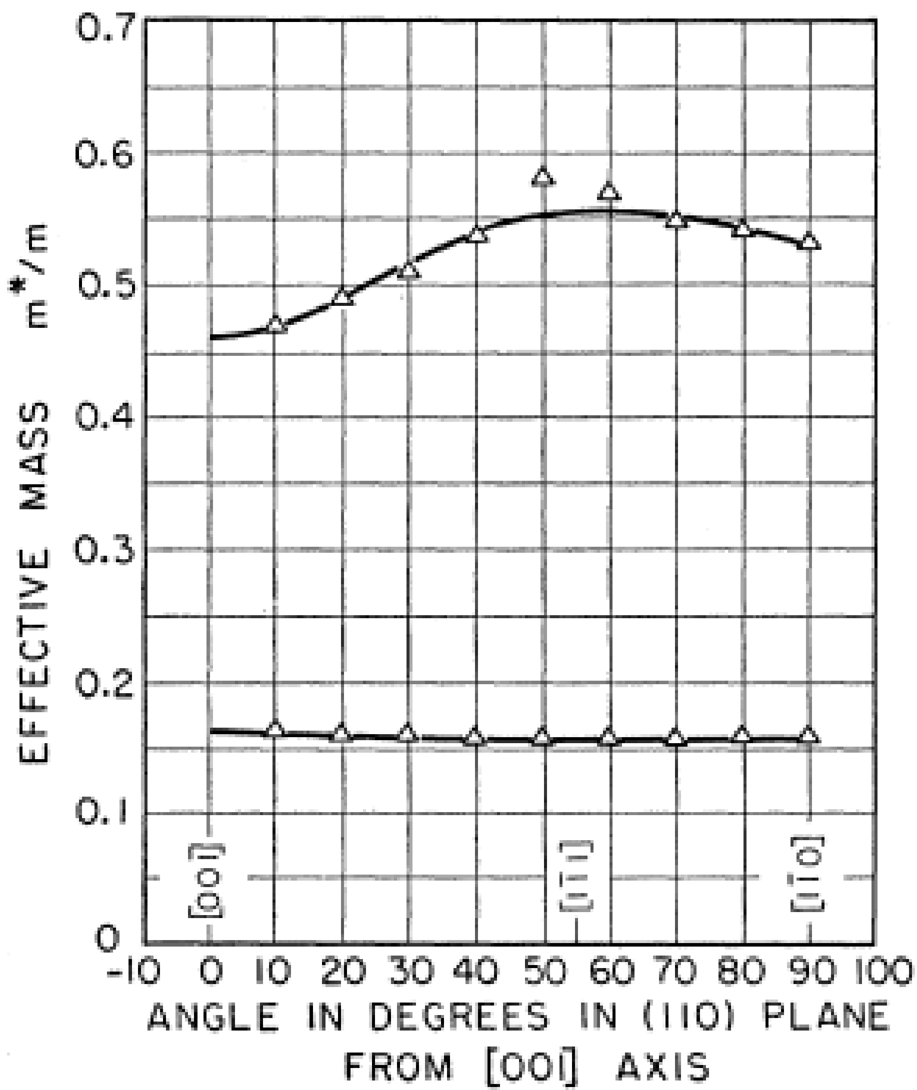
Effective masses of electrons in Ge; from Dresselhaus, Kip, Kittel, Phys. Rev. 1955. Different symbols are results of independent measurements.


EFFECTIVE MASS m*/m
0.44
0.40
0.36
0.32
0.28
0.24
0.20
0.16
0.12
0.08
0.04
0
[001]
[111]
[110]
-10 0 10 20 30 40 50 60 70 80 90 100
ANGLE IN DEGREES IN 110 PLANE
FROM [001] AXIS

Effective masses of electrons in Si; from Dresselhaus, Kip, Kittel, Phys. Rev. 1955. Different symbols are results of independent measurements.



Effective masses of holes in Ge; from Dresselhaus, Kip, Kittel, Phys. Rev. 1955. Different symbols are results of independent measurements.



Effective masses of holews in Si; from Dresselhaus, Kip, Kittel, Phys. Rev. 1955.

Summary of results from Dresselhaus, Kip, Kittel, Phys. Rev. 1955:

An experimental and theoretical discussion is given of the results of cyclotron resonance experiments on charge carriers in silicon and germanium single crystals near 4°K. A description is given of the light-modulation technique which gives good signal-to-noise ratios. Experiments with circularly polarized microwave radiation are described. A complete study of anisotropy effects is reported. The electron energy surfaces in germanium near the band edge are prolate spheroids oriented along ⟨111⟩ axes with longitudinal mass parameter $m_l = (1.58 \pm 0.04)m$ and transverse mass parameter $m_t = (0.082 \pm 0.001)m$. The electron energy surfaces in silicon are prolate spheroids oriented along ⟨100⟩ axes with $m_l = (0.97 \pm 0.02)m$; $m_t = (0.19 \pm 0.01)m$. The energy surfaces for holes in both germanium and silicon have the form

$$E(k) = Ak^2 \pm [B^2k^4 + C^2(k_x^2k_y^2 + k_y^2k_z^2 + k_z^2k_x^2)]^{\frac{1}{2}}.$$

We find, for germanium, $A = -(13.0 \pm 0.2)(\hbar^2/2m)$, $|B| = (8.9 \pm 0.1)(\hbar^2/2m)$, $|C| = (10.3 \pm 0.2)(\hbar^2/2m)$; and for silicon, $A = -(4.1 \pm 0.2)(\hbar^2/2m)$, $|B| = (1.6 \pm 0.2)(\hbar^2/2m)$, $|C| = (3.3 \pm 0.5)(\hbar^2/2m)$. A discussion of possible systematic errors in these constants is given in the paper.